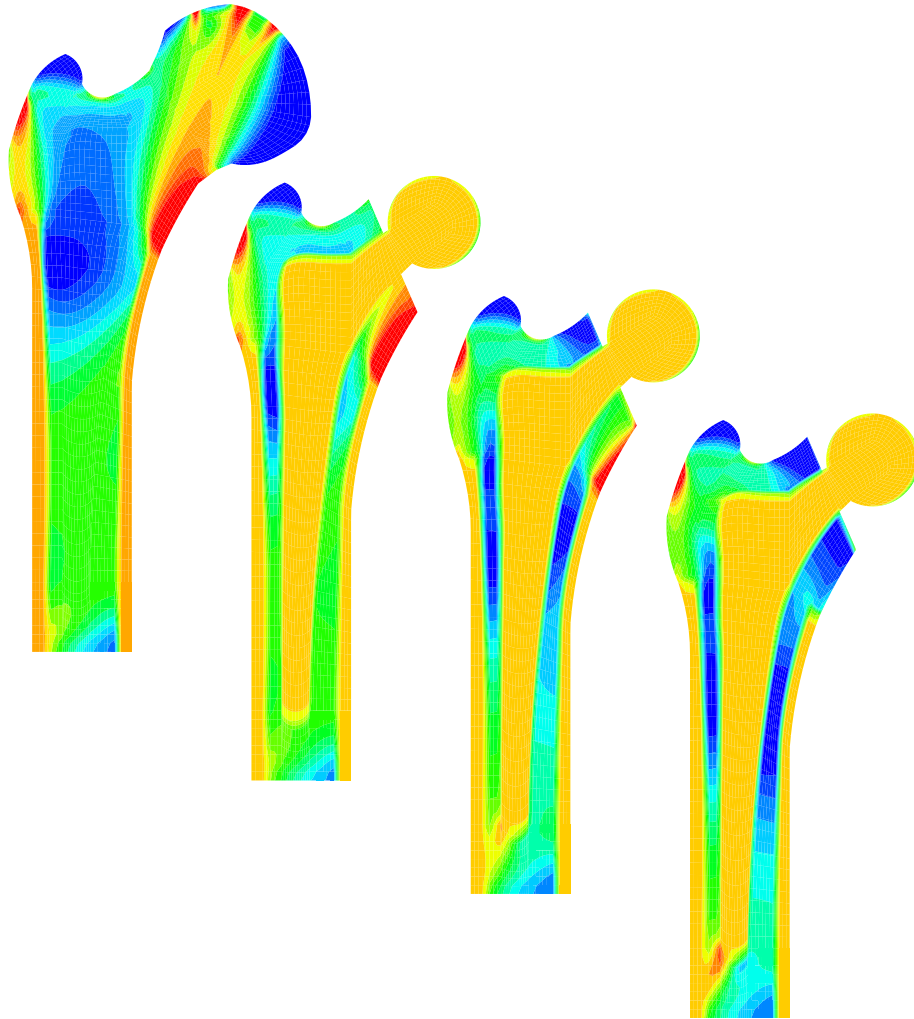


continuum biomechanics

– pantha psiloni –

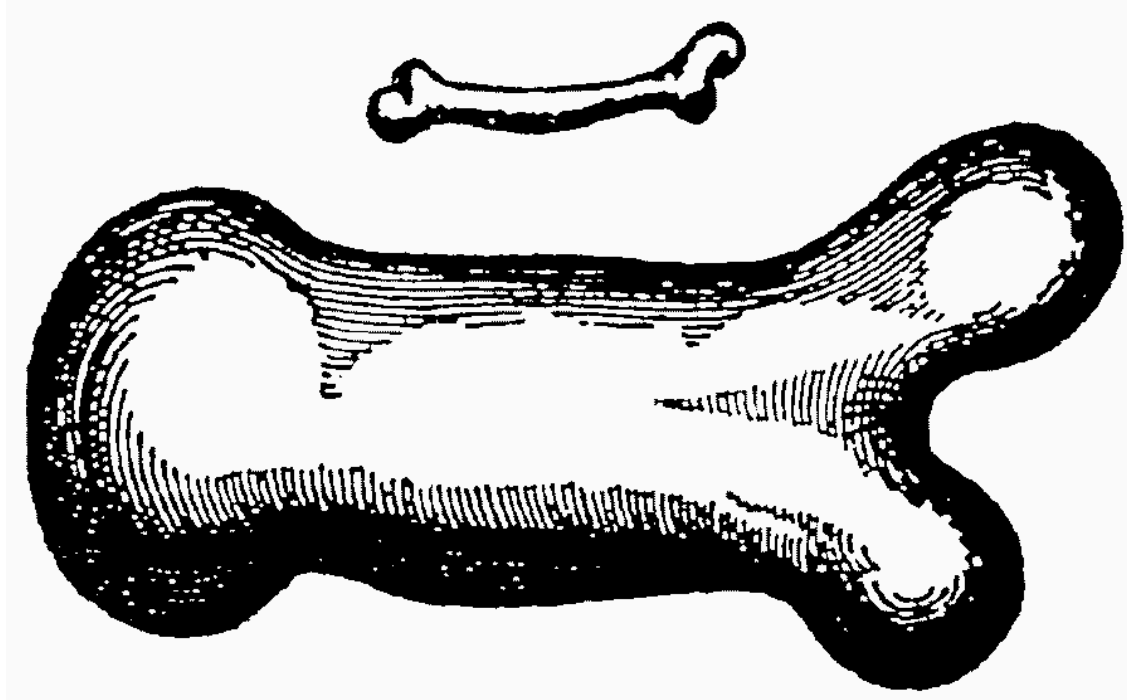


ellen kuhl

assistant professor of biomechanics
faculty of mechanical engineering
tu kaiserslautern - germany

- **motivation**
history of biomechanics
- **density growth**
adaptation in bone
wound healing
- **volume growth**
adaptation in arteries
tube in tension
- **remodelling**
simulation of skin
tissue engineering
- **outlook**

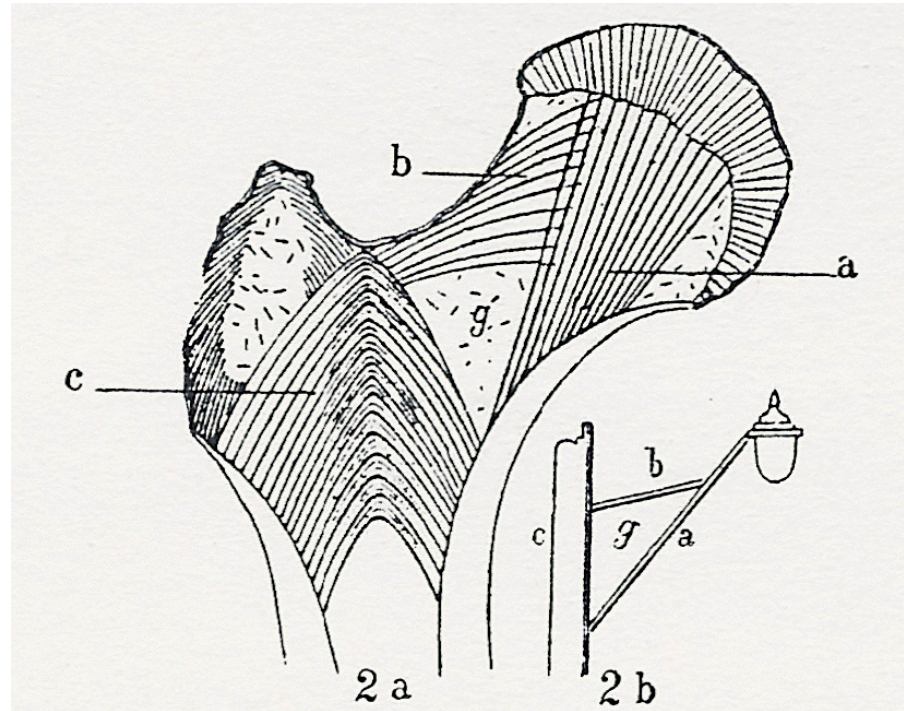
discorsi e dimostrazioni matematiche



”...dal che e manifesto, che chi volesse mantener in un vastissimo gigante le proporzioni, che hanno le membra in un huomo ordinario, bisognerebbe o trouar materia molto piu dura, e resistente per formarne l’ossa o vero ammettere, che **la robustezza sua fusse a proporzione assai piu fiacca, che negli huomini de statura mediocre**; altrimenti crescendogli a smisurata altezza si vedrebbero dal proprio peso opprimere, e cadere...”

GALILEI [1638]

law of bone remodelling

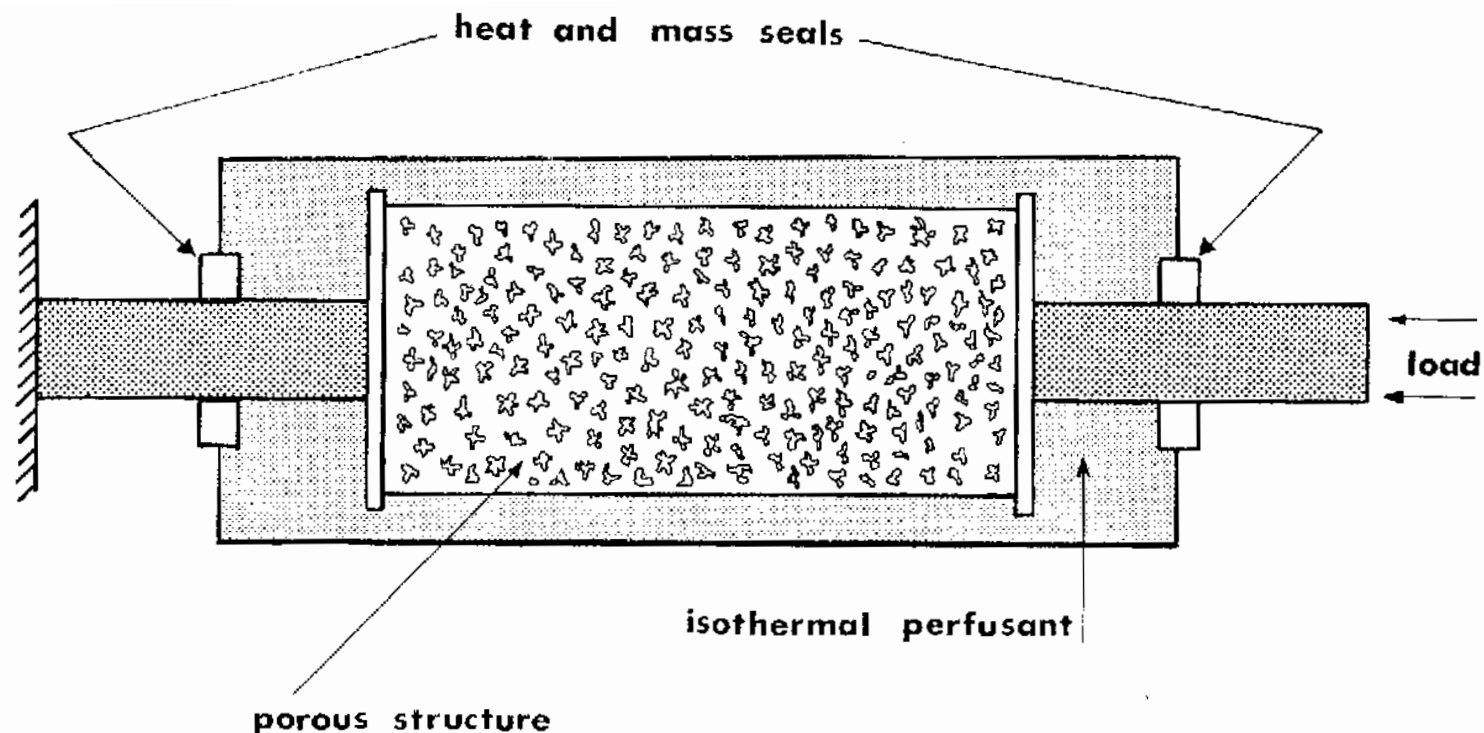


WARD [1832]

"...es ist demnach unter dem gesetze der transformation der knochen dasjenige gesetz zu verstehen, nach welchem im gefolge primärer abänderungen der form und inanspruchnahme bestimmte **umwandlungen der inneren architectur** und **umwandlungen der äusseren form** sich vollziehen..."

WOLFF [1892]

theory of adaptive elasticity



"...the system of only the porous structure without its entrained perfusant is **open with respect to momentum transfer as well as mass, energy, and entropy transfer**. we shall write balance and constitutive equations for only the bone matrix..." COWIN & HEGEDUS [1976]

density growth

example – rocket propulsion

- balance of mass

$$d_t m = \mathfrak{R} \quad \text{with ejection} \quad \mathfrak{R} \leq 0$$

- balance of momentum – volume specific

$$d_t [m \mathbf{v}] = \mathbf{f} + \mathbf{v} \mathfrak{R}$$

- balance of momentum – mass specific

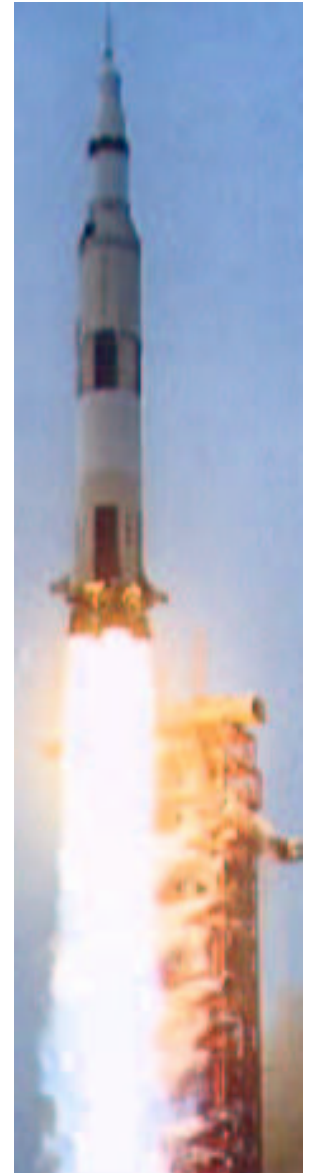
$$m d_t \mathbf{v} = \mathbf{f} \quad \text{with} \quad \mathbf{f} := \mathbf{f}^{\text{closed}} + \mathbf{f}^{\text{open}}$$

- balance of momentum of rocket & ejection

$$D_t [m \mathbf{v}] - \bar{\mathbf{v}} \mathfrak{R} = \mathbf{f}^{\text{closed}}$$

- propulsive force

$$\bar{\mathbf{f}}^{\text{open}} = [\bar{\mathbf{v}} - \mathbf{v}] \mathfrak{R}$$



• open system thermodynamics – balance of mass

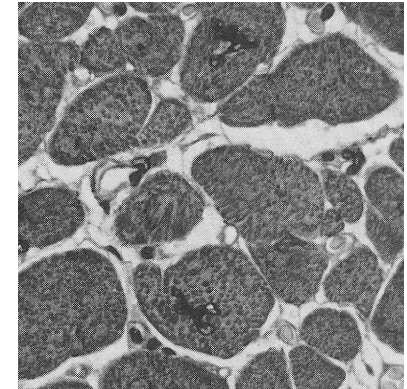
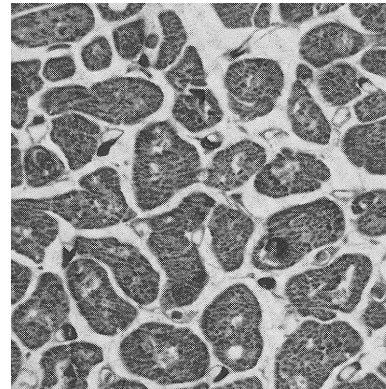
$$d_t \rho_0 = \text{Div} (\mathbf{R}) + \mathcal{R}_0$$

MASS FLUX $\mathbf{R}$

- migration (cell movement)

MASS SOURCE $\mathcal{R}_0$

- proliferation (cell growth)
- hyperplasia (cell division)
- hypertrophie (cell enlargement)



FUNG [1990]

'biological equilibrium'

COWIN & HEGEDUS [1976], BEAUPRÉ, ORR & CARTER [1990], WEINANS, HUISKES & GROETENBOER [1992], HARRIGAN & HAMILTON [1992], NACKENHORST [1997], HUISKES [2000], EPSTEIN & MAUGIN [2000], CARTER & BEAUPRÉ [2001], KUHLE & STEINMANN [2002]

density growth - balance equations

open system thermodynamics – balance of momentum

- volume specific version

$$d_t(\rho_0 \mathbf{v}) = \text{Div} (\mathbf{\Pi}^t + \mathbf{v} \otimes \mathbf{R}) + [\mathbf{b}_0 + \mathbf{v} \mathcal{R}_0 - \nabla_X \mathbf{v} \cdot \mathbf{R}]$$

subtraction of balance of mass weighted by $\mathbf{v}$

$$\mathbf{v} d_t \rho_0 = \text{Div} (\mathbf{v} \otimes \mathbf{R}) + \mathbf{v} \mathcal{R}_0 - \nabla_X \mathbf{v} \cdot \mathbf{R}$$

- mass specific version

$$\rho_0 d_t \mathbf{v} = \text{Div} (\mathbf{\Pi}^t) + \mathbf{b}_0$$

'mechanical equilibrium'

density growth – balance equations

open system thermodynamics – dissipation inequality

- free energy

$$\psi = \psi(\mathbf{F}, \rho_0, \dots)$$

- definition of stress $\mathbf{\Pi}^t$

$$\mathbf{\Pi}^t = \rho_0 \partial_{\mathbf{F}} \psi$$

- reduced dissipation inequality

$$\mathcal{D}_0^{\text{red}} = -\rho_0 \, \text{d}_{\rho_0} \psi [\text{Div}(\mathbf{R}) + \mathcal{R}_0] + \theta [\text{Div}(\mathbf{S}) + \mathcal{S}_0] + \dots \geq 0$$

- thermodynamic restrictions

$$\mathbf{R} \leq \frac{\theta}{\rho_0 \, \text{d}_{\rho_0} \psi} \mathbf{S} \qquad \mathcal{R}_0 \leq \frac{\theta}{\rho_0 \, \text{d}_{\rho_0} \psi} \mathcal{S}_0$$

”...a living organism can only keep alive by continually **drawing from its environment negative entropy**. it feeds upon negative entropy to compensate the entropy increase it produces by living...”

SCHRÖDINGER [1944]

theory of open systems

- constituents are **spatially seperated**
- overall behavior primarily determined by **one single constituent**
- **exchange** of mass, momentum, energy and entropy **with environment**

theory of porous media

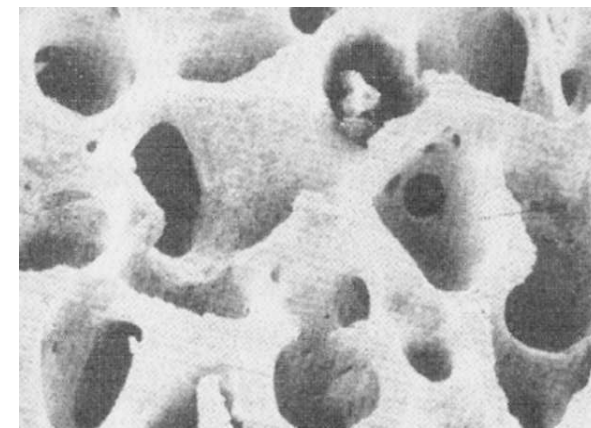
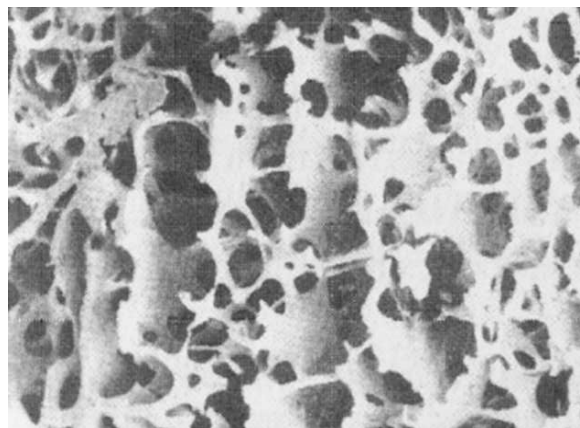
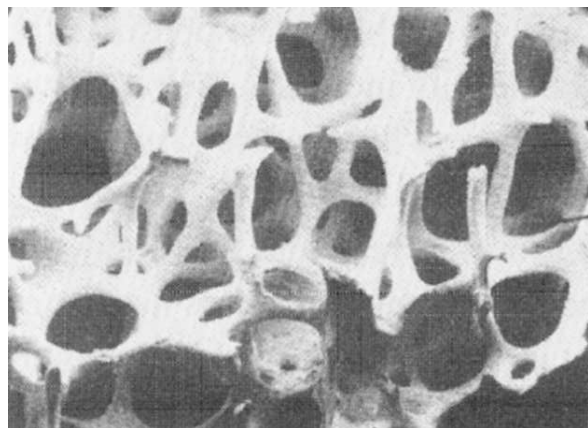
- local **superposition** of constituents
- consideration of mixture of **multiple constituents**
- **exchange** of mass, momentum, energy and entropy **amongst constituents**

BIOT [1941], [1955], DE GROOT [1951], TRUESDELL & TOUPIN [1960], BOWEN [1976], COWIN & HEGEDUS [1976], EHLERS & MARKERT [1998], EPSTEIN & MAUGIN [2000], DE BOER [2000], EHLERS [2002], HUMPHREY & RAJAGOPAL [2002], KUHL & STEINMANN [2002], [2003]

comparison – open systems vs. porous media

density growth at constant volume

- free energy $\psi_0 = \left[\frac{\rho_0}{\rho_0^*} \right]^n \psi^{\text{neo}}$
- stress $\mathbf{\Pi}^t = \left[\frac{\rho_0}{\rho_0^*} \right]^n \mathbf{\Pi}^{\text{tneo}}$
- mass flux $\mathbf{R} = R_0 \nabla \rho_0$
- mass source $\mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*} \right]^{-m} \psi_0 - \psi_0^*$



constitutive coupling of growth and deformation

HARRIGAN & HAMILTON [1992], WEINANS ET AL. [1992], JACOBS ET AL. [1995]

density growth – constitutive equations

biomechanics

- **living** tissues
- **density** ρ_0
- free energy $\psi = [\rho_0 / \rho_0^*]^n \psi^{\text{neo}}$
- **increase** in stiffness $d_t \rho_0 > 0$
- external entropy supply $\mathcal{S}_0 \geq 0, \mathbf{S} \geq 0$

damage mechanics

- **engineering** materials
- **damage variable** $d = 1 - \rho_0 / \rho_0^*$
- free energy $\psi = [1 - d] \psi^{\text{neo}}$
- **decrease** in stiffness $d_t d > 0$
- no external entropy supply $\mathcal{S}_0 = 0, \mathbf{S} = 0$

KACHANOV [1958], CHABOCHE [1982], LEMAÎTRE [1984], [1992], LEMAÎTRE & CHABOCHE [1985], SIMO & JU [1987], KRAJCINOVIC & LEMAÎTRE [1987], JU [1990], KRAJCINOVIC [1996]

continuous initial boundary value problem

$$d_t \rho_0 - \text{Div} (\mathbf{R}) - \mathcal{R}_0 = 0$$

$$\rho_0 d_t \mathbf{v} - \text{Div} (\mathbf{\Pi}^t) - \mathbf{b}_0 = 0$$

- temporal discretization

implicit EULER backward scheme
semi-discrete boundary value problem

- spatial discretization

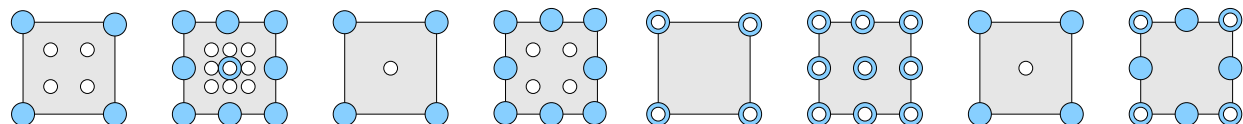
finite element method
discrete boundary value problem

- consistent linearization

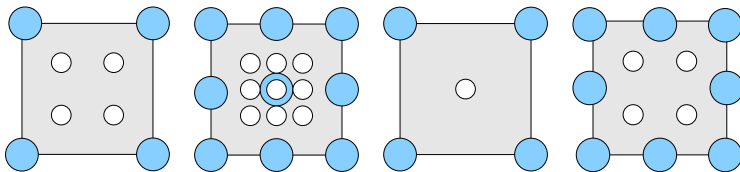
GATEAUX derivative
linearized discrete boundary value problem

- solution

incremental iterative NEWTON-RAPHSON scheme
integration point based vs. node point based approach

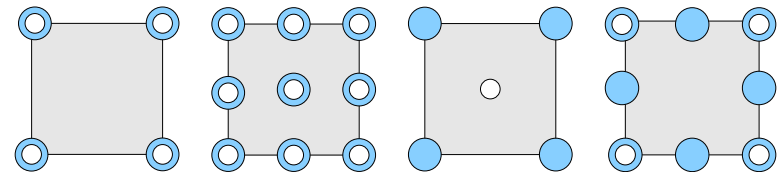


integration point based



- $\mathcal{C}^{-1}$ –**continuous** density interpolation designed for discontinuous problems
- number of global dofs unaffected
- **additional local newton iteration**
- convergence upon mesh refinement
- restricted to **flux-free** formulations

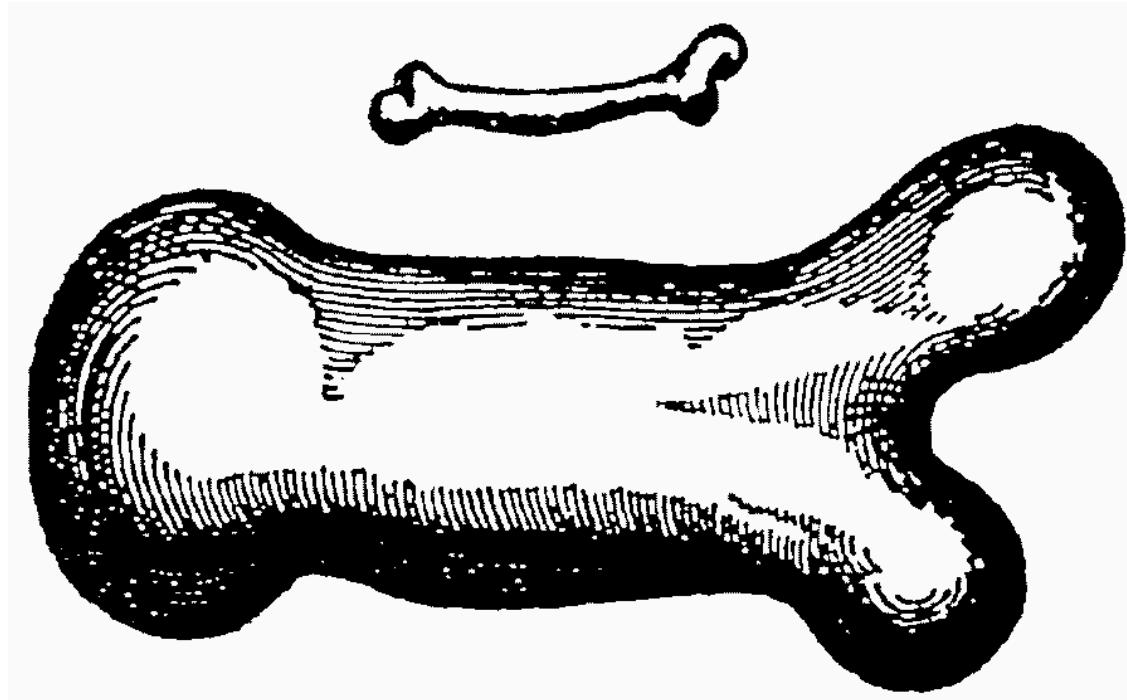
node point based



- $\mathcal{C}^0$ –**continuous** density interpolation designed for continuous problems
- **additional global dofs**
- no local newton iteration
- convergence upon mesh refinement
- mandatory in case of **mass flux** $\mathbf{R} \propto \nabla \rho_0$

comparison – integration vs. node point based

discorsi e dimostrazioni matematiche

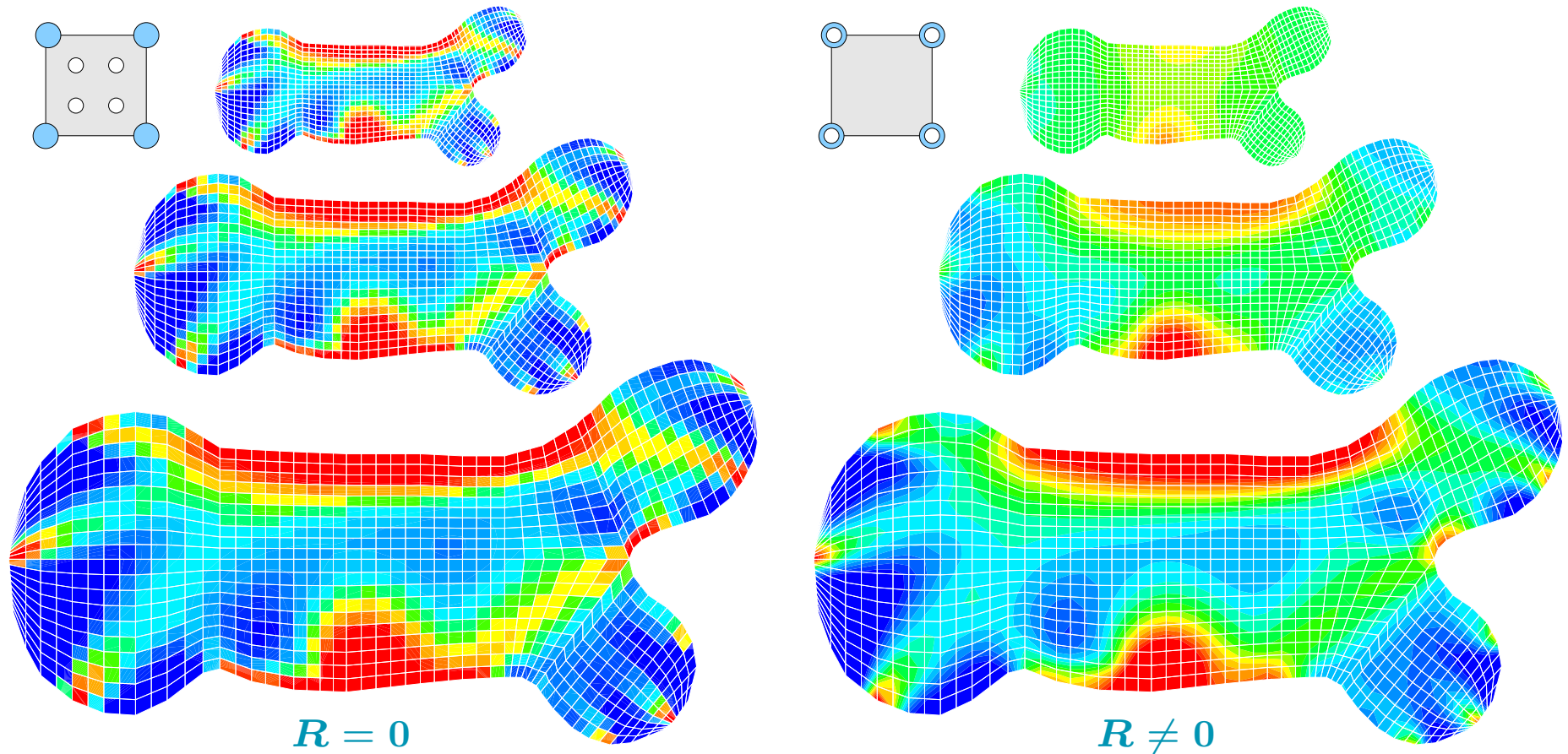


”...dal che e manifesto, che chi volesse mantener in un vastissimo gigante le proporzioni, che hanno le membra in un huomo ordinario, bisognerebbe o trouar materia molto piu dura, e resistente per formarne l’ossa o vero ammettere, che **la robustezza sua fusse a proporzione assai piu fiacca, che negli huomini de statura mediocre**; altrimenti crescendogli a smisurata altezza si vedrebbero dal proprio peso opprimere, e cadere...”

GALILEI [1638]

example – size effect in bone

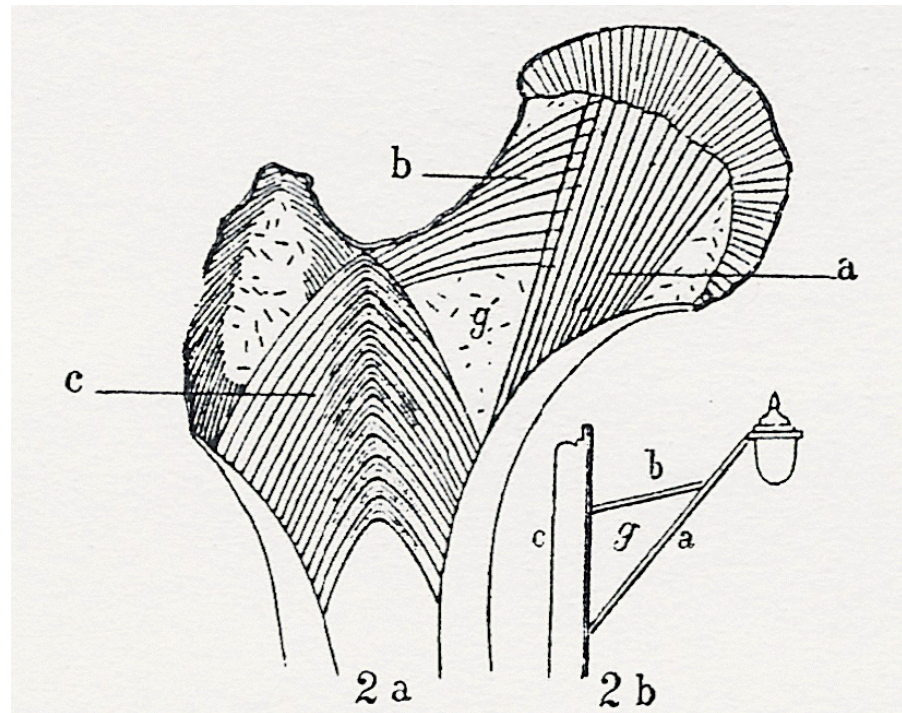
integration point based vs. node point based



incorporation of **mass flux** enables simulation of **galileo's size effect**

example - size effect in bone

law of bone remodelling



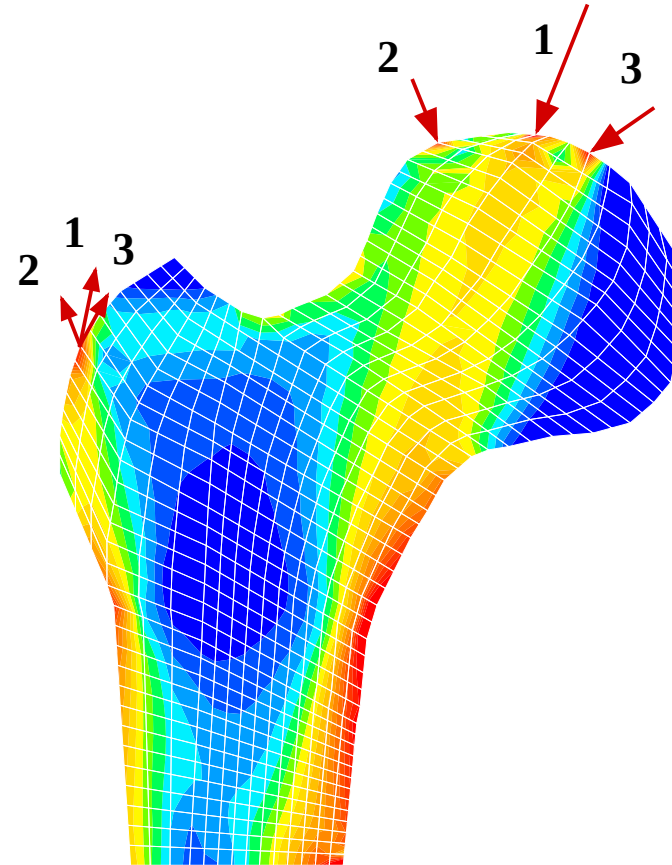
WARD [1832]

"...es ist demnach unter dem gesetze der transformation der knochen dasjenige gesetz zu verstehen, nach welchem im gefolge primärer abänderungen der form und inanspruchnahme bestimmte **umwandlungen der inneren architectur** und **umwandlungen der äusseren form** sich vollziehen..."

WOLFF [1892]

example – adaptation in bone

experimental observation vs. numerical simulation



- **dense system of compressive trabeculae** carrying stress into calcar region
- **secondary arcuate system** form medial joint surface to lateral metaphyseal region
- **ward's triangle** low-density region contrasting **dense cortical shaft**

example – adaptation in bone

total hip replacement vs. hip resurfacing

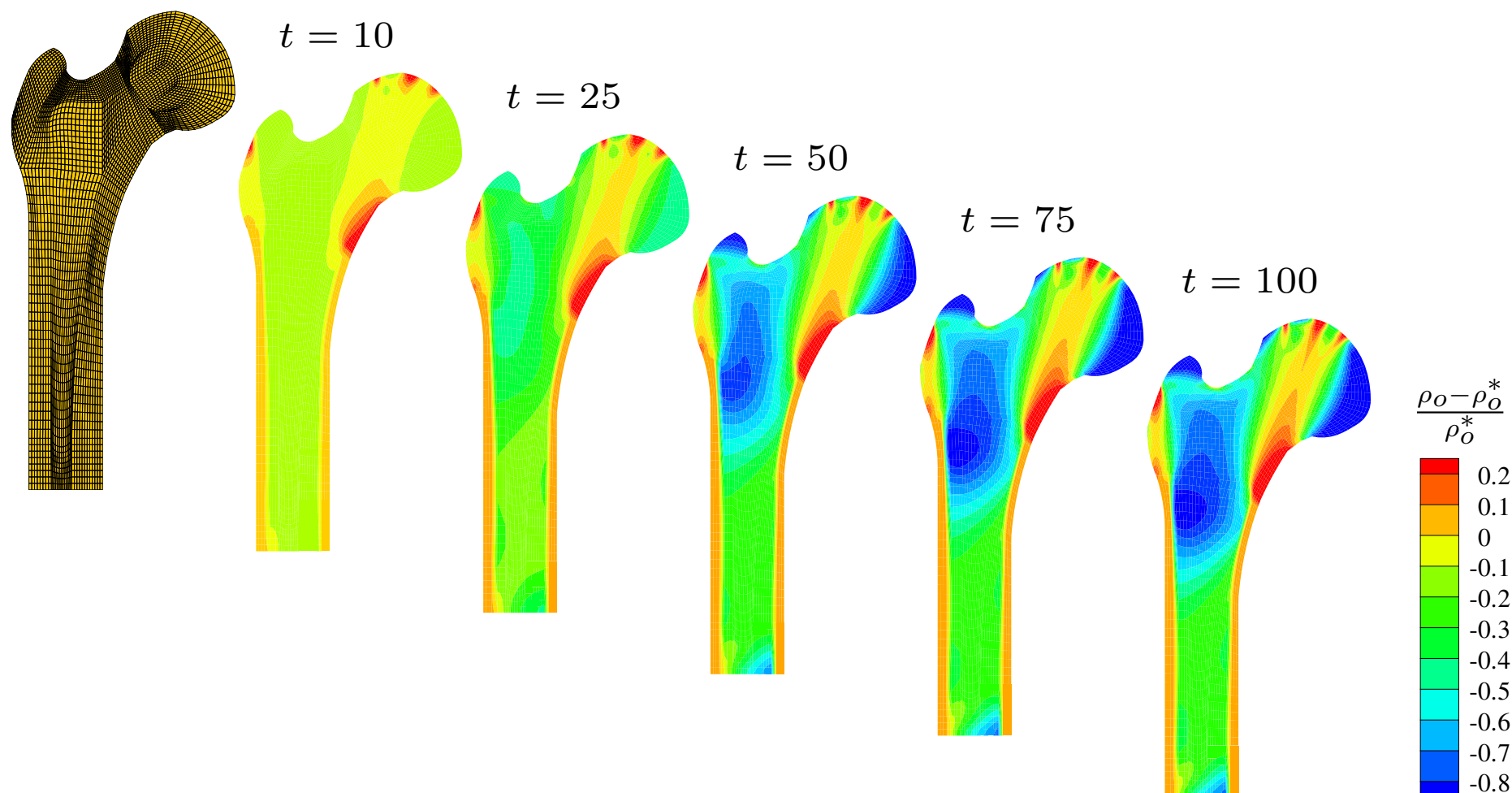


- about 1 000 000 artificial hip replacements per year
- **aseptic loosening** caused by **adaptive bone remodelling**
- goal: prediction of **redistribution of bone density**

KUHL & BALLE [2004]

example – hip replacement

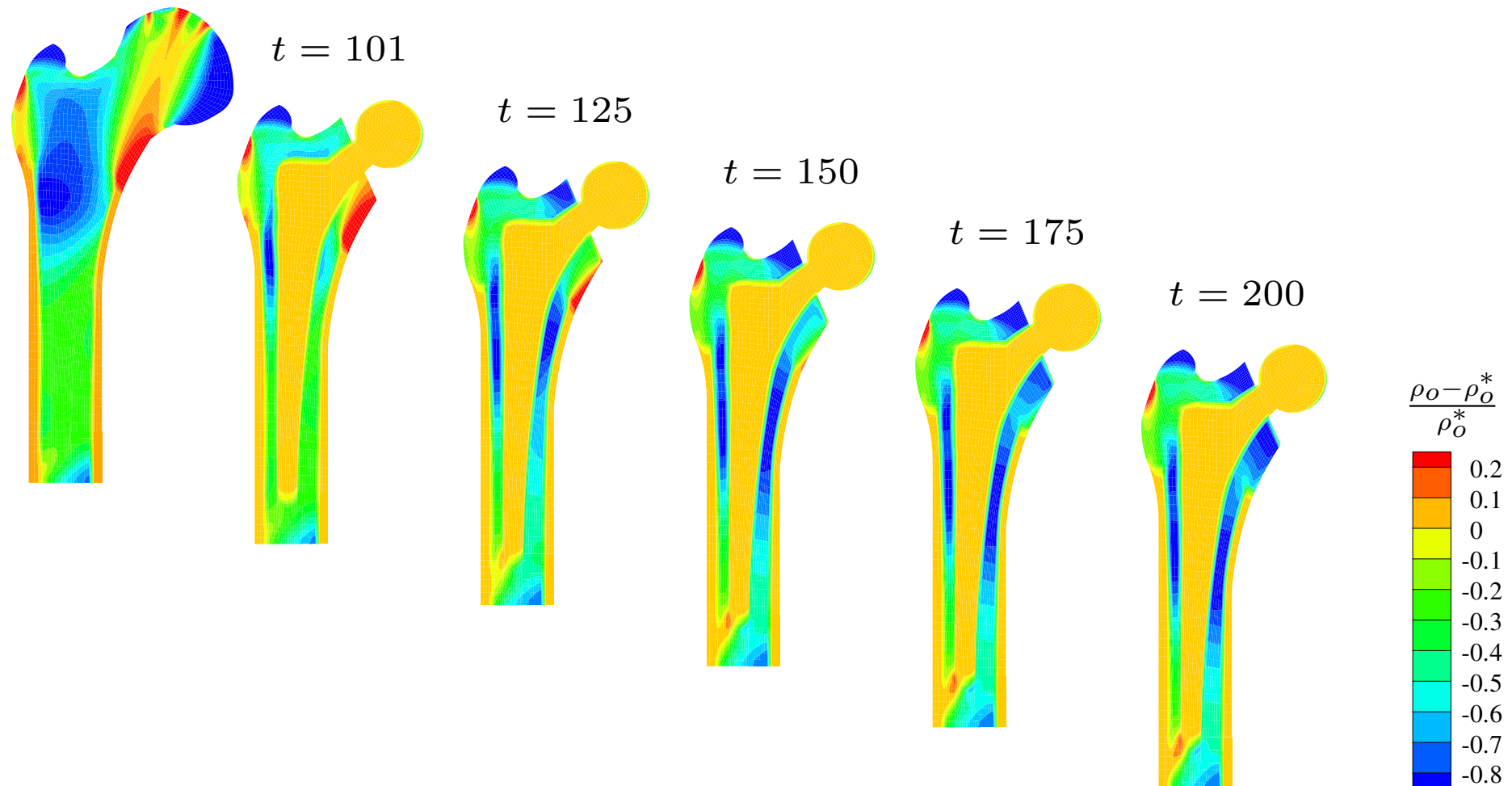
- density evolution – before implantation



ward's triangle – tensile & compressive trabeculae - dense cortical shaft

example – hip replacement

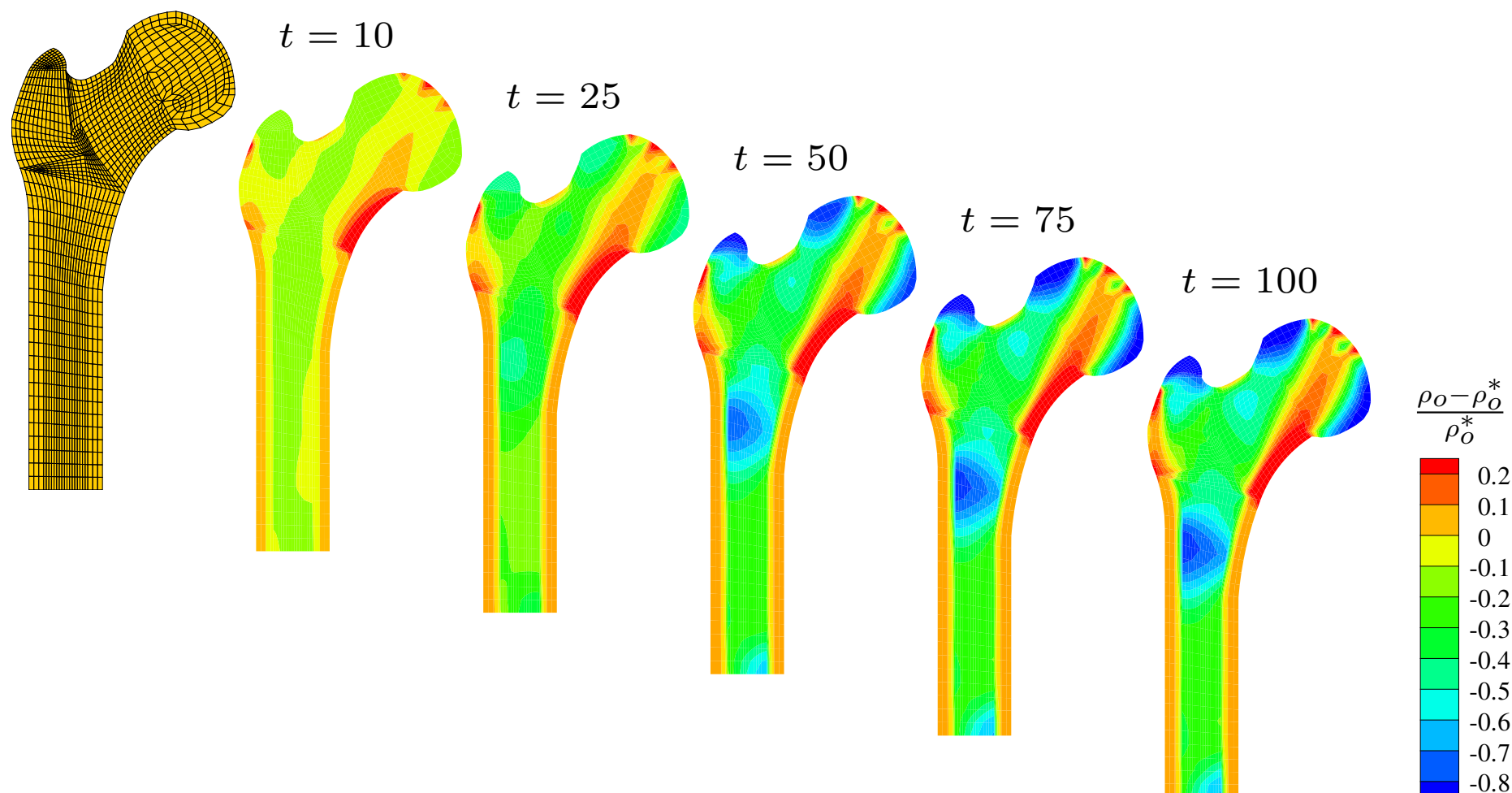
• density evolution – conventional total hip replacement



stiff prosthesis – stress shielding – bone resorption – implant loosening

example – hip replacement

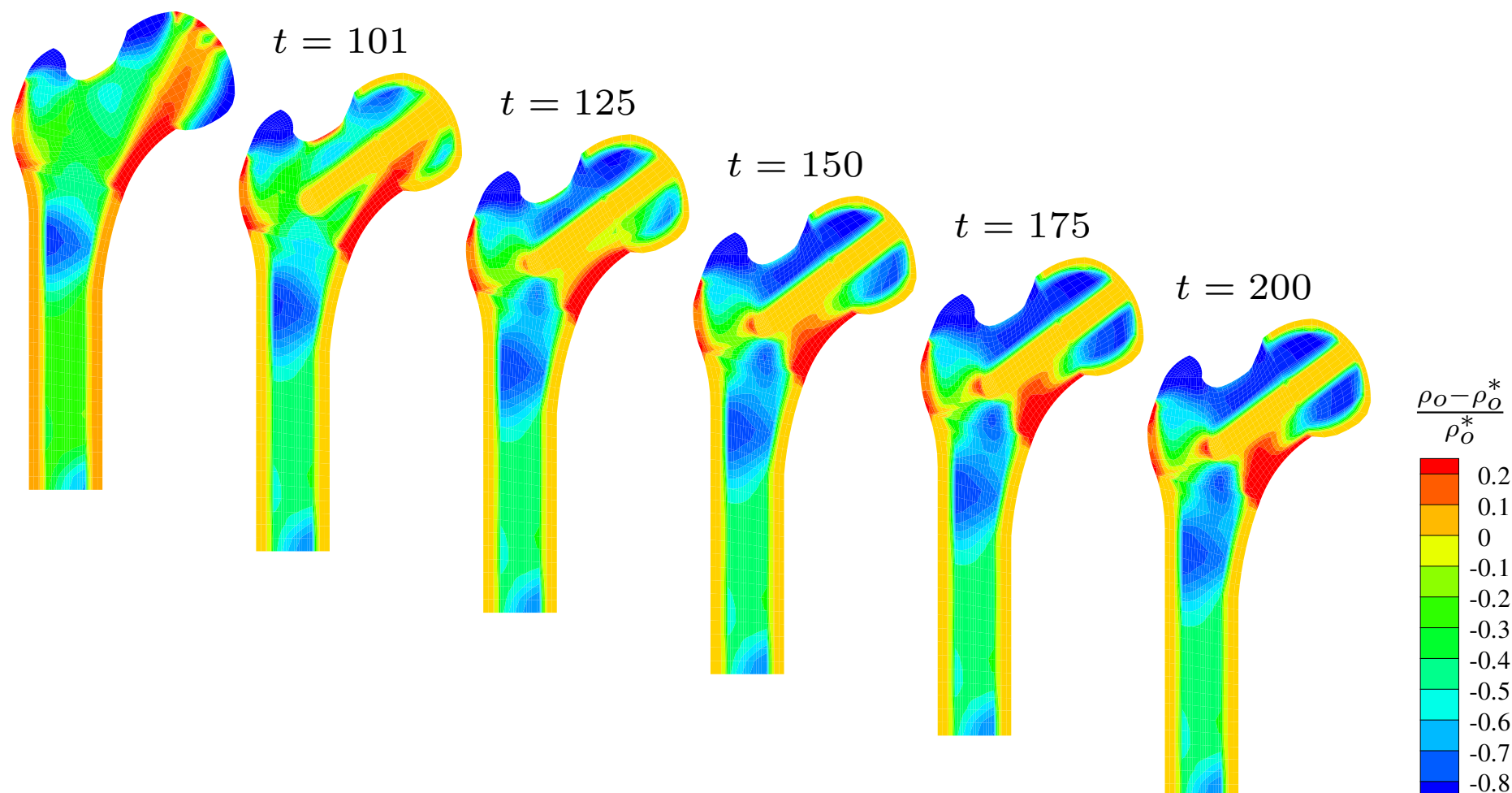
- density evolution – before implantation



ward's triangle – tensile & compressive trabeculae - dense cortical shaft

example – hip replacement

- density evolution – new birmingham hip resurfacing



improved ingrowth - anatomic loading situation preserved - less resorption

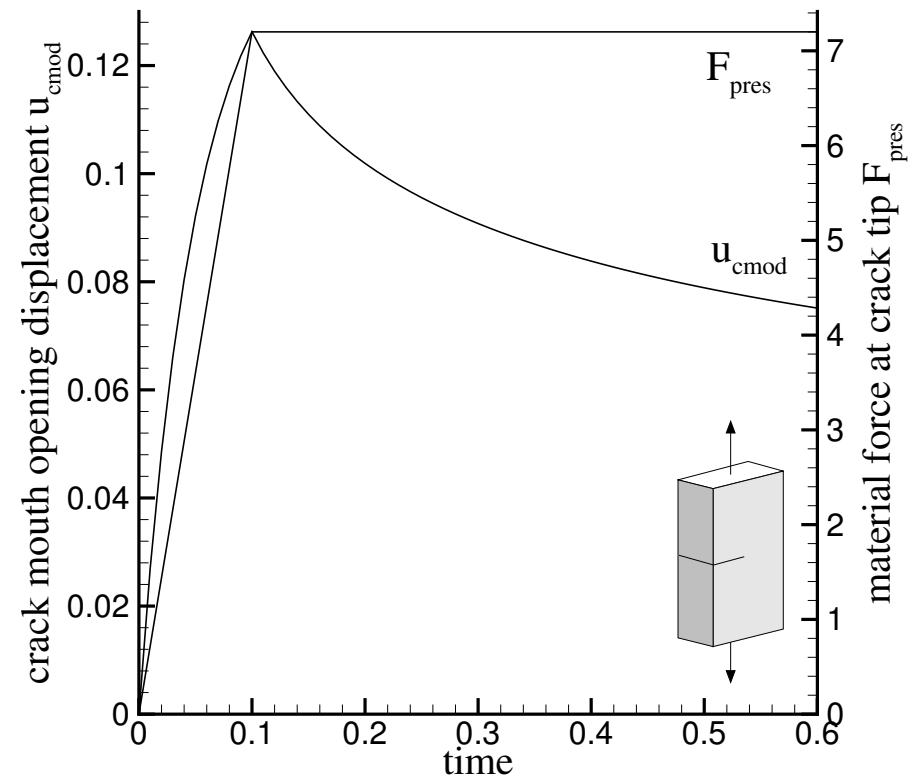
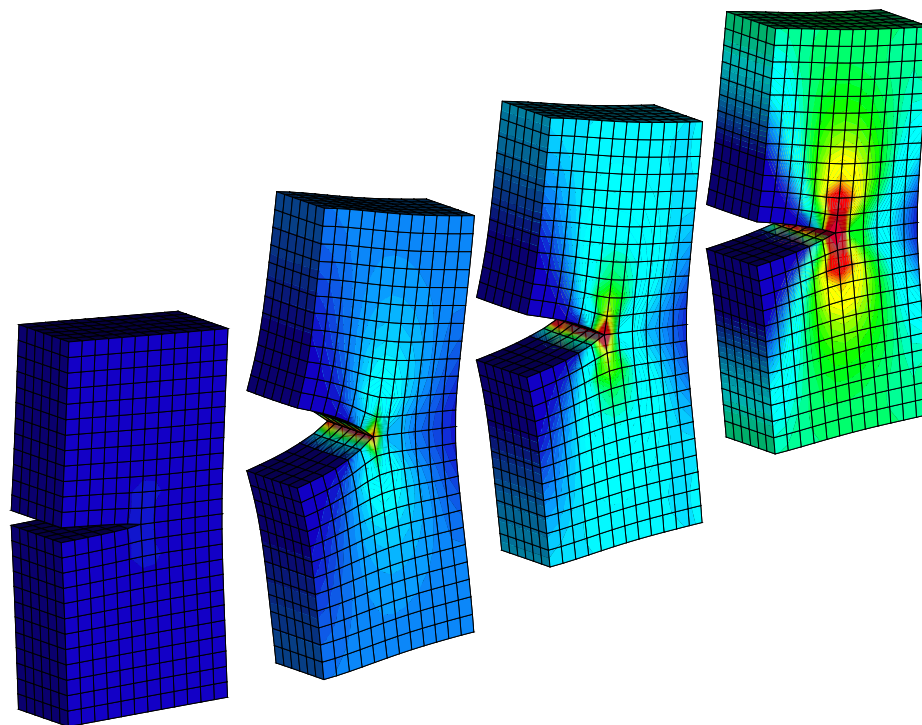
example – hip replacement

EDWIN SMITH SURGICAL PAPYRUS [13th

”...thou shouldst bind it with fresh meet the first day, and thou shouldst treat afterword with grease and honey every day until he recovers...”

24

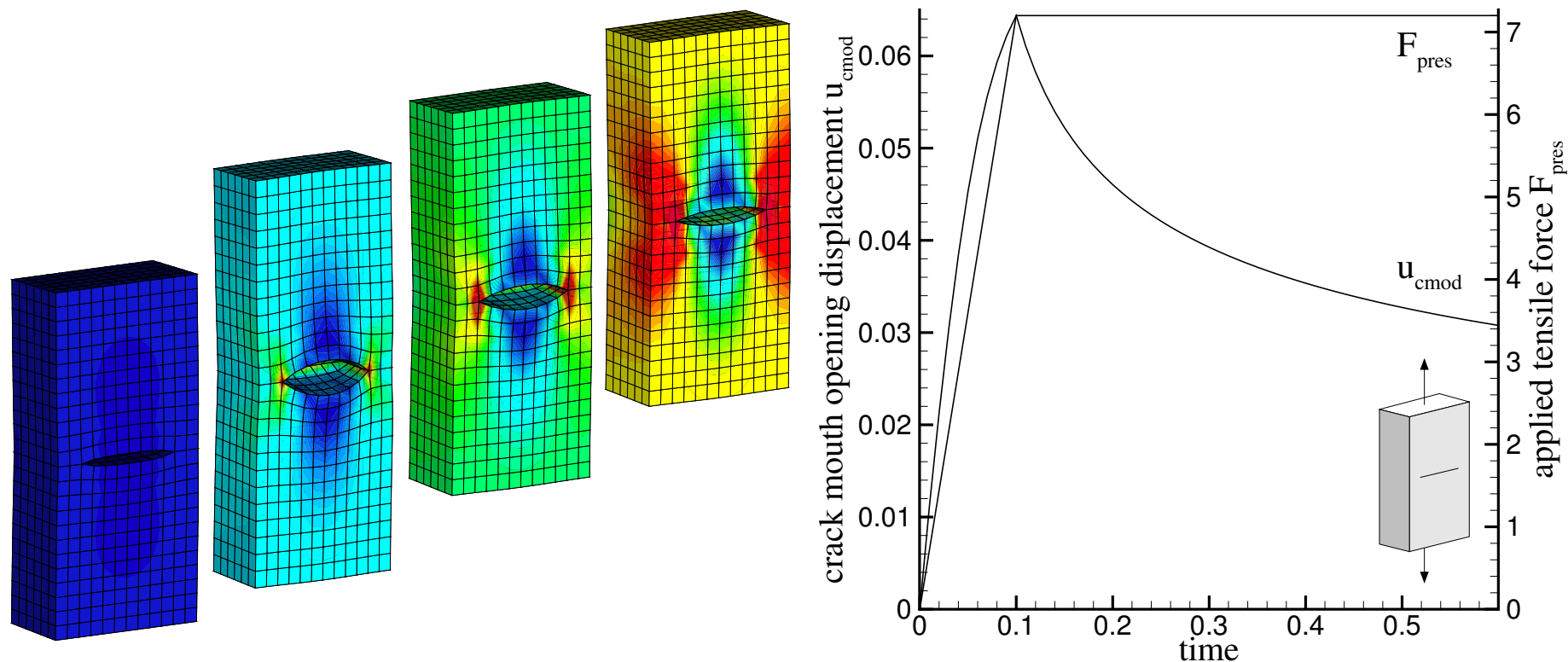
• self healing of notch under tension



- **cell migration** & **cell growth** stimulated by surface traction at wound boundary
- remarkable **reduction of overall deformation** accompanied by **wound closure**

example – wound healing

• self healing of surface cut under tension

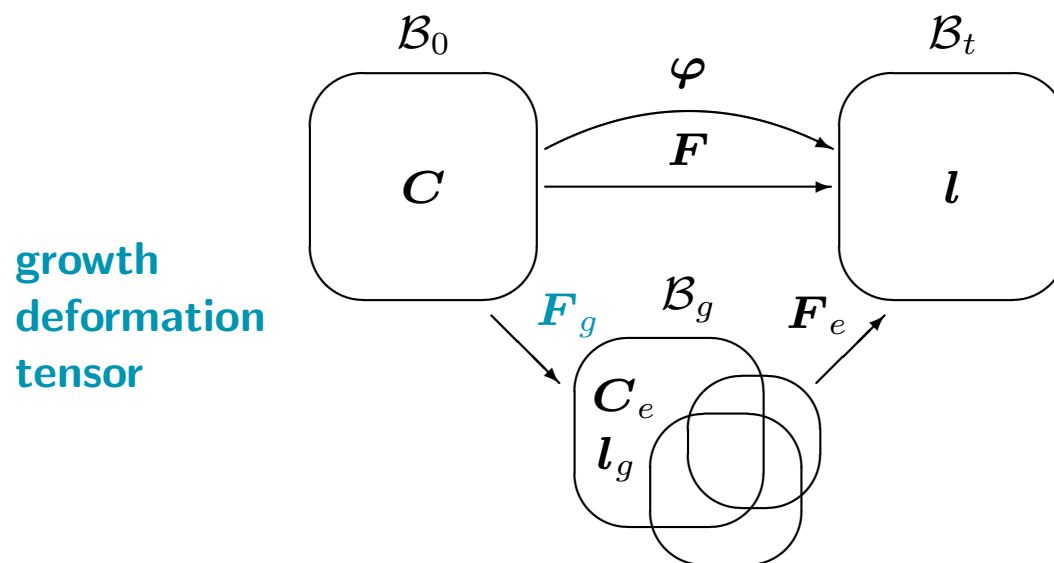


- **cell migration** & **cell growth** stimulated by surface traction at wound boundary
- remarkable **reduction of overall deformation** accompanied by **wound closure**

example – wound healing

volume growth

multiplicative decomposition



- deformation gradient

$$\mathbf{F} = \nabla \varphi = \mathbf{F}_e \cdot \mathbf{F}_g$$

- elastic right cauchy green tensor

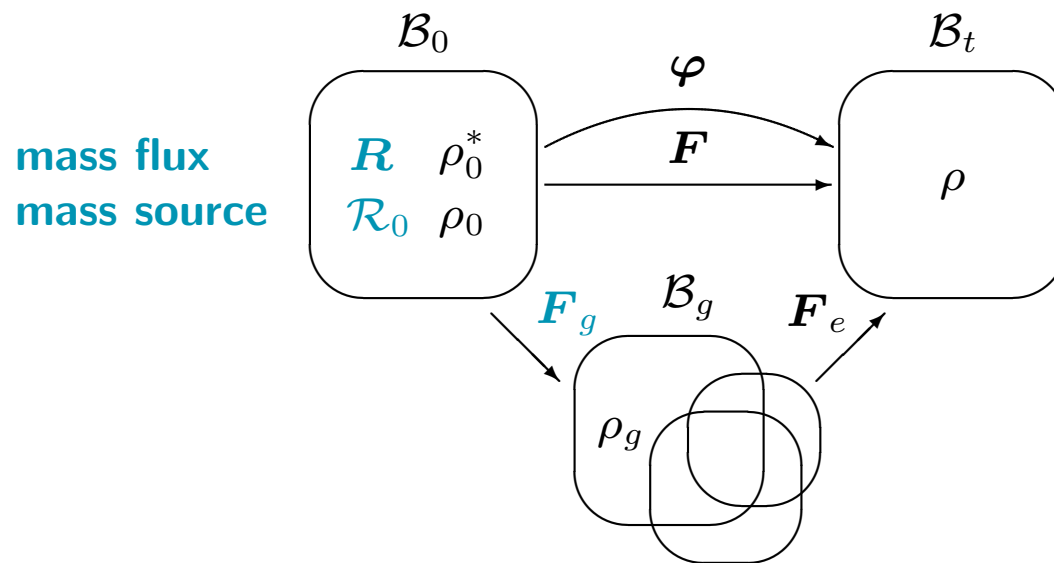
$$\mathbf{C}_e = \mathbf{F}_e^t \cdot \mathbf{F}_e$$

- growth velocity gradient

$$\mathbf{l}_g = \text{d}_t \mathbf{F}_g^t \cdot \mathbf{F}_g^{-1}$$

LEE [1969], RODRIGUEZ, HOGER & MCCULLOCH [1994], EPSTEIN & MAUGIN [2000], CHEN & HOGER [2000], AMBROSI & MOLLIKA [2002], HUMPHREY [2002], IMATANI & MAUGIN [2002], LUBARDA [2004], GARIKIPATI, ARRUDA, GROSH, NARAYANAN & CALVÉ [2004]

change in mass



- change in **density**
- change in **volume**

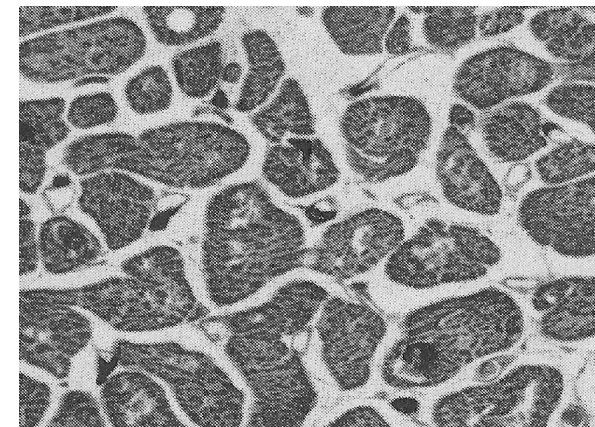
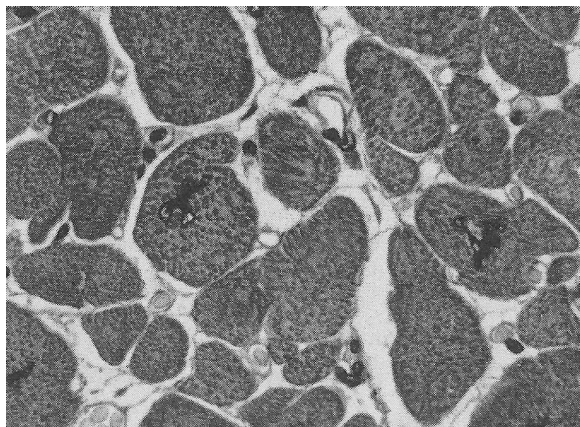
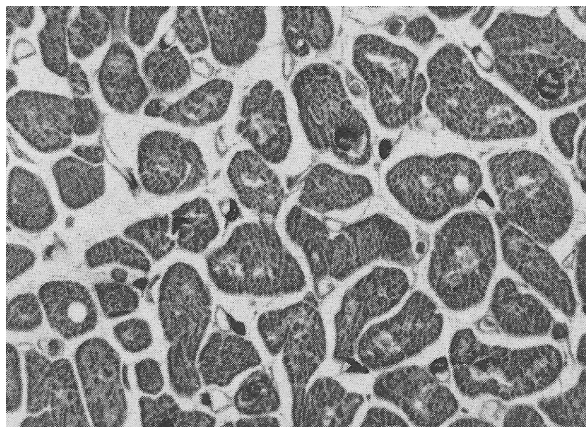
$$\rho_0 = \rho_0^* + \int_0^t \text{Div}(\mathbf{R}) + \mathcal{R}_0 \, d\tau$$

$$dV_g = \det(\mathbf{F}_g) \, dV_0$$

LUBARDA & HOGER [2002], HIMPEL, KUHL, MENZEL & STEINMANN [2005]

volume growth at constant density

- free energy $\psi_0 = \psi_0^{\text{neo}}(\mathbf{F}_e)$
- stress $\mathbf{\Pi}_e^t = \mathbf{\Pi}_e^{\text{tneo}}(\mathbf{F}_e)$
- growth deformation tensor $\mathbf{F}_g = \vartheta \mathbf{I} \quad \text{d}_t \vartheta = k_\vartheta(\vartheta) \text{tr}(\mathbf{C}_e \cdot \mathbf{S}_e)$
- mass source $\mathcal{R}_0 = 3 \rho_0^* \vartheta^2 \text{d}_t \vartheta$



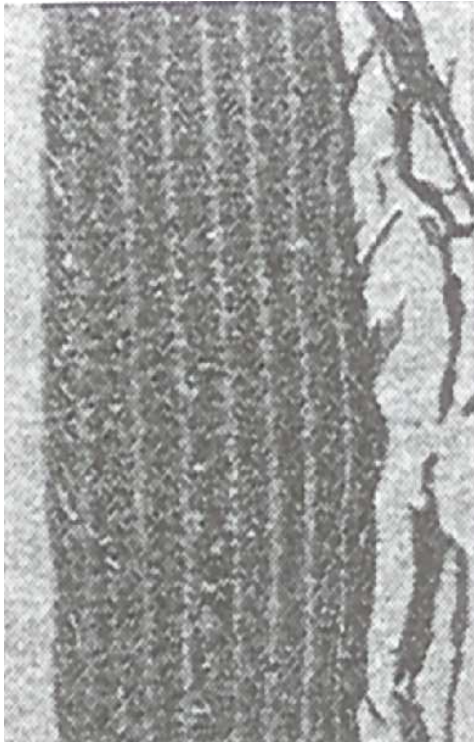
kinematic coupling of growth and deformation

RODRIGUEZ, HOGER & MCCULLOCH [1994], EPSTEIN & MAUGIN [2000], HUMPHREY [2002]

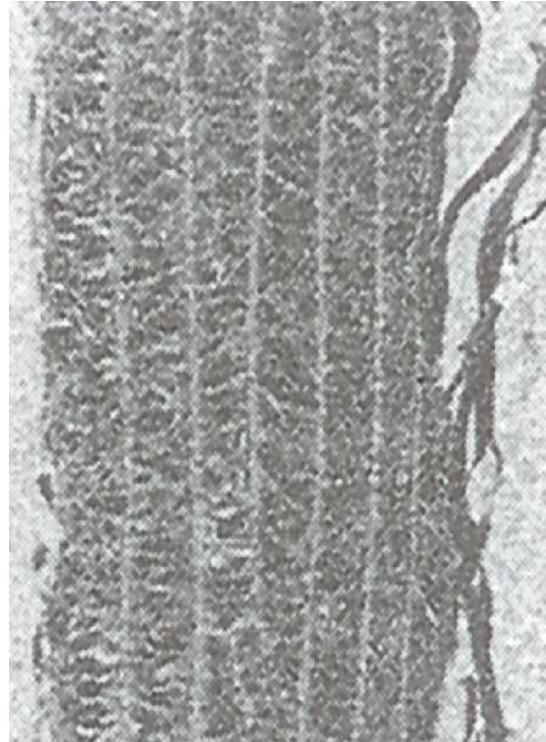
volume growth – constitutive equations

density preserving volume growth of the aortic wall

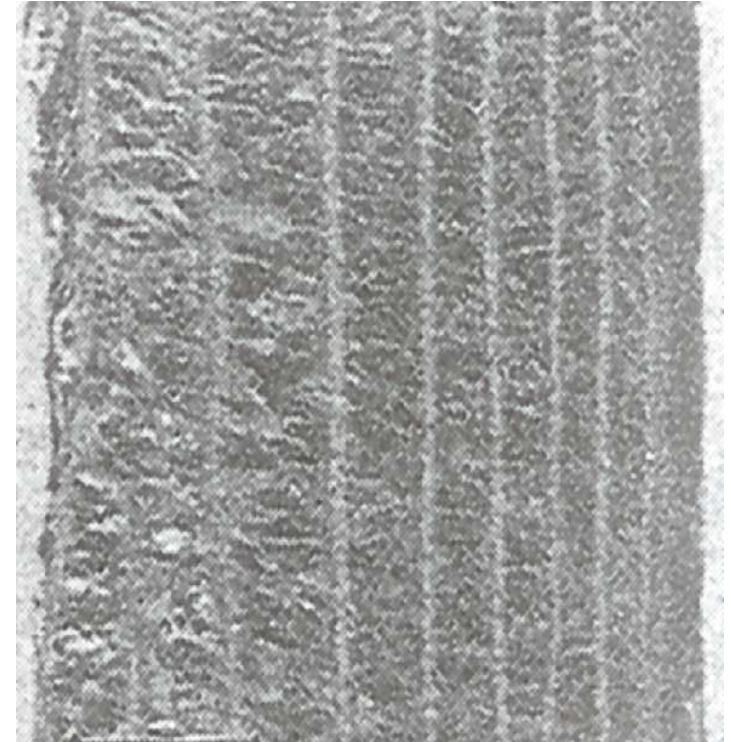
normotensive



hypertensive



severely hypertensive

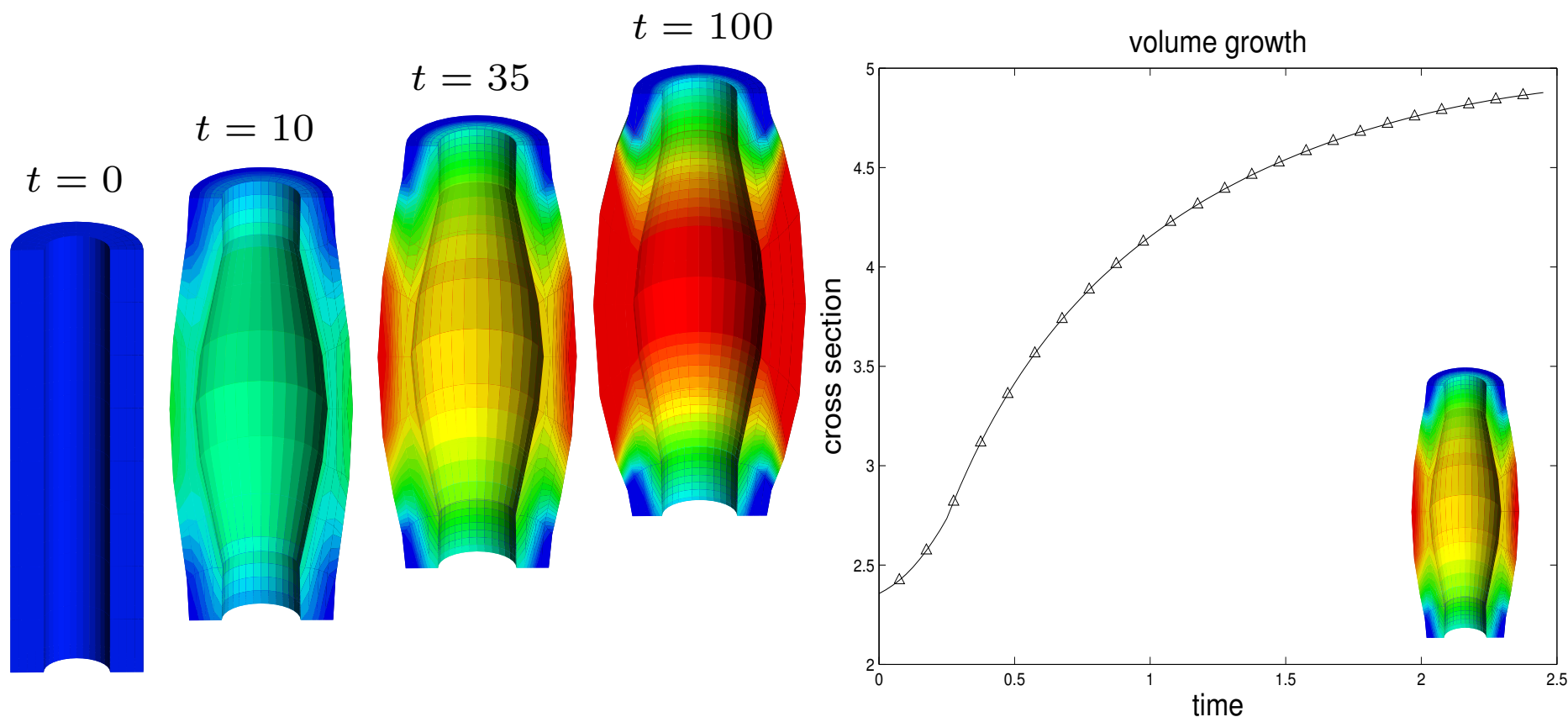


overall wall thickening via thickening of individual musculoelastic fascicles

MATSUMOTO & HAYASHI [1996], HUMPHREY [2002]

example – adaptation in aortic wall

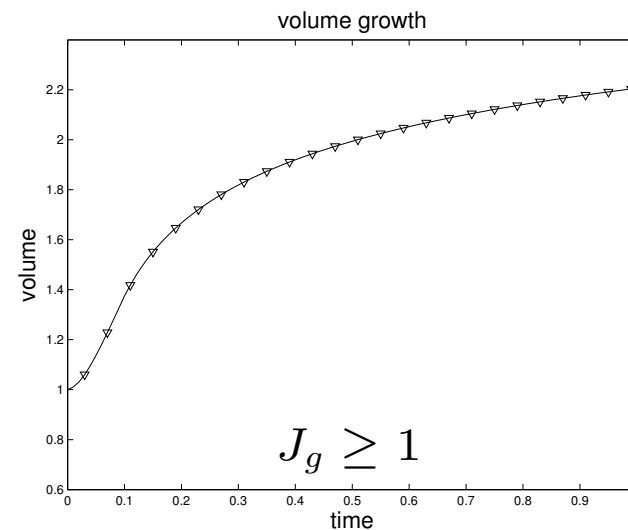
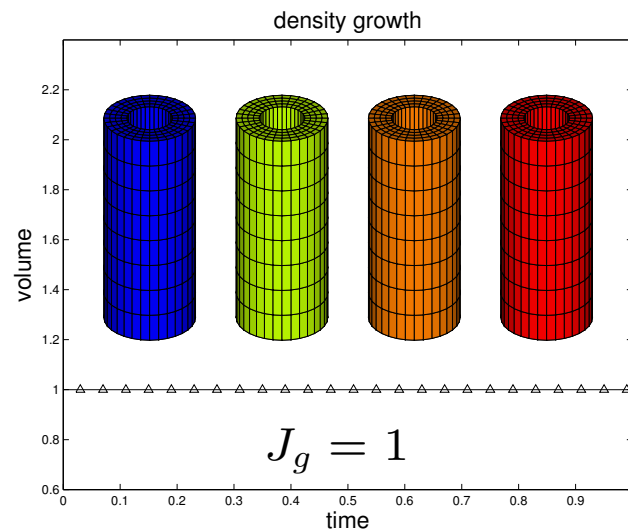
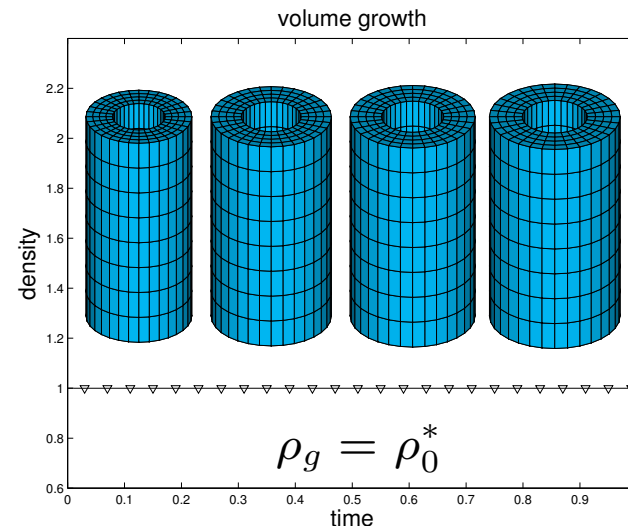
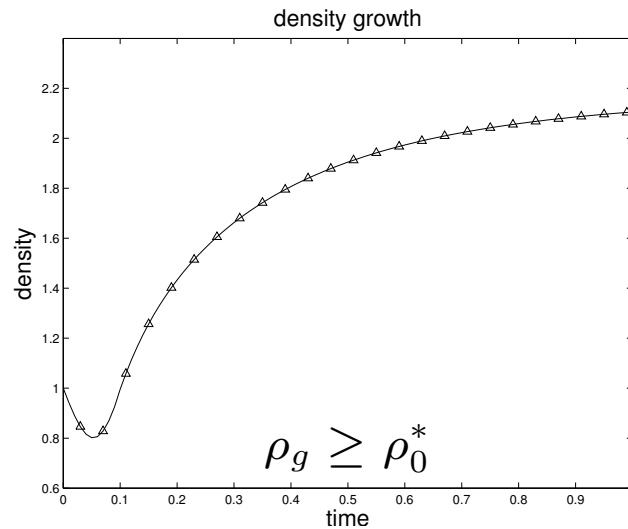
- volume evolution – qualitative simulation of stent implantation



deformation induced volume growth at constant density

example – adaptation in aortic wall

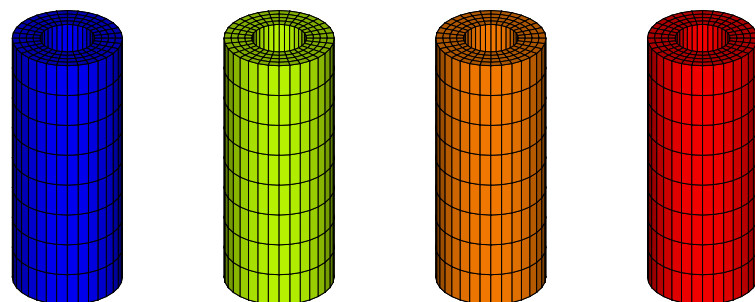
isotropic density growth vs. isotropic volume growth



comparison – density vs. volume growth

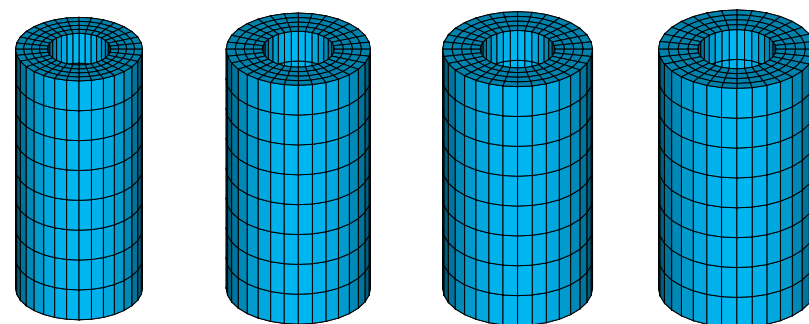
isotropic density growth

- growth in density at **constant volume**
 $\rho_g \geq \rho_0^*$ and $J_g = 1$
- $\mathbf{F}_g = \mathbf{I}$, choose $d_t \rho_0 = \mathcal{R}_0$
- **constitutive coupling**
 $\psi = \psi(\mathbf{F}, \rho_0)$
- characteristic for growth of **hard tissues**
- COWIN & HEGEDUS [1976]



isotropic volume growth

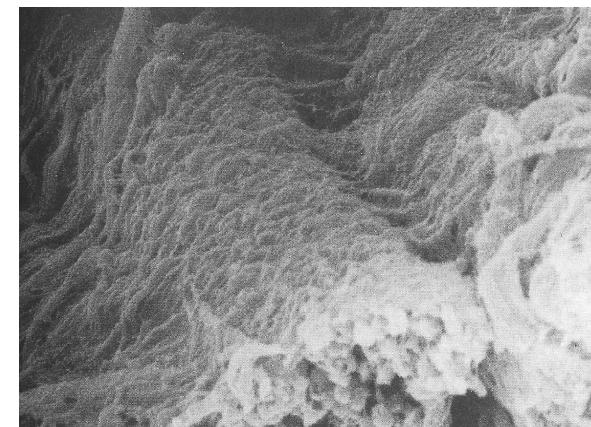
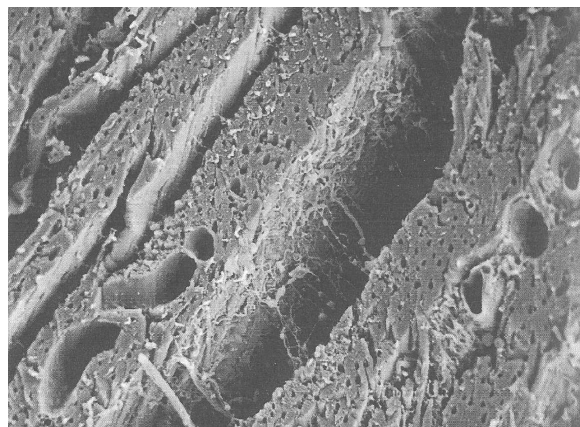
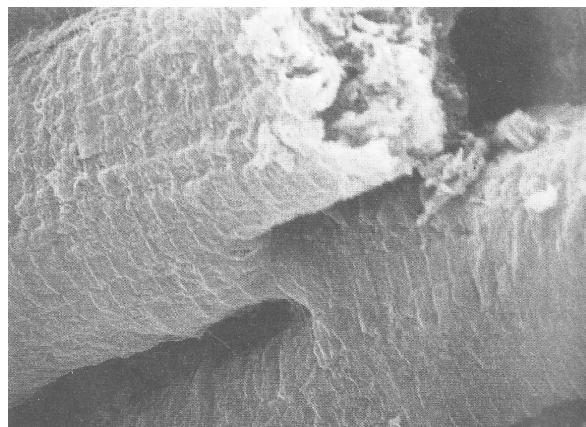
- growth in volume at **constant density**
 $\rho_g = \rho_0^*$ and $J_g \geq 1$
- $\mathcal{R}_0 = \rho_0 d_t \mathbf{F}_g : \mathbf{F}_g^{-1}$, choose $d_t \mathbf{F}_g$
- **kinematic coupling**
 $\mathbf{F}_e = \mathbf{F} \cdot \mathbf{F}_g^{-1}$
- characteristic for growth of **soft tissues**
- RODRIGUEZ, HOGER, McCULLOCH [1994]



comparison – density vs. volume growth

fiber growth at constant density

- free energy $\psi_0 = \psi_0^{\text{iso}}(\mathbf{F}_e) + \frac{1}{2} \alpha [I_4 - 1]^2$
- stress $\mathbf{\Pi}_e^t = \mathbf{\Pi}_e^{\text{iso}}(\mathbf{F}_e) + \alpha \mathbf{F}_e \cdot \mathbf{n}_0 \otimes \mathbf{n}_0$
- growth deformation tensor $\mathbf{F}_g = \vartheta \mathbf{I} + [\eta - \vartheta] \mathbf{n}_0 \otimes \mathbf{n}_0 \quad \mathbf{F}_g \cdot \mathbf{n}_0 = \eta \mathbf{n}_0$
- mass source $\mathcal{R}_0 = \rho_0^* [\eta \vartheta \, d_t \vartheta + \vartheta^2 \, d_t \eta]$

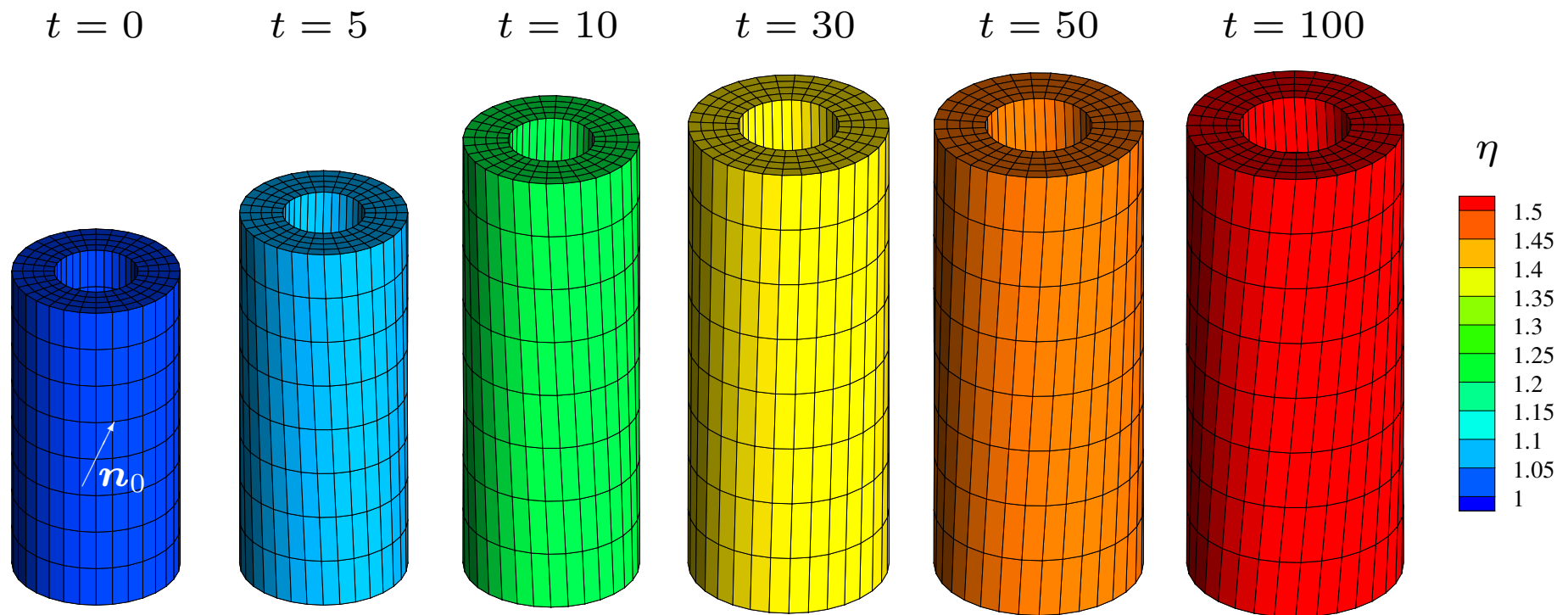


kinematic coupling of growth and deformation

LUBARDA & HOGER [2002], HIMPEL, KUHL, MENZEL & STEINMANN [2005]

fiber growth – constitutive equations

- anisotropic growth in cylindrical tube



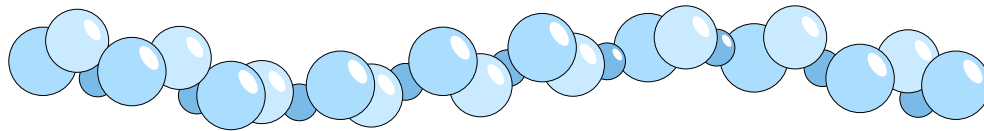
deformation induced fiber and volume growth at constant density

example – tube in tension

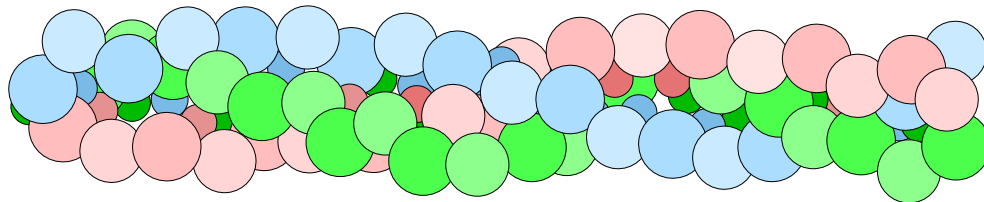
remodelling

collagen chains as major protein of extracellular matrix

● glycine ● hydroxyprolin ● prolin amino acids



about 1000 amino acids
form a collagen α chain



three α chains
form collagen triple helix
1.5 nm diameter

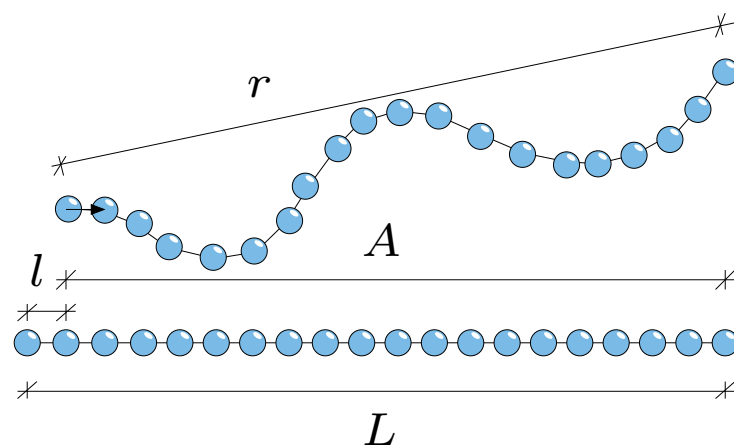


collagen fibrils
form a collagen fiber
10-300 nm diameter

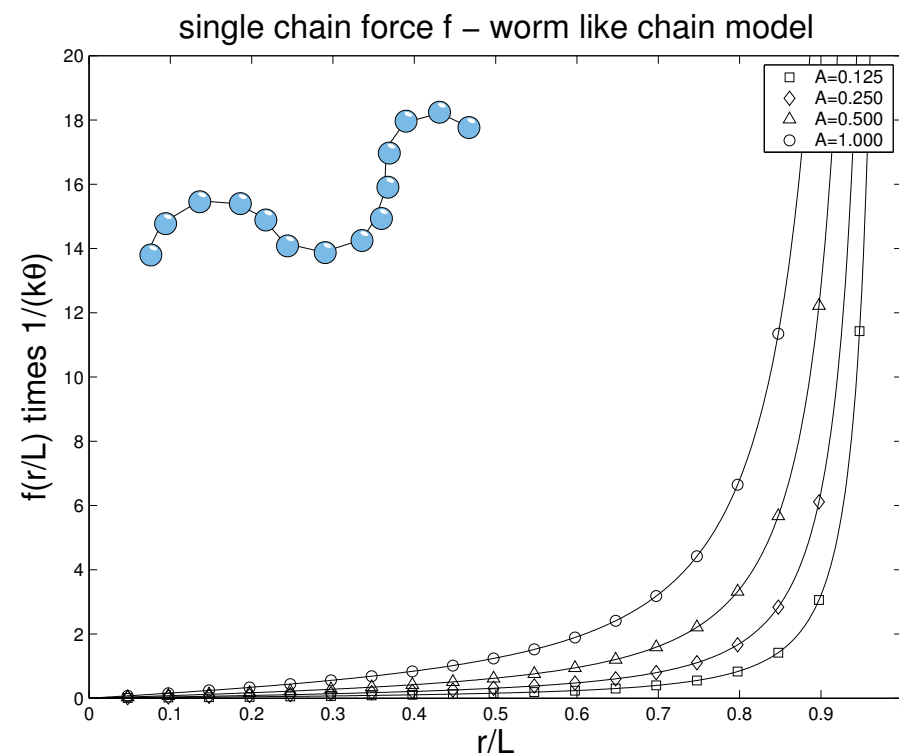
transverse isotropy – collagen fibers provide directional strengthening

remodelling – theory

microscale – statistical mechanics of long chain molecules



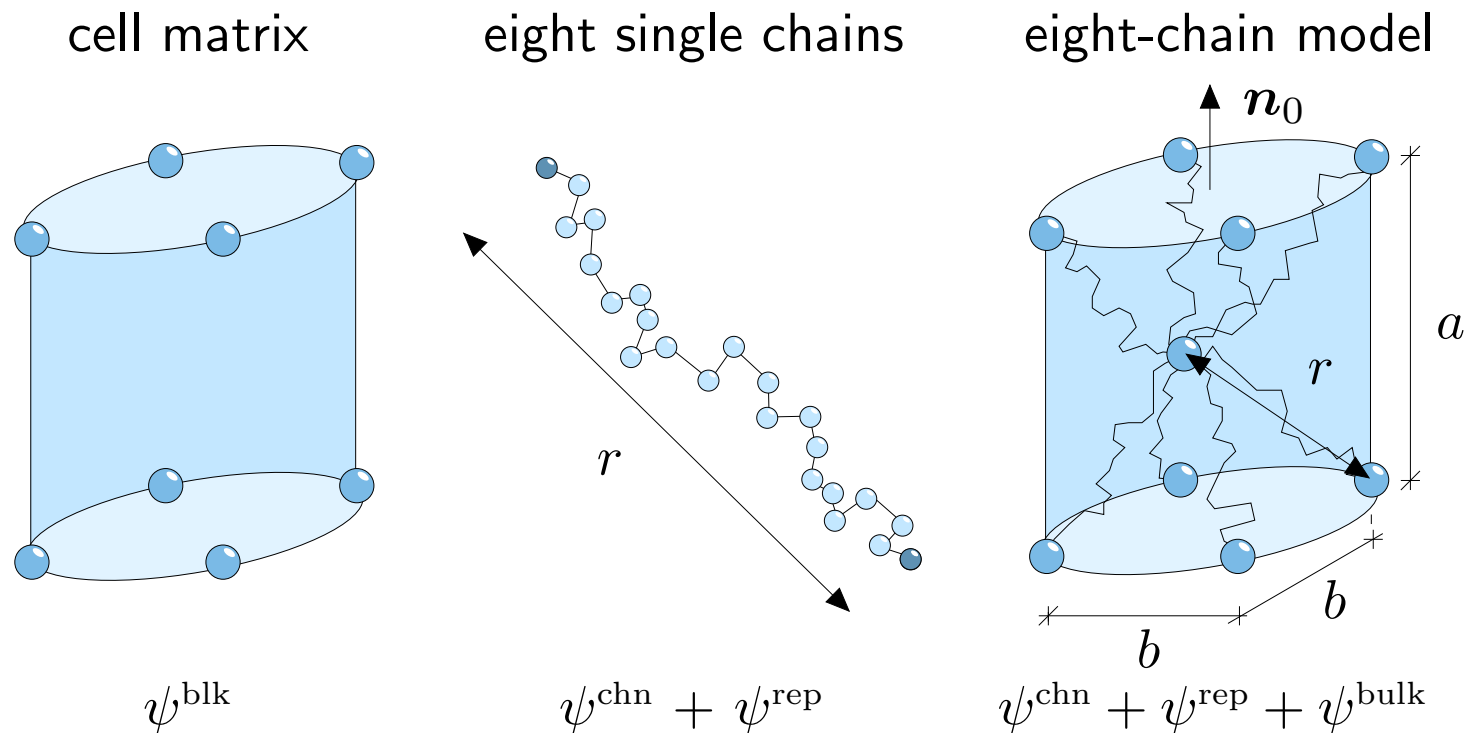
$$\psi^{\text{chn}} = \theta \frac{k L}{4A} \left[2 \frac{r^2}{L^2} + \frac{1}{[1 - r/L]} - \frac{r}{L} \right]$$



- **wormlike chain model** for biomaterials, e.g. dna or collagen fibers
- characteristic **locking behavior** towards full elongation as $r \rightarrow L$

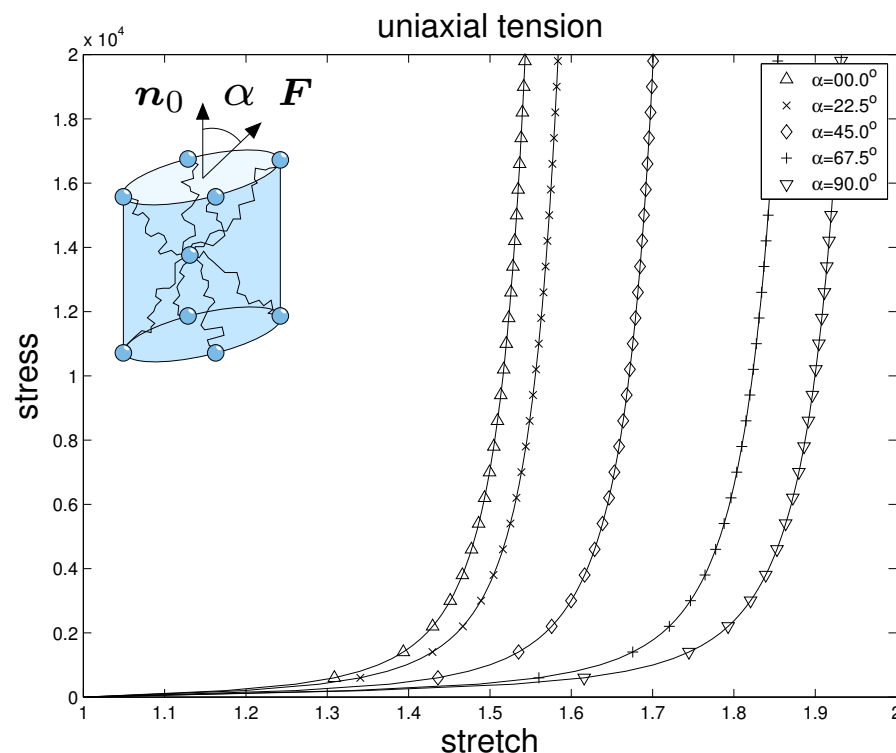
KUHN [1934], [1936], KUHN & GRÜN [1942], POROD [1949], KRATKY & POROD [1949], FLORY [1969], TRELOAR [1975], BUSTAMANTE, SMITH, MARKO & SIGGIA [1994], BOYCE [1996]

macroscale – transversely isotropic chain network



KUHL, GARIKIPATI, ARRUDA & GROSH [2005]

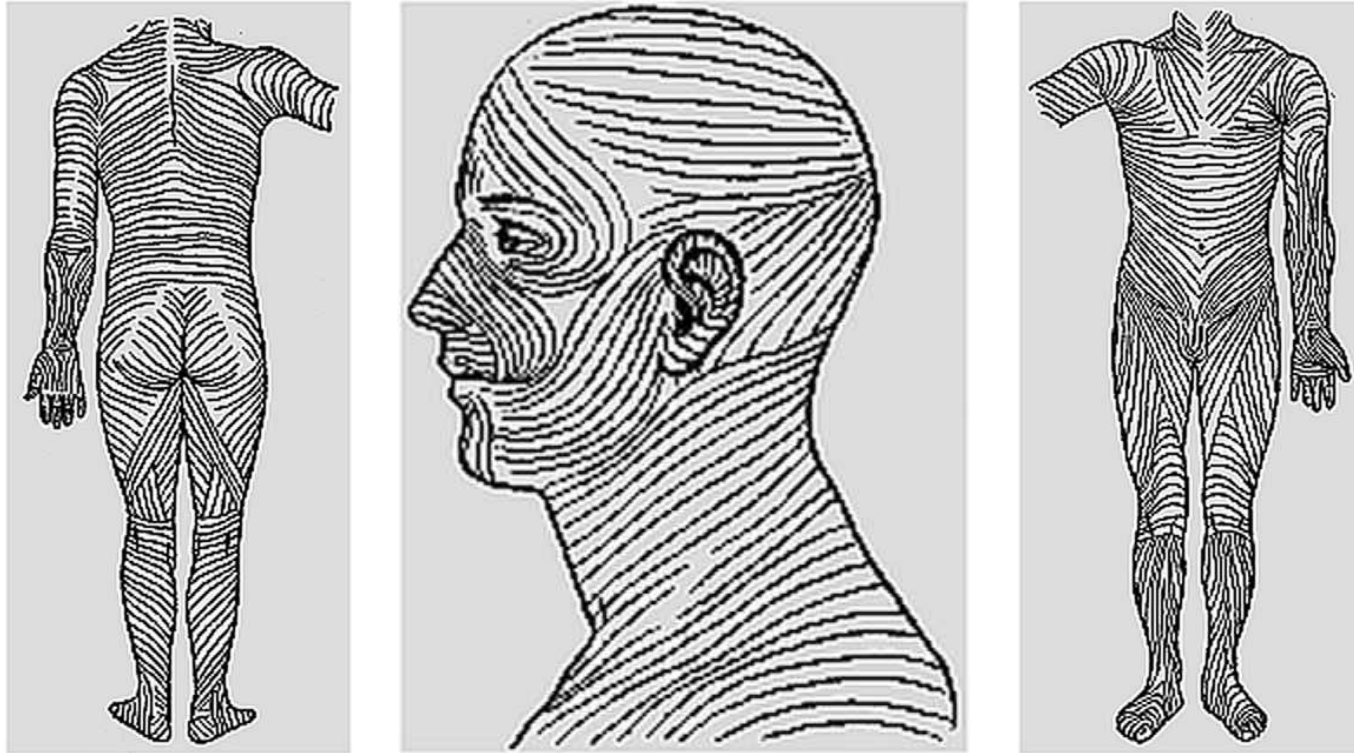
macroscale – transversely isotropic chain network



- increasing stiffness for decreasing fiber load angle

example – influence of fiber load angle

langer's lines on skin

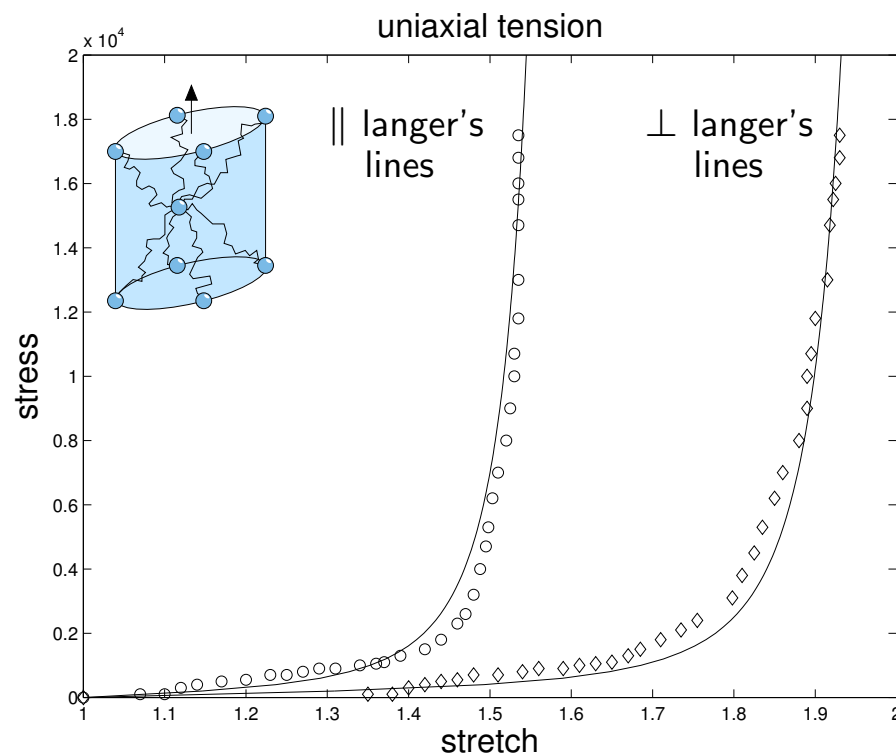


- **lines of tension** or cleavage within skin
- orientation of **collagen fiber bundles** || to langer's lines
- surgery – incisions || to langer's lines heal with finer scar

CARL RITTER VON LANGER [1819-1887]

example – langer's lines in skin

uniaxial tests on rabbit skin – experiment vs. simulation

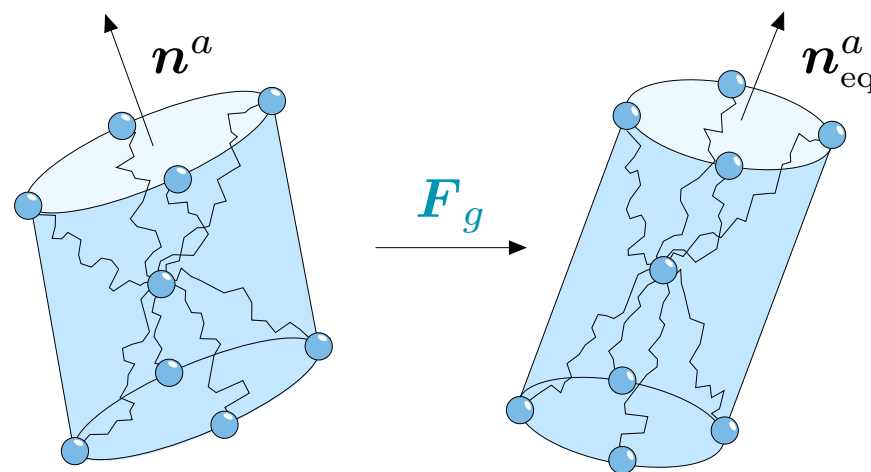


- stiffer along langer's lines, characteristic **locking behavior**
- **good agreement** of eight-chain-model and experimental results

LANIR & FUNG [1974], KUHLMAN, GARIKIPATI, ARRUDA & GROSH [2005]

example – langer's lines in skin

functional adaptation of collagen fibers



- general idea: introduction of fiber direction $\mathbf{n}^a$ as internal variable

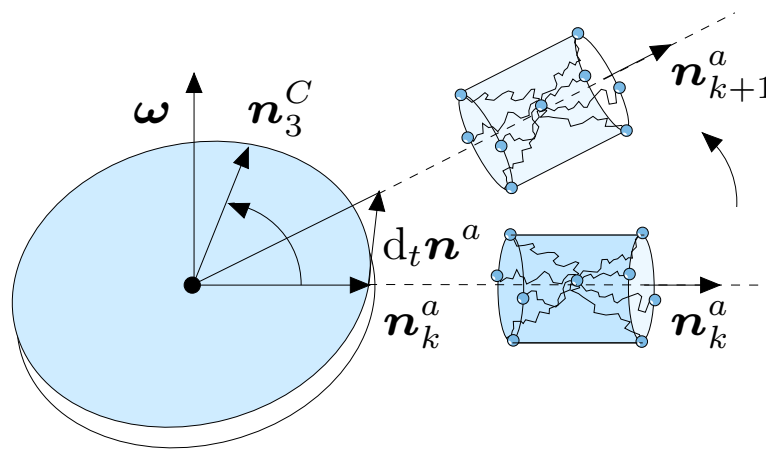
$$\psi = \psi(\mathbf{F}, \mathbf{n}^a, \dots)$$

- reorientation with respect to stimulus $\mathbf{n}_{\text{eq}}^a$, e.g. strain or stress based

$$\mathrm{d}_t \mathbf{n}^a = \frac{1}{\tau_\omega} [\mathbf{n}^a - \mathbf{n}_{\text{eq}}^a]$$

FYRHIE & CARTER [1986], COWIN [1989], PEDERSEN [1989], VIANELLO [1996], SGARRA & VIANELLO [1997], DRIESSEN, PETERS, HUYGHE, BOUTEN & BAAIJENS [2003], MENZEL [2004]

functional adaptation of collagen fibers



- gradual alignment of fiber direction $\mathbf{n}^a$ with principal strain direction $\mathbf{n}_3^C$

$$\mathbf{C} = \sum_{I=1}^3 \lambda_I^C \mathbf{n}_I^C \otimes \mathbf{n}_I^C$$

- evolution of fiber direction $\mathbf{n}^a$ with rotation vector $\boldsymbol{\omega}$ orthogonal to $\mathbf{n}^a$ and $\mathbf{n}_3^C$

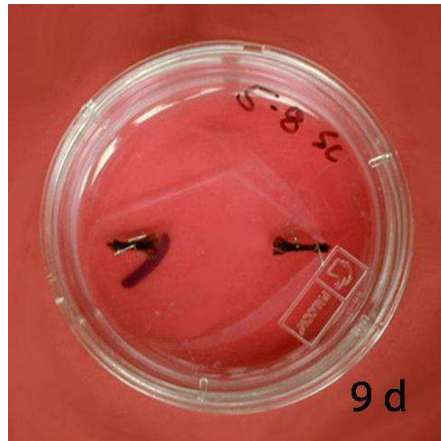
$$d_t \mathbf{n}_a = \frac{1}{\tau_{\boldsymbol{\omega}}} [\mathbf{n}_3^C - [\mathbf{n}_3^C \cdot \mathbf{n}^a] \mathbf{n}^a]$$

one single material parameter $\tau_{\boldsymbol{\omega}}$, remodelling time, can be measured optically

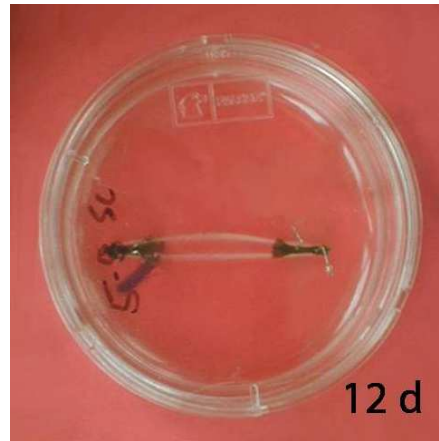
remodelling – theory

fiber reorientation in living tendon – experiment

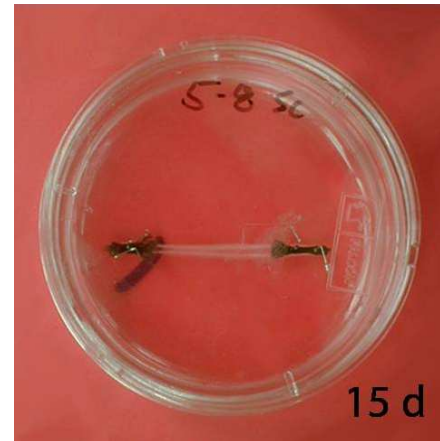
after 9 days



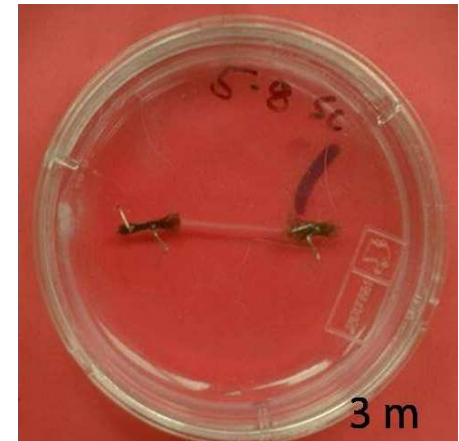
after 12 days



after 15 days



after 3 months



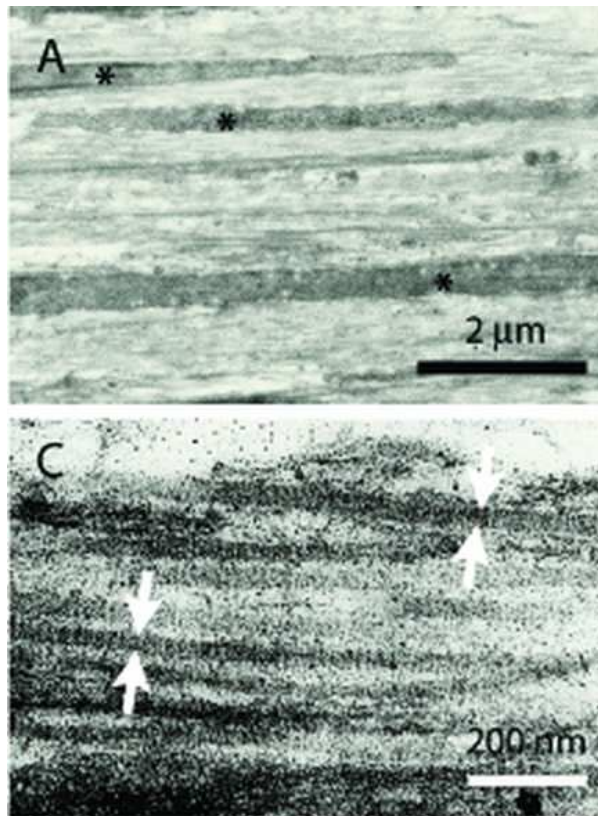
- ex vivo engineered tendon showing **characteristics of embryonic tendon**
- **remodelling of collagen fibers** upon mechanical loading
- long term goal – **mechanically stimulated tissue engineering**

CALVE, DENNIS, KOSNIK, BAAR, GROSH & ARRUDA [2004]

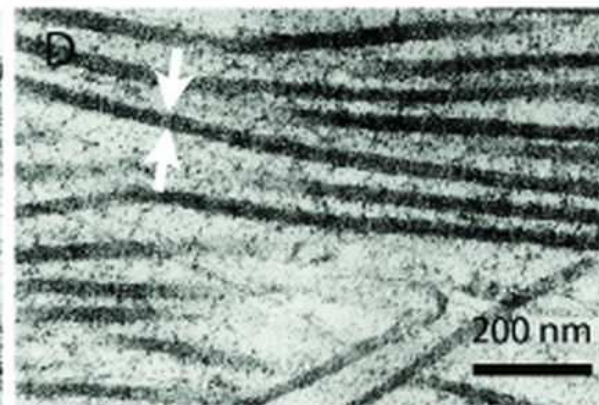
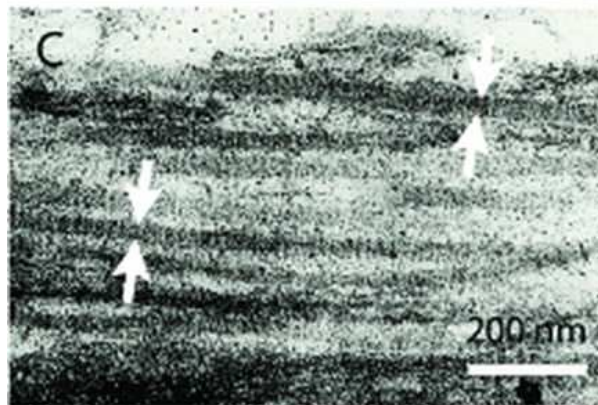
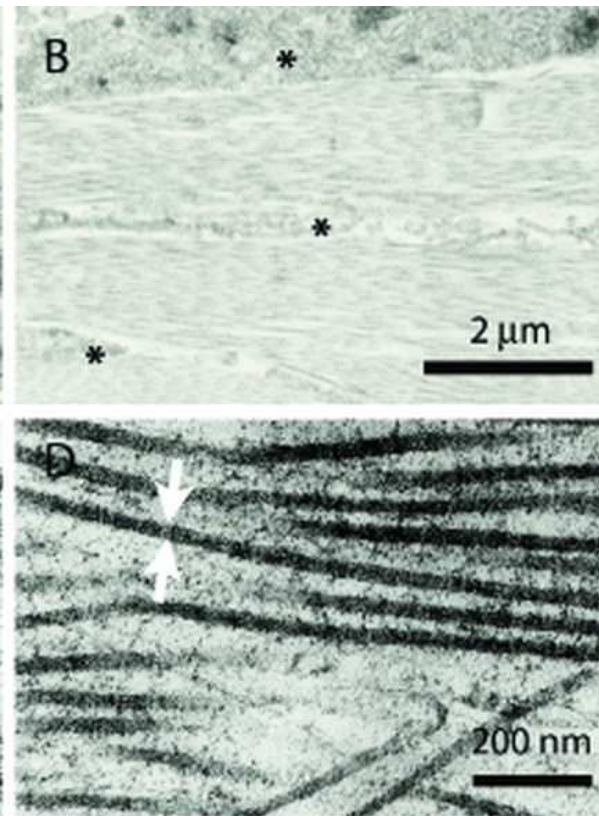
example – tissue engineering

fiber reorientation in living tendon – experiment

ex vivo grown tendon



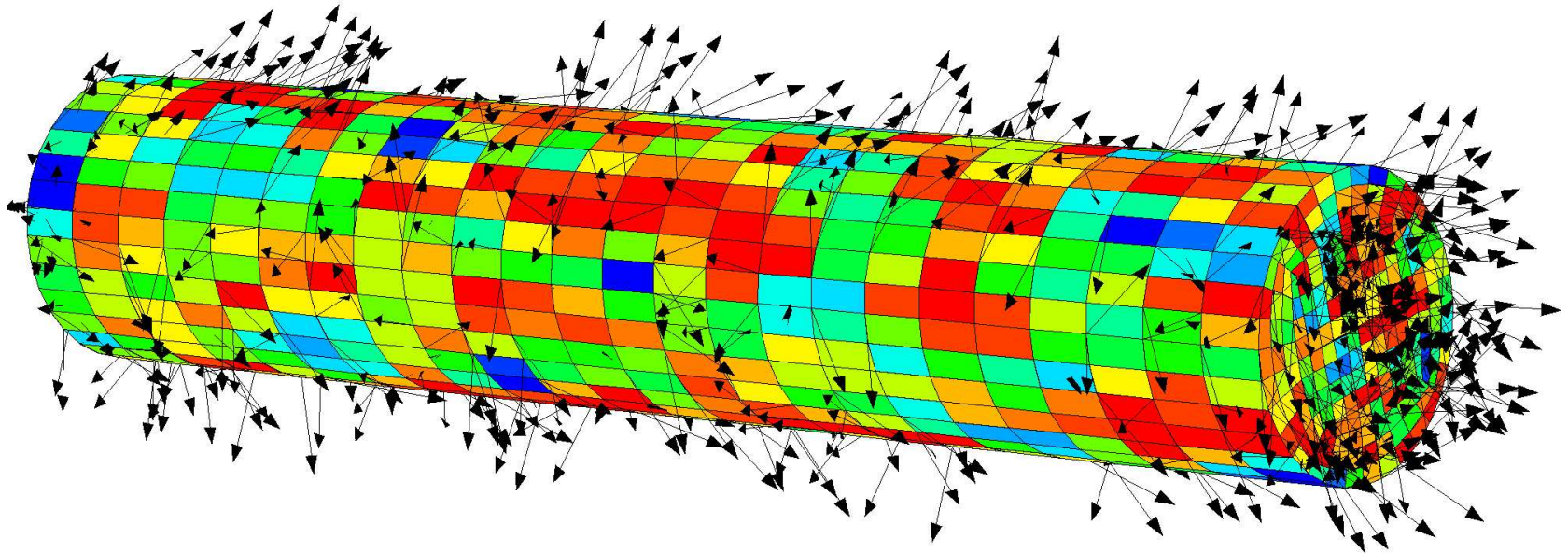
embryonic tendon



- **absence of collagenaceous scaffold** in ex vivo grown
- **typical characteristics of embryonic tendon**

example – tissue engineering

fiber reorientation in living tendon – simulation

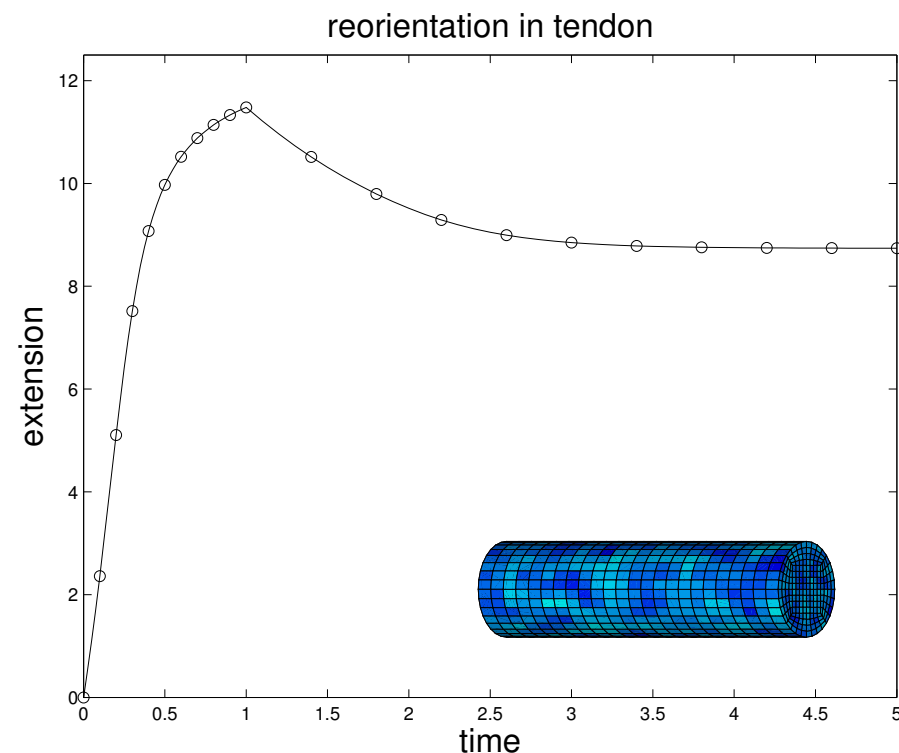
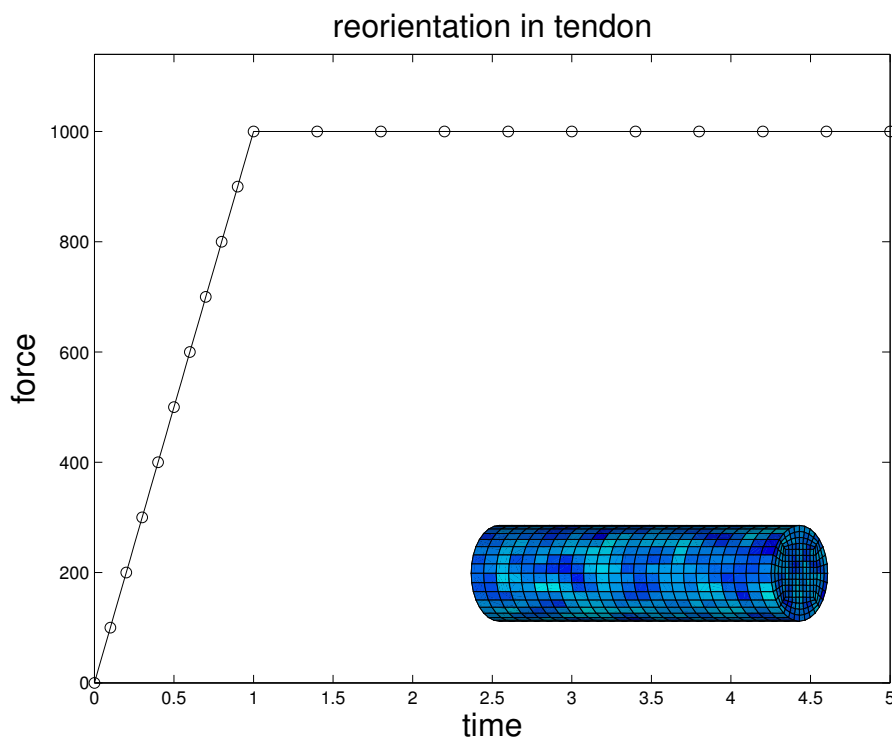


- finite element simulation of **functional adaptation in tendons**
- transversely isotropic **wormlike chain model**, **initially random** fiber orientation
- analysis of **fiber reorientation in uniaxial tension**

KUHL, GARIKIPATI, ARRUDA & GROSH [2004]

example – tissue engineering

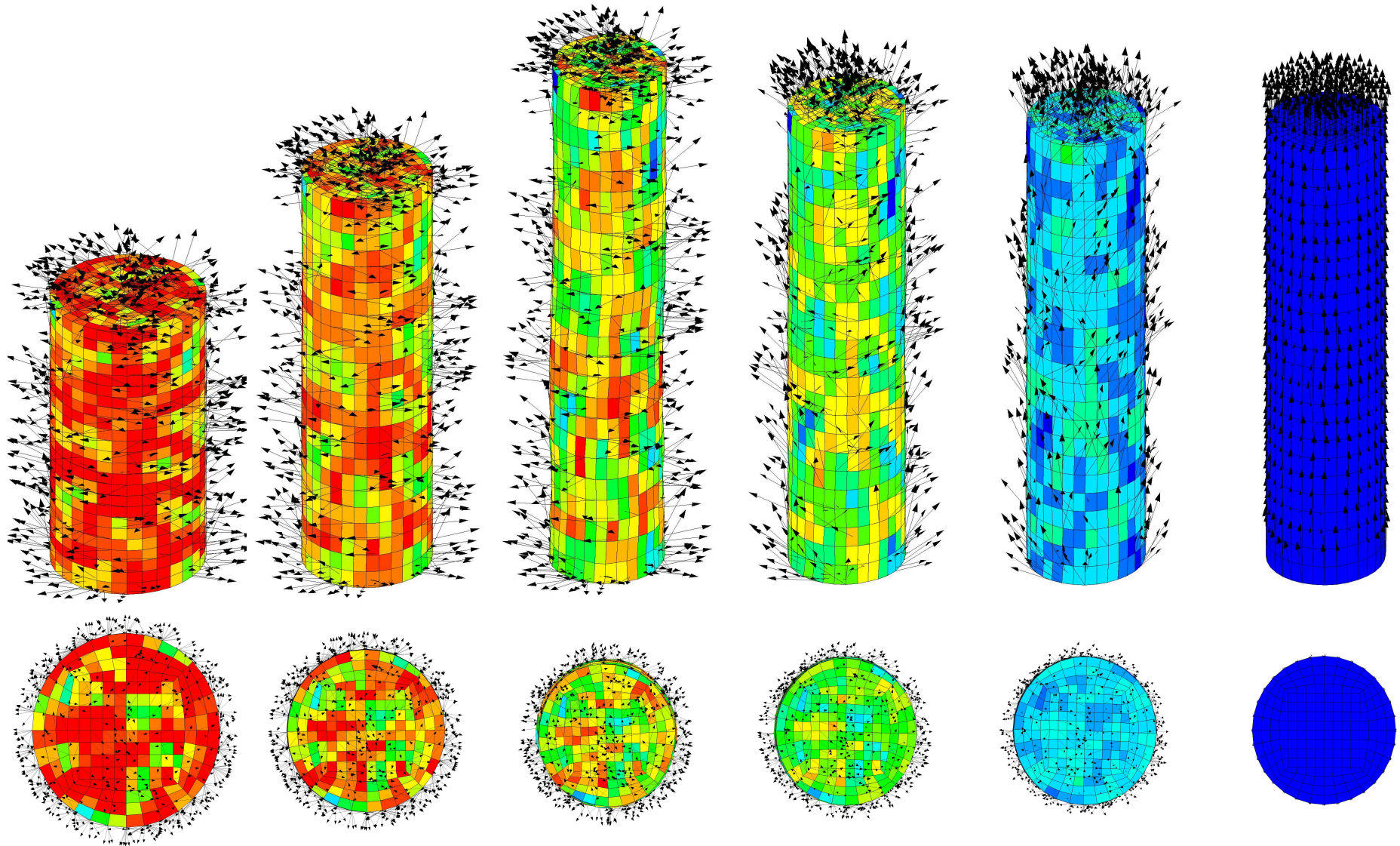
fiber reorientation in living tendon – simulation



- characteristic **locking behavior** upon loading
- **remodelling** and **stiffening** – **biological equilibrium**

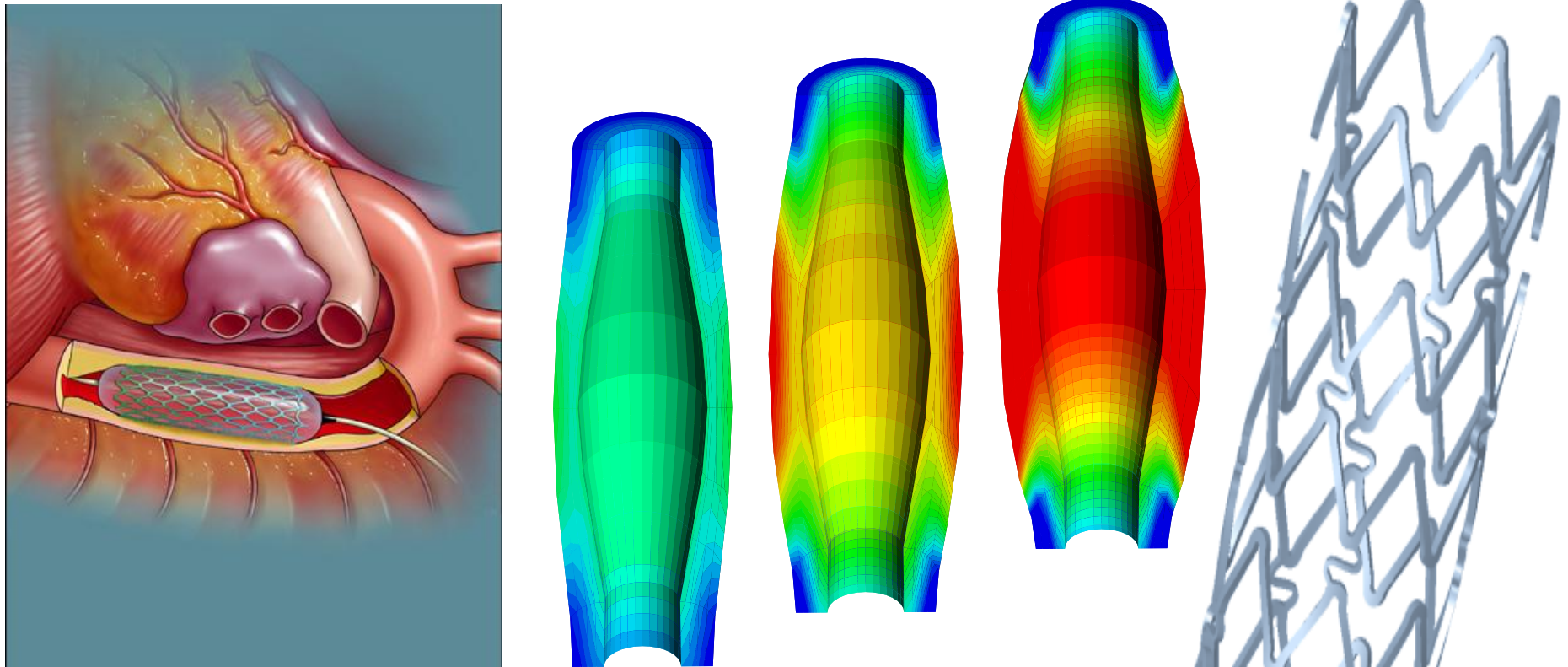
example – tissue engineering

- fiber reorientation in living tendon – simulation



example – tissue engineering

simulation of stent implantation



improved long term reliability

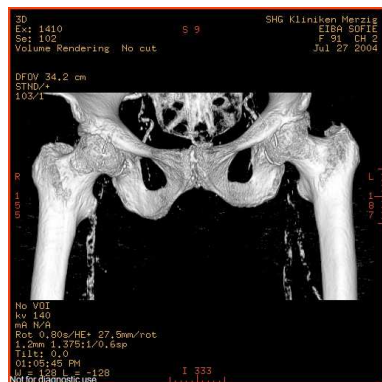
currently working on this project ramona maas & grieta himpel



visions – optimization of medical implants

generation of patient specific model

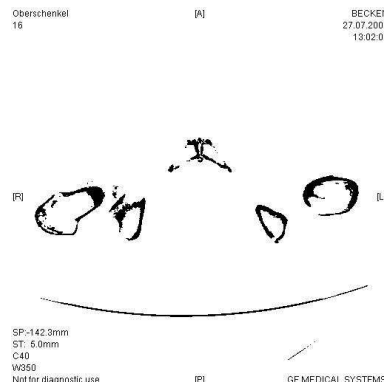
ct of overall hip



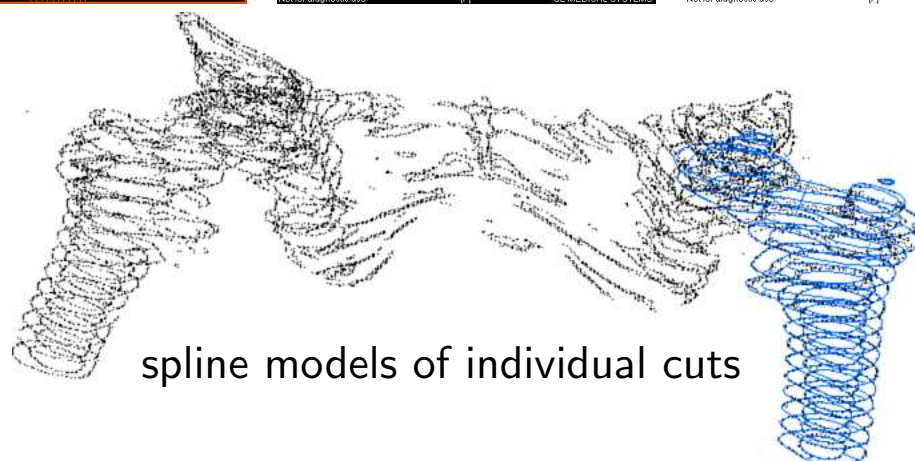
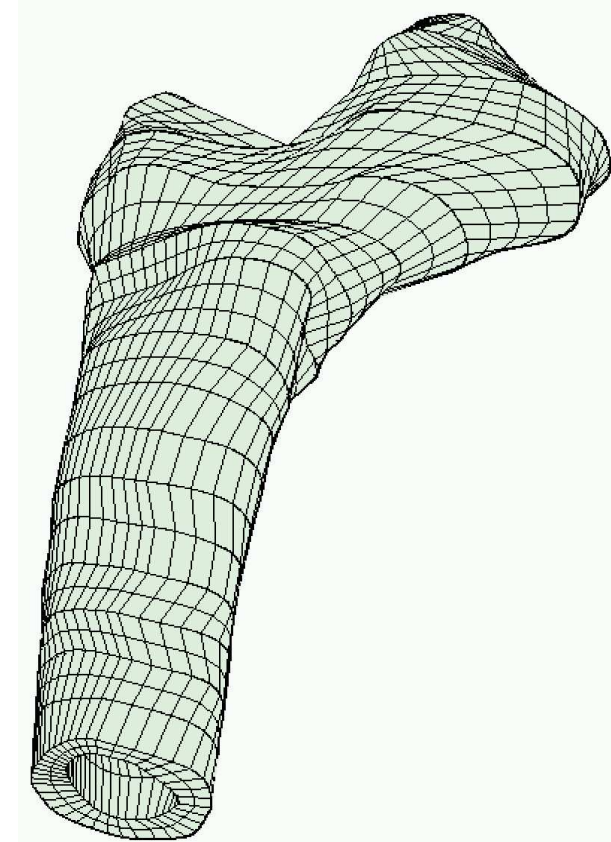
representative cut



binary data model



solid model of proxima femur



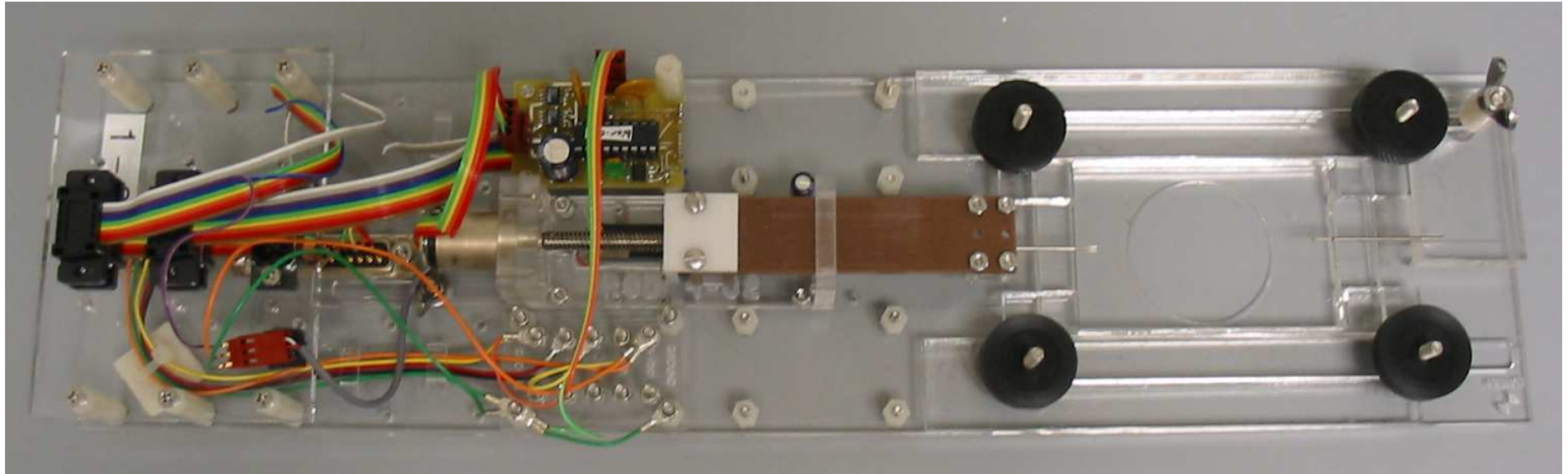
spline models of individual cuts

computer tomography of human femur

in cooperation with **Klinik Merzig** [2004]

visions – patient specific medical treatment

optimal stimulation of growth and remodelling



- analysis of **influence of** constant or cyclic **mechanical loading**
- long term goal – **mechanically stimulated tissue engineering**

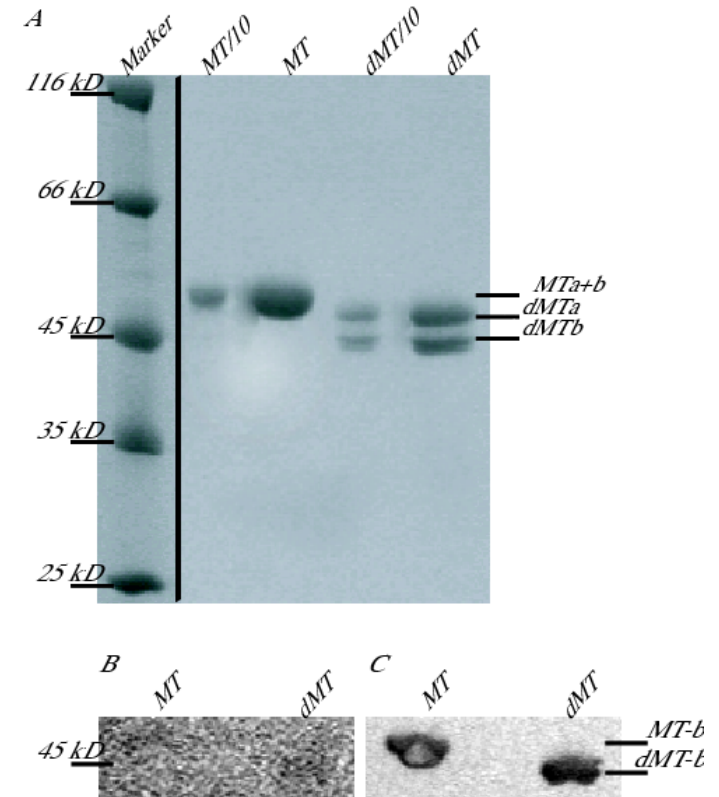
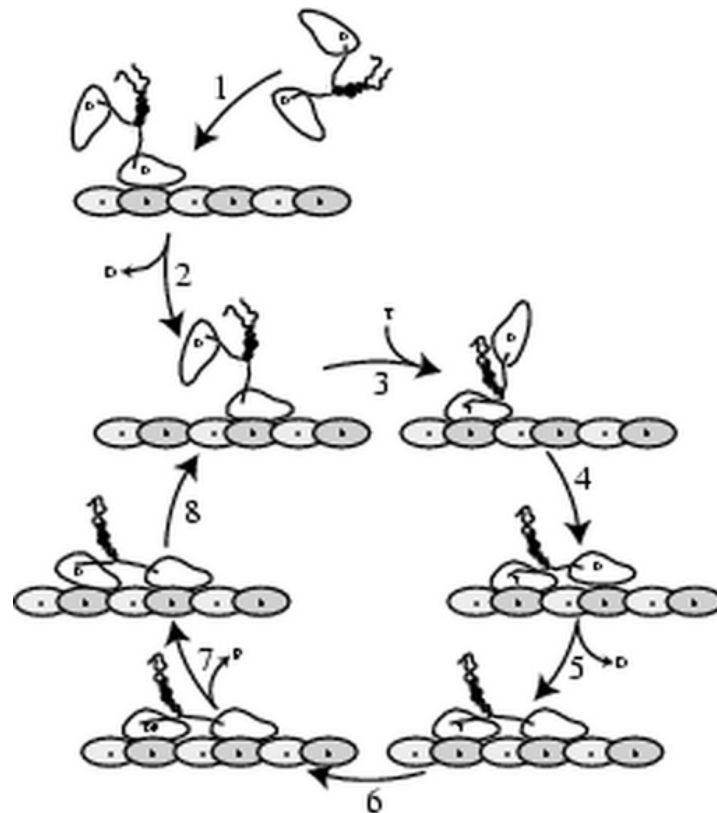
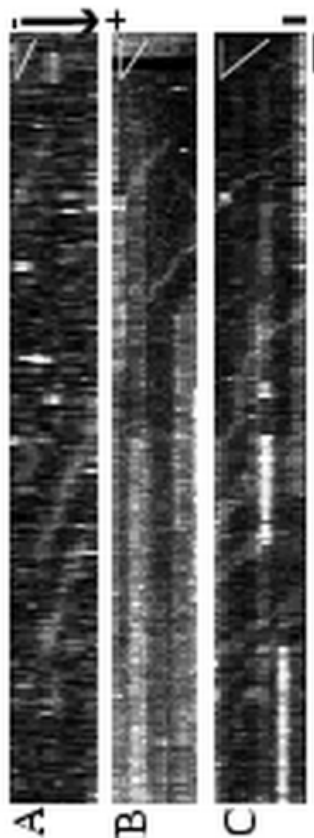
in cooperation with krishna garikipati & ellen m. arruda



[2004]

visions – tissue engineering

simulation of kinesin moving along microtubules



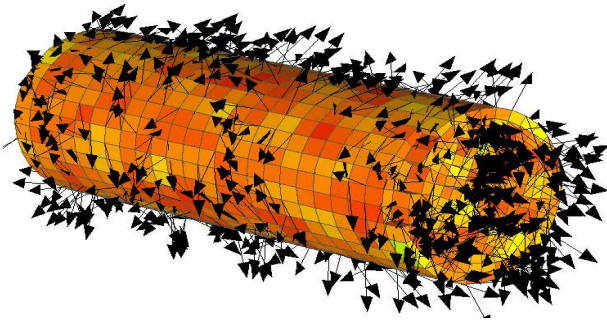
influence of mass changes & electrostatic interaction on processivity

joint work with stefan lakämper [vrije Universiteit amsterdam](http://www.vrije.universiteit.nl)



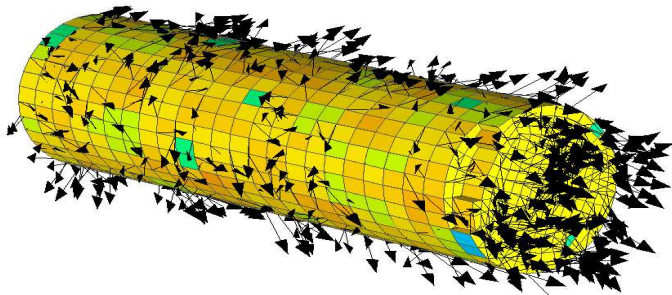
visions – improved understanding of molecular motors

... everything grows ...



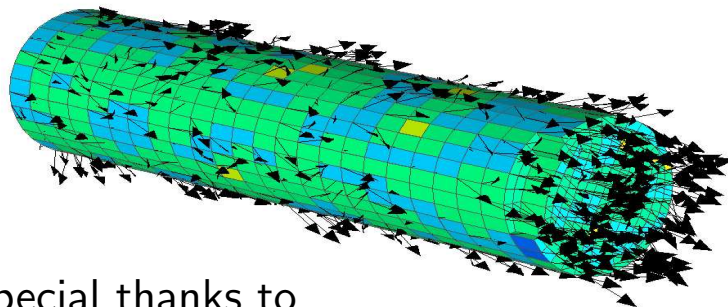
- **isotropic density growth**

changes in density at constant volume
example: artificial hip replacement



- **isotropic volume growth**

changes in volume at constant density
example: stent implantation



- **anisotropic fiber growth**

changes in fiber volume at constant orientation
example: blood vessel

- **anisotropic fiber remodelling**

changes in fiber orientation at constant volume
example: tissue engineering of functional tendon

with special thanks to

ELLEN ARRUDA, FRANK BALLE, SARAH CALVÉ, GRIETA HIMPEL, KRISHNA GARIKIPATI,
KARL GROSH, STEFAN LAKÄMPER, ANDREAS MENZEL, PAUL STEINMANN