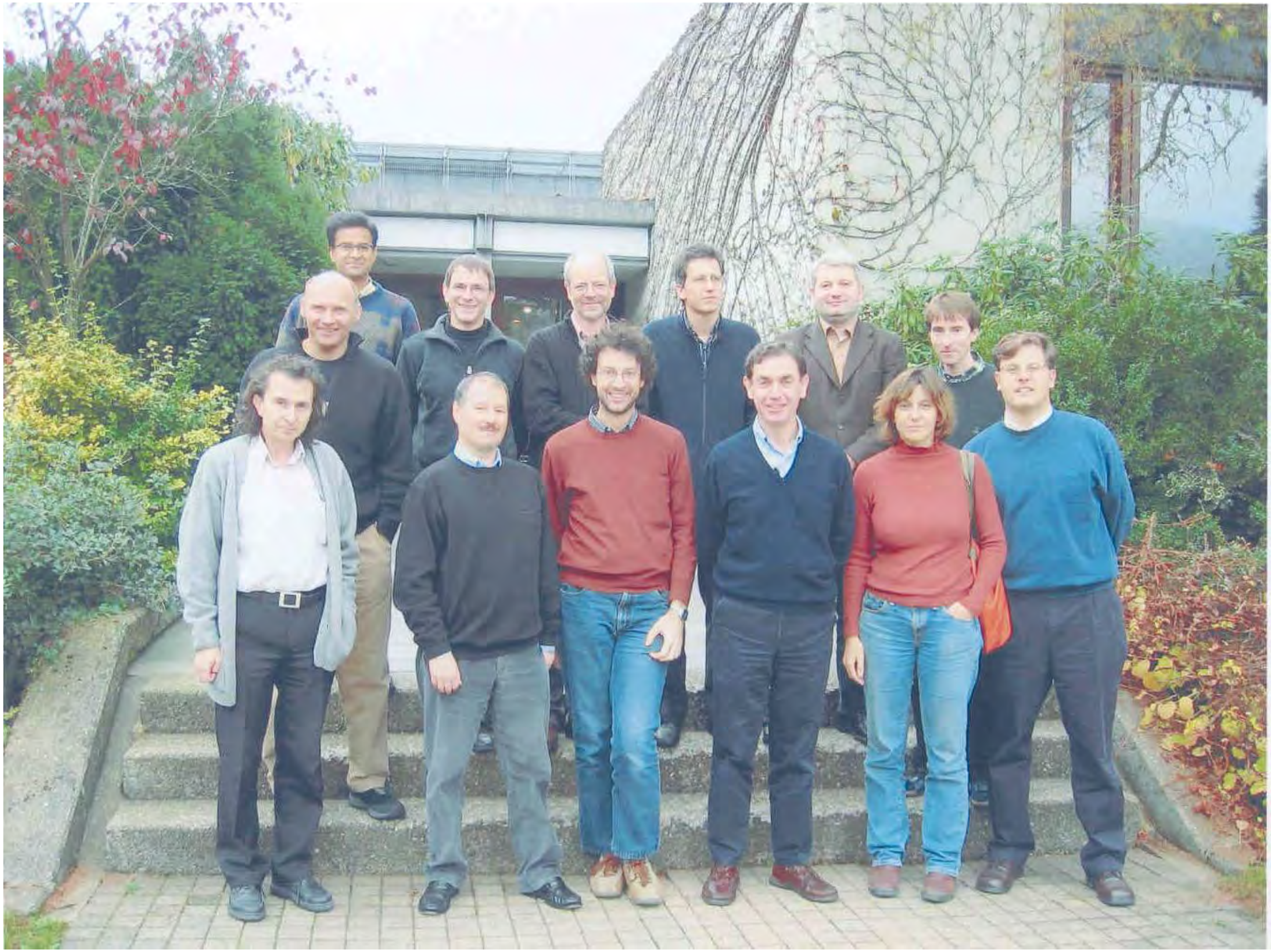
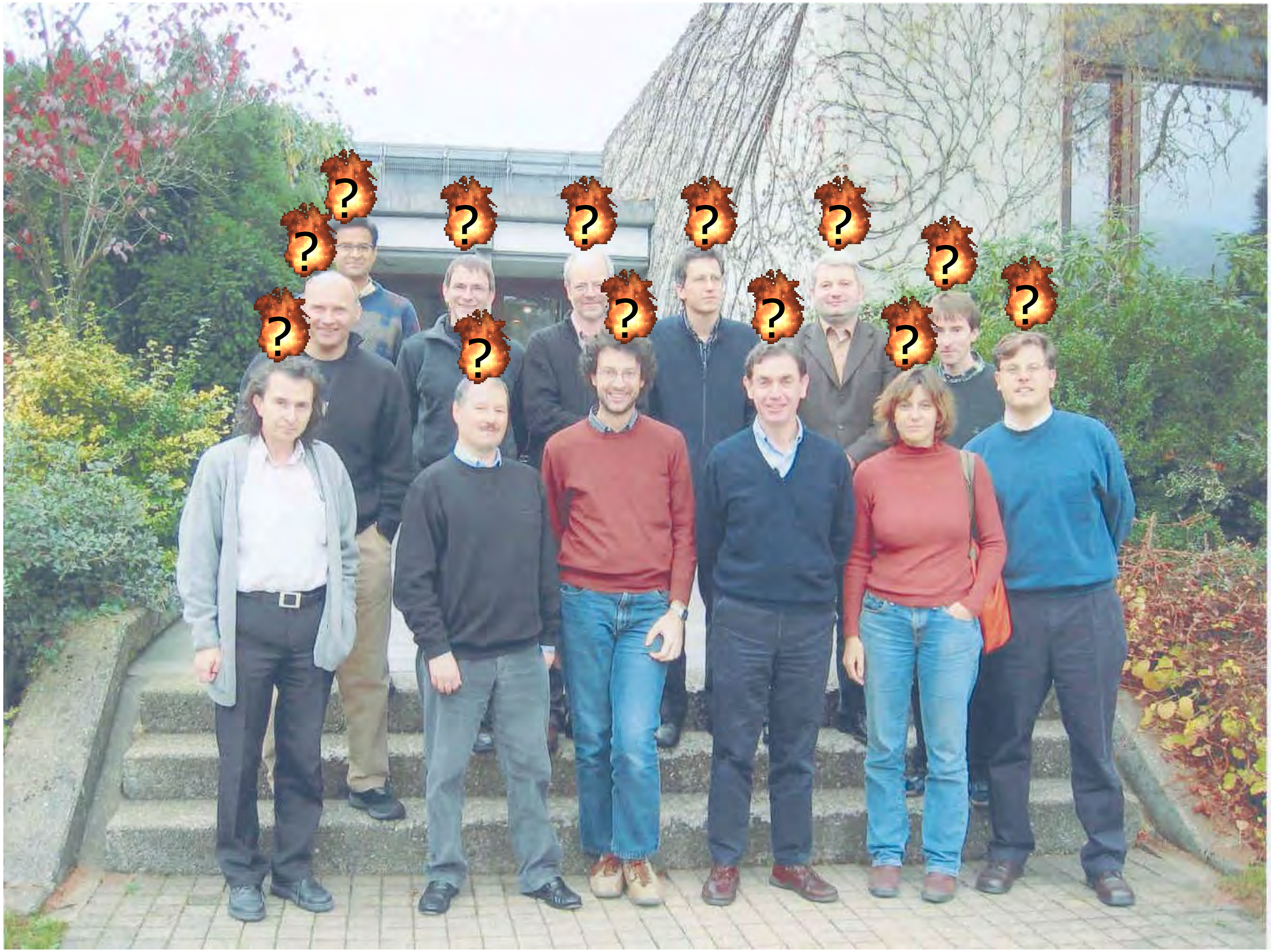
Optimal scaling in plasticity and fracture

M. Ortiz
California Institute of Technology

Workshop on Analysis and Computation of Microstructure in Finite Plasticity
Bonn, Germany, 4-8 May 2015

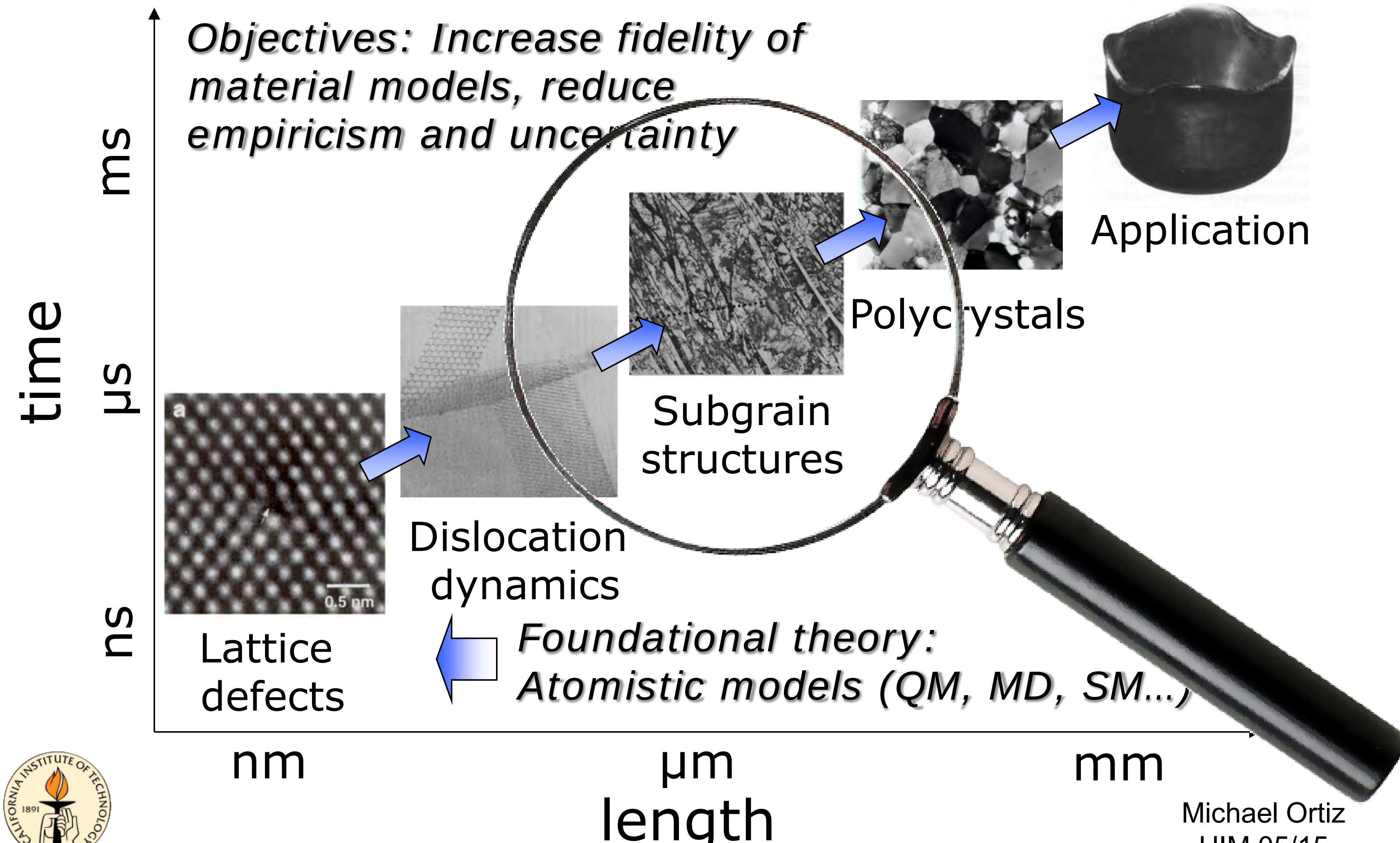


Mini-Workshop on Analysis and Computation of Microstructures in Finite Plasticity
Mathematisches Forschungsinstitut Oberwolfach
Oberwolfach, Germany, Nov. 14-18, 2005



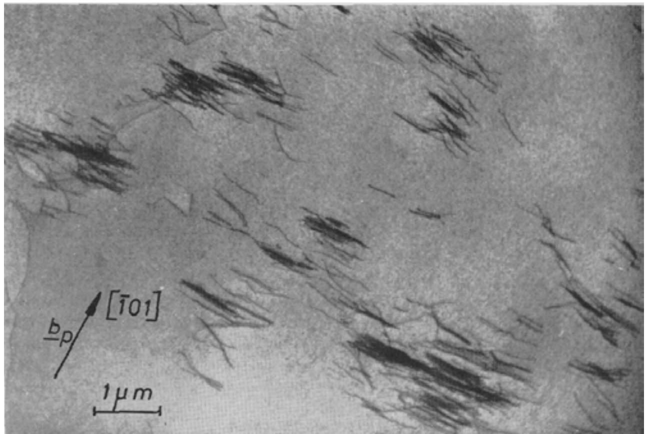
The Genesis of the program: Mathematicians and engineers puzzle over microplasticity…

The framework: Multiscale physics



The question: Evolving microstructures

Copper single crystal
(Mughrabi, Phil. Mag. **23**, 869, 1971)



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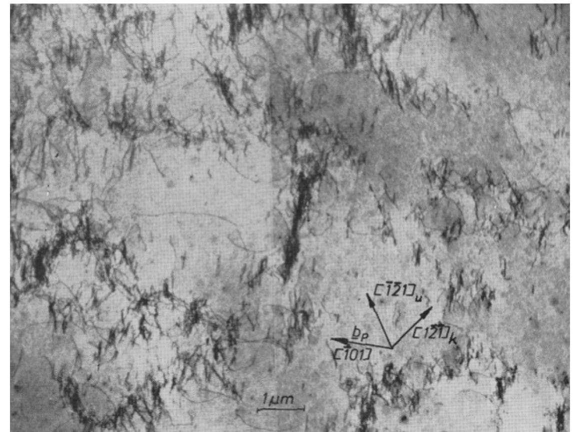
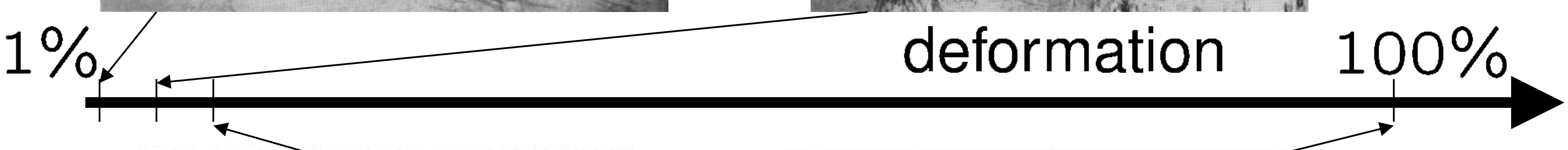
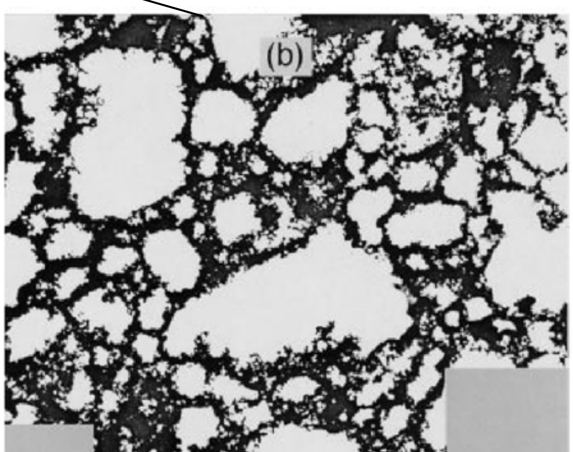
(Mughrabi, Phil. Mag. **23**, 869, 1971)







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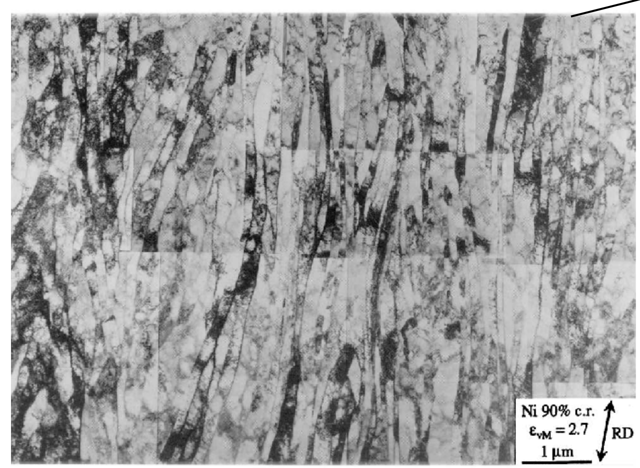


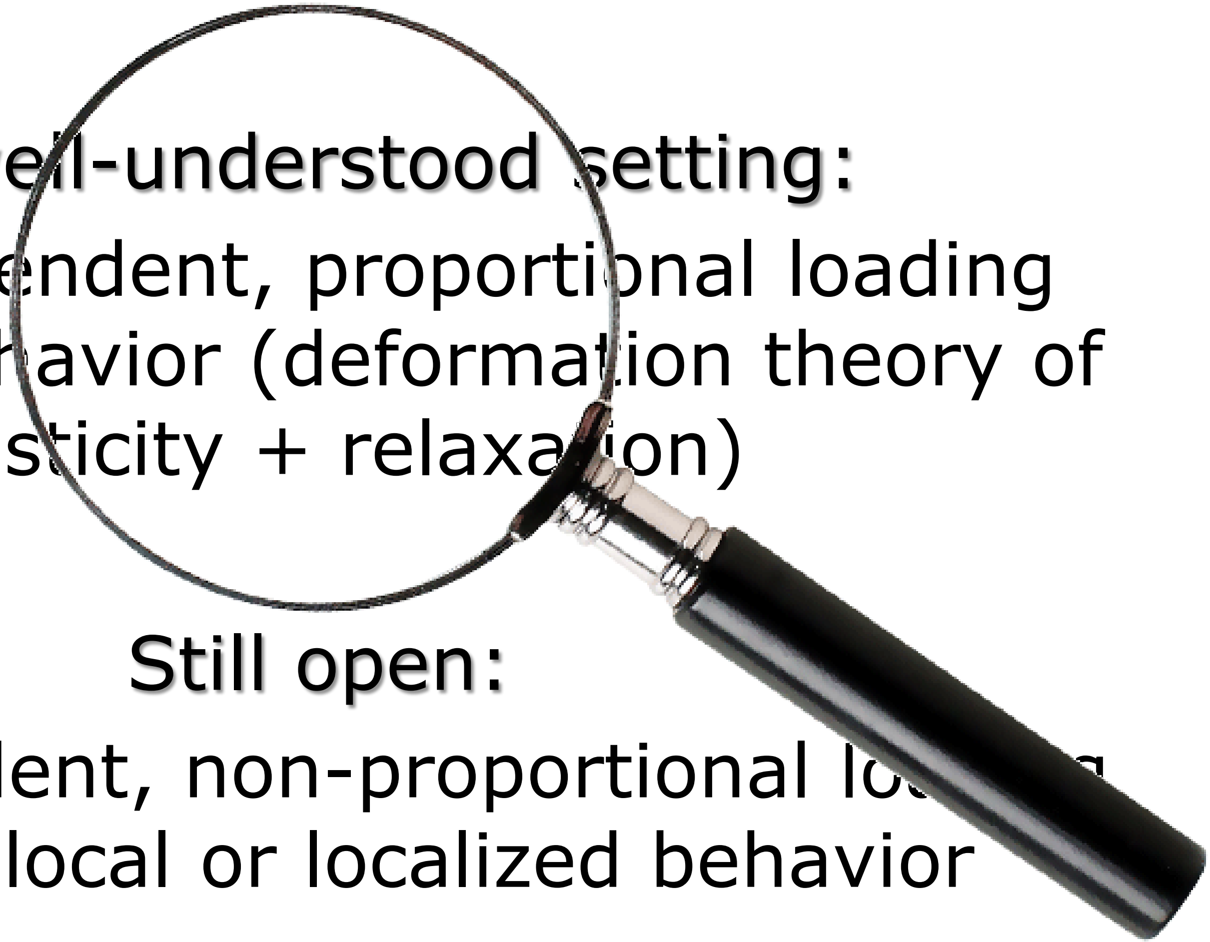
90% cold-rolled Ni (Hansen, Huang and Hughes, Mat. Sci. Engin. A **317**, 3, 2001)

The promise: Nonlinear analysis

Can analysis shed light on the experimental record? (e.g., can some of the observed microstructures be understood as energy minimizers?)

Can analysis inform modeling and simulation? (e.g., homogeneization, multiscale modeling, relaxation, acceleration…)

Ten Years Later…



The well-understood setting:

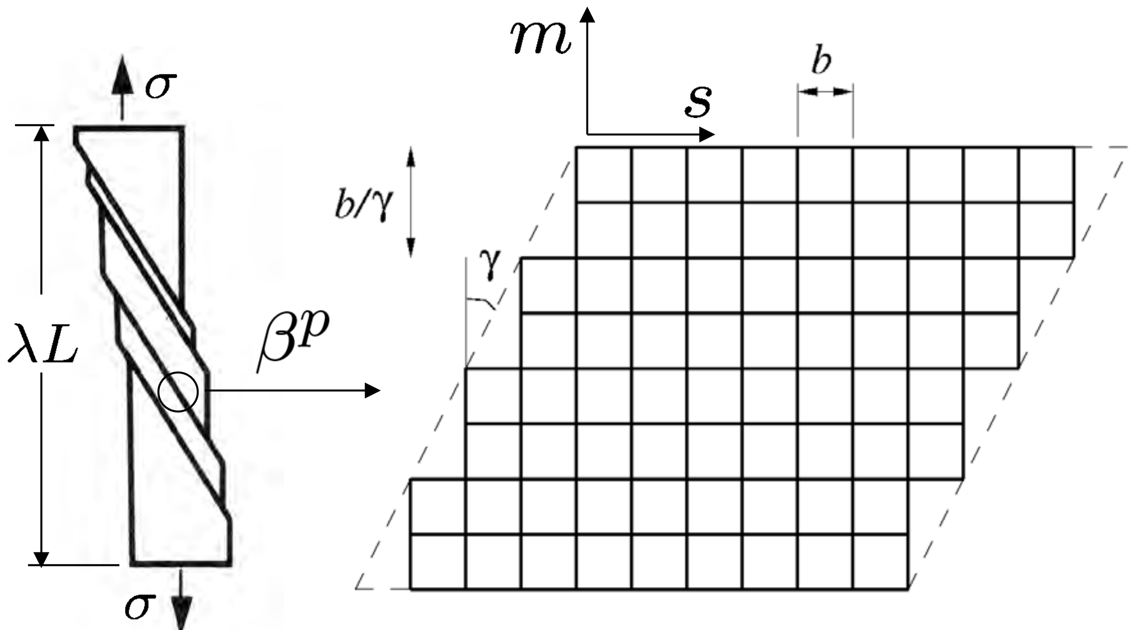
Rate-independent, proportional loading and local behavior (deformation theory of plasticity + relaxation)

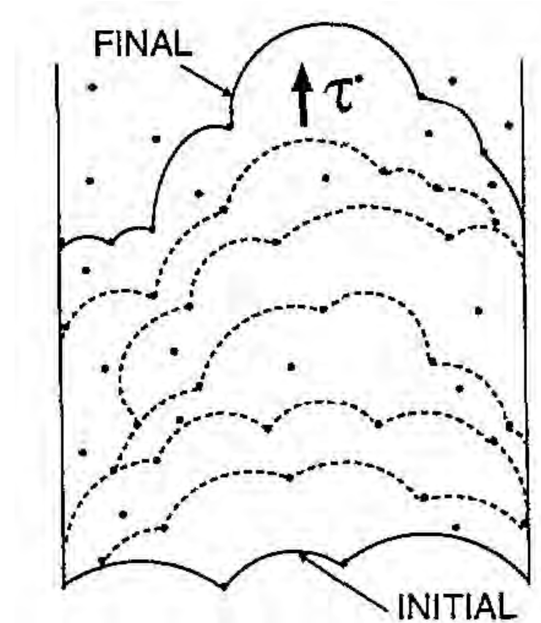
Still open:

Rate-dependent, non-proportional loading and non-local or localized behavior

Michael Ortiz
HIM 05/15

Crystal plasticity – Linearized kinematics





- Kinematics: $\epsilon^p(\gamma) = \frac{1}{|\Omega|}\int_{J_u} [\![u]\!]\odot m \, d\mathcal{H}^2 \equiv \sum \gamma s \odot m$
- Energy: $E(u,\gamma) = \int_\Omega [\, W^e(\nabla u - \epsilon^p(\gamma)) + \cancel{T\,|\nabla\gamma\times m|}\,]\, dx$
- Dissipation: $\psi(\dot\gamma) = \begin{cases} \tau_c|\dot\gamma|, & \text{single slip,} \\ +\infty, & \text{otherwise.} \end{cases}$

Crystal plasticity – Deformation theory

- Energy-dissipation functional[1]:

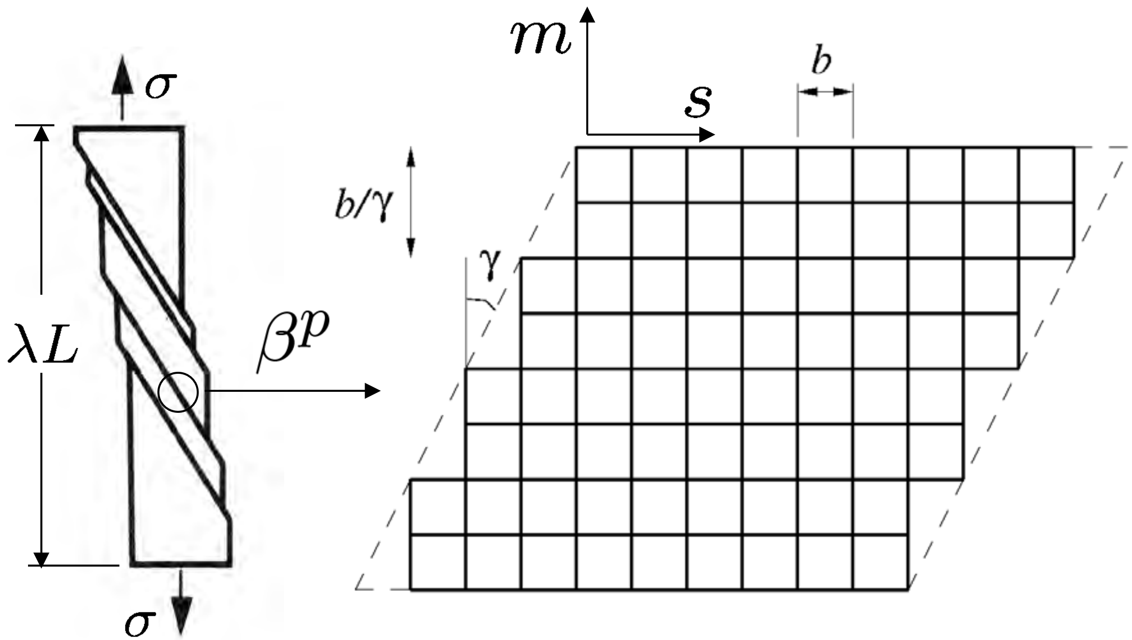
$$F_\epsilon(u,\gamma) = \int_0^T \mathrm{e}^{-t/\epsilon}[\Psi(\dot\gamma(t)) + \frac{1}{\epsilon} E(u(t),\gamma(t))]\,dt \to \text{inf!}$$

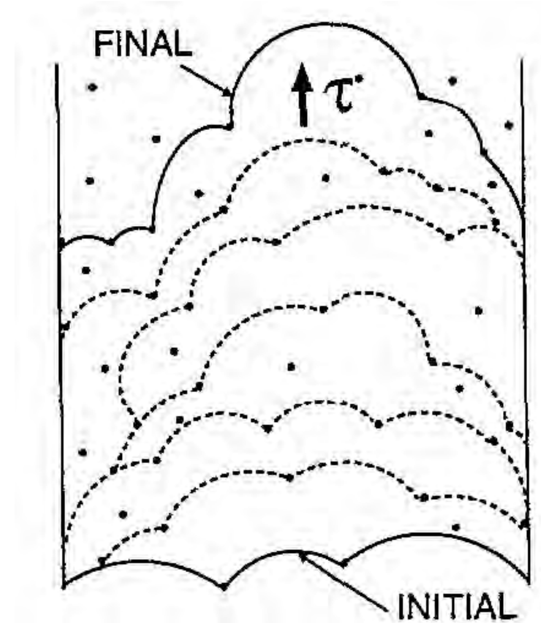
- Assume: $\gamma(t)$ monotonic (proportional loading).
- Plastic work density: $W^p(\gamma) = \begin{cases} \sum \tau_c \gamma, & \text{single slip,} \\ +\infty, & \text{otherwise.} \end{cases}$
- Then: $\Psi(\dot\gamma(t)) = \frac{d}{dt}\int_\Omega W^p(\gamma(t))\,dx = \frac{d}{dt} P(\gamma(t))$
- Energy-dissipation functional: minimize pointwise!

$$F_\epsilon(u,\gamma) = \int_0^T \frac{\mathrm{e}^{-t/\epsilon}}{\epsilon}[E(u(t),\gamma(t)) + P(\gamma(t))]\,dt \to \text{inf!}$$

[1]A. Mielke and M. Ortiz, *ESAIM COCV*, **14** (2008) 494.

Crystal plasticity – Deformation theory





- Incremental flow rule: $\epsilon^p(\gamma) = \sum \gamma s \odot m$
- Pseudo-elastic strain energy density:

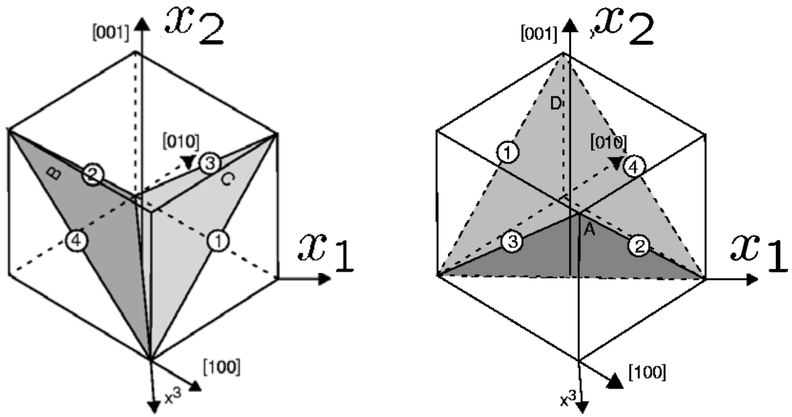
$$W(\epsilon) = \inf_{\gamma} \{W^e(\epsilon - \epsilon^p(\gamma)) + W^p(\gamma)\}$$

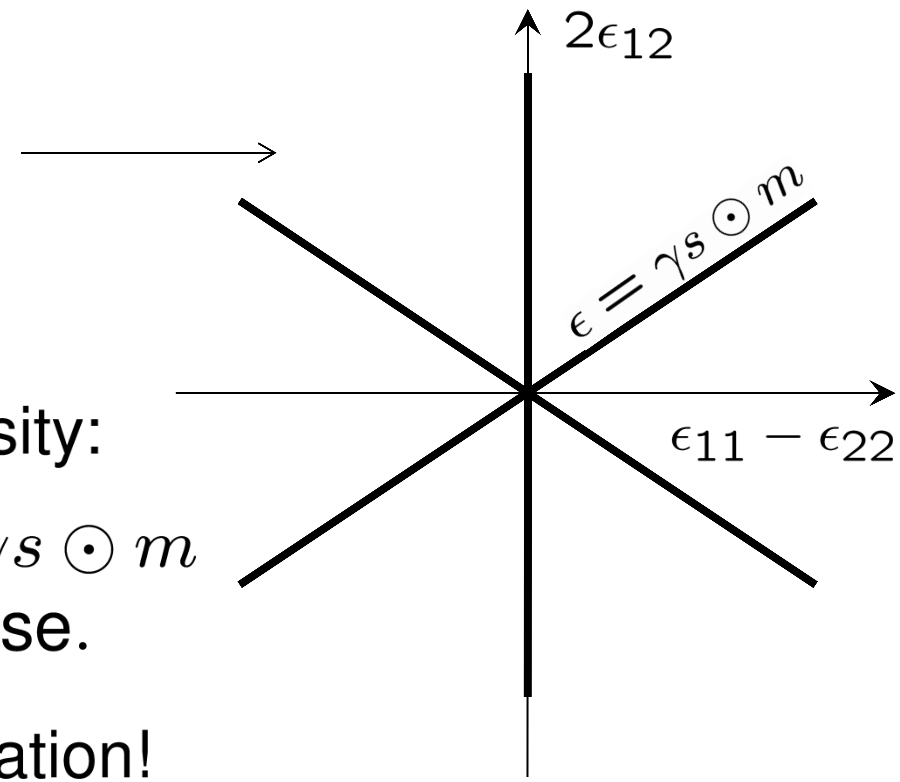
- Variational problem (static equilibrium):

$$F(u) = \int_{\Omega} W(\epsilon(u))\, dx \to \text{inf!}$$

Crystal plasticity – Non-convexity[1]

- Example: FCC crystal deforming on $(1\bar{1}0)$-plane





- Pseudo-elastic energy density:

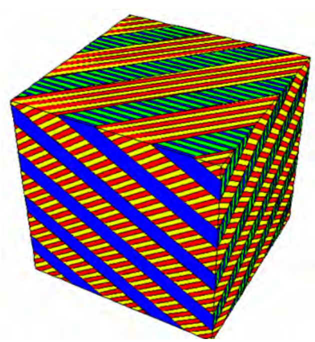
$$W = \begin{cases} \text{linear,} & \text{if } \epsilon = \gamma s \odot m \\ \text{quadratic,} & \text{otherwise.} \end{cases}$$

- $W(\epsilon)$ non-convex! $\Rightarrow$ Relaxation!

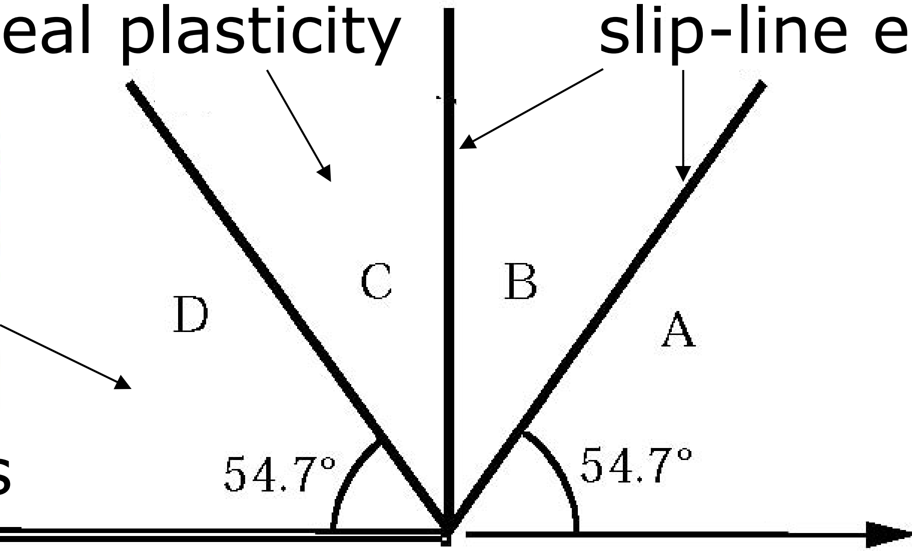
[1] M. Ortiz and E. A. Repetto, *JMPS*, **47**(2) 1999, p. 397.

Crystal plasticity – Relaxation[1]

$$sc^- F(u) = \underbrace{\int_\Omega W^{**}(\epsilon(u))dx}_{\text{ideal plasticity}} + \underbrace{\int_\Omega W^\infty\left(\frac{E_s u}{|E_s u|}\right)}_{\text{slip-line energy}} d|E_s u|$$

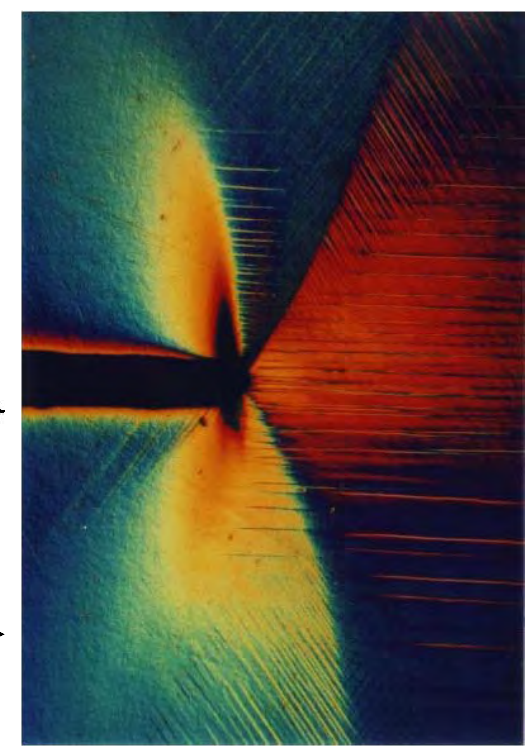


laminates



(Rice, *Mech. Mat.*, 1987)

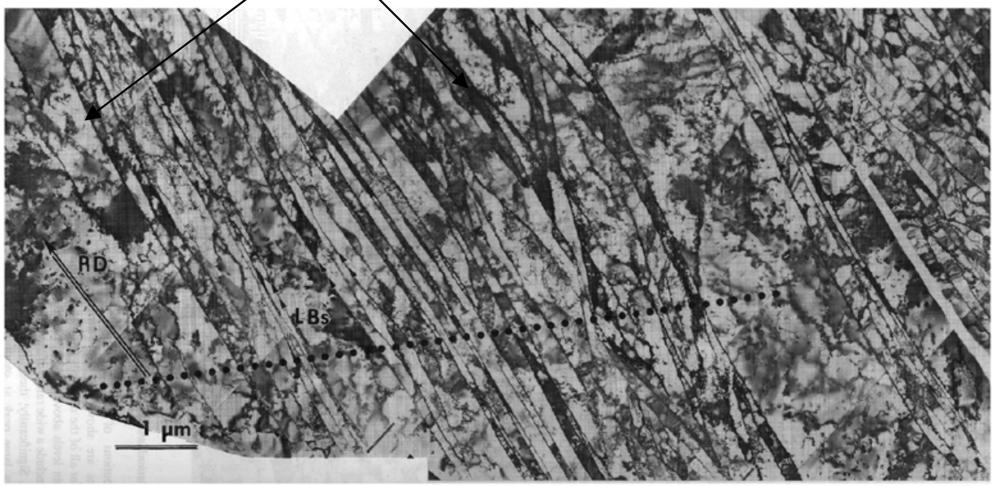
(Crone and Shield, *JMPS*, 2002)



[1]S. Conti and M. Ortiz, *ARMA*, **176** (2005), pp. 103–147

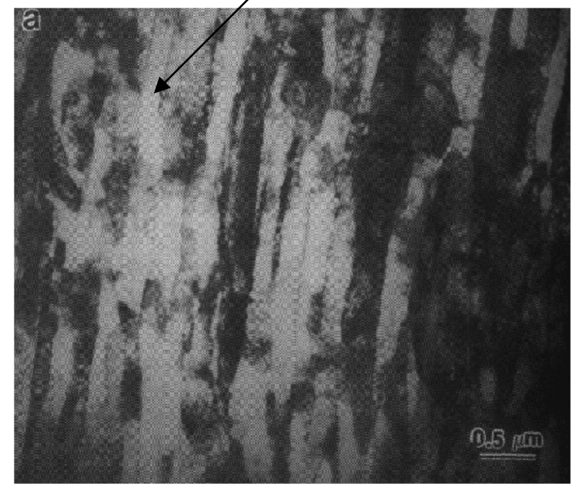
Crystal plasticity – Lamellar structures

Dislocation walls



Lamellar dislocation structure
in 90% cold-rolled Ta
(DA Hughes and N Hansen, Acta Materialia,
44 (1) 1997, pp. 105-112)

Dislocation walls

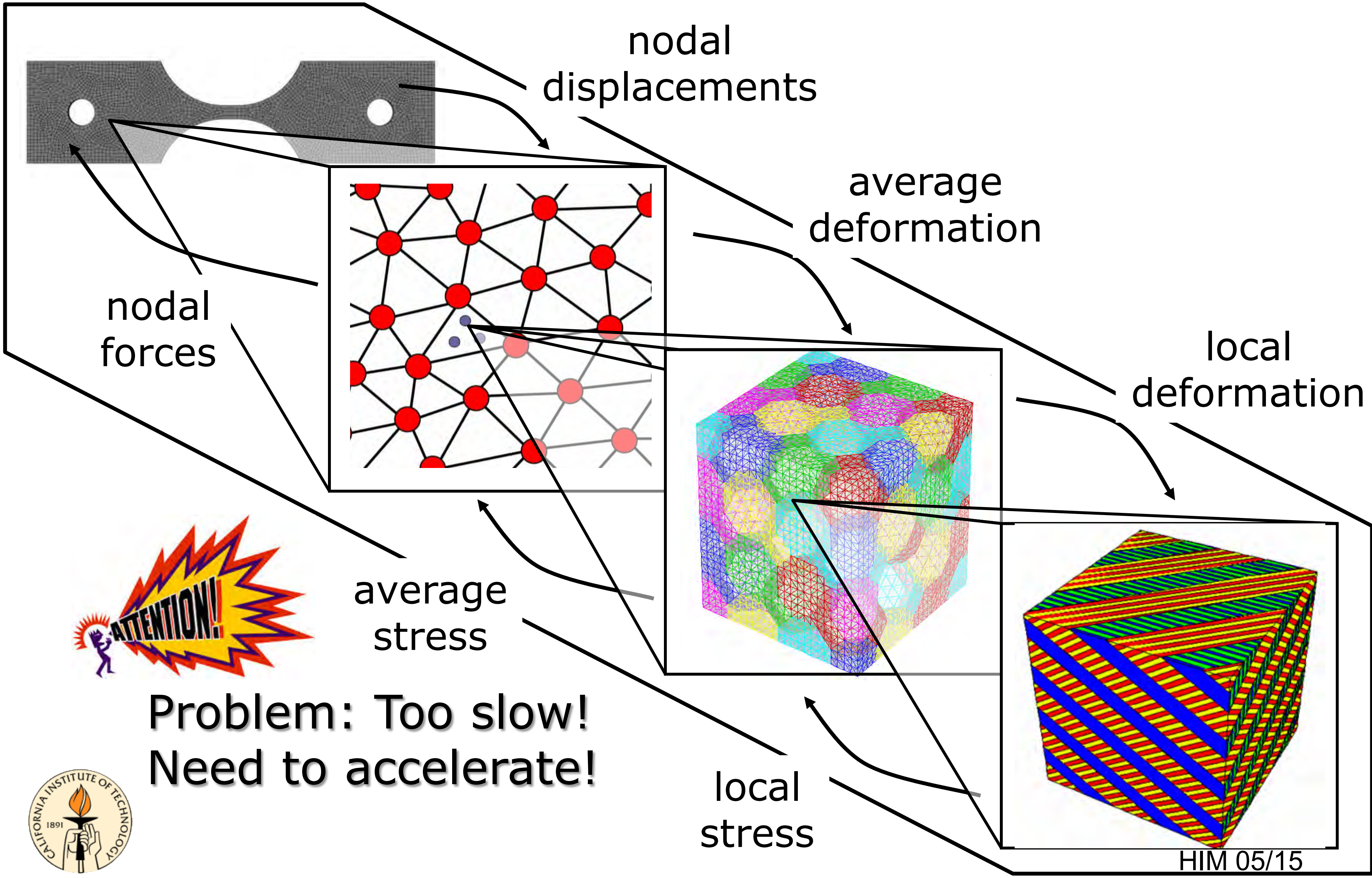


Lamellar structure
in shocked Ta
(MA Meyers et al.,
Metall. Mater. Trans.,
26 (10) 1995, pp. 2493-2501)

Lamellar dislocation structures at large strains

S. Conti, G. Dolzmann and C. Kreisbeck, *Math. Models Methods Appl. Sci.*, **23**(11) (2013) 2111.

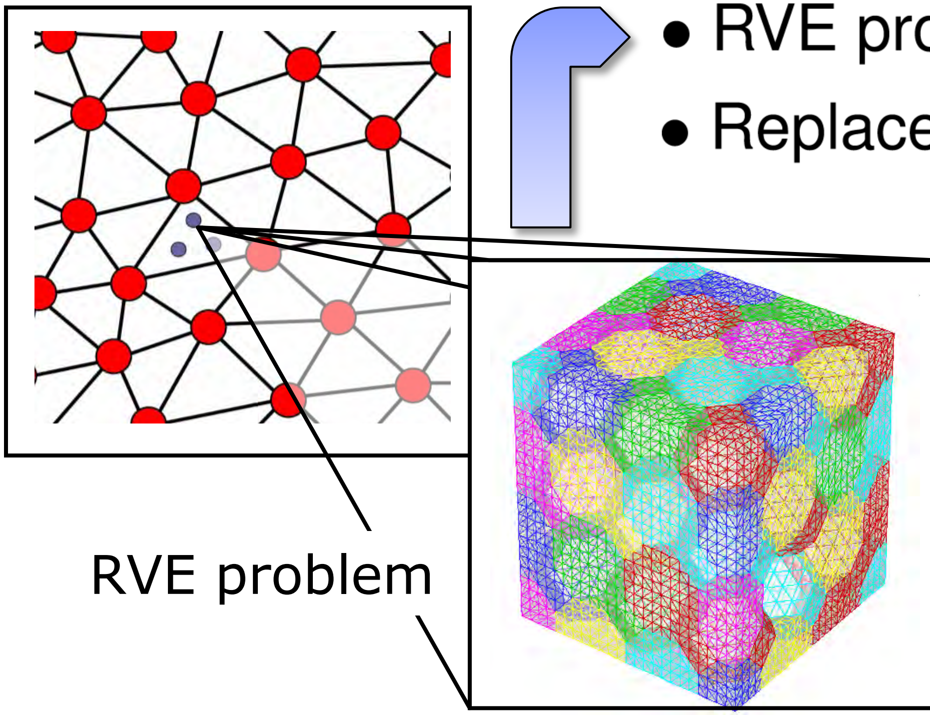
Polycrystals – Concurrent multiscale (C^3)

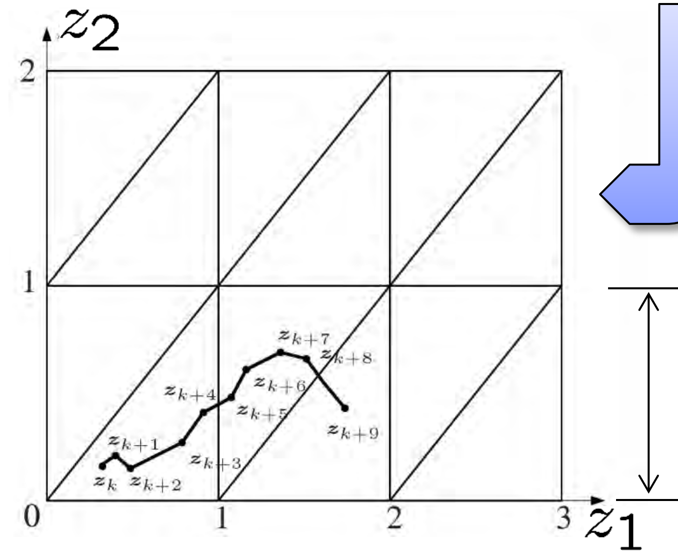


Problem: Too slow!
Need to accelerate!

Acceleration: Phase-space interpolation

- RVE problem must supply $\boldsymbol{P}(\overbrace{\boldsymbol{F}, \dot{\boldsymbol{F}}, T}^{\boldsymbol{z}})$
- Replace by interpolant $\boldsymbol{P}_h(\boldsymbol{F}, \dot{\boldsymbol{F}}, T)$!

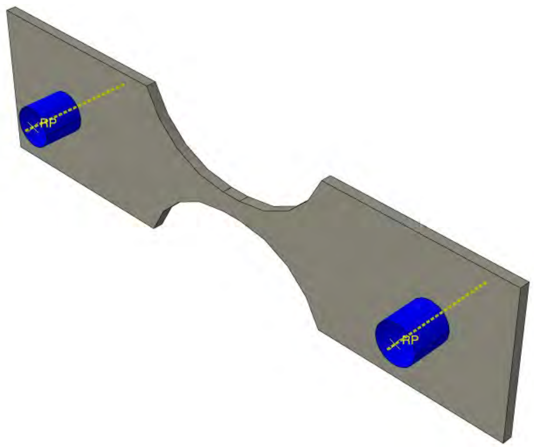




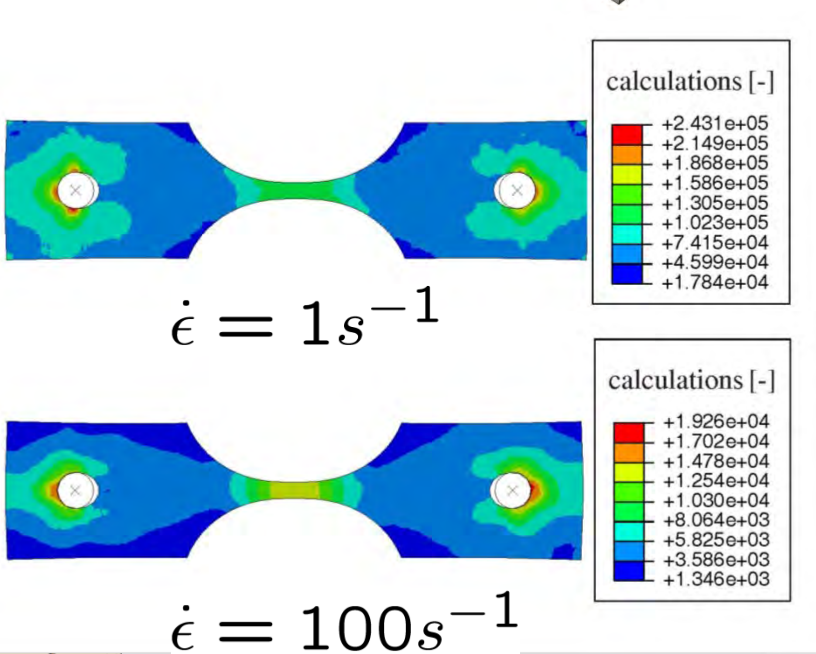
- Simplicial interpolation in high-dimensional spaces[1]
- One single RVE calculation per boundary crossing
- Speed-up = #steps/simplex @ constant accuracy

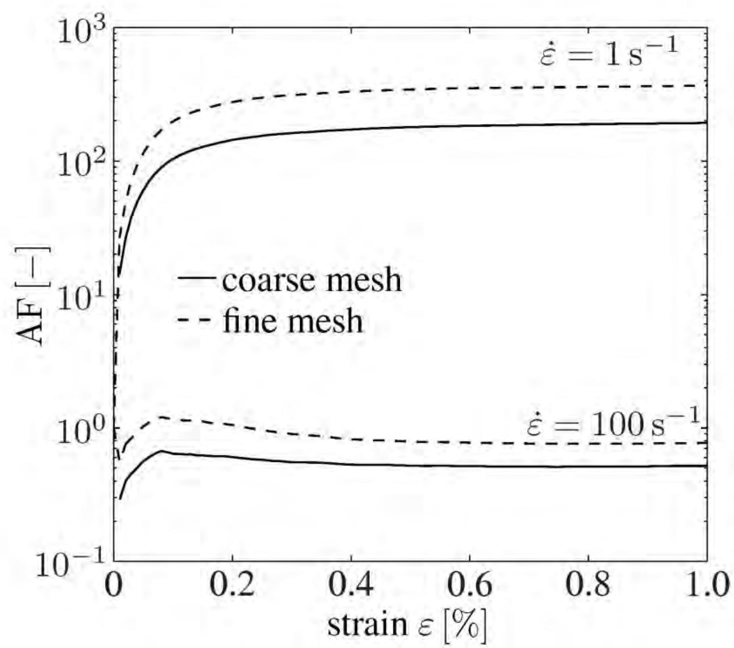
[1] Chien, M.J. and Kuh, E., *IEEE Transactions*, 1978; **25**(11):938–940.
Klusemann, B. and Ortiz, M., IJNME, 10.1002/nme.4887, 2015.

Acceleration: Phase-space interpolation



- Dynamic extension of tensile neo-Hookean specimen
- Explicit Newmark integration
- Hexahedral finite elements
- Quadratic: $W(F) \rightarrow W_h(F)$





Klusemann, B. and Ortiz, M., IJNME, 10.1002/nme.4887, 2015.

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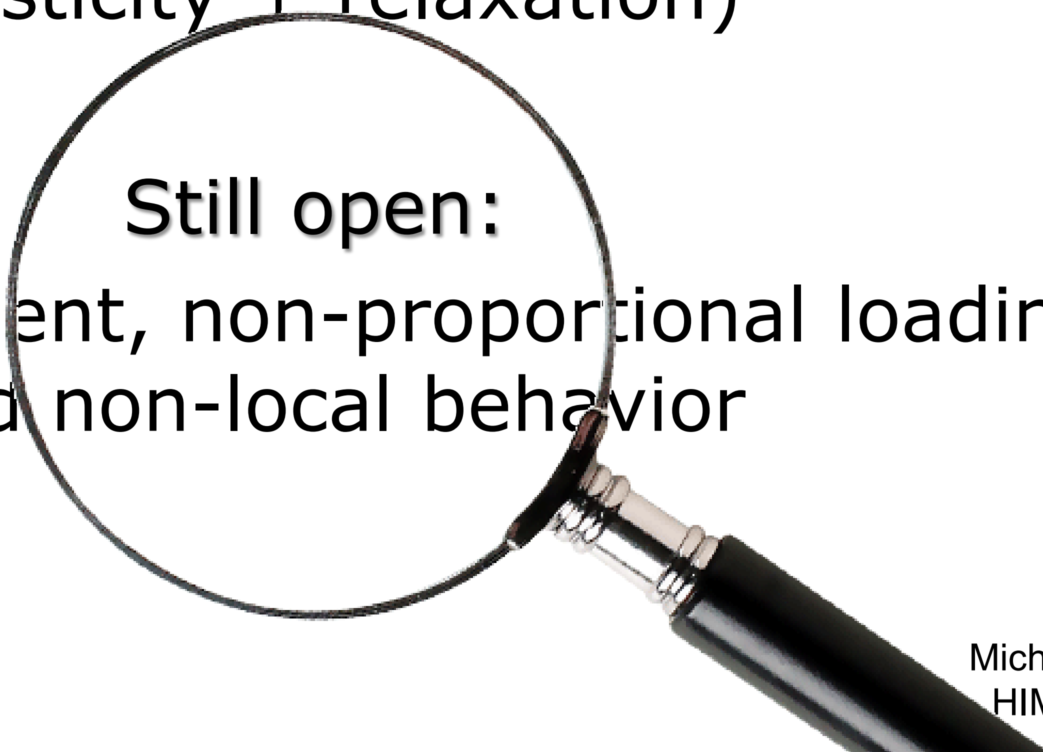
Still open:

Rate-dependent, non-proportional loading and non-local behavior



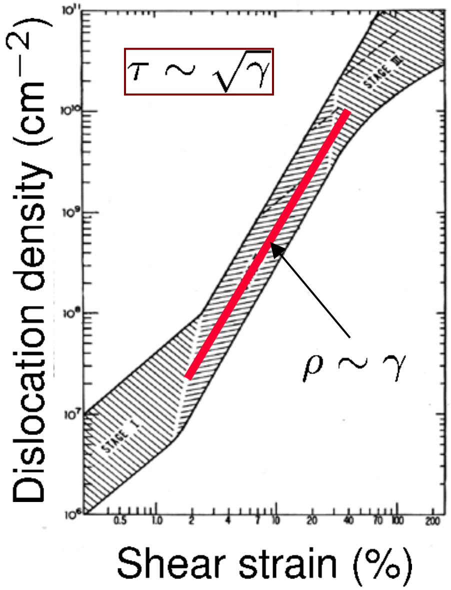
Pitfalls

'Standard program' may fail due to:

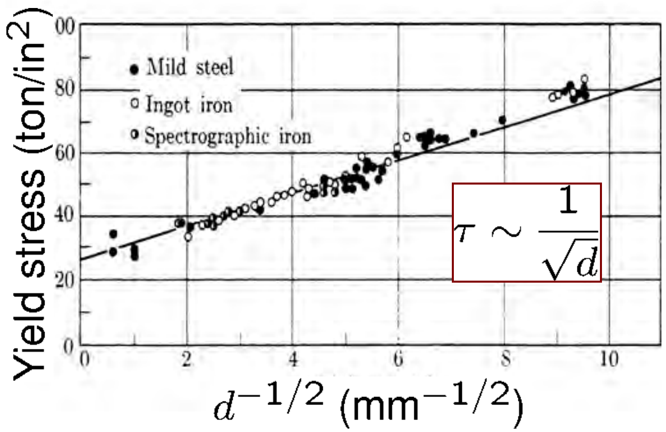
Non-proportional loading (unloading, cycling loading, change of loading path direction) leading to microstructure evolution

Departures from volume scaling (size effect, domain dependence, localization) leading to failure of homogeneization and relaxation

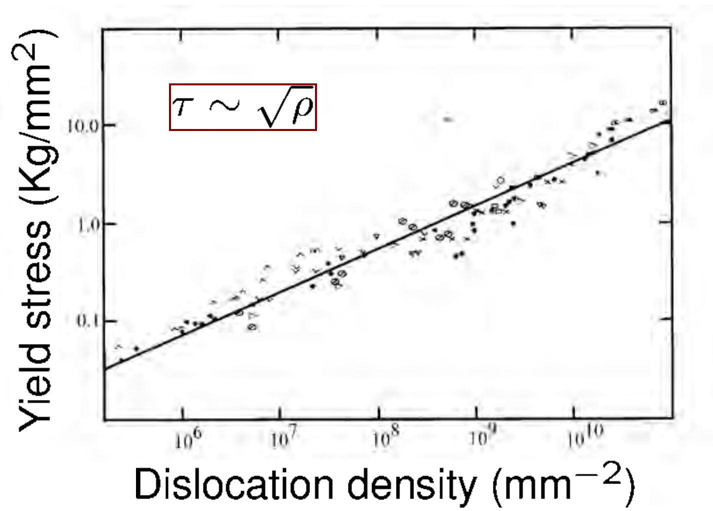
Crystal plasticity – Scaling laws



Taylor hardening
(RJ Asaro,
Adv. Appl. Mech.,
23, 1983, p. 1.)



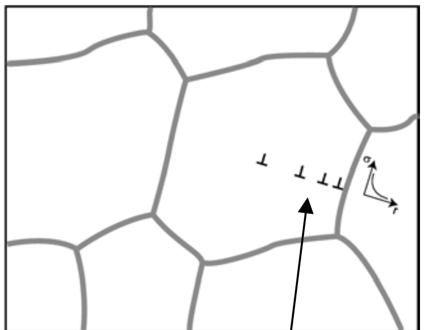
Hall-Petch scaling
(NJ Petch,
J. Iron and Steel Inst.,
174, 1953, pp. 25-28.)

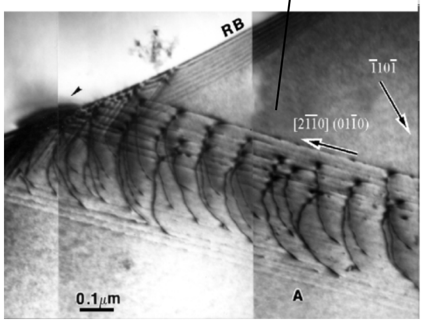


Taylor scaling
(SJ Basinski and ZS Basinski,
Dislocations in Solids,
FRN Nabarro (ed.)
North-Holland, 1979.)

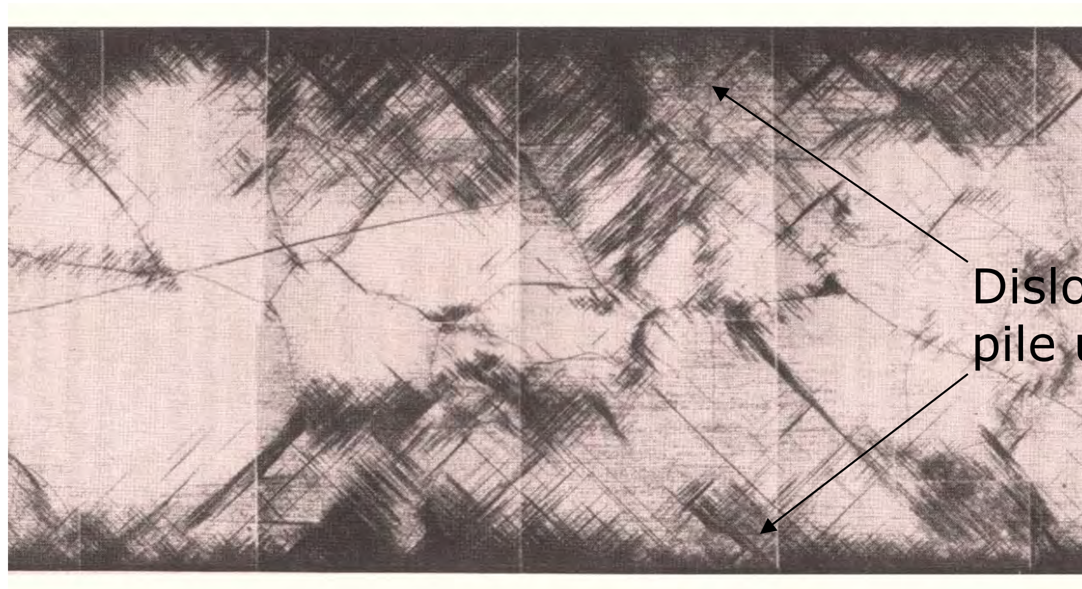
Classical scaling laws of crystal plasticity

Crystal plasticity – Effect of boundaries





Dislocation pile-up
at Ti grain boundary
(I. Robertson)



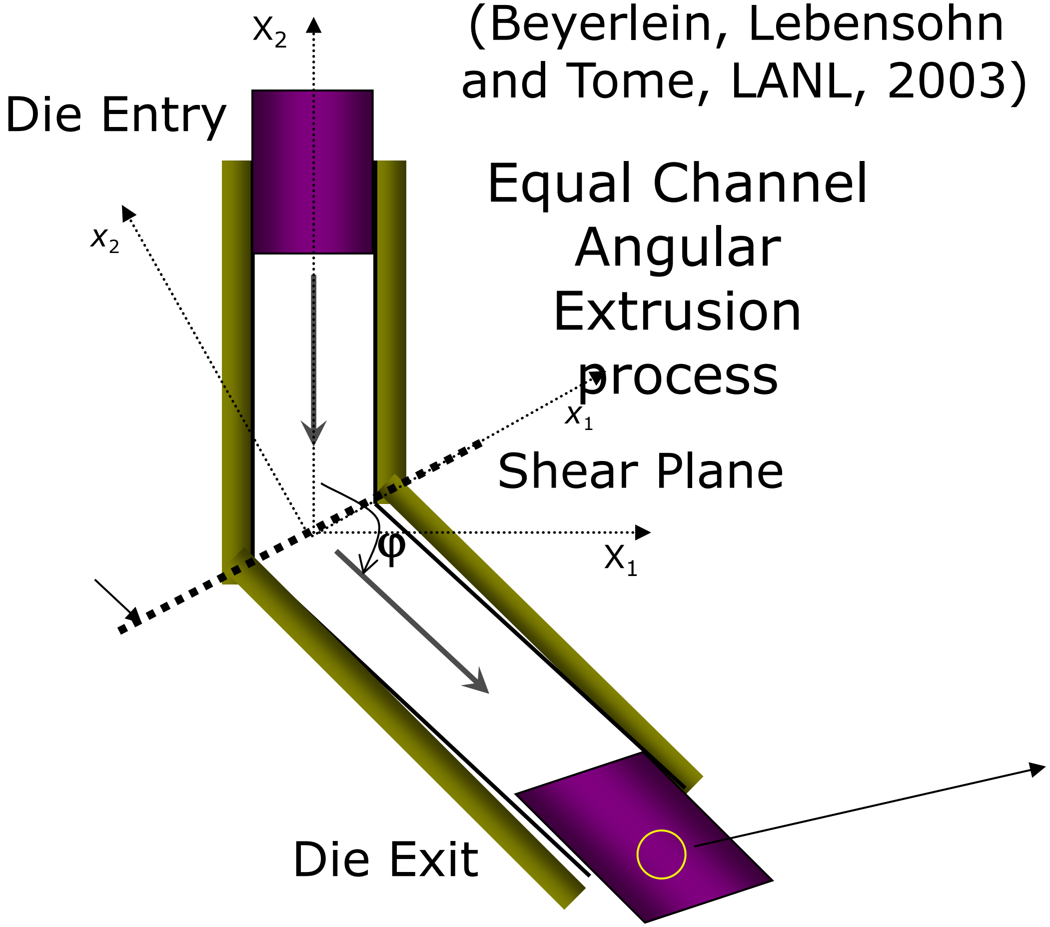
LiF plate impact experiment.
Dislocation pile-ups at surfaces
and grain boundaries
(G Meir and RJ Clifton, J. Appl. Phys.,
59 (1) 1986, pp. 124-148)

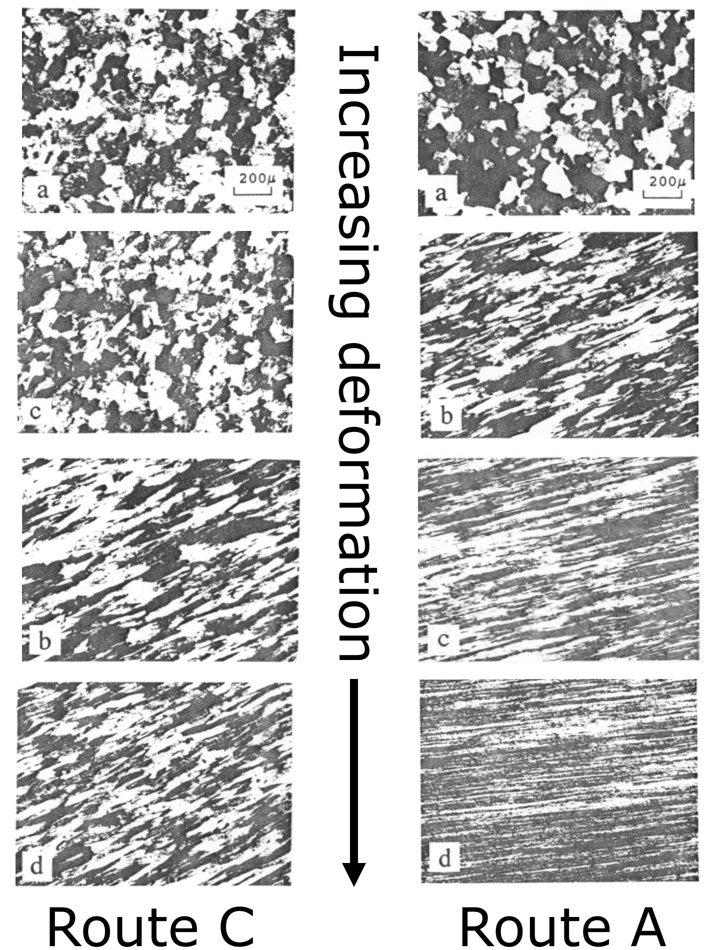
Dislocation pile-ups
at grain boundaries, surfaces

Crystal plasticity – Size effect

(Beyerlein, Lebensohn and Tome, LANL, 2003)

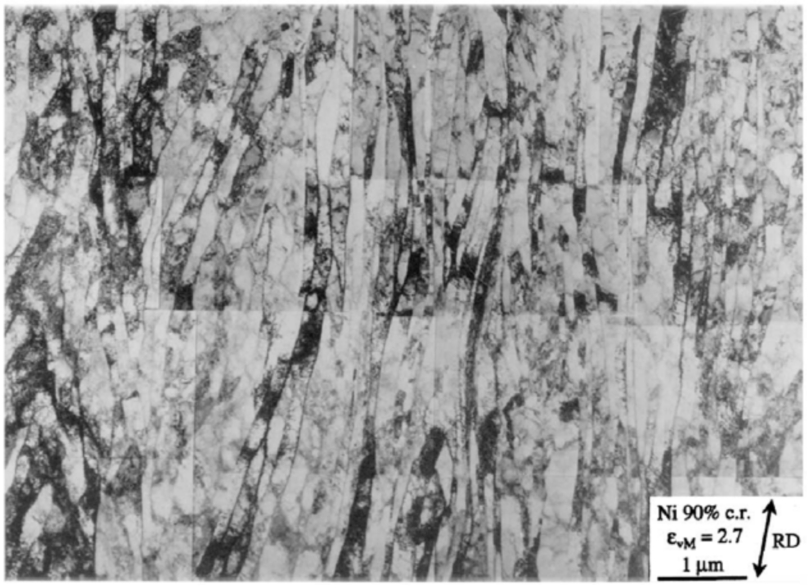
Equal Channel Angular Extrusion process



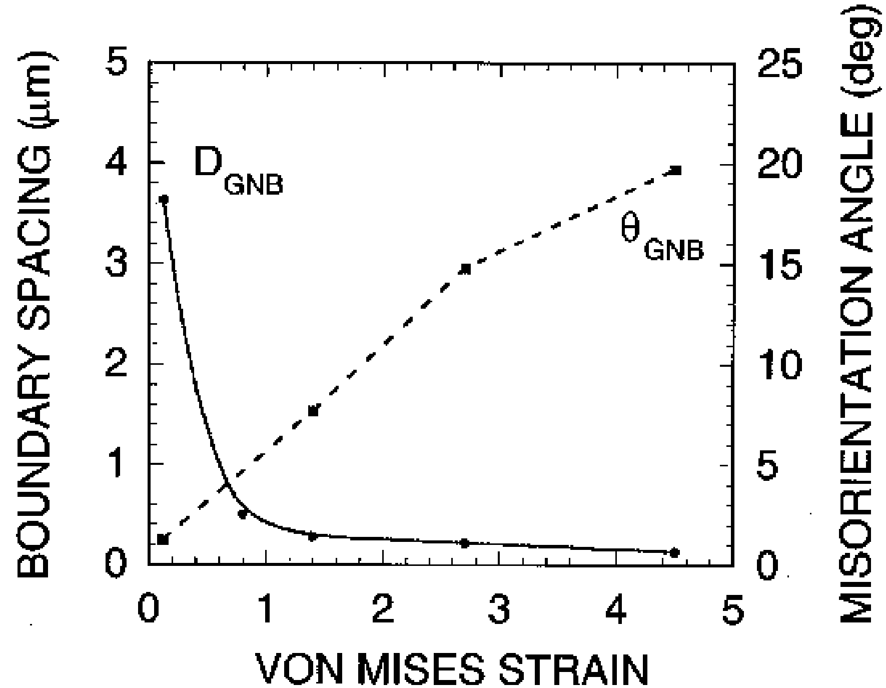


Evolution of dislocation structures in Cu specimen. Lamellar width: $l \sim \gamma^{-0.65}$

Crystal plasticity – Size effect



Pure nickel cold rolled to 90%
Hansen *et al.* Mat. Sci. Engin. A317 (2001).



Lamellar width and misorientation angle as a function of deformatation
Hansen *et al.* Mat. Sci. Engin. A317 (2001).

Scaling of lamellar width and misorientation angle with deformation

Non-local microplasticity

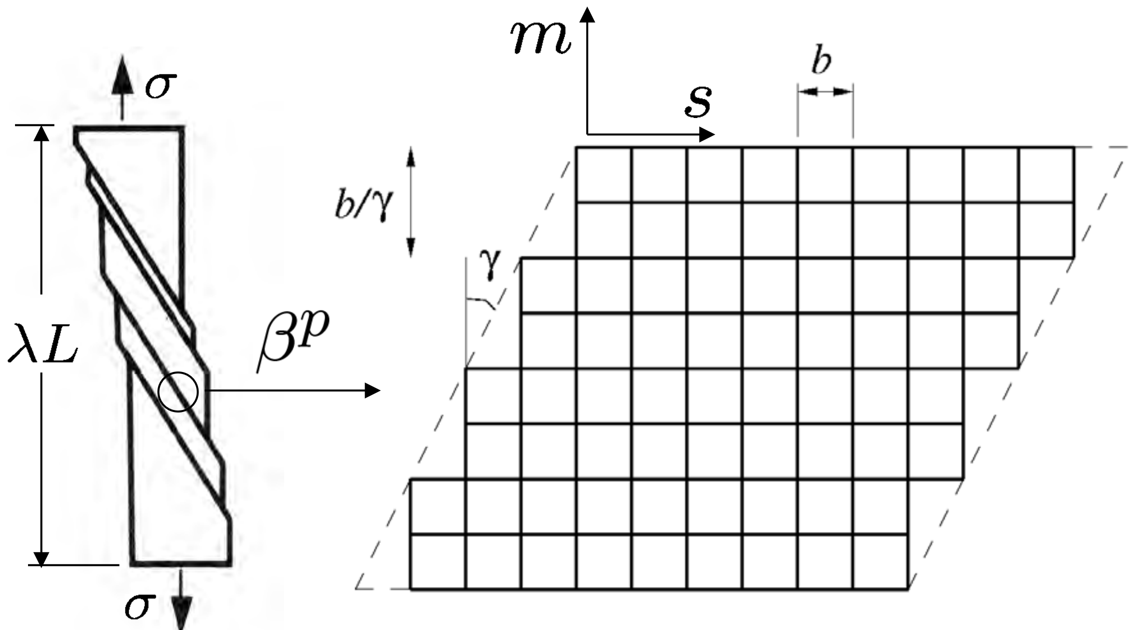
Scaling laws such as Hall-Petch suggest the existence of an intrinsic material length scale

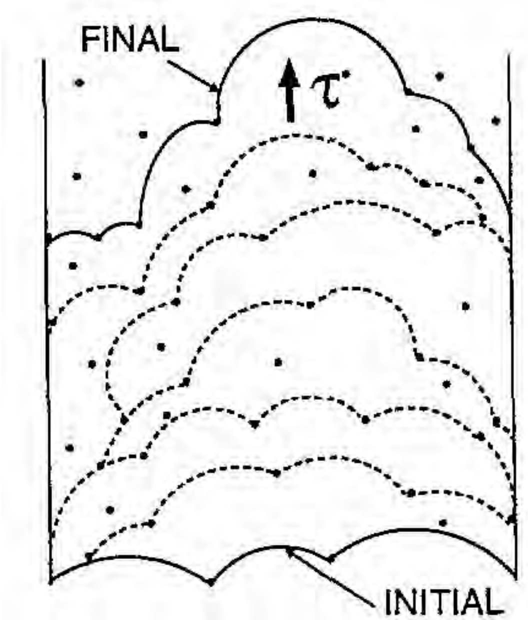
Modeling assumption: Account for dislocation self-energy using a line-tension approximation

The resulting deformation-theoretical energy is non-local (specifically, depends on $\nabla\gamma$)

Intrinsic length-scale: Burgers vector

Crystal plasticity – Linearized kinematics





- Kinematics: $\epsilon^p(\gamma) = \frac{1}{|\Omega|}\int_{J_u} [\![u]\!]\odot m \, d\mathcal{H}^2 \equiv \sum \gamma s \odot m$
- Energy: $E(u,\gamma) = \int_\Omega [\, W^e(\nabla u - \epsilon^p(\gamma)) + T\,|\nabla\gamma \times m| \,]\, dx$
- Dissipation: $\psi(\dot\gamma) = \begin{cases} \tau_c |\dot\gamma|, & \text{single slip,} \\ +\infty, & \text{otherwise.} \end{cases}$

Crystal plasticity – Optimal scaling

Theorem [Conti & MO, ARMA, 2005] *There are constants c and c' such that*

$$cE_0(T,\gamma,\tau_0,\mu,d) \leq \inf E \leq c' E_0(T,\gamma,\tau_0,\mu,d)$$

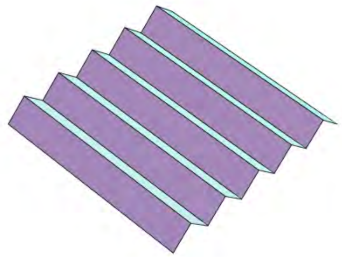
where $E_0(T,\gamma,\tau_0,\mu,d)\Big/G\gamma^2 d^3 =$

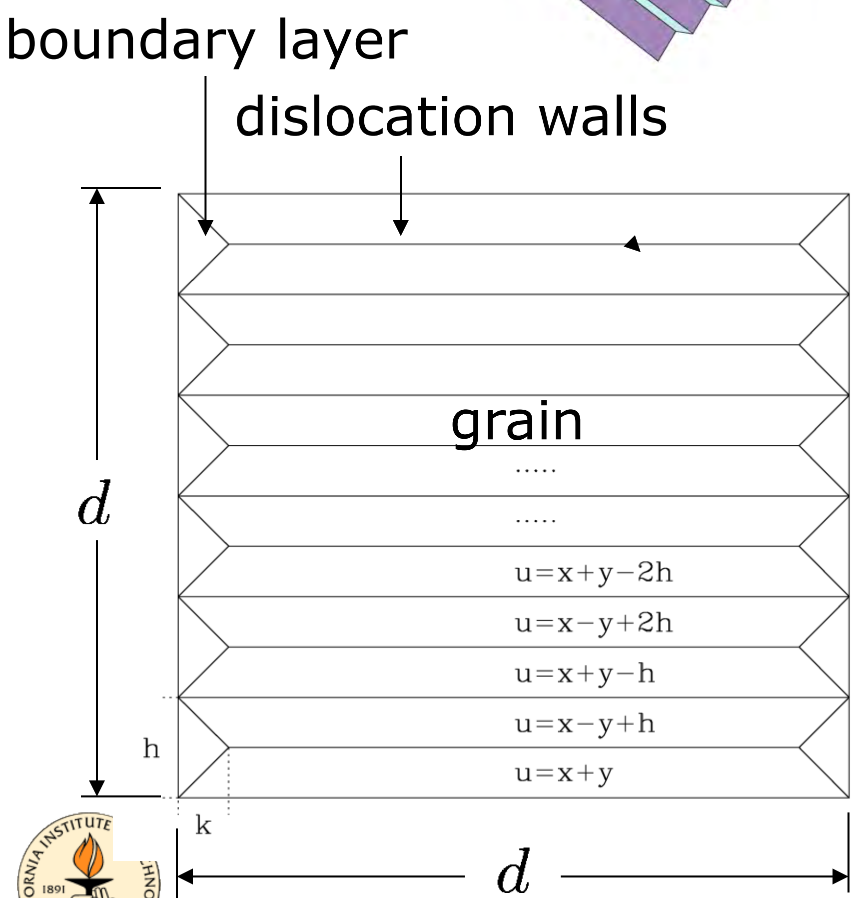
$$\min\left\{1, \frac{\mu}{G}, \frac{\tau_0}{G\gamma} + \left(\frac{\mu}{G}\right)^{1/2}\left(\frac{T}{G\gamma bd}\right)^{1/2}, \frac{\tau_0}{G\gamma} + \left(\frac{T}{G\gamma bd}\right)^{2/3}\right\}$$

- Upper bounds determined by construction
- Lower bounds: Rigidity estimates, ansatz-free lower bound inequalities (Kohn and Müller '92, '94; Conti '00)

S. Conti and M. Ortiz, *ARMA*, **176** (2005), pp. 103–147

Optimal scaling – Laminate construction





- Energy:

$$W \equiv \frac{E_0}{d^3} \sim \tau_0 \gamma + \left(\frac{\mu T \gamma^3}{bd} \right)^{1/2}$$

- Yield stress:

$$\tau \equiv \frac{\partial W}{\partial \gamma} \sim \tau_0 + \frac{1}{2} \left(\frac{\mu T \gamma}{bd} \right)^{1/2}$$

parabolic hardening +

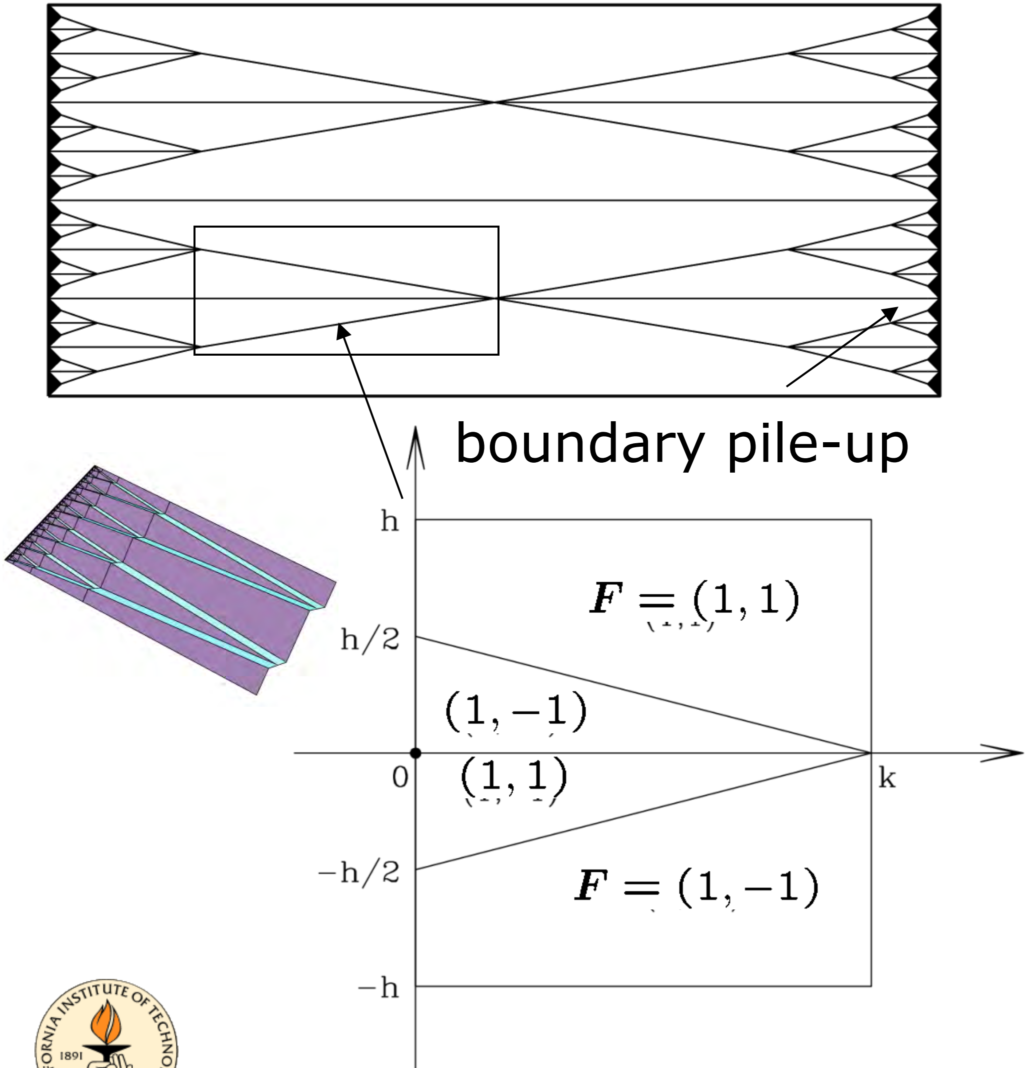
Hall-Petch scaling

- Lamellar width:

$$l \sim \left(\frac{\mu T d}{\mu \gamma b} \right)^{1/2}$$

S. Conti and M. Ortiz, *ARMA*, **176** (2005), pp. 103–147

Optimal scaling – Branching construction



- Energy:

$$W \sim \tau_0 \gamma + G \left(\frac{T\gamma^2}{Gbd} \right)^{2/3}$$

- Yield stress:

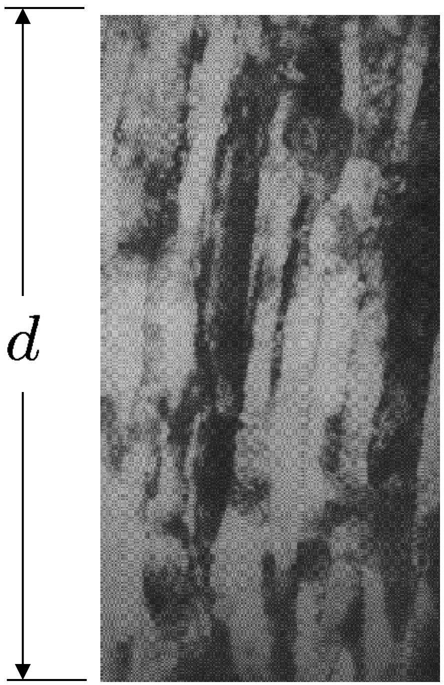
$$\tau \sim \tau_0 + \left(\frac{T}{bd} \right)^{2/3} (G\gamma)^{1/3}$$

- Microstructure size:

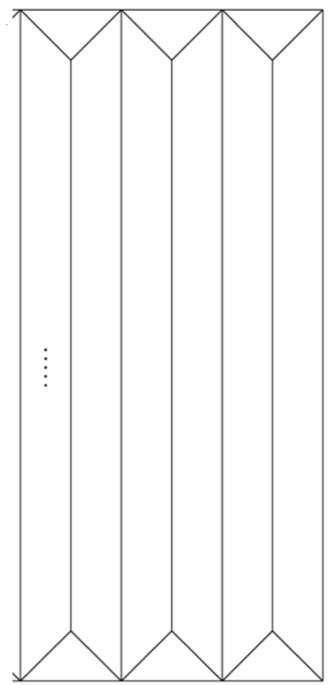
$$l \sim \left(\frac{Td^2}{G\gamma b} \right)^{1/3}$$

S. Conti and M. Ortiz, *ARMA*, **176** (2005), pp. 103–147

Optimal scaling – Microstructures

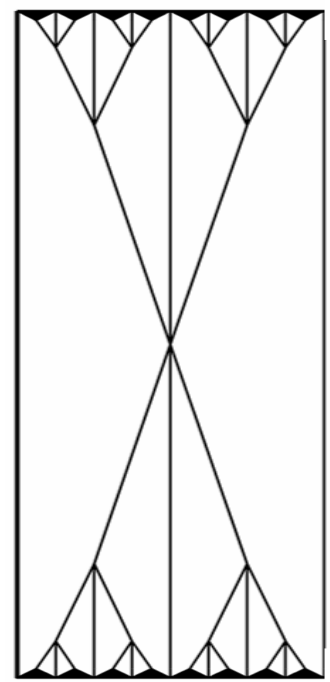


Shocked Ta
(Meyers et al ´95)



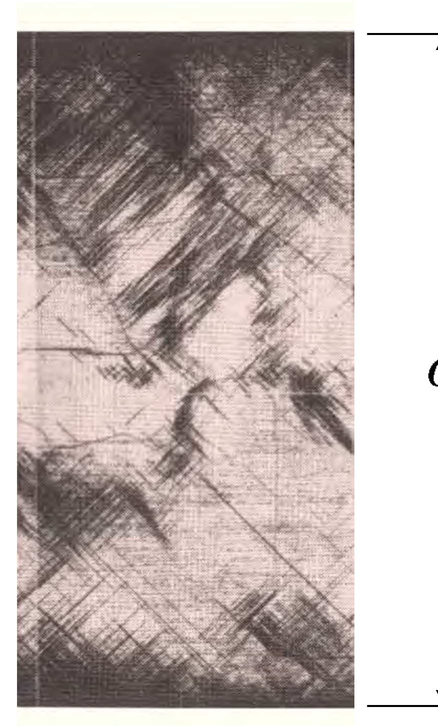
Laminate

$\tau \sim d^{-1/2}$



Branching

$\tau \sim d^{-2/3}$

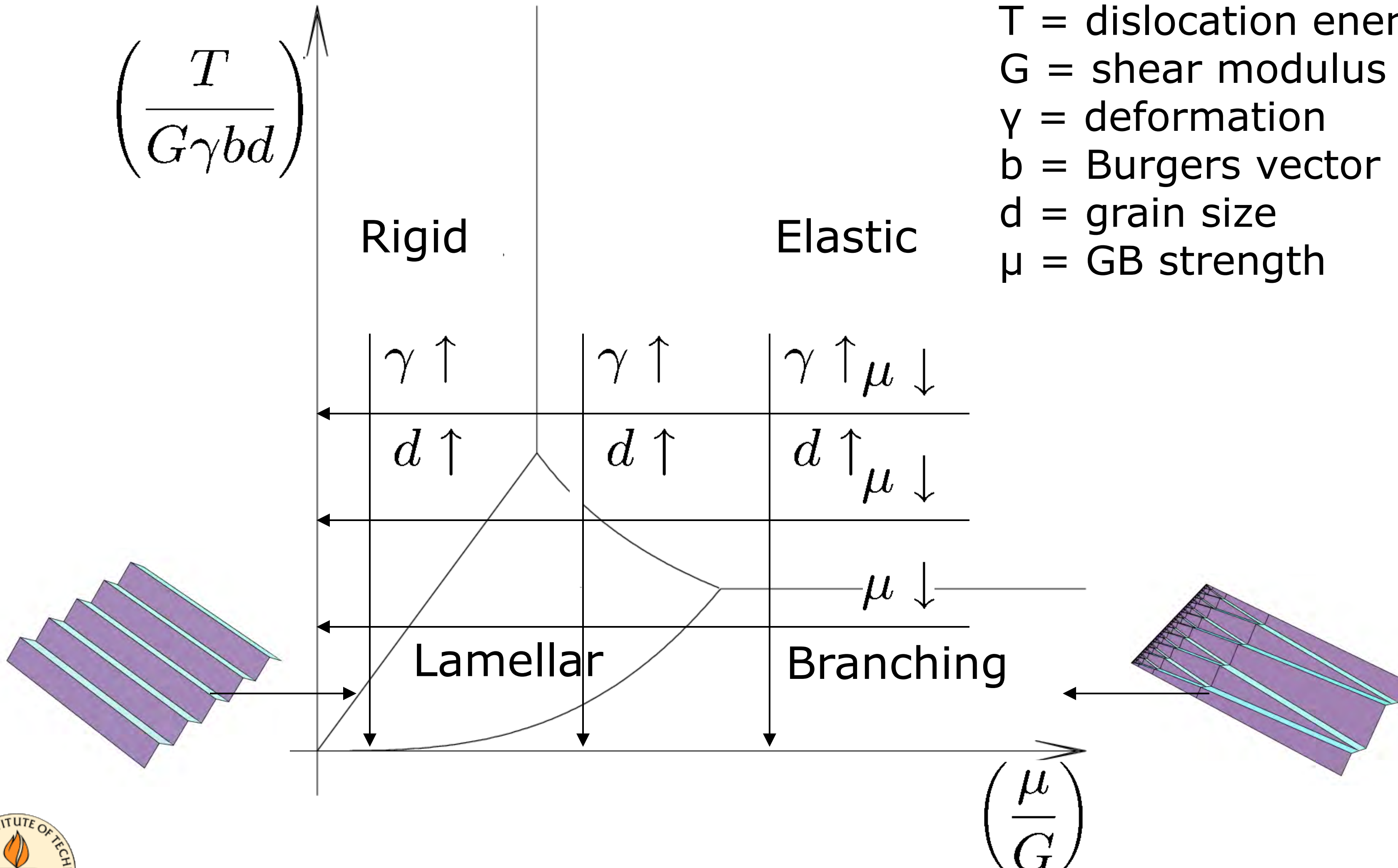


LiF impact
(Meir and Clifton ´86)

Dislocation structures corresponding to the lamination and branching constructions

S. Conti and M. Ortiz, *ARMA*, **176** (2005), pp. 103–147

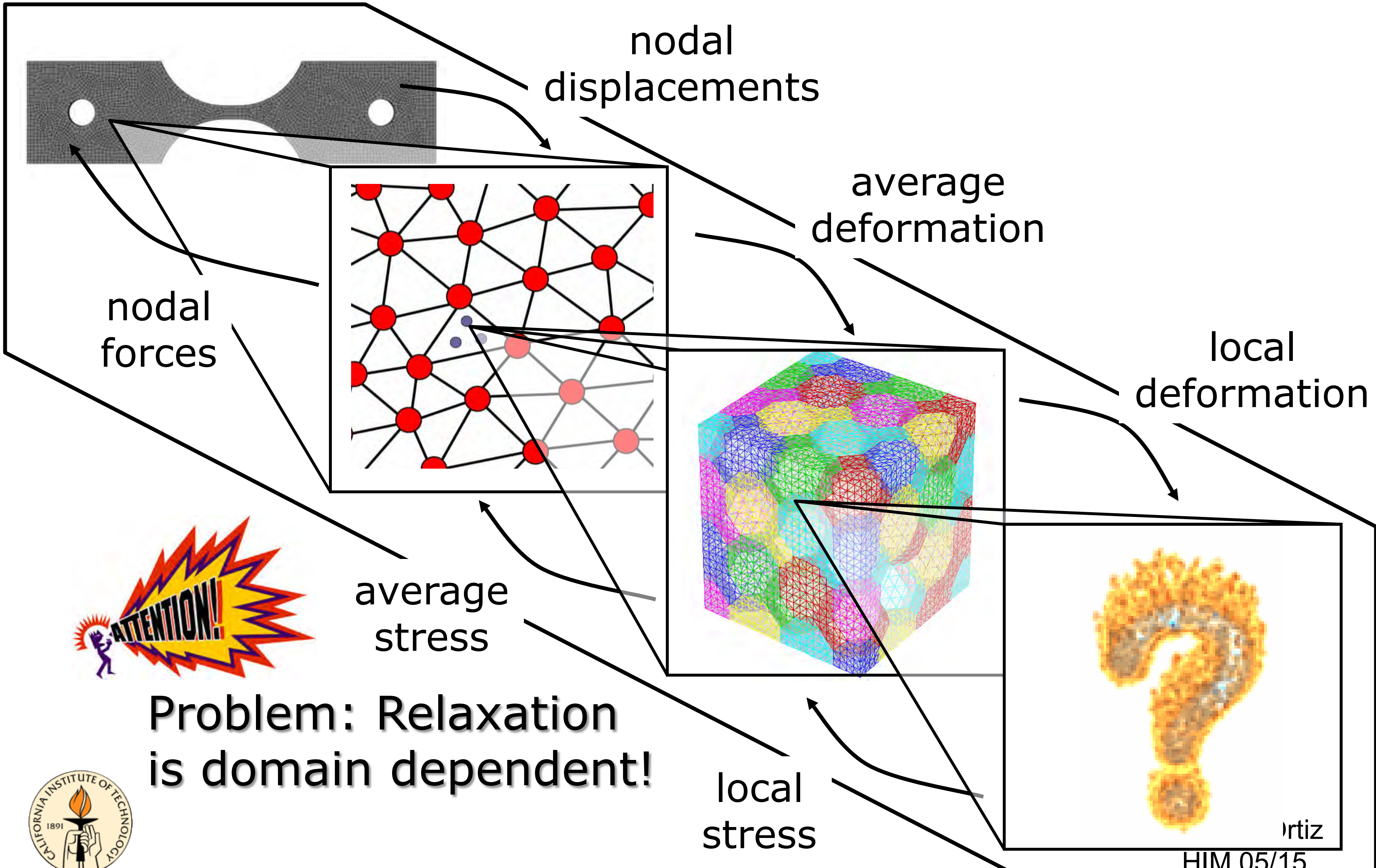
Optimal scaling – Phase diagram



T = dislocation energy
G = shear modulus
γ = deformation
b = Burgers vector
d = grain size
μ = GB strength

S. Conti and M. Ortiz, *ARMA*, **176** (2005), pp. 103–147

Polycrystals – Concurrent multiscale (C^3)



Problem: Relaxation is domain dependent!

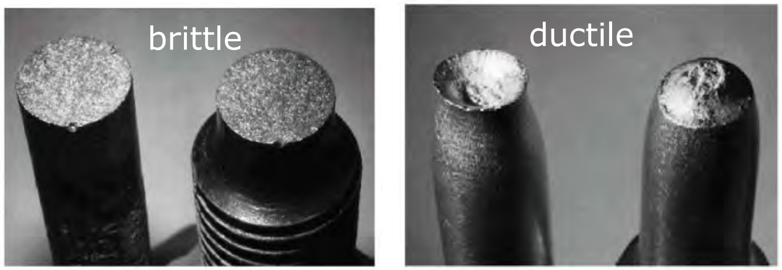
Pitfalls

'Standard program' mail fail due to:

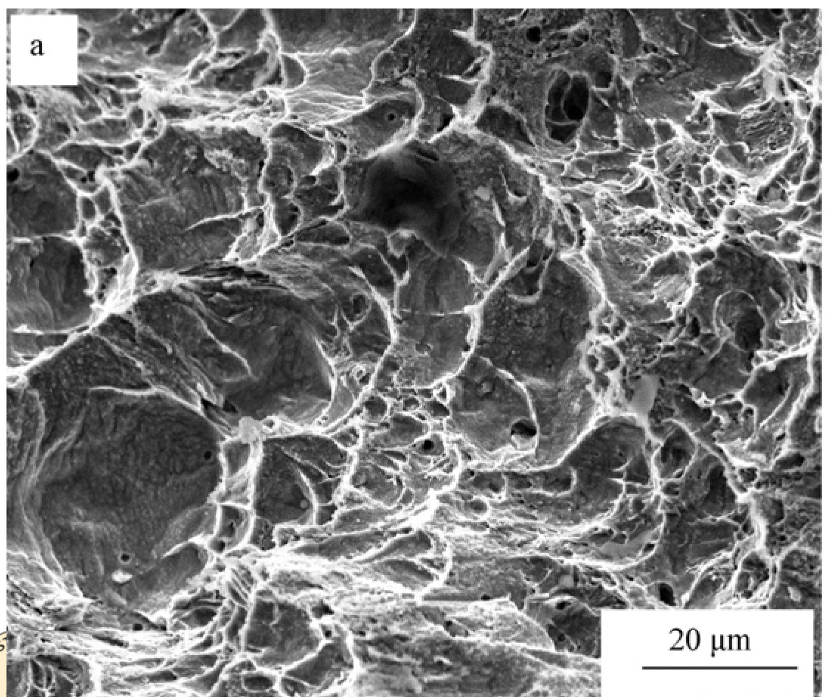
Non-proportional loading (unloading, cycling loading, change of loading path direction) leading to microstructure evolution

Departures from volume scaling (size effect, domain dependence, localization) leading to failure of homogeneization and relaxation

Localization – Fracture scaling



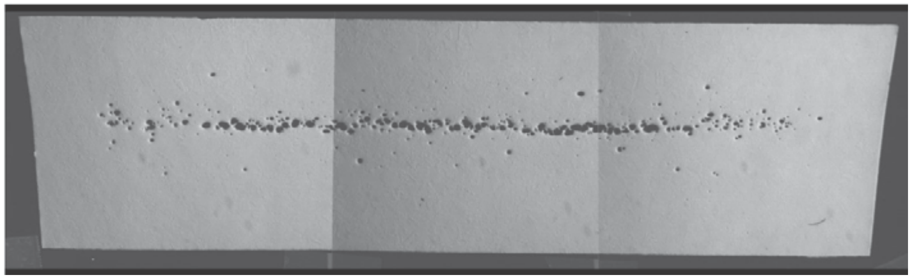
(Courtesy NSW HSC online)



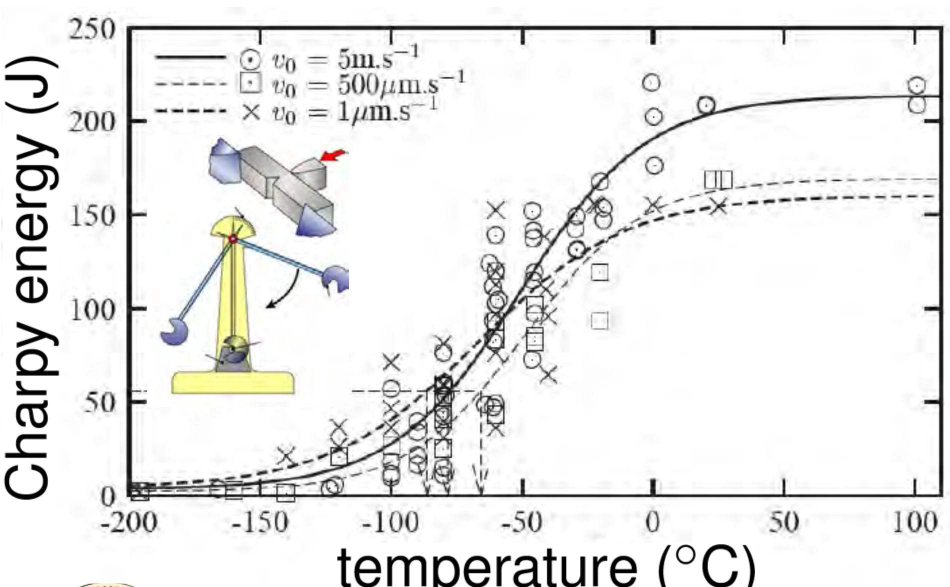
- Ductile fracture in metals occurs by *void nucleation, growth and coalescence*
- Fractography of ductile-fracture surfaces exhibits profuse *dimpling*, vestige of microvoids
- Ductile fracture entails large amounts of *plastic deformation* (*vs*. surface energy) and dissipation.

Fracture surface in SA333 steel, room temp., $d\varepsilon/dt=3\times10^{-3}s^{-1}$ (S.V. Kamata, M. Srinivasa and P.R. Rao, Mater. Sci. Engr. A, ***528*** *(2011) 4141–4146)*

Localization – Fracture scaling



Void sheet in copper disk[1]

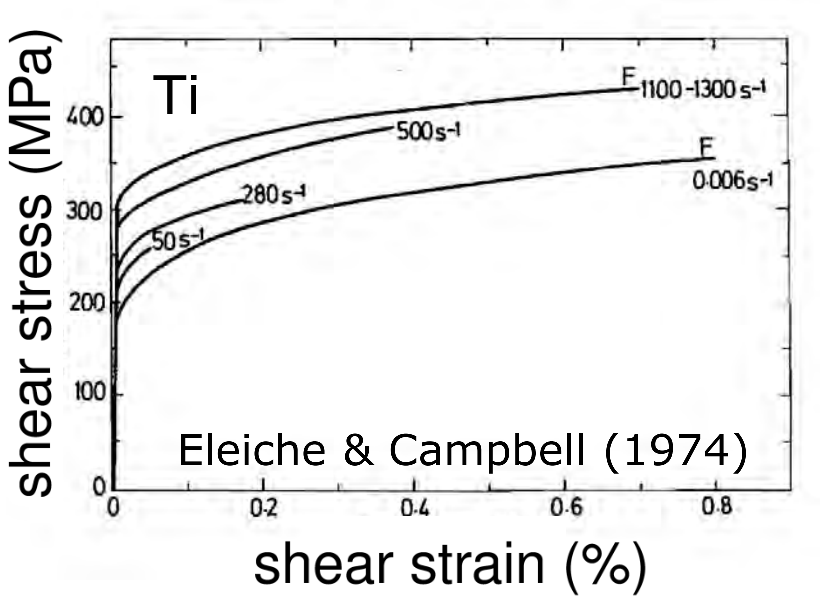


Charpy energy of A508 steel[2]

- Fracture energy scales with crack area: $E \sim L^2$
- A number of ASTM engineering standards are in place to characterize ductile fracture properties (J-testing, Charpy test)
- In general, the specific fracture energy for ductile fracture is greatly in excess of that required for brittle fracture…

[1]Heller, A., Science & Technology, LLNL, pp. 13-20, July/August, 2002
[2]Tanguy *et al.*, *Eng. Frac. Mechanics*, 2005

Naïve model: Local plasticity

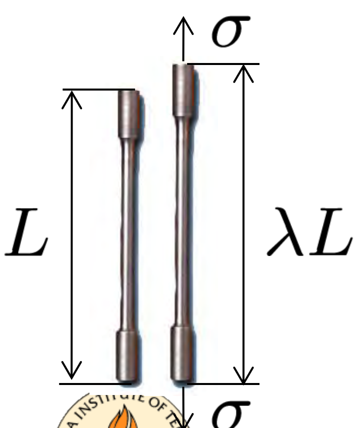


- Deformation theory: Minimize

$$E(y) = \int_\Omega W\Big(Dy(x)\Big)\,dx$$

- Growth of $W(F)$?
- Asume power-law hardening:

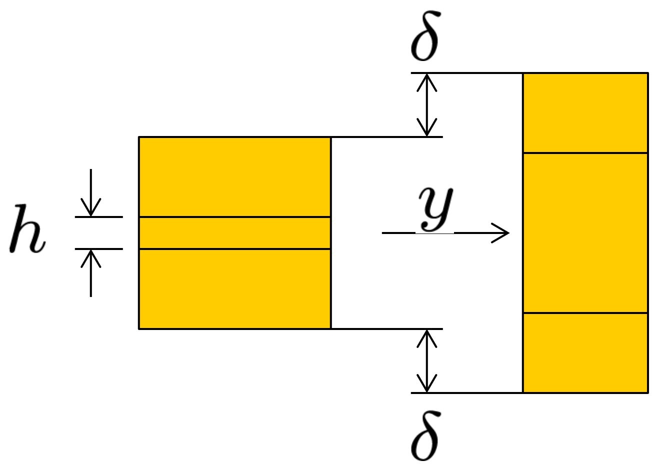
$$\sigma \sim K\epsilon^n = K(\lambda - 1)^n$$



- Nominal stress: $\partial_\lambda W = \sigma/\lambda = K(\lambda-1)^n/\lambda$
- For large λ: $\partial_\lambda W \sim K\lambda^{n-1} \Rightarrow W \sim K\lambda^n$
- In general: $W(F) \sim |F|^p,\ p = n \in (0,1)$

$\Rightarrow$ Sublinear growth!

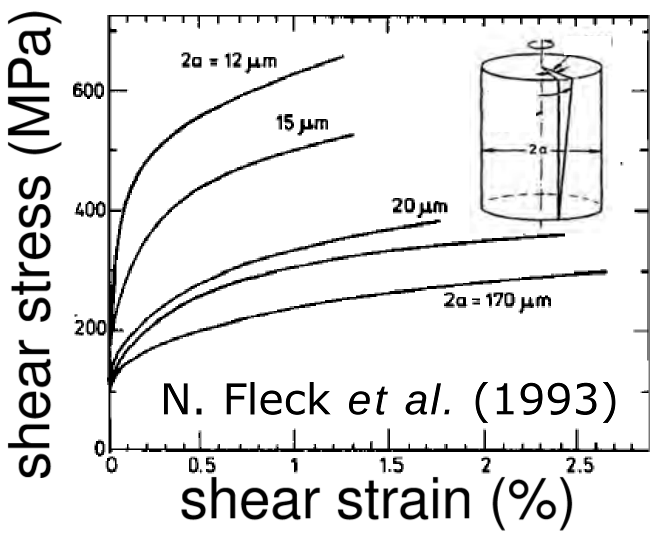
Naïve model: Local plasticity

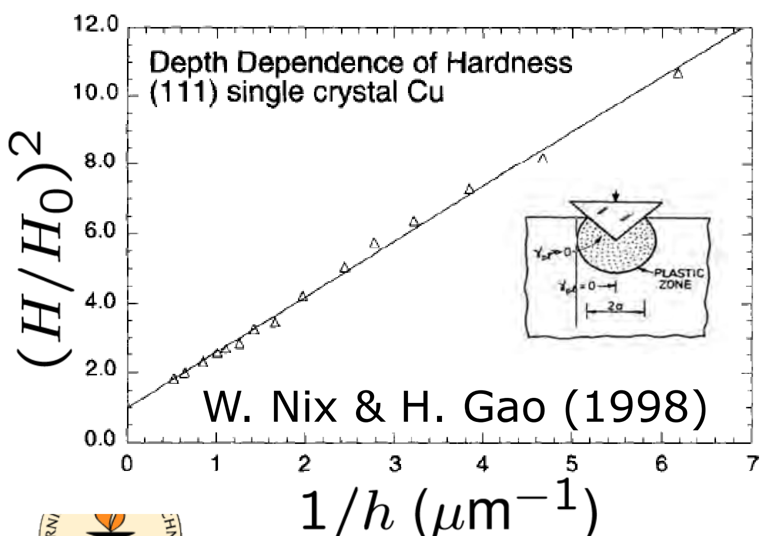


- Example: Uniaxial extension
- Energy: $E_h \sim h \left(\frac{2\delta}{h}\right)^p$
- For $p < 1$: $\lim_{h \to 0} E_h = 0$

- Energies with sublinear growth relax to 0.
- *For hardening exponents in the range of experimental observation, local plasticity yields no useful information regarding ductile fracture properties of materials!*
- Need additional physics, structure…

Strain-gradient plasticity





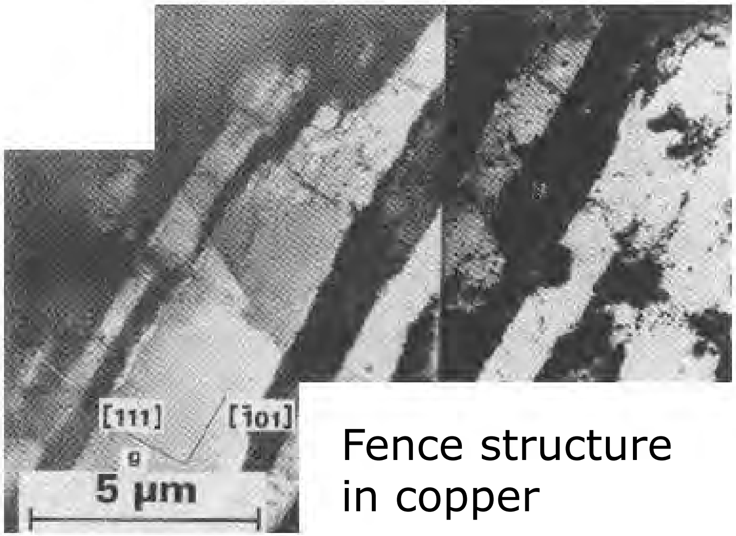
- The yield stress of metals is observed to increase in the presence of strain gradients
- Deformation theory of strain-gradient plasticity:

$$E(y) = \int_\Omega W\Big(Dy(x), D^2y(x)\Big)\,dx$$

$y : \Omega \to \mathbb{R}^n$, volume preserving

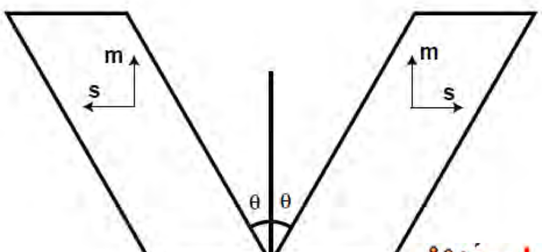
- Strain-gradient effects may be expected to oppose localization
- Question: Can fracture scaling be understood as the result of strain-gradient plasticity?

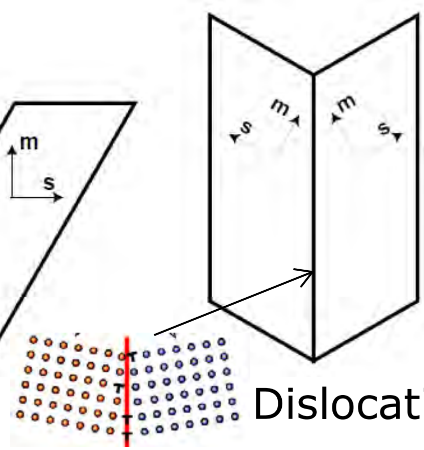
Strain-gradient plasticity



Fence structure in copper

(J.W. Steeds, *Proc. Roy. Soc. London*, **A292**, 1966, p. 343)





Dislocation wall

- Growth of $W(F,\cdot)$?
- For fence structure:

$$F^{\pm} = R^{\pm}(I \pm \tan\theta\, s \otimes m)$$

- Across jump planes:

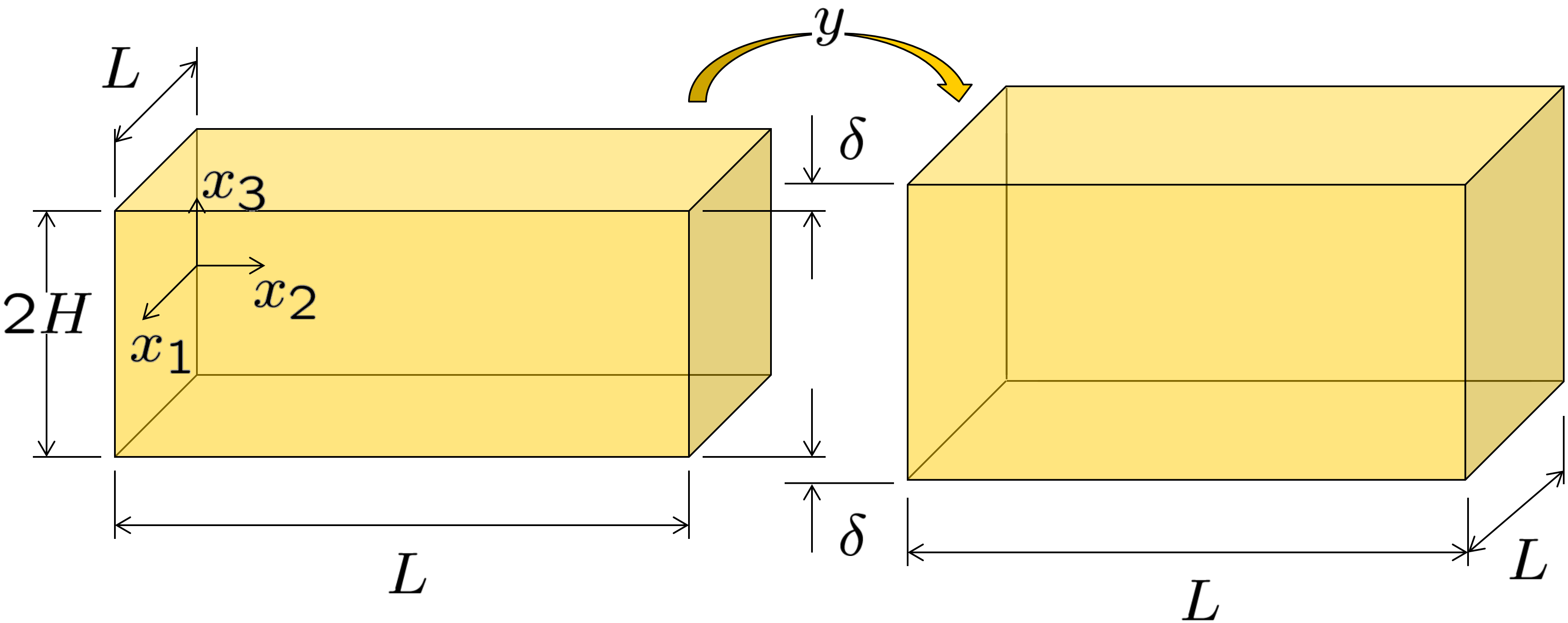
$$|[\![F]\!]| = 2\sin\theta$$

- Dislocation-wall energy:

$$E = \frac{T}{b} 2\sin\theta = \frac{T}{b}|[\![F]\!]|$$

$\Rightarrow W(F,\cdot)$ has linear growth!

Optimal scaling – Ductile fracture



- Approach: Optimal scaling
- Slab: $\Omega = [0, L]^2 \times [-H, H]$, periodic
- Uniaxial extension: $y_3(x_1, x_2, \pm H) = x_3 \pm \delta$

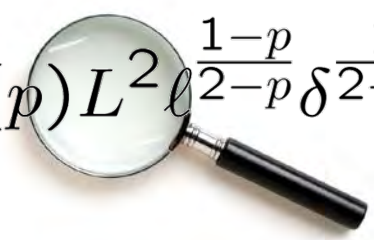
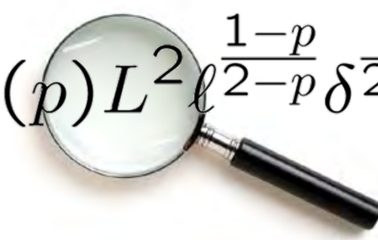
Optimal scaling – Ductile fracture

- $y : \Omega \to \mathbb{R}^3$, $[0, L]^2$-periodic, volume preserving
- $y \in W^{1,1}(\Omega; \mathbb{R}^3)$, $Dy \in BV(\Omega; \mathbb{R}^{3\times 3})$
- Growth: For $p \in (0, 1)$, *intrinsic length* $\ell > 0$,

$$E(y) \sim K \left(\int_\Omega (|Dy|^p - 3^{p/2})\, dx + \ell \int_\Omega |D^2 y|\, dx \right)$$

Theorem [Fokoua, Conti & MO, ARMA, 2013]. For ℓ sufficiently small, $p \in (0, 1)$, $0 < C_L(p) < C_U(p)$,

$$C_L(p) L^2 \ell^{\frac{1-p}{2-p}} \delta^{\frac{1}{2-p}} \leq \inf E \leq C_U(p) L^2 \ell^{\frac{1-p}{2-p}} \delta^{\frac{1}{2-p}}$$

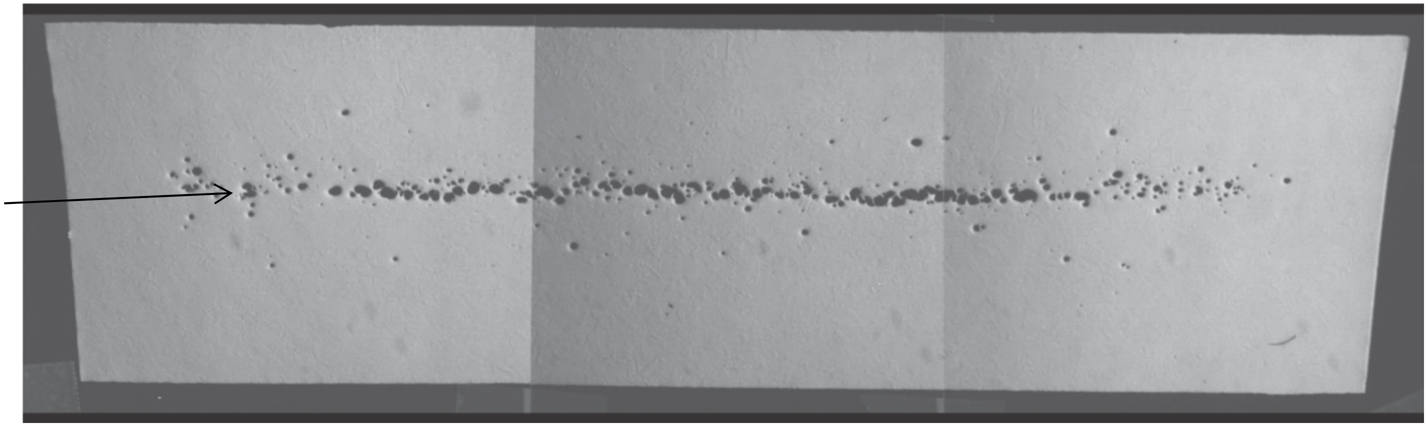



Fracture!

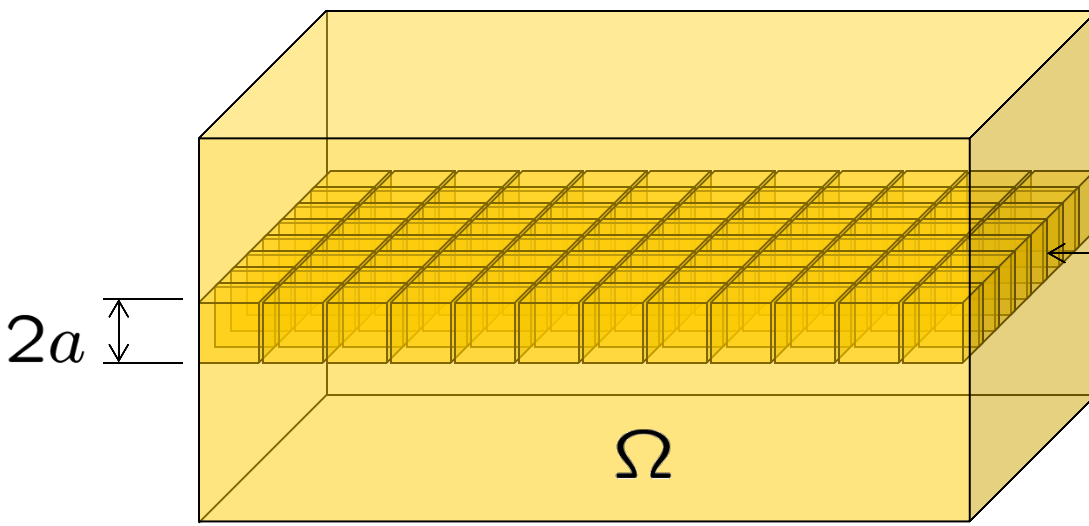
L. Fokoua, S. Conti and M. Ortiz, *ARMA*, **212** (2014) pp. 331-357.

Optimal scaling – Upper bound

void
sheet



Heller, A., Science & Technology Review Magazine,
LLNL, pp. 13-20, July/August, 2002

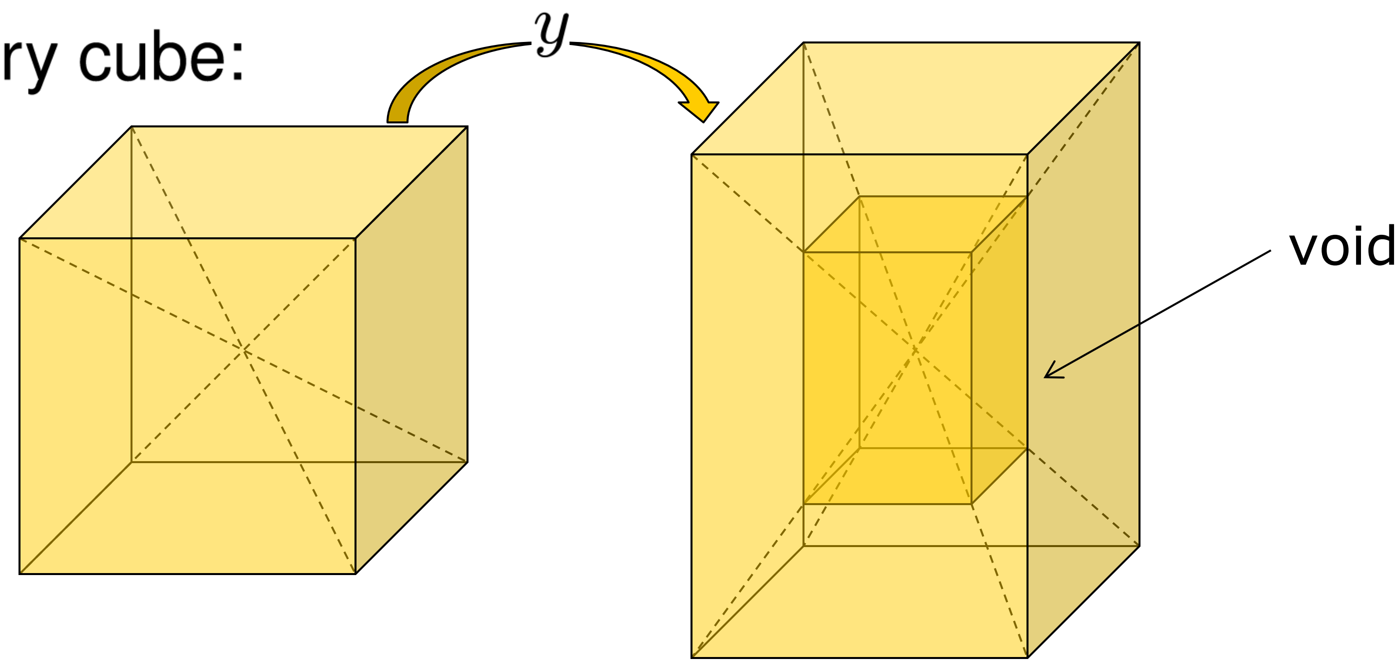


void
sheet

L. Fokoua, S. Conti and M. Ortiz, *ARMA*, **212** (2014) pp. 331-357.

Optimal scaling – Upper bound

- In every cube:

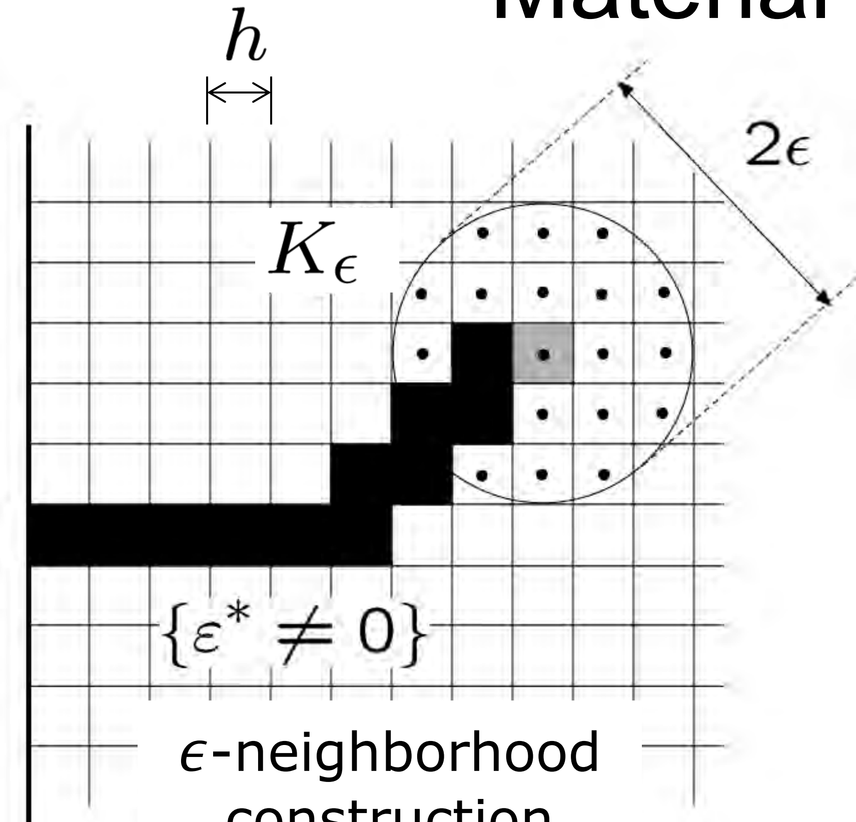


- Calculate, estimate: $E \leq CL^2 \left(a^{1-p}\delta^p + \ell\delta/a \right)$
- Optimize: $a = (\ell\delta^{1-p})^{1/(2-p)} \Rightarrow E \leq C_U L^2 \ell^{\frac{1-p}{2-p}} \delta^{\frac{1}{2-p}}$

void growth! fracture!

L. Fokoua, S. Conti and M. Ortiz, *ARMA*, **212** (2014) pp. 331-357.

Numerical implementation
Material-point erosion



ϵ-neighborhood construction

- ϵ-neighborhood construction: Choose $h \ll \epsilon \ll L$
- Erode material point if

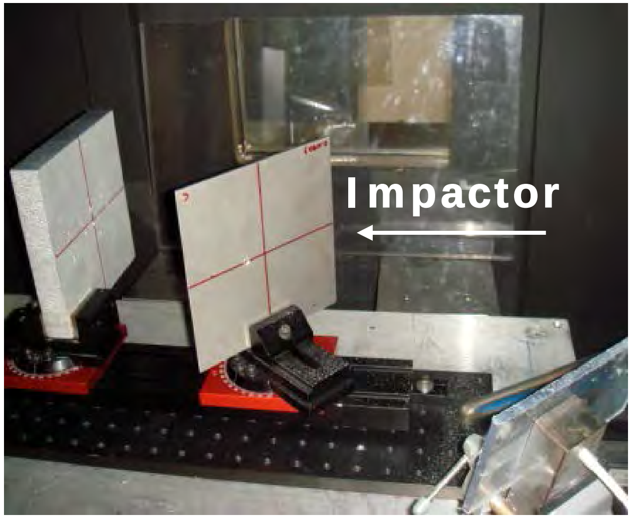
$$\frac{h^2}{|K_\epsilon|}\int_{K_\epsilon} W(\nabla u)\, dx \geq J_c$$

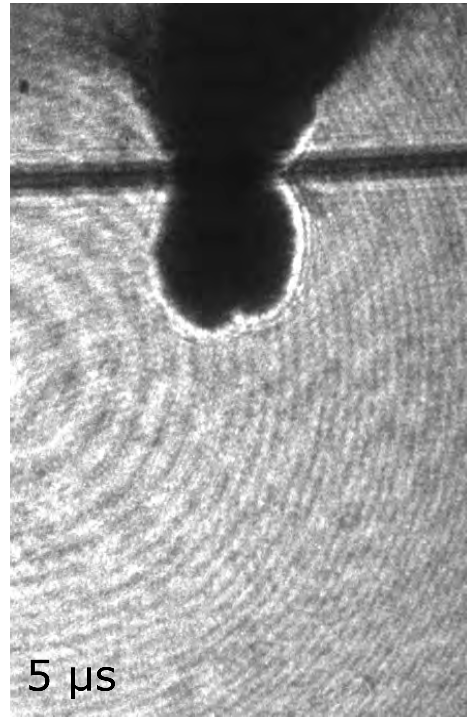
- For linear elasticity, proof of Γ-convergence to Griffith fracture

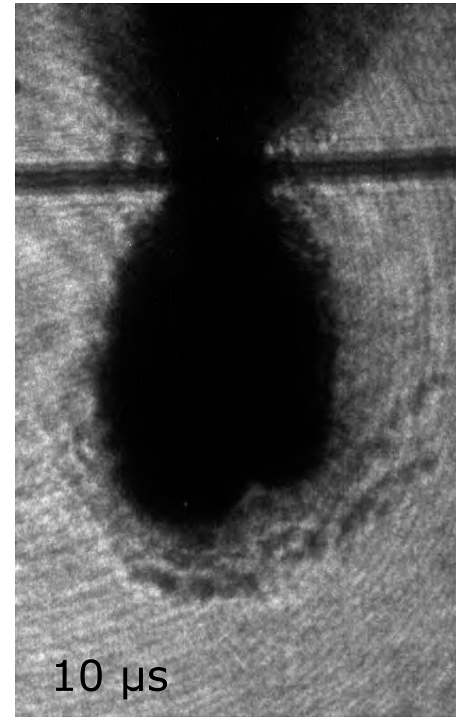
Theorem[1]: Suppose $\epsilon = \epsilon(h)$ and $h/\epsilon(h) \to 0$ as $h \to 0$. Then, $\Gamma - \lim_{h\to 0} E_{h,\epsilon(h)} =$ Griffith energy

[1]Schmidt, B., *et al.*, *SIAM Multi. Model.*, **7** (2009) 1237.

Application to hypervelocity impact



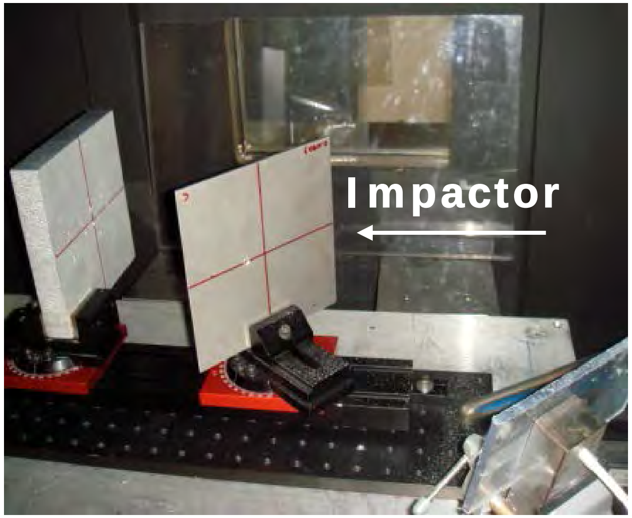


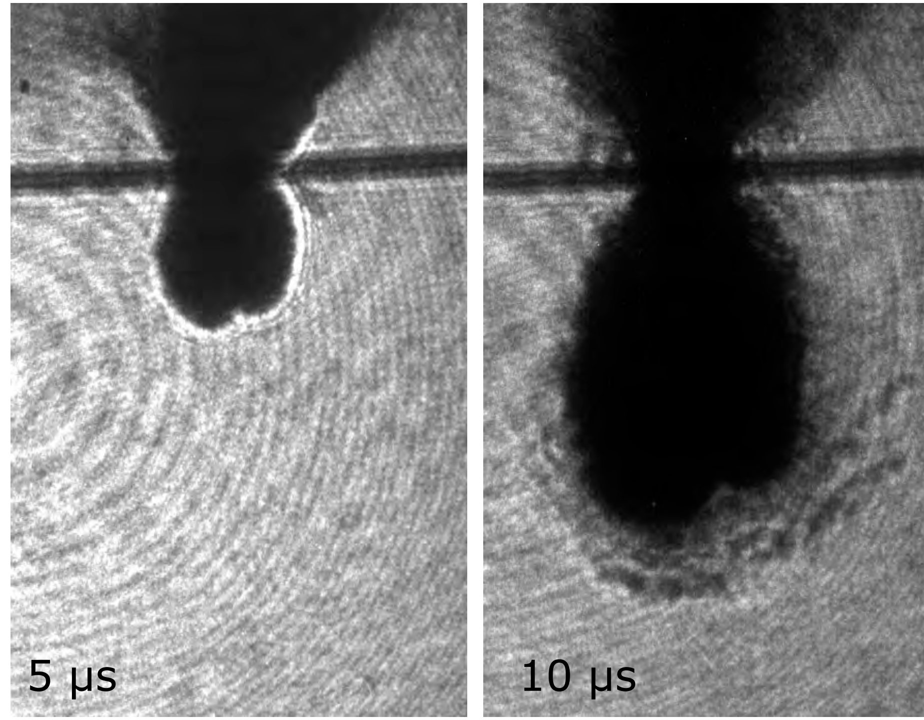


Hypervelocity impact (5.7 Km/s) of 0.96 mm thick aluminum plates by 5.5 mg nylon 6/6 cylinders (Caltech)

Pandolfi, A. & Ortiz, M. , *IJNME*, **92** *(*2012) 694.
Pandolfi, A., Li, B. & Ortiz, M. , *Int. J. Fract.*, **184** *(*2013) 3.
Li, B., Stalzer, M. & Ortiz, M., *IJNME*, **100** (2014) 40.

Application to hypervelocity impact





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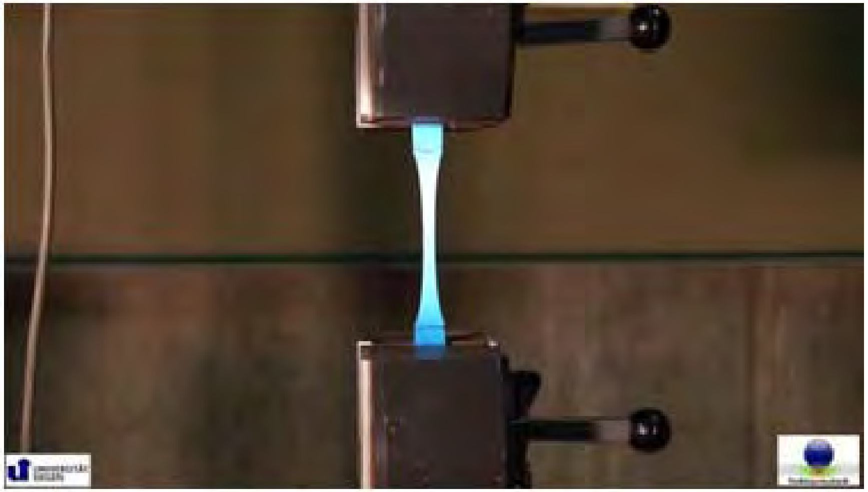
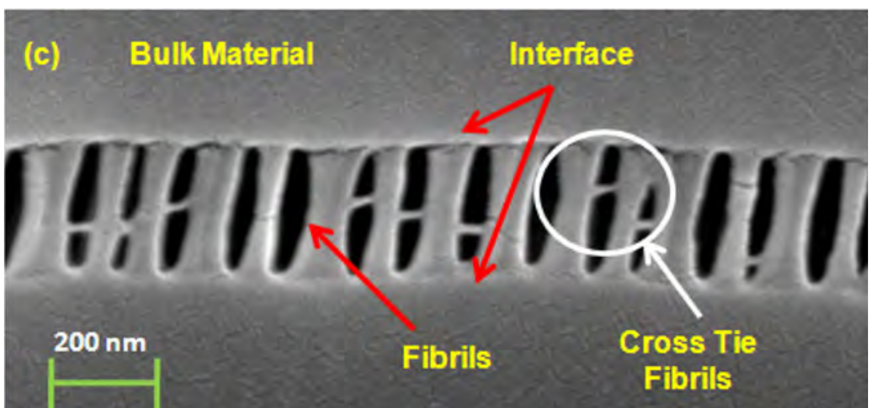
Fracture of polymers



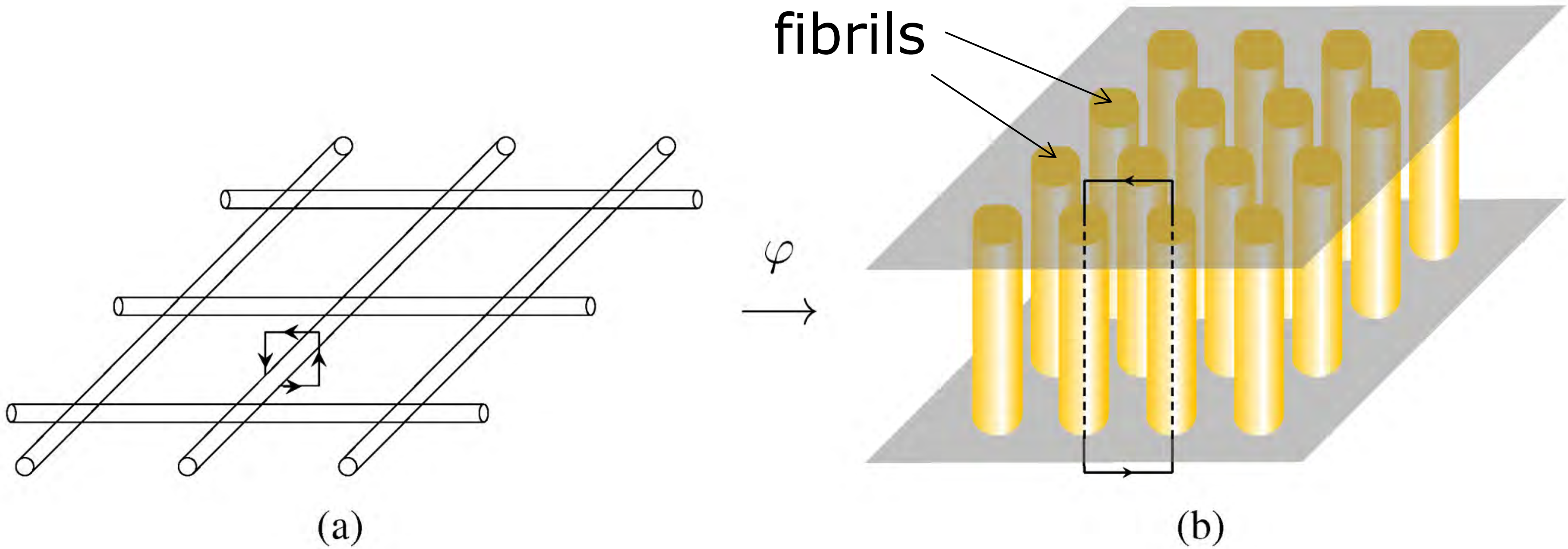
T. Reppel, T. Dally, T. and K. Weinberg, Technische Mechanik, 33 (2012) 19-33.



Crazing in 800 nm polystyrene thin film (C. K. Desai *et al*., 2011)

- Polymers undergo entropic elasticity and damage due to chain stretching and failure
- Polymers fracture by means of the crazing mechanism consisting of fibril nucleation, stretching and failure
- The free energy density of polymers saturates in tension once the majority of chains are failed: p=0!
- Crazing mechanism is incompatible with strain-gradient elasticity…

Fracture of polymers - Topology



Formation of fibers from solid polymer entails a topological transition

S. Heyden *el al.*, *JMPS*, **74** (2015) 175.

Fracture of polymers

- Suppose: For $K_U > 0$, *intrinsic length* $\ell > 0$,

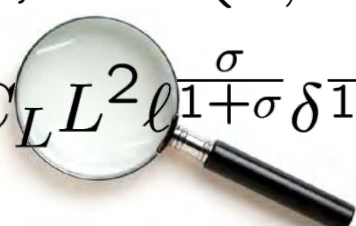
$$E(y) \leq K_U \left(\int_\Omega (|Dy|^p - 3^{p/2})\, dx + \ell \int_\Omega |D^2 y|\, dx \right)$$

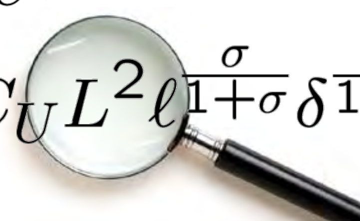
- If $E(y) < +\infty \Rightarrow y$ continuous on a.e. plane!
- Crazing is precluded by the continuity of y!
- Instead suppose: For $\sigma \in (0,1)$,

$$E(y) \leq K_U \left(\int_\Omega (|Dy|^p - 3^{p/2})\, dx + \ell^\sigma |y|_{W^{1+\sigma,1}(\Omega)} \right)$$

Theorem [Conti & MO, ARMA]. For ℓ sufficiently small, $p = 0$, $\sigma \in (0,1)$, $0 < C_L < C_U$,

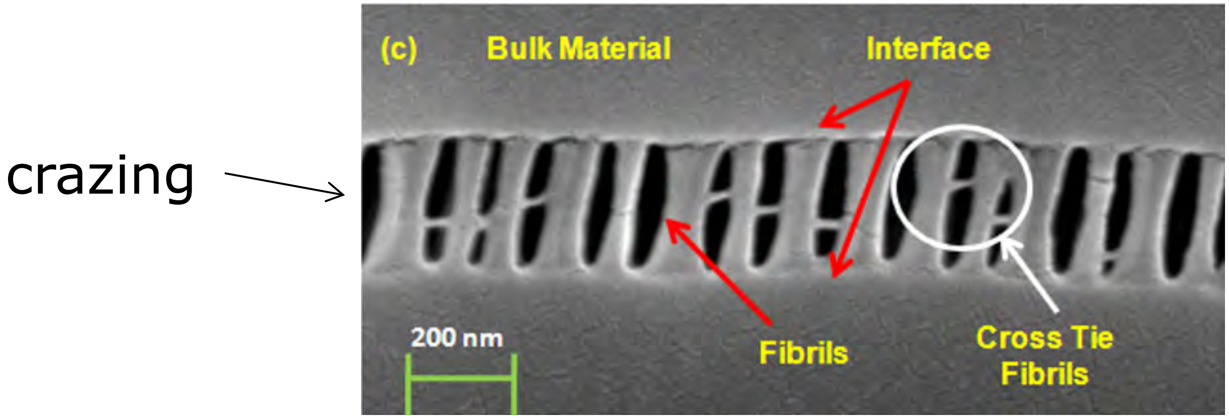
$$C_L L^2 \ell^{\frac{\sigma}{1+\sigma}} \delta^{\frac{1}{1+\sigma}} \leq \inf E \leq C_U L^2 \ell^{\frac{\sigma}{1+\sigma}} \delta^{\frac{1}{1+\sigma}}$$





Ortiz, *ARMA* (submitt).

Optimal scaling – Upper bound



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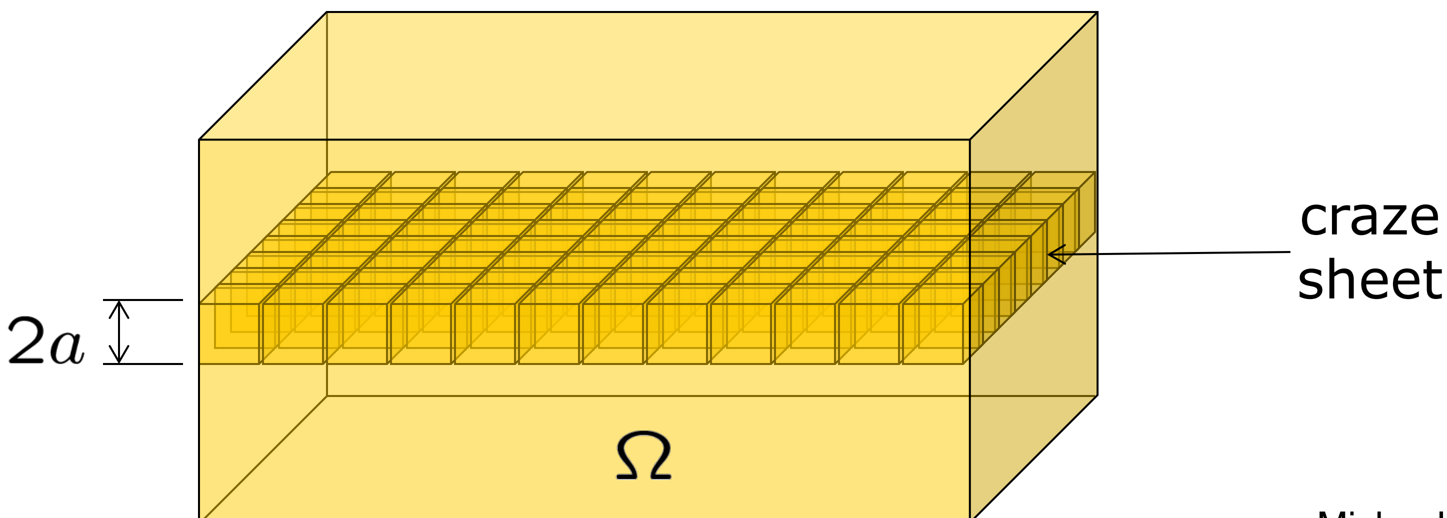
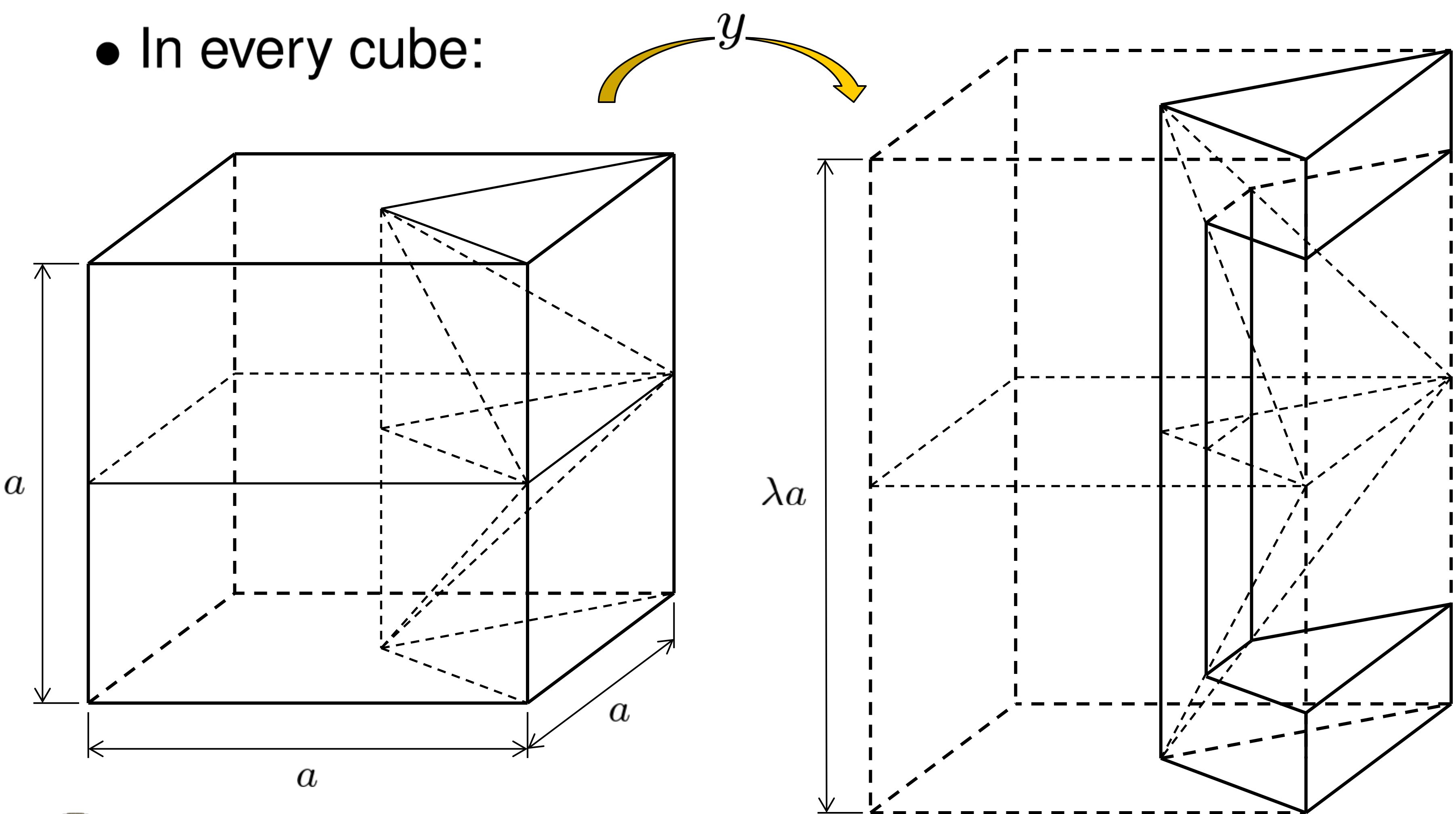
thin film (C. K. Desai *et al*., 2011)



S. Conti and M. Ortiz, *ARMA* (submitted for publication).

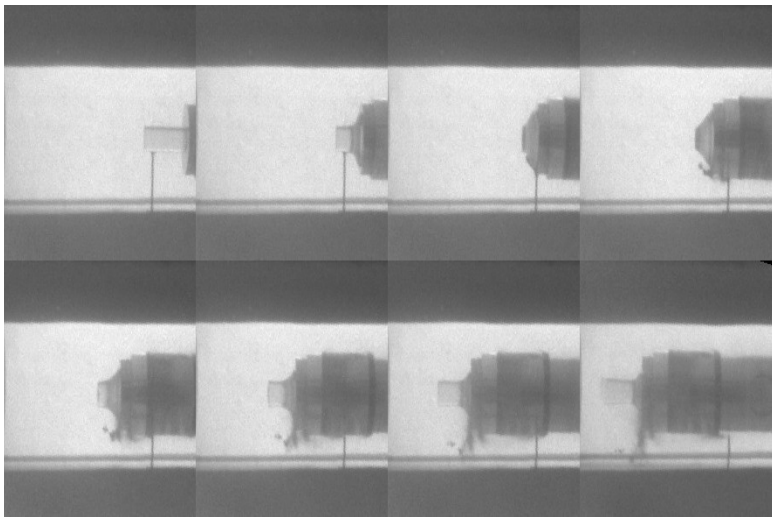
Optimal scaling – Upper bound

- In every cube:

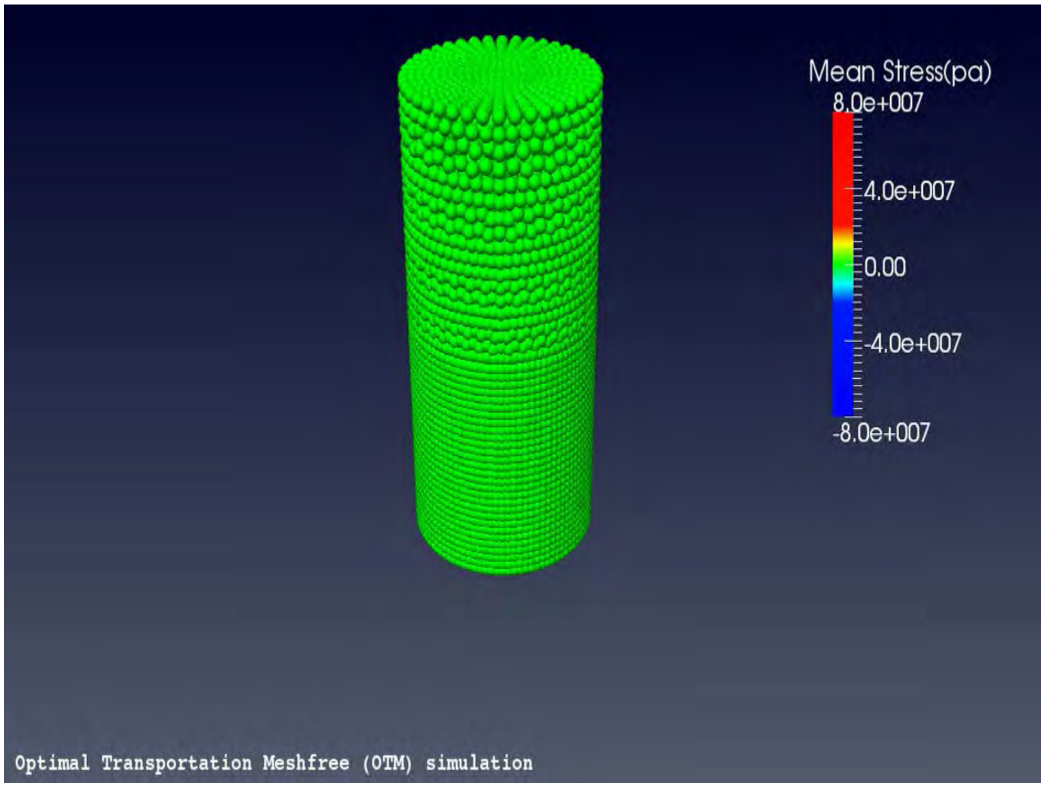


S. Conti and M. Ortiz, *ARMA* (submitted for publication).

Taylor-anvil tests on polyurea



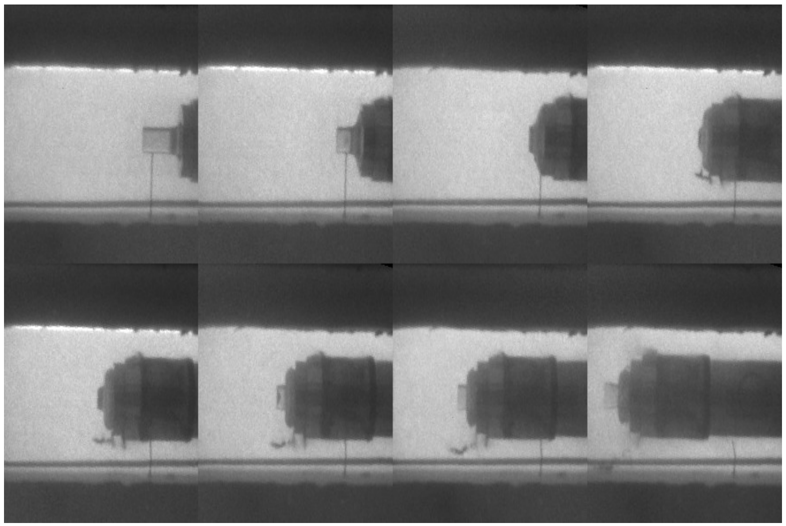
Shot #854:
R0 = 6.3075 mm,
L0 = 27.6897 mm,
v = 332 m/s



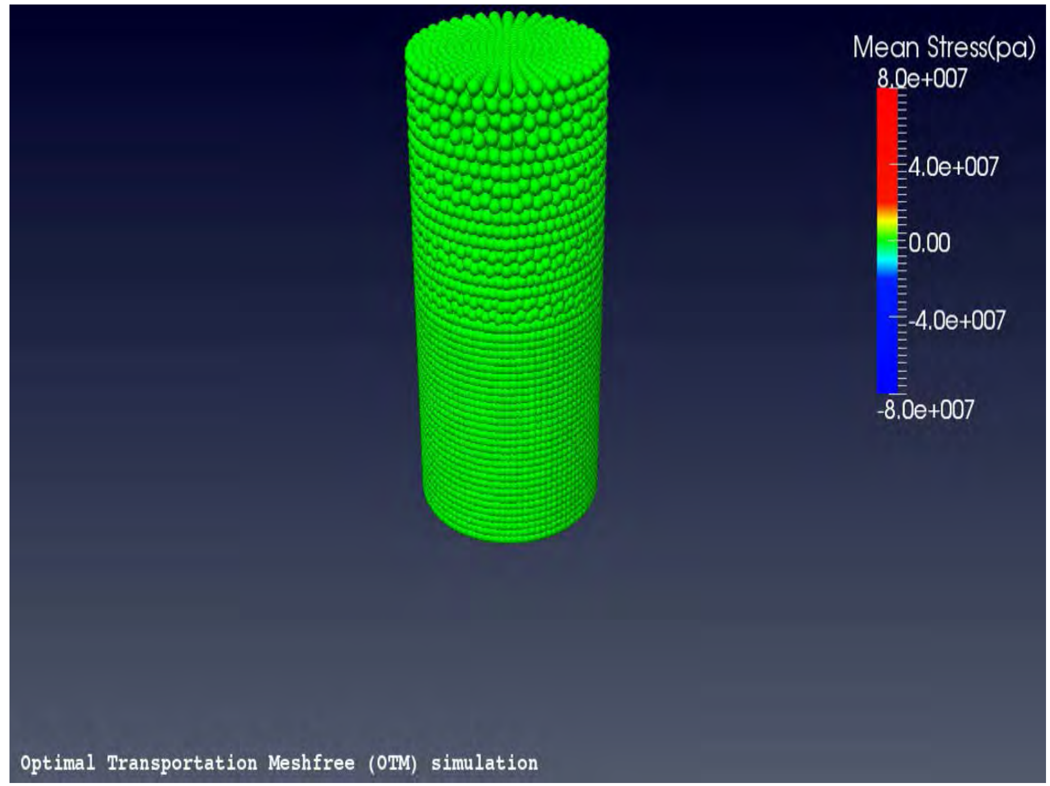
Experiments conducted by W. Mock, Jr. and J. Drotar,
at the Naval Surface Warfare Center (Dahlgren Division)
Research Gas Gun Facility, Dahlgren, VA 22448-5100, USA

S. Heyden *el al.*, *JMPS*, **74** (2015) 175.

Experiments and simulations



Shot #861:
R0 = 6.3039 mm,
L0 = 27.1698 mm,
v = 424 m/s



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S. Heyden *el al*., *JMPS*, **74** (2015) 175.

Concluding remarks

Can analysis shed light on the experimental record? (e.g., can some of the observed microstructures be understood as energy minimizers?)

Can analysis inform modeling and simulation? (e.g., homogeneization, multiscale modeling, relaxation, acceleration…)

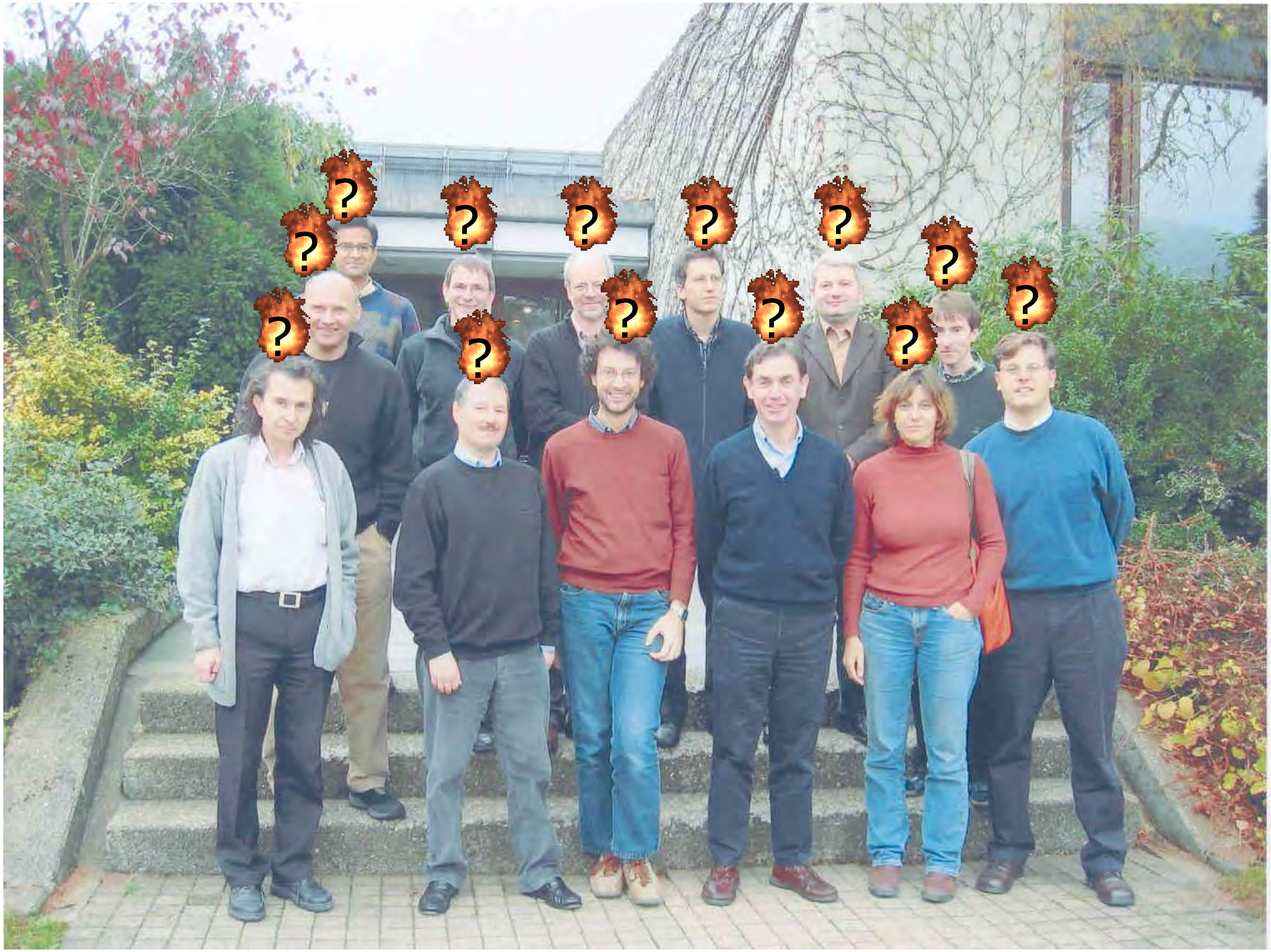
Ten Years Later…

The well-understood setting:

Rate-independent, proportional loading and local behavior (deformation theory of plasticity + relaxation)

Still open:

Rate-dependent, non-proportional loading and non-local or localized behavior



The Apotheosis of the program: Mathematicians and engineers still puzzle over microplasticity…

Thanks!