Optimal scaling laws in ductile fracture

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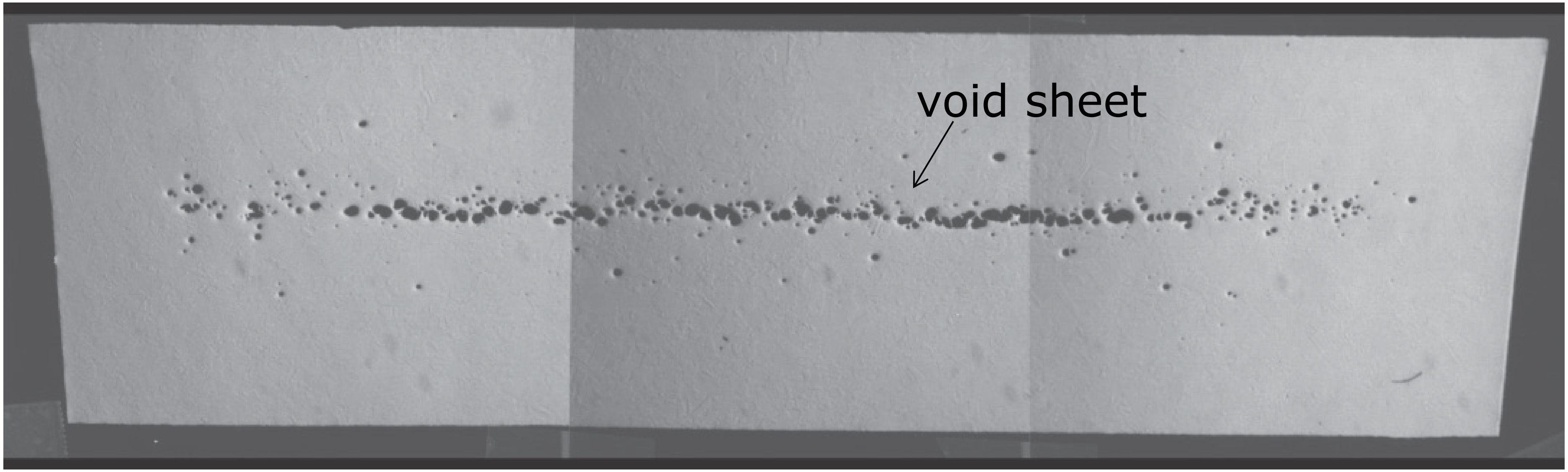
Background on ductile fracture



Photomicrograph of a copper disk tested in a gas-gun experiment showing the formation of voids and their coalescence into a fracture plane

Heller, A., How Metals Fail,
Science & Technology Review Magazine,
Lawrence Livermore National Laboratory,
pp. 13-20, July/August, 2002

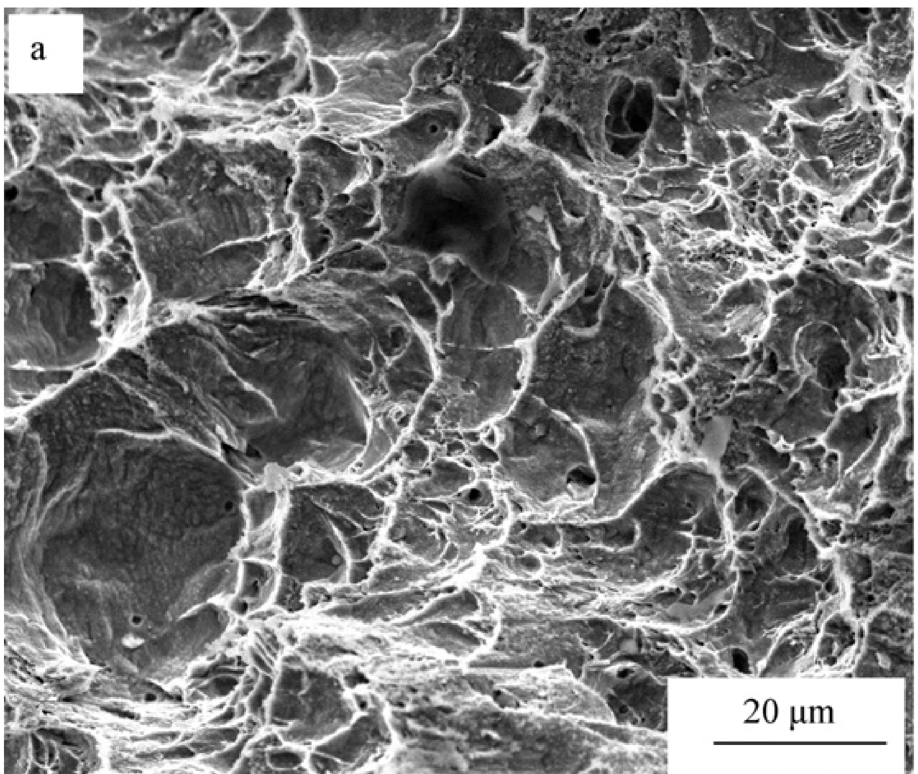
Scope

- Micro-macro relations for ductile fracture
- (Universal) scaling relations in ductile fracture?
- Application of optimal scaling to ductile fracture
- Results for metals and polymers

Background on ductile fracture

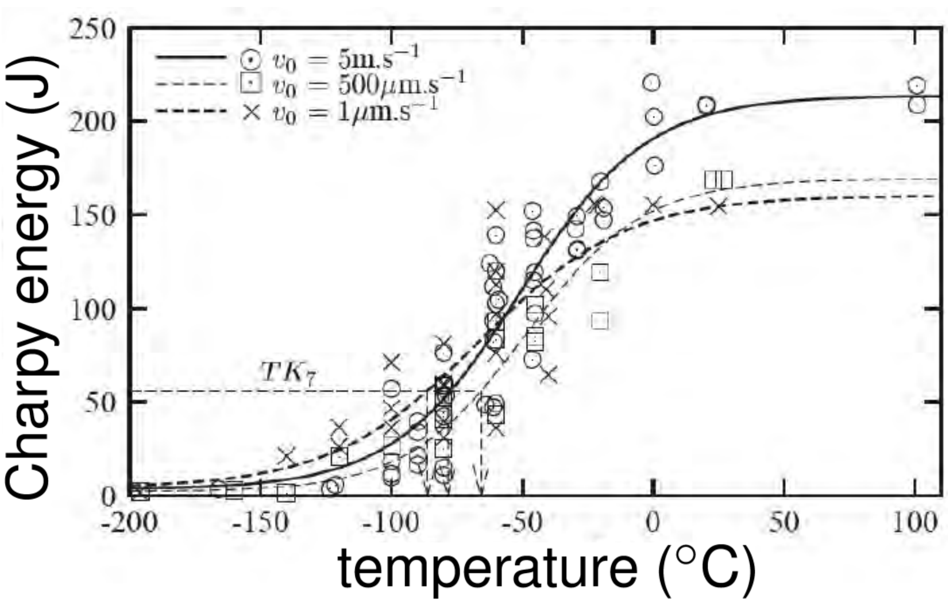
(Courtesy NSW HSC online)

- Ductile fracture in metals occurs by *void nucleation, growth and coalescence*
- Fractography of ductile-fracture surfaces exhibits profuse *dimpling*, vestige of microvoids
- Ductile fracture entails large amounts of *plastic deformation* (*vs*. surface energy) and dissipation.

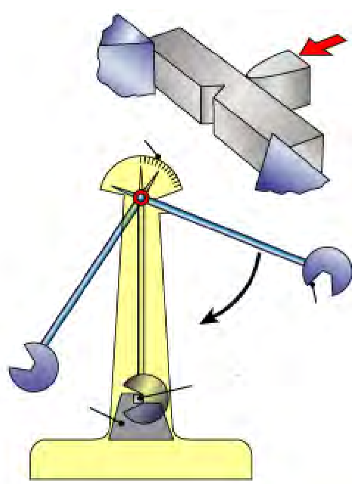


Fracture surface in SA333 steel, room temp., $d\varepsilon/dt=3\times10^{-3}s^{-1}$ (S.V. Kamata, M. Srinivasa and P.R. Rao, Mater. Sci. Engr. A, ***528*** *(2011) 4141–4146)*

Background on ductile fracture



Charpy energy of
A508 steel
(Tanguy *et al.*, *Eng.
Frac. Mechanics*, 2005)



- A number of ASTM engineering standards are in place to characterize ductile fracture properties (J-testing, Charpy test)
- The Charpy test data reveals a brittle-to-ductile transition temperature
- In general, the specific fracture energy for ductile fracture is greatly in excess of that required for brittle fracture…

Micromechanics of ductile fracture

- Objective: Elucidate microstructure/property relations (voids to specific fracture energy)
- Traditional ‘micromechanics’ approach:
  - *Select a specific microscale model (crystal plasticity, porous plasticity, strain-gradient plasticity…)*
  - *Select a ‘representative microstructure’ (void in periodic cell, shear/damage localization band…)*
  - *Perform ‘unit-cell’ calculations, parametric studies…*
- Critique:
  - *Pros: Calculations ‘exact’ (within numerical precision)*
  - *Cons: Model-specific results, non-optimal static microstructures, numerical (vs. epistemic) results…*
- Alternative: Analysis (e.g., optimal scaling)

Scaling laws in science

- A broad variety of physical phenomena obey power laws over wide ranges of parameters
- Scale invariance: If $y = C\, x^q$, then (x,y) iff $(\lambda x, \lambda^q y)$, law of corresponding states
- Universality:
  - *Exponents are material-independent ('universal')*
  - *Systems displaying identical scaling behavior are likely to obey the same fundamental dynamics*
- Experimental master curves, data collapse
- Examples:
  - *Critical phenomena (second-order transitions)*
  - *Materials science (Taylor, Hall-Petch, creep laws…)*
  - *Continuum mechanics (hydrodynamic, fracture…)*

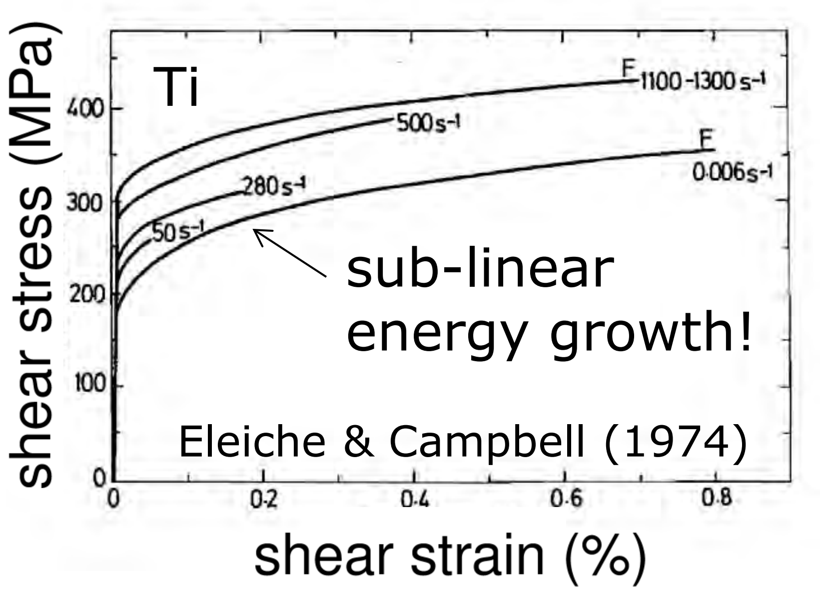
Optimal scaling

- Suppose: Energy $= E(u, \epsilon_1, \dots, \epsilon_N)$
- Optimal (matching) upper and lower bounds:

$$C_L \epsilon_1^{\alpha_1} \dots \epsilon_N^{\alpha_N} \leq \inf E(\cdot, \epsilon_1, \dots, \epsilon_N) \leq C_U \epsilon_1^{\alpha_1} \dots \epsilon_N^{\alpha_N}$$

- The exponents $\alpha_1, \dots, \alpha_N$ are *sharp*, *universal*
- The constants C_L and C_U are often lax, imprecise...
- Upper bound by construction, *ansatz*-free lower bound
- Originally applied to branched microstructures in martensite (Kohn-Müller 92, 94; Conti 00)
- Applications to micromagnetics (Choksi-Kohn-Otto 99), thin films (Belgacem *et al* 00)...

Naïve model: Local plasticity

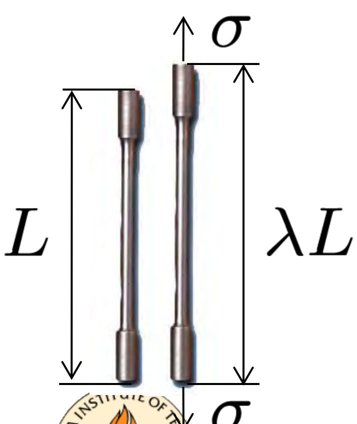


- Deformation theory: Minimize

$$E(y) = \int_\Omega W\Big(Dy(x)\Big)\, dx$$

- Growth of $W(F)$?
- Asume power-law hardening:

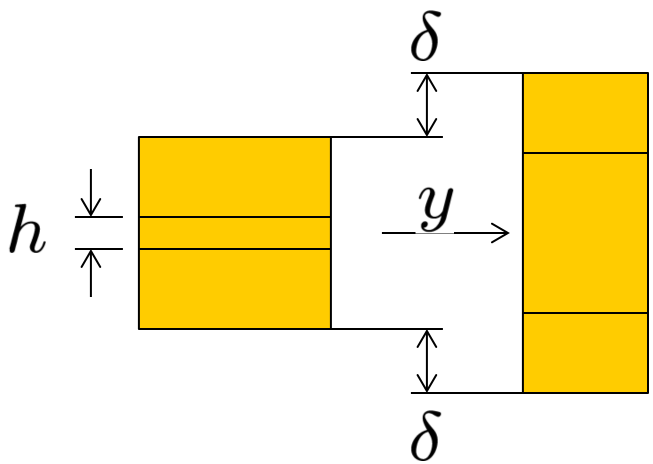
$$\sigma \sim K\epsilon^n = K(\lambda - 1)^n$$



- Nominal stress: $\partial_\lambda W = \sigma/\lambda = K(\lambda - 1)^n/\lambda$
- For large λ: $\partial_\lambda W \sim K\lambda^{n-1} \Rightarrow W \sim K\lambda^n$
- In general: $W(F) \sim |F|^p,\ p = n \in (0,1)$

$\Rightarrow$ Sublinear growth!

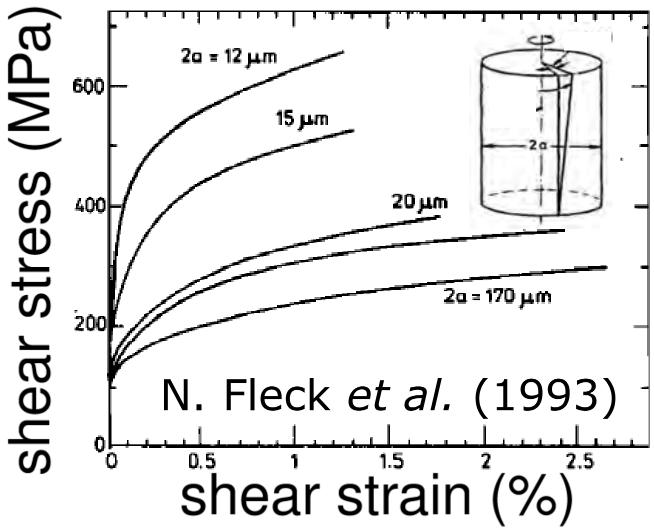
Naïve model: Local plasticity

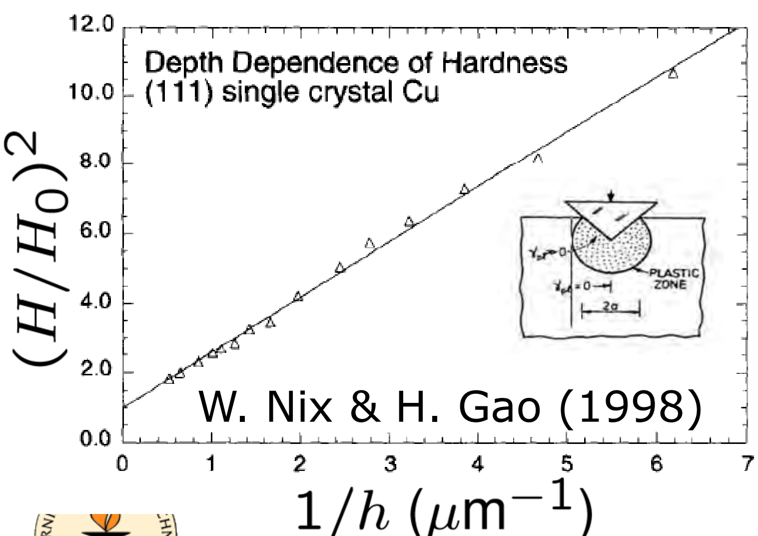


- Example: Uniaxial extension
- Energy: $E_h \sim h \left(\frac{2\delta}{h}\right)^p$
- For $p < 1$: $\lim_{h \to 0} E_h = 0$

- Energies with sublinear growth relax to 0.
- *For hardening exponents in the range of experimental observation, local plasticity yields no useful information regarding ductile fracture properties of materials!*
- Need additional physics, structure…

Strain-gradient plasticity





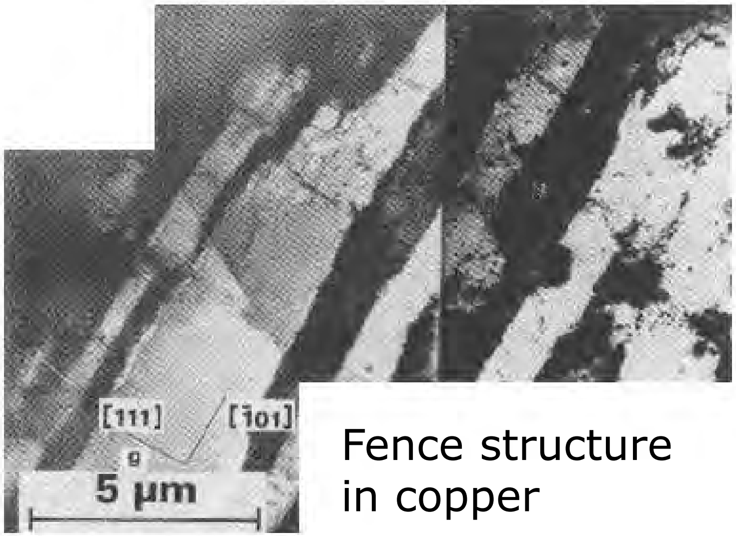
- The yield stress of metals is observed to increase in the presence of strain gradients
- Deformation theory of strain-gradient plasticity:

$$E(y) = \int_\Omega W\Big(Dy(x), D^2y(x)\Big)\,dx$$

$y : \Omega \to \mathbb{R}^n$, volume preserving

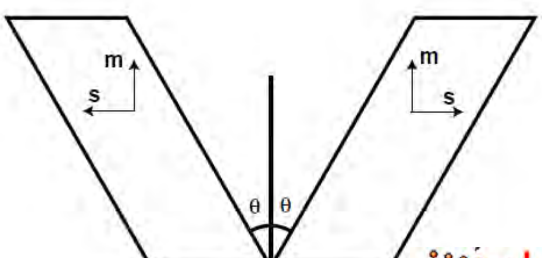
- Strain-gradient effects may be expected to oppose localization
- Growth of *W* with respect to the second deformation gradient?

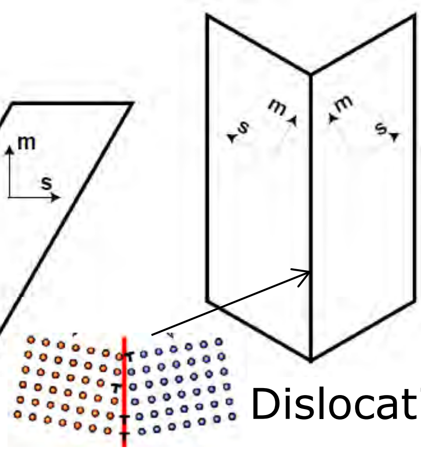
Strain-gradient plasticity



Fence structure in copper

(J.W. Steeds, *Proc. Roy. Soc. London*, **A292**, 1966, p. 343)





Dislocation wall

- Growth of $W(F,\cdot)$?
- For fence structure:

$$F^{\pm} = R^{\pm}(I \pm \tan\theta\, s \otimes m)$$

- Across jump planes:

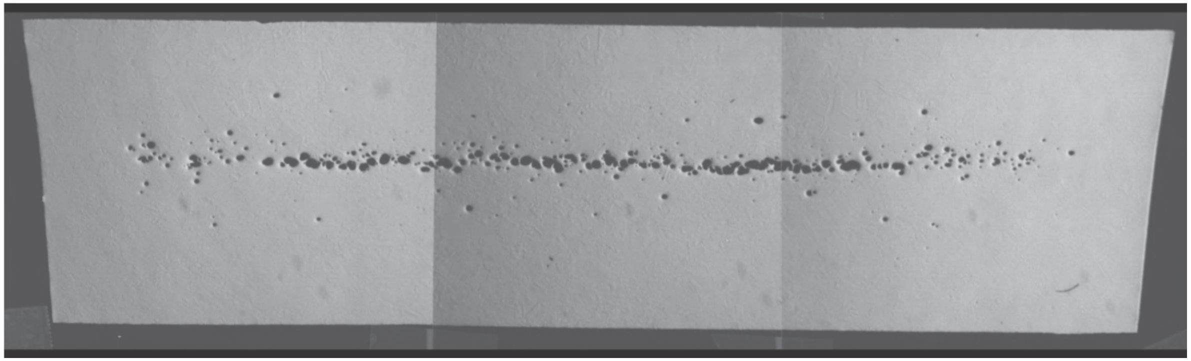
$$|[\![F]\!]| = 2\sin\theta$$

- Dislocation-wall energy:

$$E = \frac{T}{b} 2\sin\theta = \frac{T}{b}|[\![F]\!]|$$

$\Rightarrow W(F,\cdot)$ has linear growth!

Strain-gradient plasticity & fracture



Heller, A., How Metals Fail, Science & Technology Review Magazine, Lawrence Livermore National Laboratory, pp. 13-20, July/August, 2002

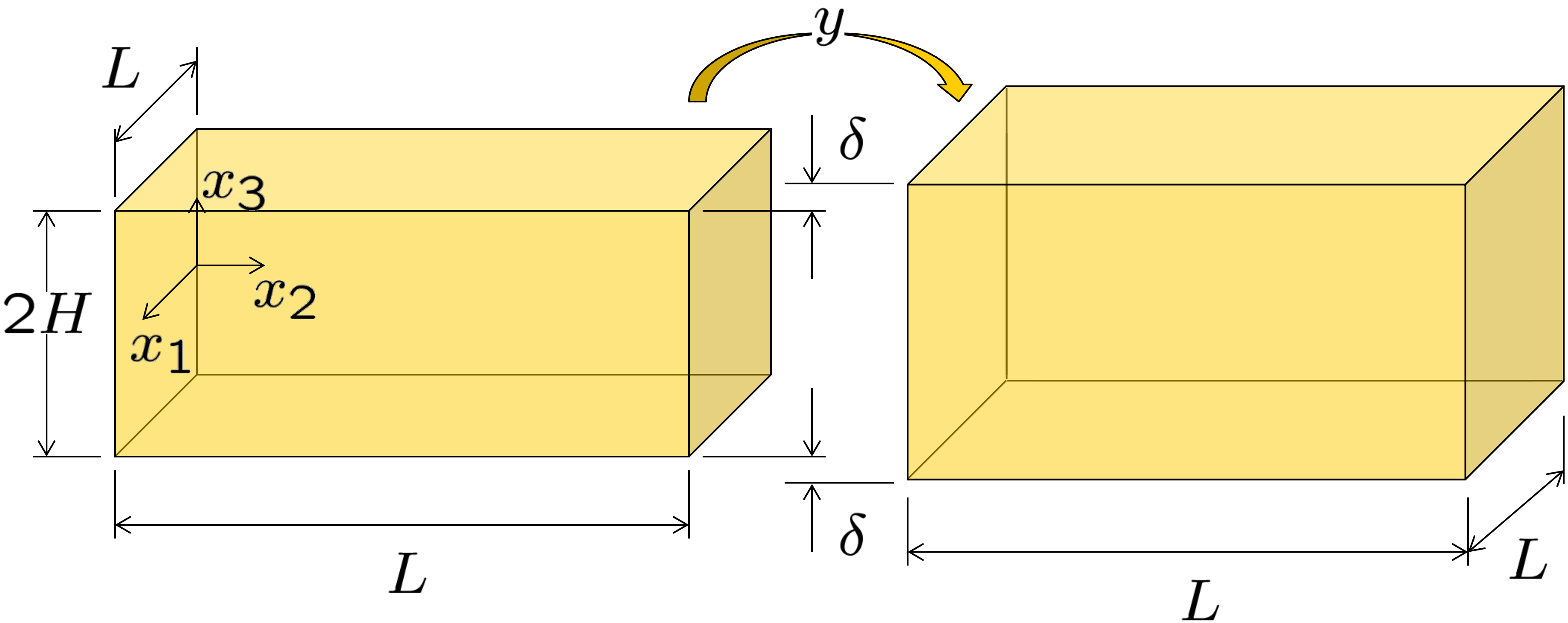
- Mathematical model: Minimize

$$E(y) = \int_\Omega W\Big(Dy(x), D^2y(x)\Big)\,dx$$

$y : \Omega \to \mathbb{R}^n$, volume preserving

- For metals, local plasticity exhibits sub-linear growth, strain-gradient plasticity linear growth
- *Question: Can ductile fracture be understood as the result of a competition between sublinear growth and strain-gradient plasticity?*

Optimal scaling – Ductile fracture



- Approach: Deformation theory SG-plasticity
- Slab, $[0, L]^2$-periodic, volume-preserving
- Uniaxial extension + voids

Optimal scaling – Ductile fracture

- $E(y) \equiv$ general deformation-theoretical energy

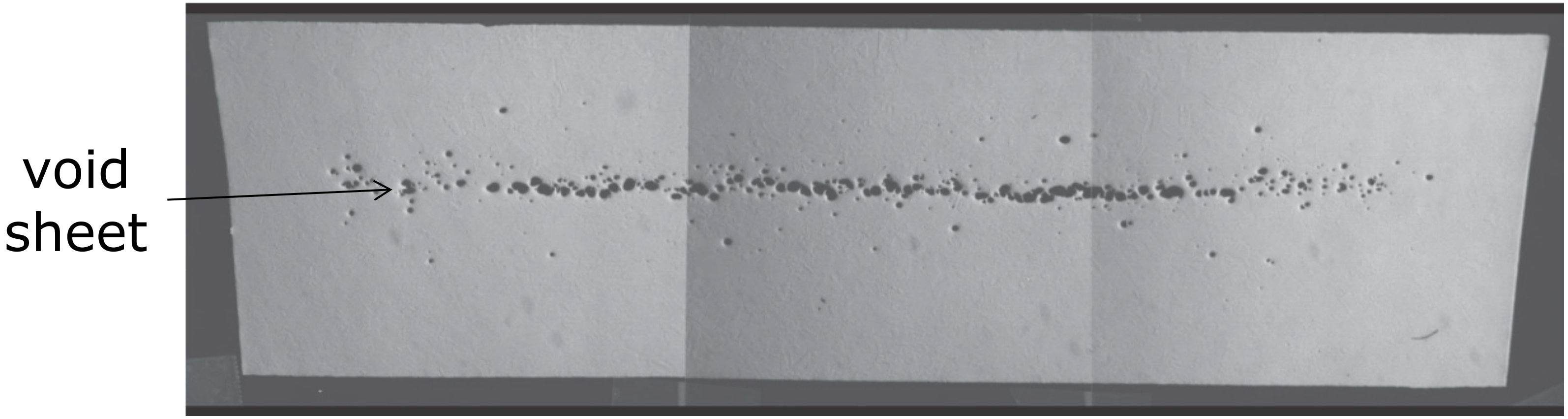
- Growth: For $0 < K_L < K_U$, *intrinsic length* $\ell > 0$,

$$E(y) \geq K_L \left(\int_\Omega (|Dy|^p - 3^{p/2})\, dx + \ell \int_\Omega |D^2 y|\, dx \right)$$
$$E(y) \leq K_U \left(\int_\Omega (|Dy|^p - 3^{p/2})\, dx + \ell \int_\Omega |D^2 y|\, dx \right)$$

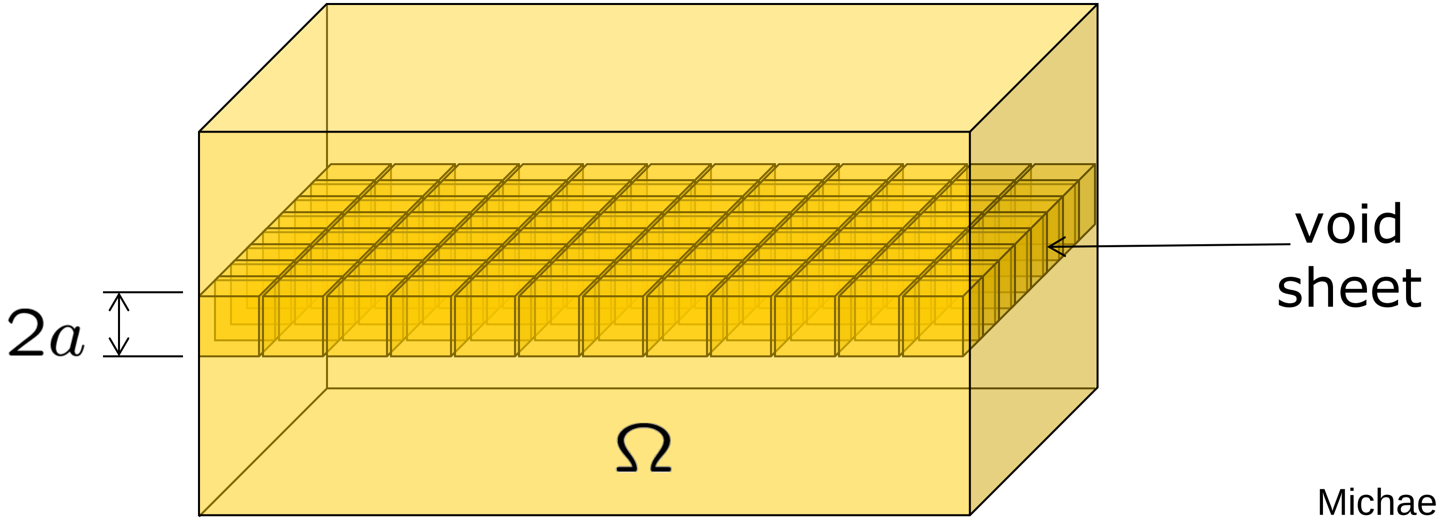
Theorem [Fokoua *et al.*, ARMA, 2013]. For ℓ sufficiently small, $p \in (0,1)$, $0 < C_L(p) < C_U(p)$,

$$C_L(p) L^2 \ell^{\frac{1-p}{2-p}} \delta^{\frac{1}{2-p}} \leq \inf E \leq C_U(p) L^2 \ell^{\frac{1-p}{2-p}} \delta^{\frac{1}{2-p}}$$

Sketch of proof – Upper bound

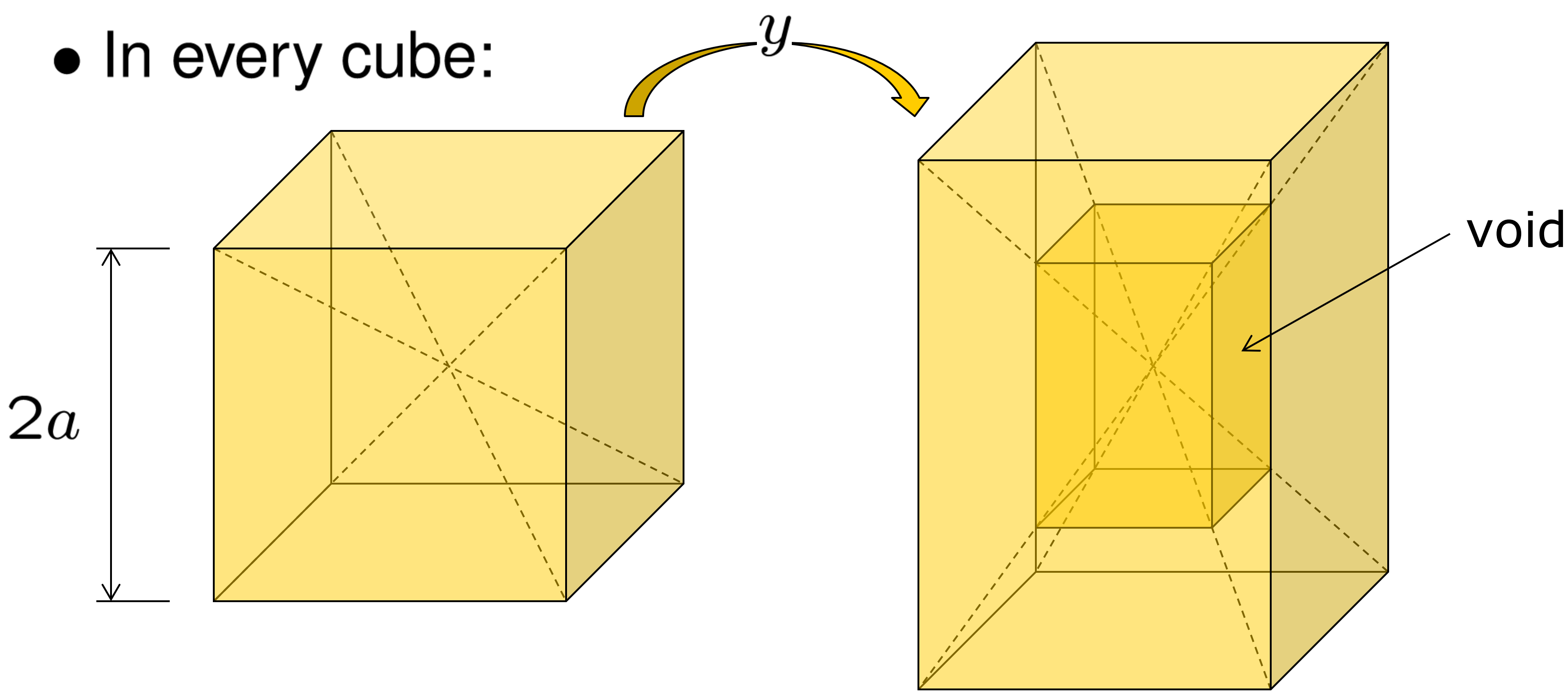


Heller, A., Science & Technology Review Magazine, LLNL, pp. 13-20, July/August, 2002



Sketch of proof – Upper bound

- In every cube:



- Calculate, estimate: $E \leq CL^2 \left(a^{1-p}\delta^p + \ell\delta/a \right)$
- Optimize: $a = (\ell\delta^{1-p})^{1/(2-p)} \Rightarrow E \leq C_U L^2 \ell^{\frac{1-p}{2-p}} \delta^{\frac{1}{2-p}}$

↑ void growth!

Optimal scaling – Ductile fracture

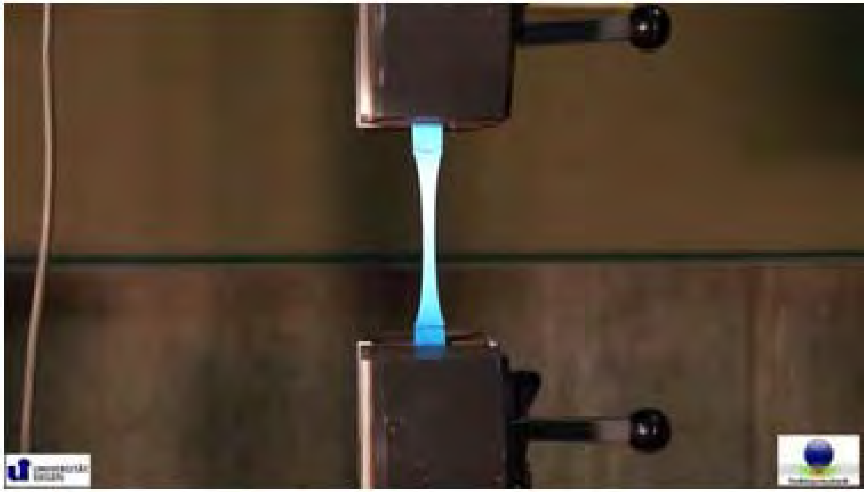
- Optimal (matching) upper and lower bounds:

$$C_L(p) L^2 \ell^{\frac{1-p}{2-p}} \delta^{\frac{1}{2-p}} \leq \inf E \leq C_U(p) L^2 \ell^{\frac{1-p}{2-p}} \delta^{\frac{1}{2-p}}$$

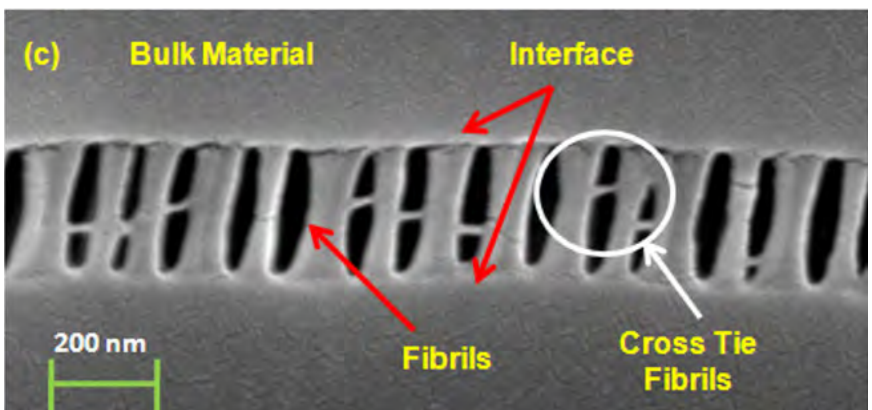
- Bounds apply to *classes of materials* having the same growth, specific model details immaterial
- Energy scales with *area* (L^2): Fracture scaling!
- Energy scales with power of *opening displacement* (δ): Cohesive behavior!
- Lower bound degenerates to 0 when the intrinsic length (ℓ) decreases to zero…
- Bounds on specific fracture energy:

$$C_L(p) \ell^{\frac{1-p}{2-p}} \delta_c^{\frac{1}{2-p}} \leq G_c \leq C_U(p) \ell^{\frac{1-p}{2-p}} \delta_c^{\frac{1}{2-p}}$$

Fracture of polymers



T. Reppel, T. Dally, T. and K. Weinberg, Technische Mechanik, 33 (2012) 19-33.



Crazing in 800 nm polystyrene thin film (C. K. Desai *et al*., 2011)

- Polymers undergo entropic elasticity and damage due to chain stretching and failure
- Polymers fracture by means of the crazing mechanism consisting of fibril nucleation, stretching and failure
- The free energy density of polymers saturates in tension once the majority of chains are failed: p=0!
- Crazing mechanism is incompatible with strain-gradient elasticity…

Fracture of polymers

- Suppose: For $K_U > 0$, *intrinsic length* $\ell > 0$,

$$E(y) \leq K_U \left(\int_\Omega (|Dy|^p - 3^{p/2})\, dx + \ell \int_\Omega |D^2 y|\, dx \right)$$

- If $E(y) < +\infty \Rightarrow y$ continuous on a.e. plane!
- Crazing is precluded by the continuity of y!
- Instead, *fractional* SG elasticity: For $\sigma \in (0,1)$,

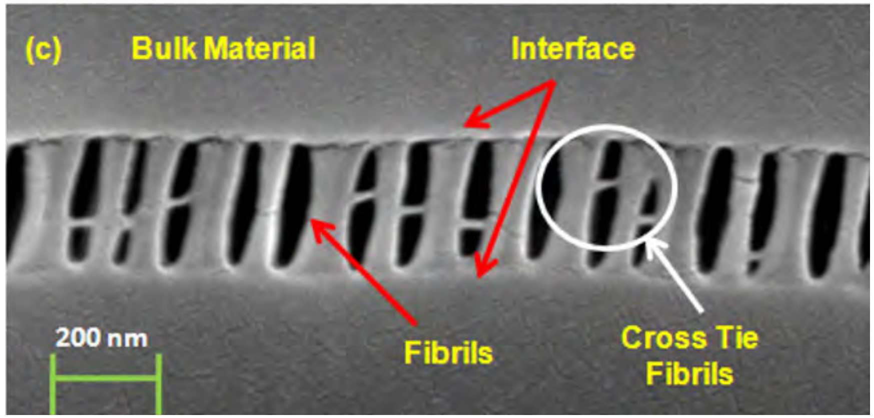
$$E(y) \leq K_U \left(\int_\Omega (|Dy|^p - 3^{p/2})\, dx + \ell^\sigma |y|_{W^{1+\sigma,1}(\Omega)} \right)$$

Theorem [Conti *et al.*, ARMA, 2014] For ℓ sufficiently small, $p = 0$, $\sigma \in (0,1)$, $0 < C_L < C_U$,

$$C_L L^2 \ell^{\frac{\sigma}{1+\sigma}} \delta^{\frac{1}{1+\sigma}} \leq \inf E \leq C_U L^2 \ell^{\frac{\sigma}{1+\sigma}} \delta^{\frac{1}{1+\sigma}}$$

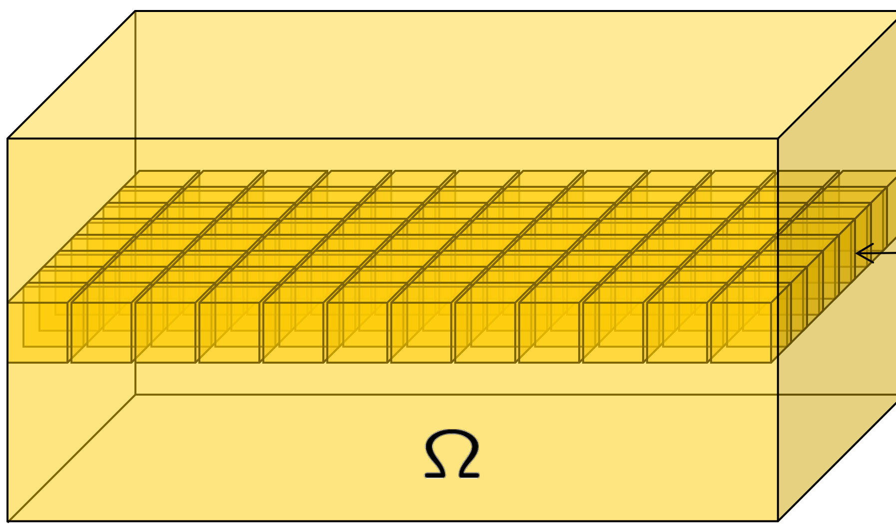
Sketch of proof – Upper bound

crazing



Crazing in 800 nm polystyrene thin film (C. K. Desai *et al*., 2011)

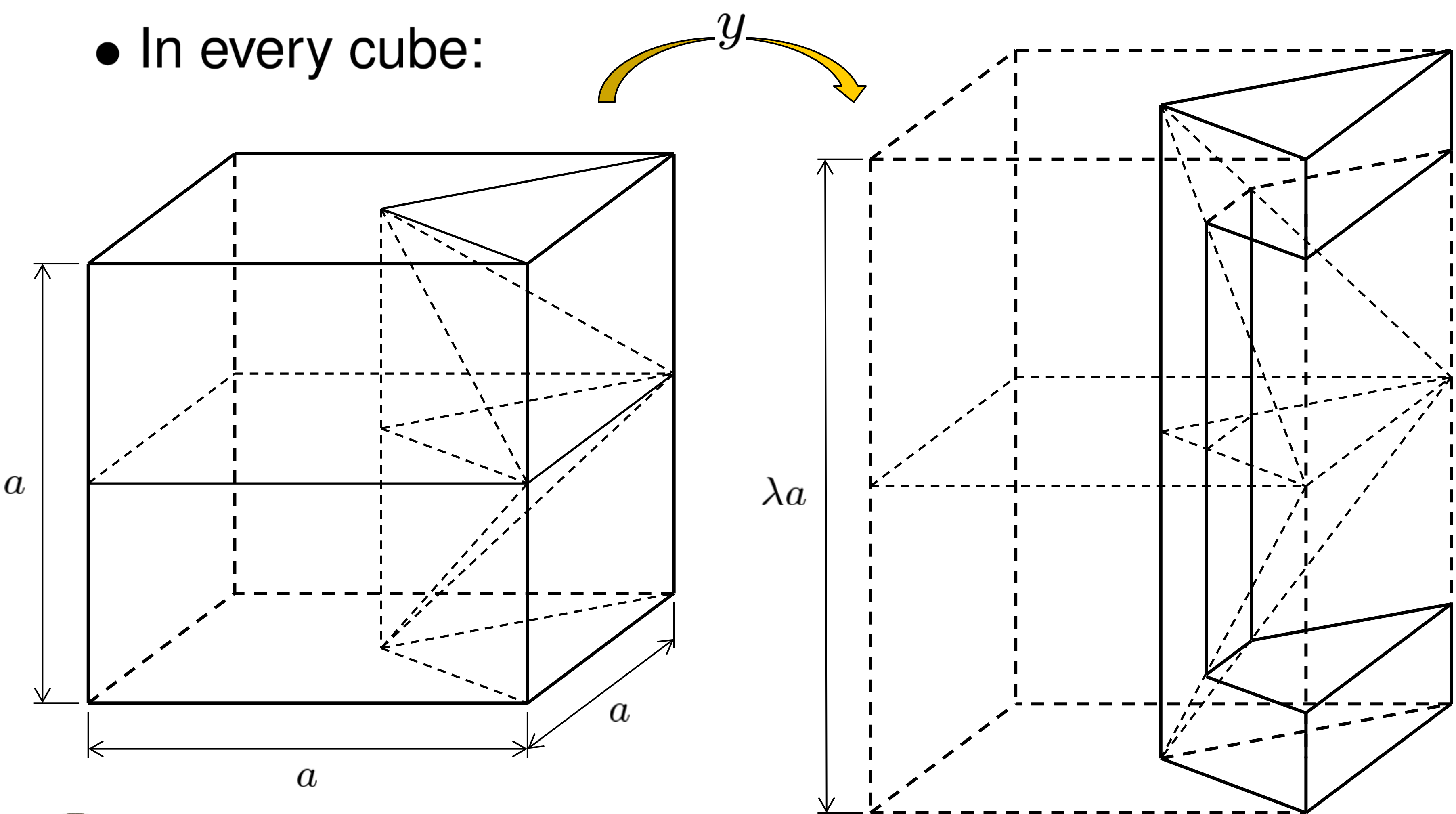
$2a$



craze sheet

Sketch of proof – Upper bound

- In every cube:



Optimal scaling – Crazing

- Optimal (matching) upper and lower bounds:

$$C_L L^2 \ell^{\frac{\sigma}{1+\sigma}} \delta^{\frac{1}{1+\sigma}} \leq \inf E \leq C_U L^2 \ell^{\frac{\sigma}{1+\sigma}} \delta^{\frac{1}{1+\sigma}}$$

- *Fractional* strain-gradient elasticity supplies bounded energies for crazing mechanism
- Energy scales with *area* (*L^2*): *Fracture scaling*!
- Energy scales with power of opening displacement (δ): *Cohesive behavior*!
- Lower bound degenerates to 0 when the intrinsic length (ℓ) decreases to zero…
- Bounds on specific fracture energy:

$$C_L \ell^{\frac{\sigma}{1+\sigma}} \delta_c^{\frac{1}{1+\sigma}} \leq G_c \leq C_U \ell^{\frac{\sigma}{1+\sigma}} \delta_c^{\frac{1}{1+\sigma}}$$

Concluding remarks

- Ductile fracture can indeed be understood as the result of the competition between sublinear growth and (possibly fractional) strain-gradient effects
- Optimal scaling laws are indicative of a well-defined specific fracture energy, cohesive behavior, and provide a (multiscale) link between macroscopic fracture properties and micromechanics (intrinsic micromechanical length scale, void-sheet and crazing mechanisms…)
- Ductile fracture can be efficiently implemented through material-point erosion schemes…

Concluding remarks

Thanks!