Dislocations in graphene

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Symposium on Multiscale Dislocation Dynamics

UCSD, La Jolla, January 16-17, 2010

Graphene



Andre K. Geim
School of Physics
& Astronomy
Manchester University

- First fabricated by Novoselov,
 K.S., et al. "Electric field effect in atomically thin carbon films"
 Science, 306 (2004) pp. 666-669
- One-atom thick free-standing carbon sheet
- Stable under ambient conditions
- 2D crystal arranged in a chickenwire or honeycomb lattice
- Fabricated by:
 - Mechanical cleavage of graphite (manually, ultrasonic)
 - Epitaxial growth followed by chemical etching
- Exceptional properties!

THEORETICAL STUDIES OF ICOSAHEDRAL C60 AND SOME RELATED SPECIES

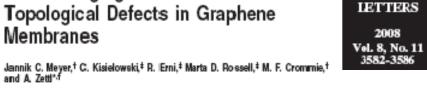
NANO

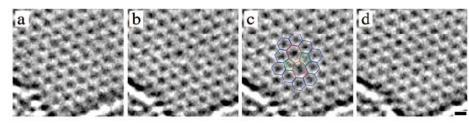
A.J. STONE and D.J. WALES

University Chemical Laboratories, Lensfield Road, Cambridge CB2 1EW, UK

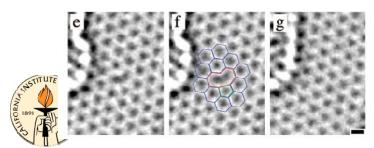
Received 1 May 1986; in final form 23 May 1986

Direct Imaging of Lattice Atoms and Topological Defects in Graphene Membranes





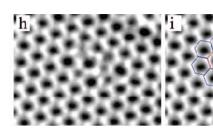
Stone-Wales defect

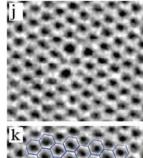


vacancy

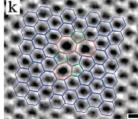


Stone-Wales defect





heptagons and pentagons

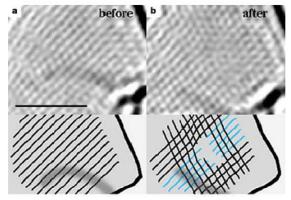


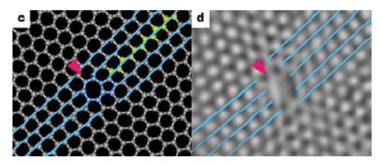
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Direct evidence for atomic defects in graphene layers

Ayako Hashimoto 1 , Kazu Suenaga 1 , Alexandre Gloter 1,2 , Koki Urita 1,3 & Sumio liiima 1

NATURE | VOL 430 | 19 AUGUST 2004 | www.nature.com/nature





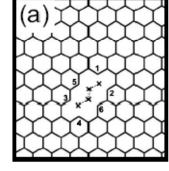
edge dislocation (missing zig-zag chain)

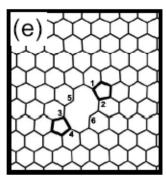
topological defects induced by electron-beam irradiation

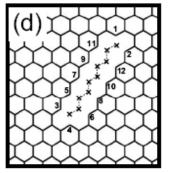
PHYSICAL REVIEW B 78, 165403 (2008)

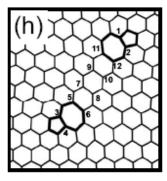
Stability of dislocation defect with two pentagon-heptagon pairs in graphene

Byoung Wook Jeong,¹ Jisoon Ihm,¹ and Gun-Do Lee²











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- Main issues of interest:
 - Properties of individual defects: Core structure, core energies, limiting behaviors (dilute, continuum...)
 - Equilibrium properties of defect ensembles: Free energy, critical temperature for spontaneous defect nucleation
- Majority of studies to date computational:
 - Ab initio: Restricted to small cells, difficult to extract thermodynamic properties...
 - Molecular dynamics: Predictiveness limited by empirical potentials, cell size, time step...
- Defect densities at critical temperature small, not accessible to direct simulation?
- Mixed continuum-atomistic:
 - Ertekin E., Chrzan D.C., Daw M.S., Phys. Rev. B, 79(15)
 155421, 2009; Phil. Mag. Let., 88(2) 159-167 2008.



In this work: Discrete lattice elasticity, discrete defects

Our approach:

- Discrete elasticity of graphene lattice
- Discrete dislocations as eigendeformations
- Discrete Fourier transform, closed-form solutions
- Limit of dilute dislocations, continuum limit
- Equilibrium statistical mechanics, mean field

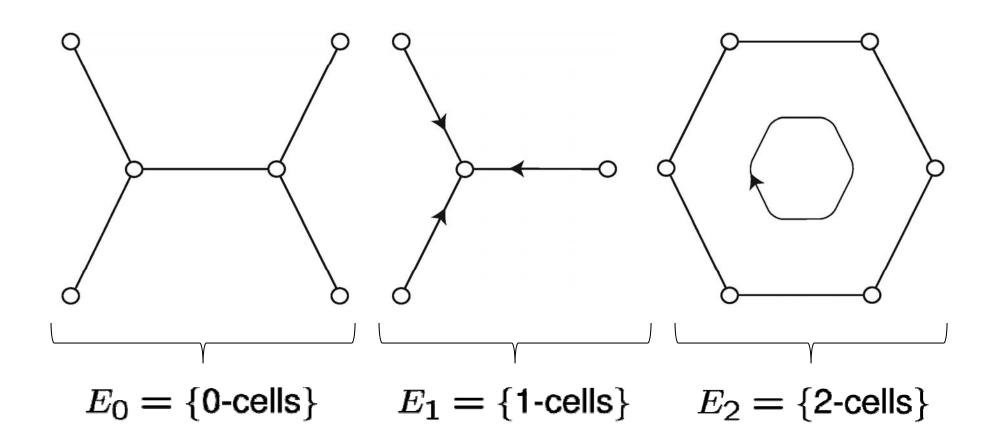
Results to date:

- Closed-form solutions: Dipoles, quadrupoles...
- Verification & Validation: Core structures, core energies, dynamic stability
- Rigorous mathematical limits: Dilute/continuum, prelogarithmic energy factors
- Equilibrium properties: Critical temperature, scaling (in progress)



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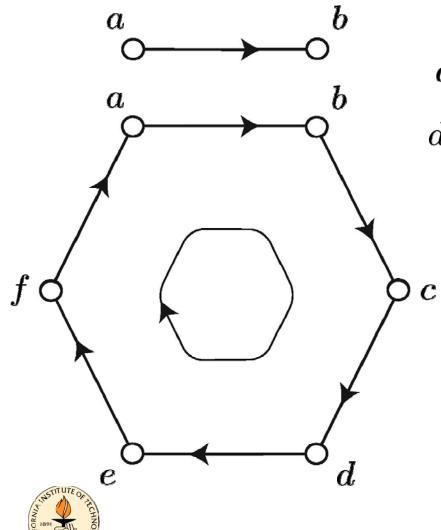
Graphene – Differential cell complex







Graphene – Differential cell complex



Differential operator:

$$d\omega(e_{ab}) = \omega(e_b) - \omega(e_a)$$

$$d\omega(e_{abcdef}) = \omega(e_{ab}) + \omega(e_{bc})$$

$$+ \omega(e_{cd}) + \omega(e_{de})$$

$$+ \omega(e_{ef}) + \omega(e_{fa})$$

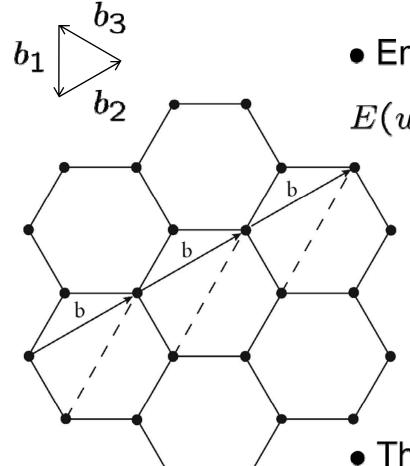
• Codifferential operator:

$$\langle \Lambda, d\omega \rangle = \langle \delta \Lambda, \omega \rangle$$

Fundamental property:

$$d^2 = 0, \quad \delta^2 = 0$$

Graphene – Discrete dislocations



Energy of defective graphene:

$$E(u,\beta) = \frac{1}{2} \langle \Psi * (du - \beta), (du - \beta) \rangle$$

 $\Psi \equiv$ bondwise force constants

$$\Omega^0 \ni u \equiv \text{displacement field}$$

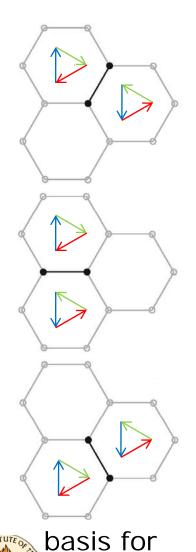
$$\Omega^1 \ni du \equiv \text{bond deformation}$$

$$\Omega^1 \ni \beta \equiv$$
 eigendeformations

Three lattice-preserving shears:

$$eta(e_1) \in b_1 \mathbb{Z} + b_2 \mathbb{Z} + b_3 \mathbb{Z}$$
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Graphene – Discrete dislocations



graphene

dislocations

Energy of defective graphene:

$$E(u,\beta) = \frac{1}{2} \langle \Psi * (du - \beta), (du - \beta) \rangle$$

• Displacement equilibrium problem:

$$E(\beta) = \min_{u} E(u, \beta)$$

• Note: $E(\beta) = 0$ if $\beta = dv$

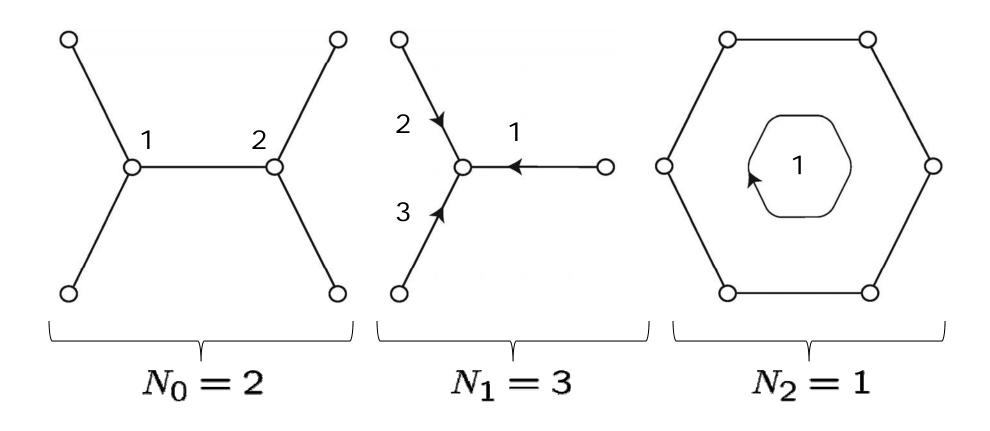
• Discrete dislocation density: $\alpha = d\beta$

• Hodge: $\alpha = 0$ iff $\beta = dv$

• Stored energy depends on α :

$$E(\alpha) = \min_{u} \{ E(u, \beta) \mid d\beta = \alpha \}$$
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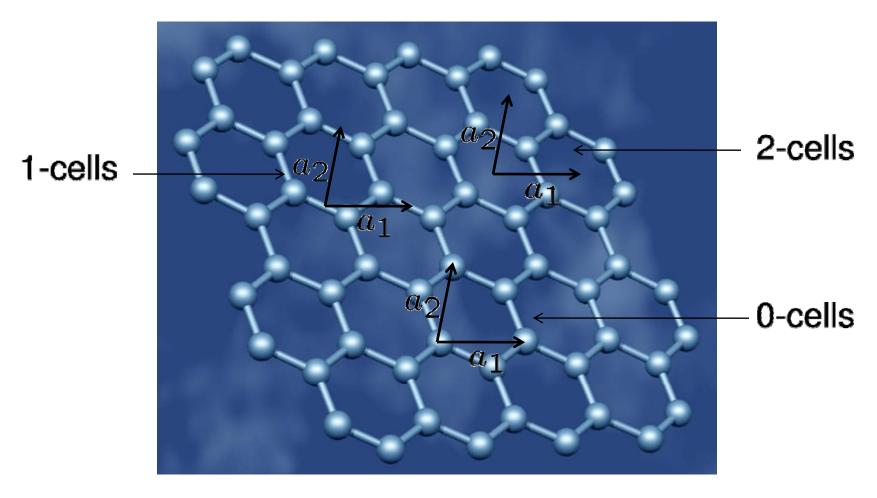
Graphene – Discrete Fourier Transform



Classification of cells by type



Graphene – Discrete Fourier Transform





cells of same type define simple Bravais lattices

Graphene - Discrete Fourier transform

• Discrete Fourier Transform:

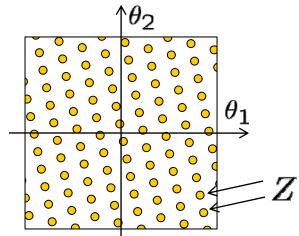
$$\widehat{\alpha}(\theta) = \sum_{l \in \mathbb{Z}^2} \alpha(l) e^{-i\theta \cdot l}, \quad \theta \in [-\pi, \pi]^2$$

• For given discrete dislocation density:

$$E(\alpha) = \frac{1}{(2\pi)^2} \int_{[-\pi,\pi]^2} \frac{1}{2} \langle \widehat{\Gamma}(\theta) \widehat{\alpha}(\theta), \widehat{\alpha}^*(\theta) \rangle d\theta$$

- Explicit expressions for $\widehat{\Gamma}(\theta)$
- Periodic densities: $E(\alpha) =$

$$\frac{1}{|Z|} \sum_{\Theta \in Z} \frac{1}{2} \langle \widehat{\Gamma}(\Theta) \widehat{\alpha}(\Theta), \widehat{\alpha}^*(\Theta) \rangle$$

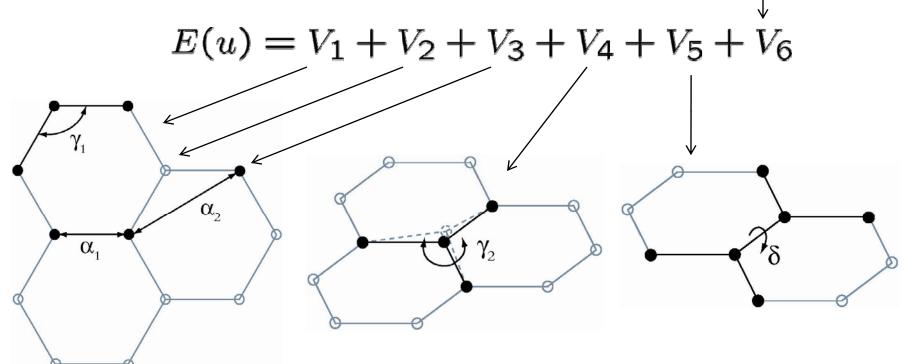


Closed form expressions for energies, displacements!

Graphene – Force constants

ASOIO Potential:

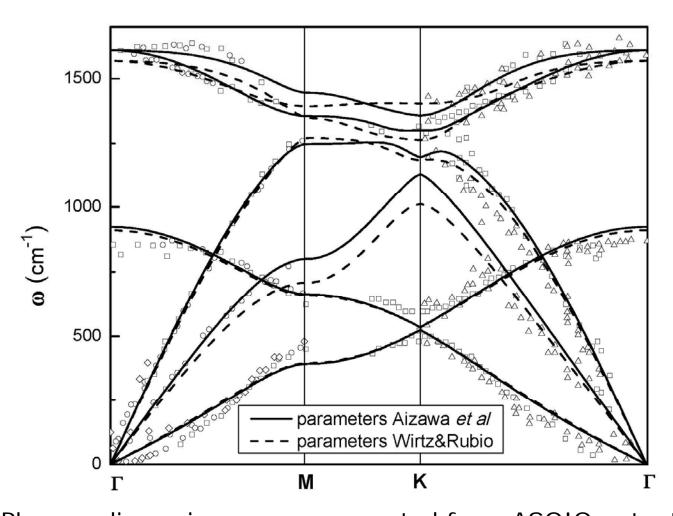
interaction with substrate





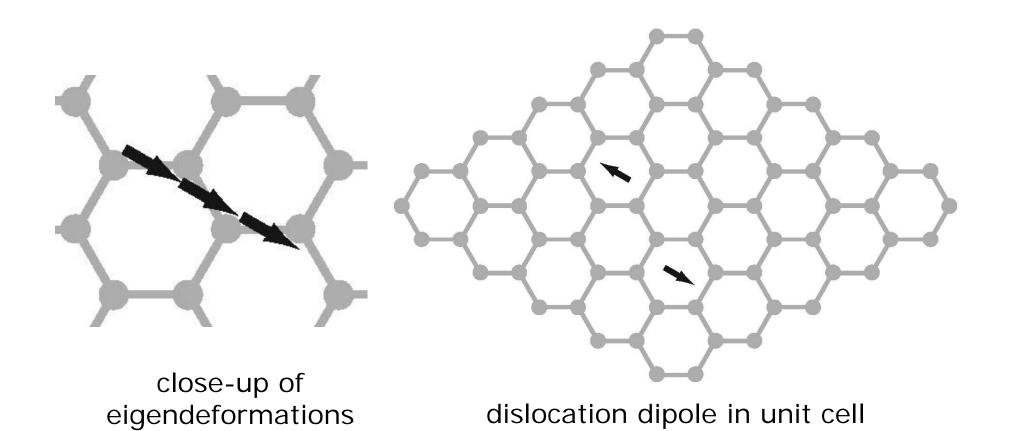
Aizawa, T. et al., "Bond Softening in Monolayer Graphite Formed on Transition-Metal Carbide Surfaces" Phys. Rev. B, **42**(18) (1990) pp. 11469--11478

Graphene – Force constants





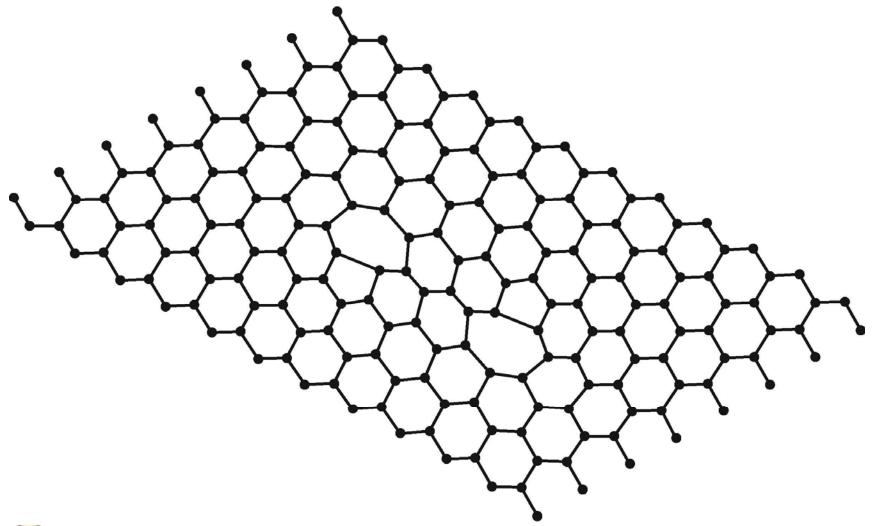
Phonon-dispersion curves computed from ASOIO potential (Aizawa *et al.* (1990) and by Wirtz & Rubio (2004)) Michael Ortiz UCSD011610





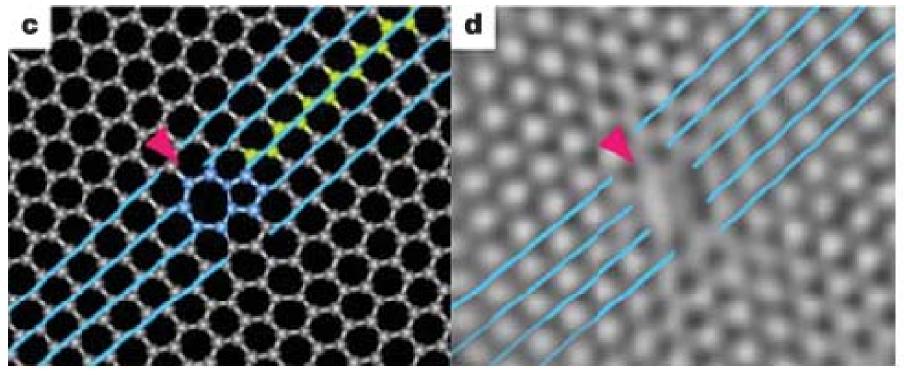
Periodic arrangement of dislocation dipoles

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Discrete dipole core structure in ASOIO graphene, exhibiting dissociated pentagon-heptagon ring (5-7-7-5) Michael Ortiz UCSD011610

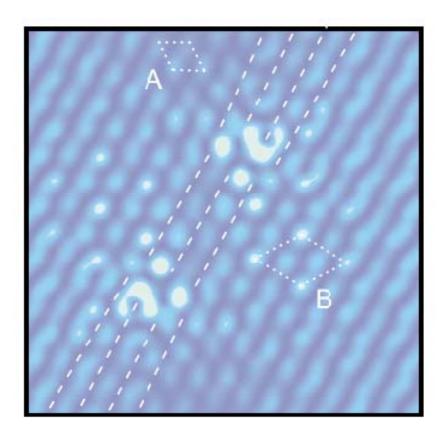


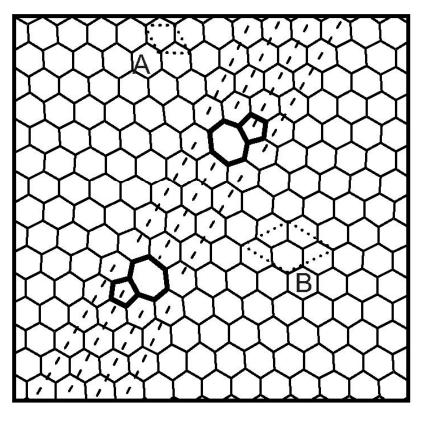
pentagon-heptagon pair in the graphitic network

simulated HR-TEM image

In situ observation of a dislocation in a graphene layer (Hasimoto et al. "Direct evidence for atomic defects in graphene layers", Letters to Nature, 430 (2004) pp.870-873)







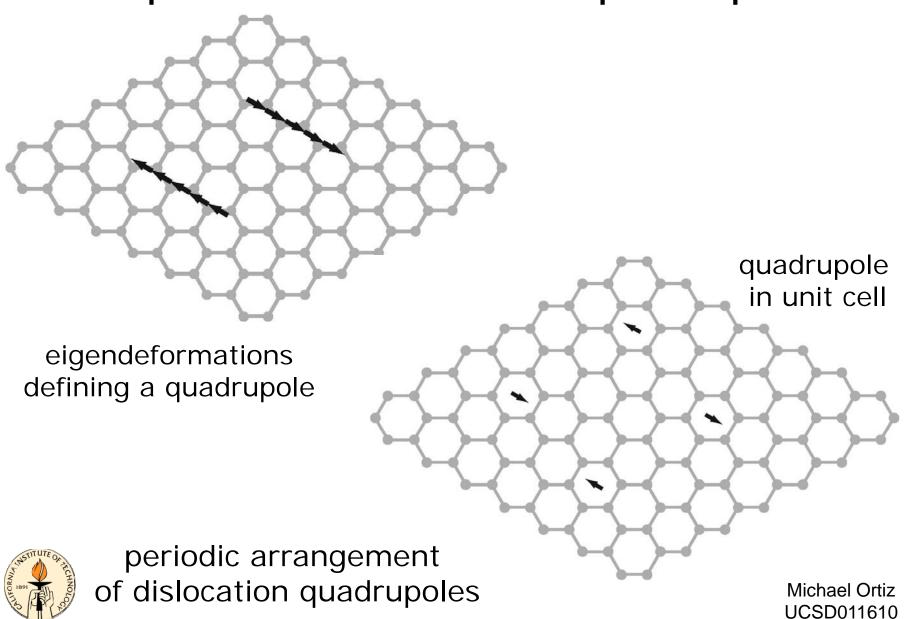
Simulated STM image (DFT)

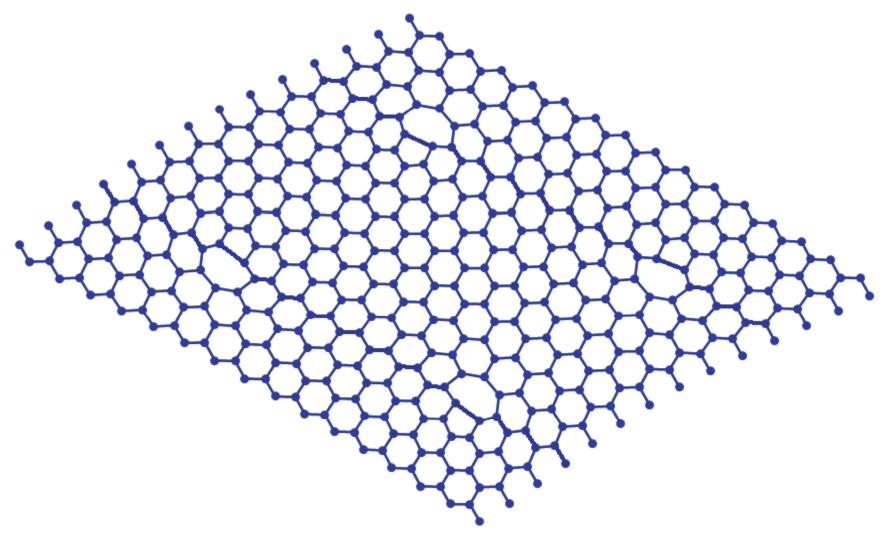
Atomic structure



Jeong, B.W. et al., Stability of dislocation defect with two pentagon-heptagon pairs in graphene,
Phys. Rev. B, **78**, 165403 (2008)

Michael Ortiz UCSD011610



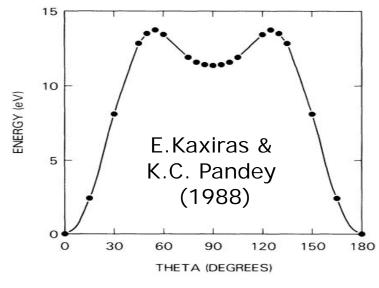


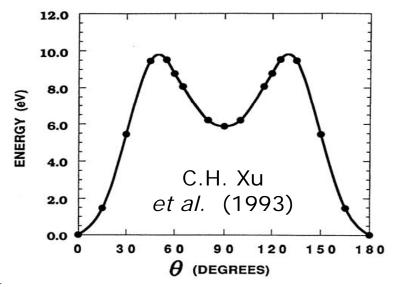


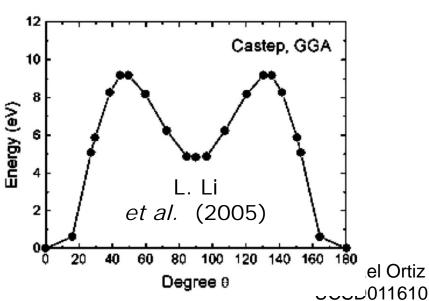
Discrete quadrupole core structure in ASOIO graphene, exhibiting dissociated pentagon-heptagon ring (5-7-7-5) Michael UCSE



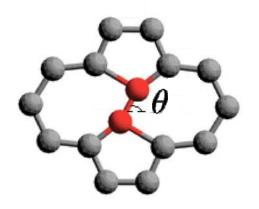
bond rotation angle









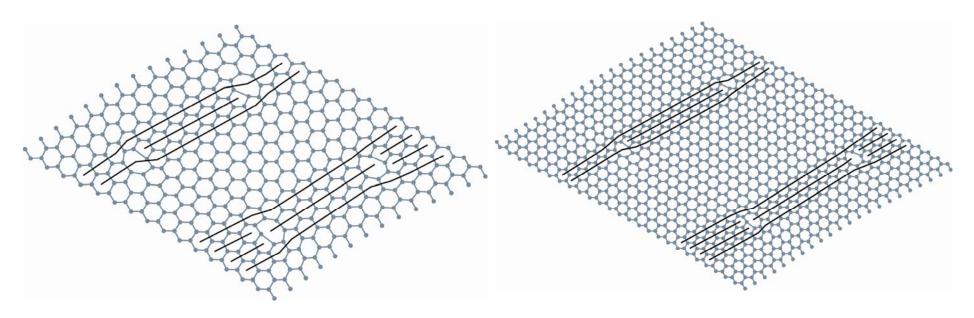


Summary of bond angles and formation energies at transition state computed from firs-principles calculations

bond rotation angle

Study	Formation angle	Formation energy
Ariza & MO	52 degrees	11.92 eV
Li et al. (2005)	45 degrees	9.2 eV
Kaxiras & Pandy (1988)	55 degrees	14.0 eV
Xu et al. (1993)	50 degrees	9.8 eV
Los et al. (2005)	50 degrees	8.0 eV
Meyer et al. (2008)	-	< 15.6 eV





- Discrete-dislocation model predicts 5-7 dislocation core structures, in agreement with observation and first-principles calculations
- Predicted unstable transition configuration and formation energies are in the range of firstprinciples calculations



Dynamic stability of core structures?

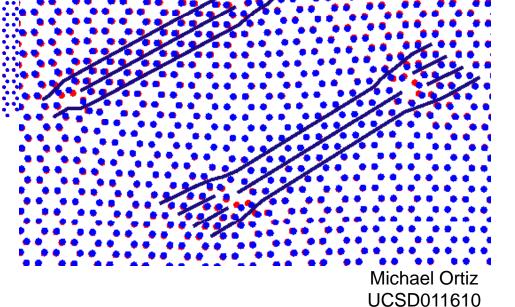
3200-atom cell Temperature: OK Max time: 4 ps Relaxed atoms (red) DD atoms (blue)

LAMMPS (SNL)

AIREBO potential

Stuart SJ, Tutein AB, Harrison JA

J Chem Phys **112**(14):6472–6486 (2000)





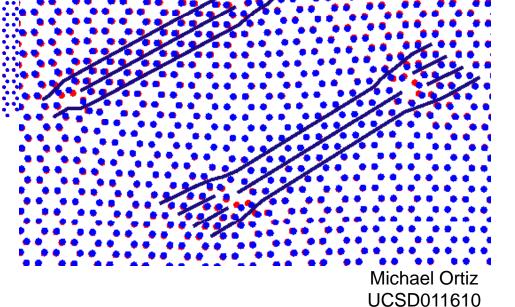
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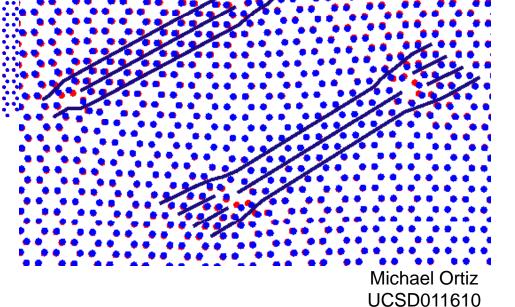
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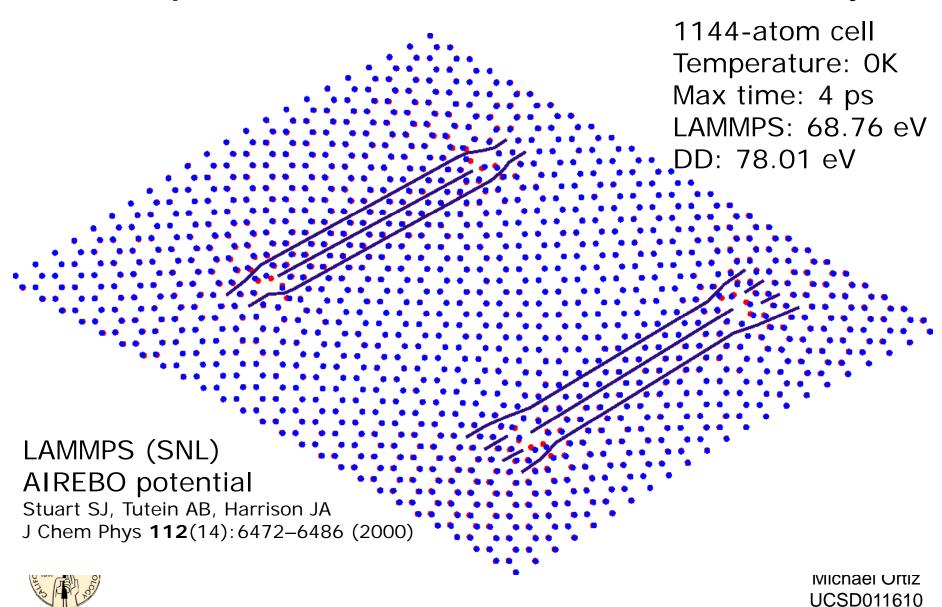
AIREBO potential

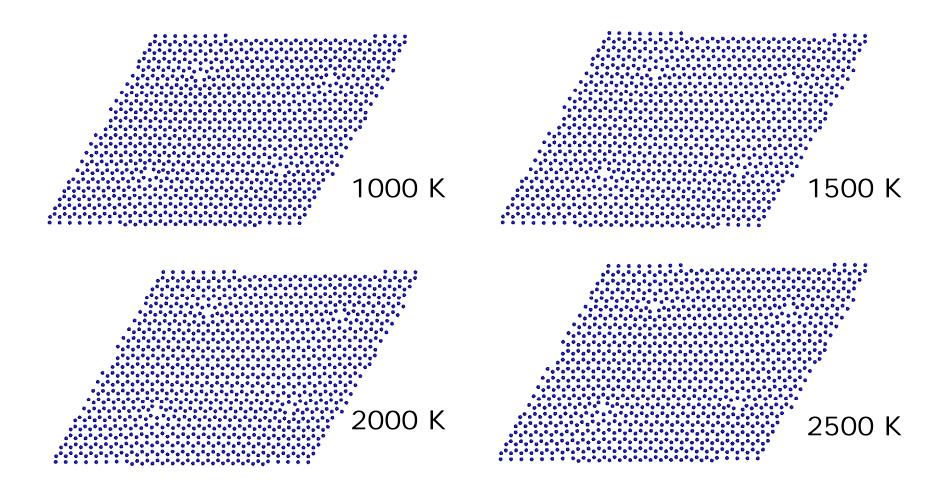
Stuart SJ, Tutein AB, Harrison JA

J Chem Phys **112**(14):6472–6486 (2000)



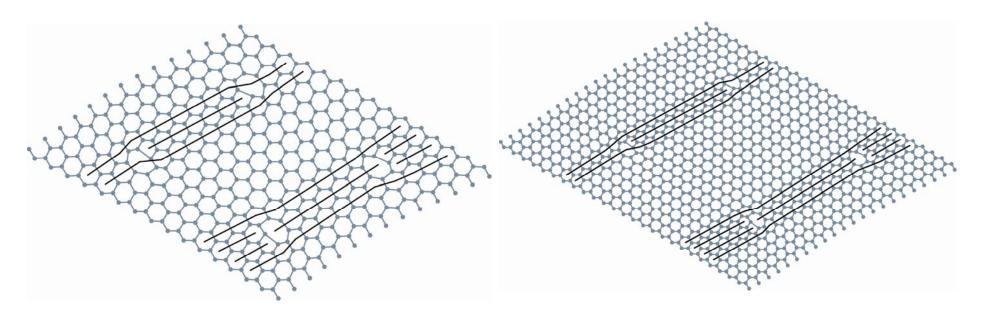








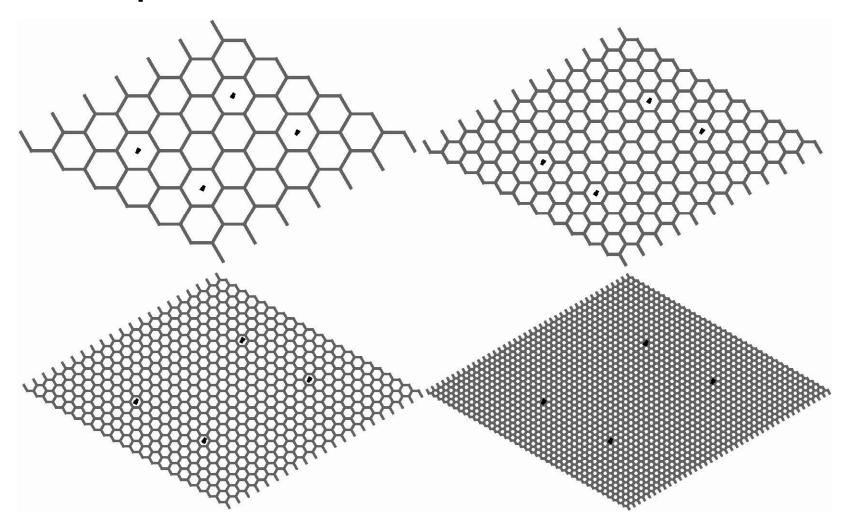
LAMMPS (SNL) AIREBO potential, 1144-atom cell



- Dislocation cores predicted by the discretedislocation are stable with respect to fully nonlinear AIREBO potential
- Dislocation cores appear stable up to high temperatures (2500 K)



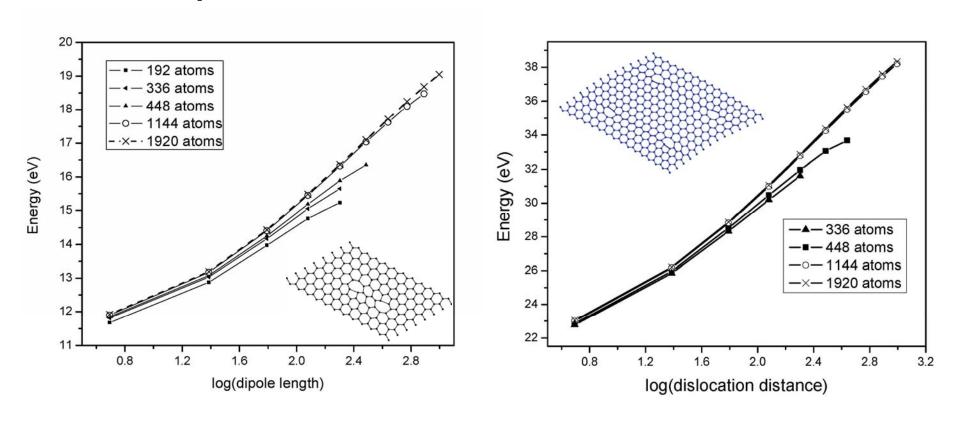
Spontaneous nucleation, critical behavior?





Limit of dilute dislocation densities (equivalently, continuum limit)

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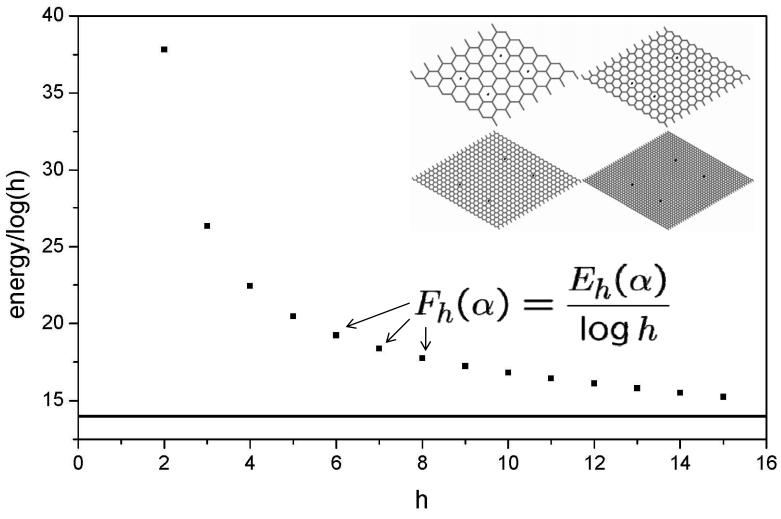


 $E = (23.07255 \text{ eV}) + (6.80 \text{ eV}) \log h$



Energy of periodic dislocation densities in ASOIO graphene as a function of dislocation separation and unit-cell size.

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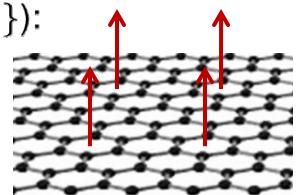


Energy sequence is decreasing!

• Limiting energy $(s(l) \in \{-1, 0, 1\})$:

$$E_0 = \frac{1}{2}Kb^2 \log \frac{R}{b} \sum_{l \in \mathbb{Z}^2} |s(l)|$$

(line-tension approximation)



- Non-interaction three-state Potts, triangular lattice!
- Application to mean-field analysis (in progress):

Free energy
$$\equiv F \sim \min_K \left(F_0 + \langle E - E_0 \rangle_0 \right)$$

where $\langle \cdot \rangle_0 \equiv$ average in E_0 -ensemble.

Spontaneous dislocation nucleation near melting

Dislocations in graphene

Thank you!