Distributed Damage and Enhanced Permeability in Confined Brittle Materials under Triaxial Compression

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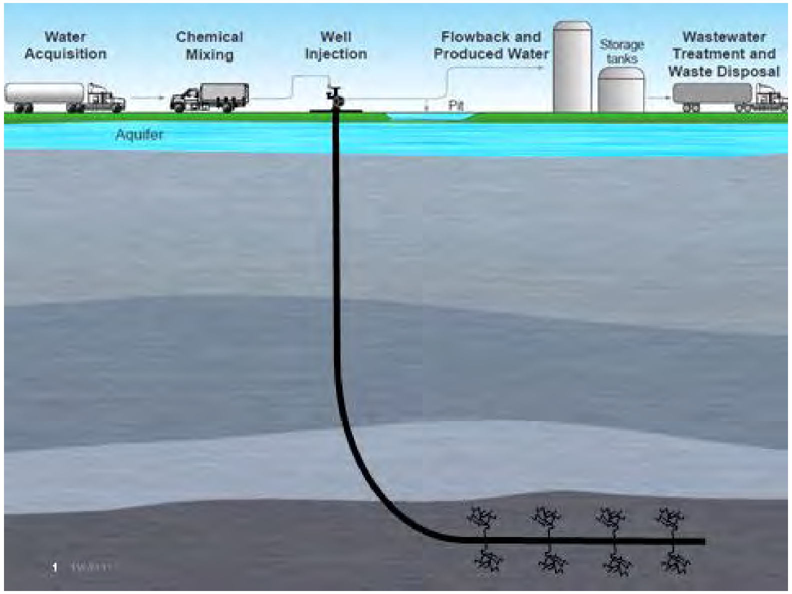
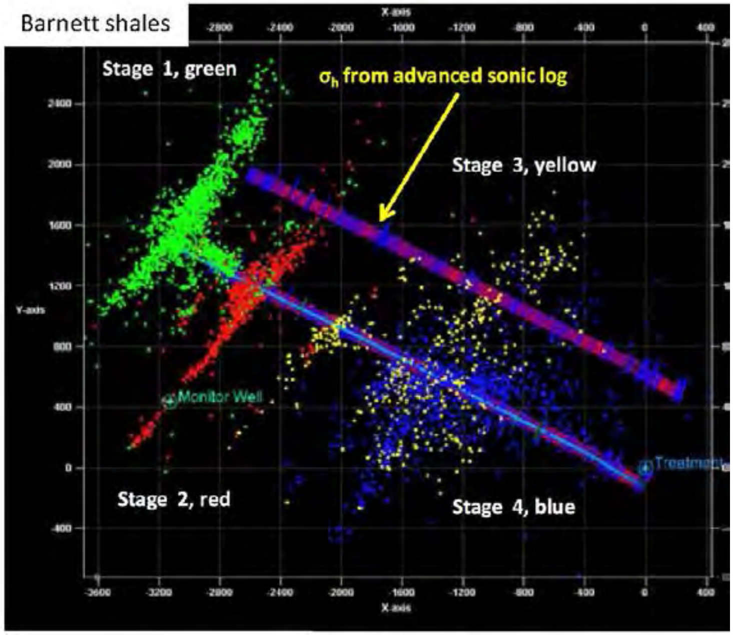
Motivation: Hydraulic Fracture (HF)

Hydraulic fracture: Example of extreme complexity, uncertainty, coupling to the environment…

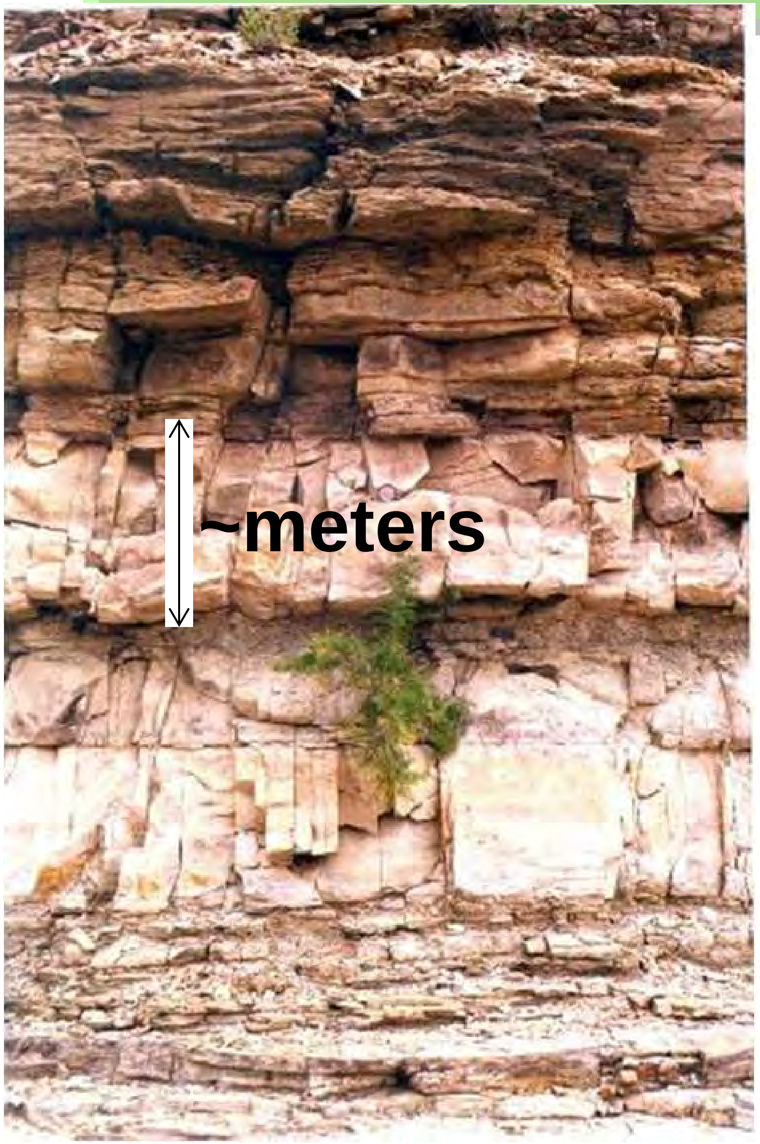


Schematic of hydraulic fracture by horizontal drilling (S. Green and R. Suarez-Rivera, AAPG Geoscience Technology Workshop, 2013)

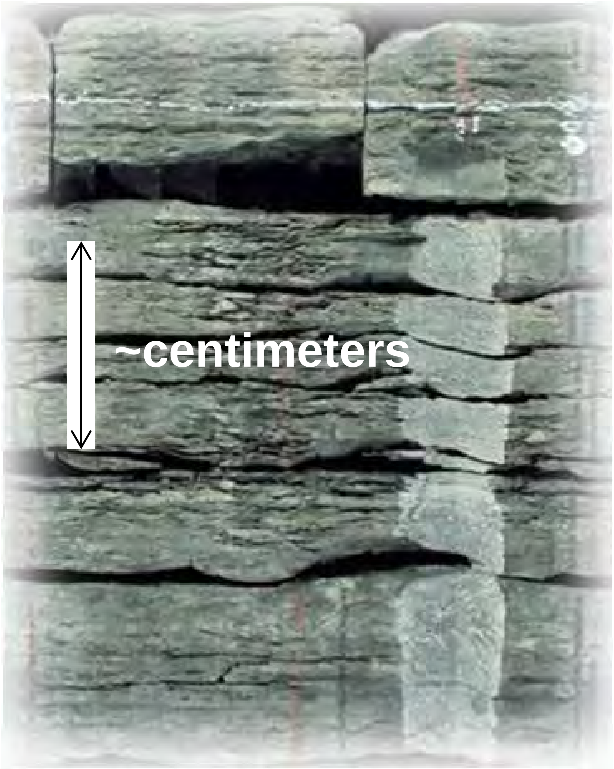


Complex pattern of hydraulic fractures generated during fracking mapped from acoustic emissions (R. Wu *et al*., SPE-152052-MS, 2012)

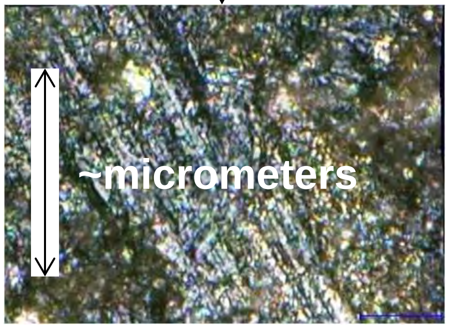
HF as a multiscale phenomenon



Subgrid length scales!



permeability



complex multiscale rock formations

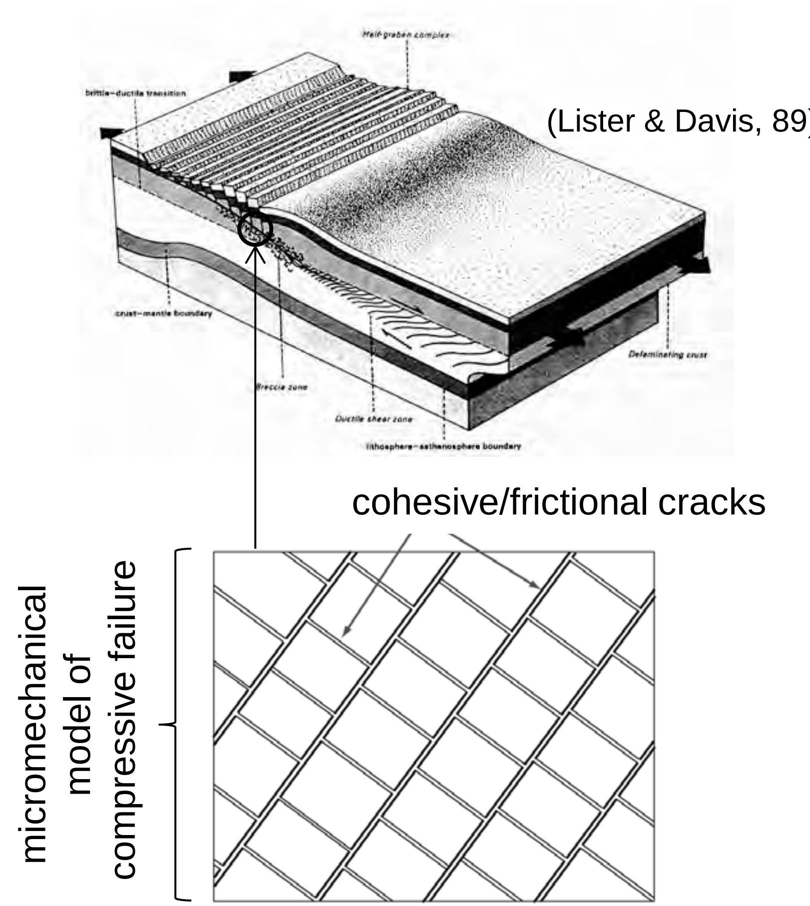
S. Green, R. Suarez-Rivera
Schlumberger Innovation Center
AAPG Geoscience Technology Workshop
July 16, 2013

Modeling and simulation challenges, objectives

- **Hydraulic fracture** is characterized by:
  - **Distributed fracture** under **triaxial compression**
  - **Complexity** of fracture patterns, geology
  - **Multiscale** phenomena: From 10^2 m to 10^{-6} m (subgrid)
  - **Multiphysics**: coupling to permeability, flow
  - **Uncertainty quantification**: limited data
- **Past models** of HF:
  - Mathematically sharp cracks in homogeneous elastic media
  - Newtonian/non-Newtonian flow through parallel planes
- **Past models** of compressive fracture:
  - 'Abstract' empirical/phenomenological distributed damage models
  - No explicit connection with microstructure geometric/evolution
  - Coupling to elasticity and permeability through empirical laws…
- **Objective**: **Multiscale model** of compressive damage/permeability based on **explicit constructions** of distributed fracture (recursive faulting)

Multiscale modeling of compressive damage

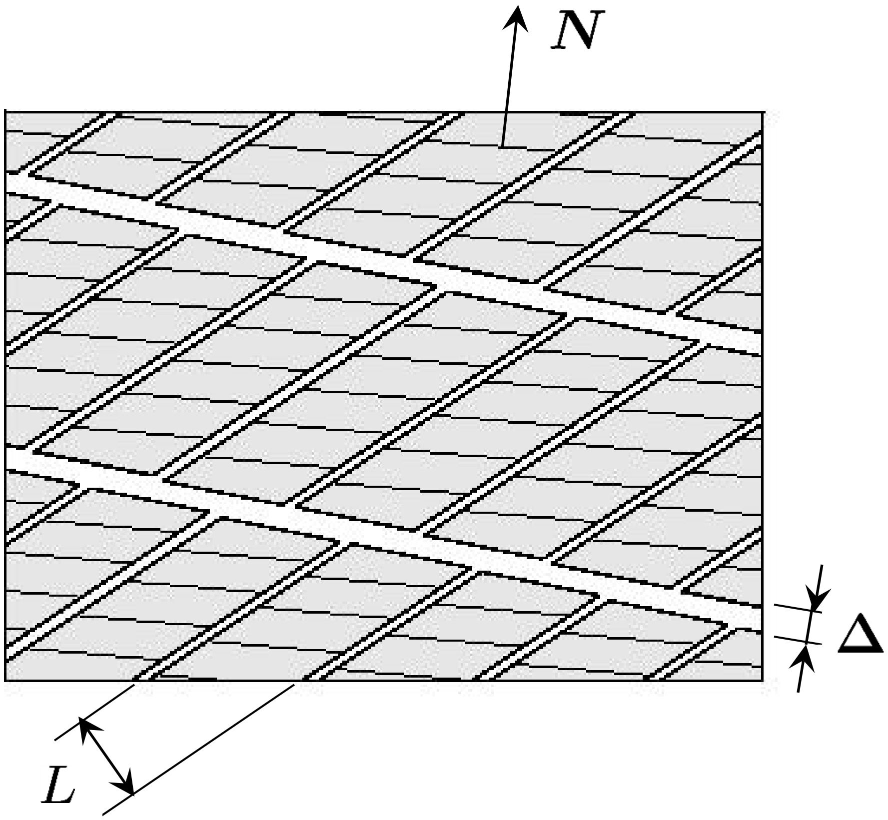
- Rocks in compression fail through complex 3D fracture patterns
- At the microscale: Multiple families of nested shear/frictional cracks
- Length scales from microscopic to geological
- Behavior of fracture system depends on behavior at both micro and macroscales:
  - Overall geometry of HF governed by macroscopic damage mechanics
  - **Permeability governed by fine detail of microstructure**
- **'Abstract' damage mechanics not enough: Multiscale modeling!**



Pandolfi, A., Conti, S. and Ortiz, M., *J. Mech. Phys. Solids*, **54**: 1972-2003, 2006

Multiscale model of compressive damage

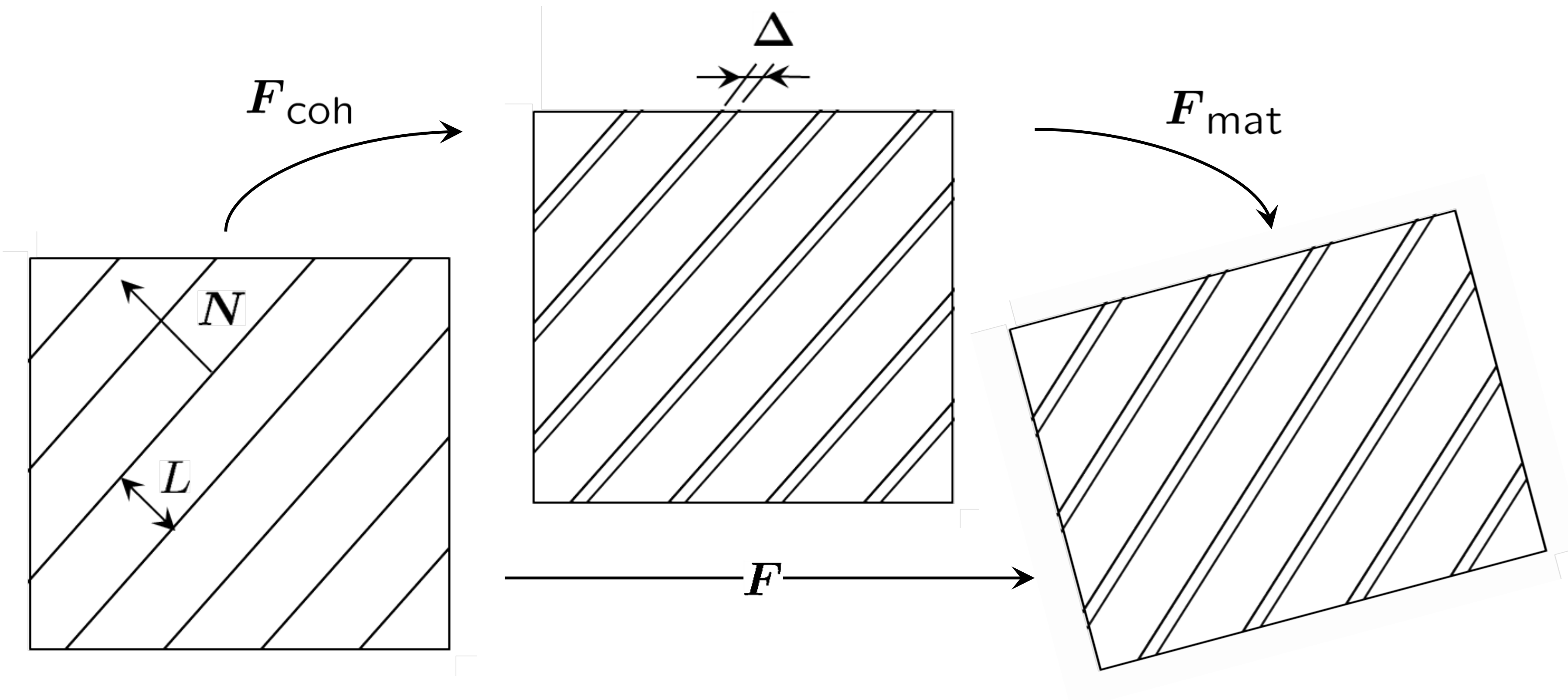
- Special class of microstructures, consisting of nested families of equi-spaced cohesive faults. The matrix material can be elastic or inelastic (e.g., plastic)
- Each family of cohesive faults is characterized by an orientation $\boldsymbol{N}$ and a spacing L (both determined by the theory)
- The behavior of the faults (opening displacement Δ) is governed by a cohesive law (open fault) and Coulomb friction (closed fault).
- The faults may be pre-existing (geological) or nucleate during deformation.



Schematic of recursive-faulting construction (two families)

Pandolfi, A., Conti, S. and Ortiz, M.,
J. Mech. Phys. Solids, **54**: 1972-2003, 2006

Single-fault model - Kinematics



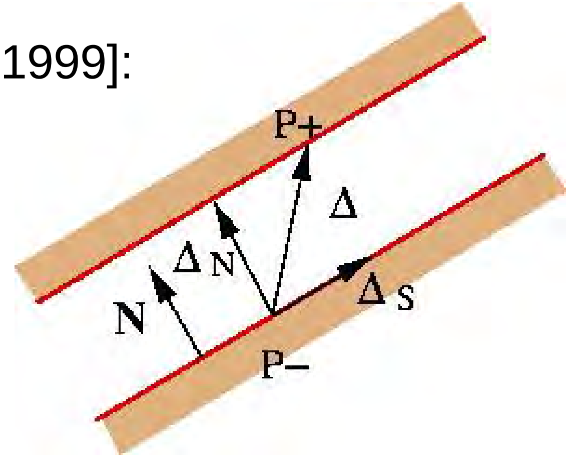
- Deformation due to faults: $\boldsymbol{F}_{\mathrm{coh}} = \boldsymbol{I} + \frac{1}{L}\boldsymbol{\Delta} \otimes \boldsymbol{N}$
- Total deformation: $\boldsymbol{F} = \boldsymbol{F}_{\mathrm{mat}}\boldsymbol{F}_{\mathrm{coh}}$

Irreversible cohesive behavior

- Effective opening displacement [Ortiz & Pandolfi, 1999]:

$$\Delta = \sqrt{\Delta_N^2 + \beta^2 \Delta_S^2}$$

$$\Delta_N = \mathbf{\Delta} \cdot \mathbf{N} > 0, \qquad \Delta_S = |\mathbf{\Delta} - \Delta_N \mathbf{N}|$$

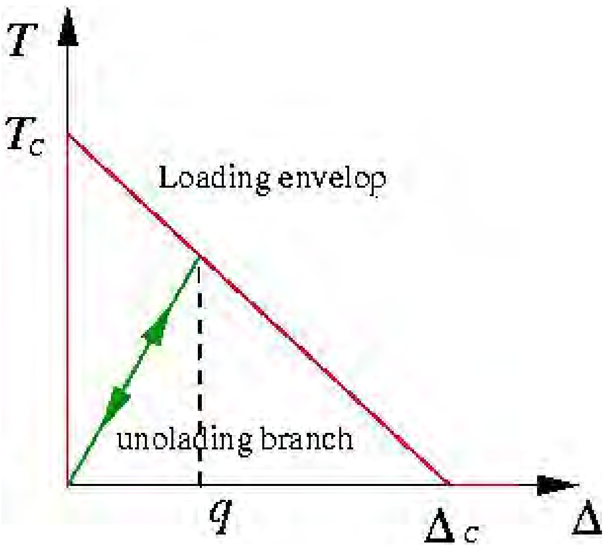


- Cohesive energy

$$\Phi = \Phi(\Delta, \mathbf{q}), \qquad T = \frac{\partial \Phi(\Delta, \mathbf{q})}{\partial \Delta}$$

- Irreversibility: unloading to origin, use the maximum attained opening displacement q as internal variable with kinetic relations

$$\dot{q} = \begin{cases} \dot{\Delta}, & \text{if } \Delta = q \text{ and } \dot{\Delta} \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



- Variational update [Ortiz & Stainier, 1999]:

$$W_n(\mathbf{F}) = \inf_{\mathbf{\Delta}, q} \left\{ W_{\text{mat}}(\mathbf{F}_{\text{mat}}) + \frac{1}{L} \Phi(\mathbf{\Delta}, q) \right\}$$

subject to:

$$\mathbf{\Delta} \cdot \mathbf{N} \geq 0\,, \qquad q \geq q_n$$

Frictional contact and sliding

- Friction is concurrently present at faults with cohesion. Once faults loose cohesion, friction remains the only dissipation mechanism.
- Dual dissipation potential Ψ^* per unit surface [Pandolfi *et al.*, IJNME, 2002]

$$\Psi^*(\dot{\boldsymbol{\Delta}}; \boldsymbol{F}, q, \boldsymbol{\Delta}) = \mu \max\{0, \ -\boldsymbol{N}\cdot\boldsymbol{S}\boldsymbol{N}\} \, |\dot{\boldsymbol{\Delta}}| \, ,$$

- where $\mathbf{S}$ is the 2nd PK stress tensor and $\mu = \tan\phi$ the friction coefficient
- Variational update [Ortiz & Stainier 1999, Pandolfi *et al.* 2006]:

$$W_n(\mathbf{F}) = \inf_{\boldsymbol{\Delta}, q} \left\{ W^{\text{mat}}(\mathbf{F}_{\text{mat}}) + \frac{1}{L}\Phi(\boldsymbol{\Delta}, q) + \frac{\Delta t}{L}\Psi^*\left(\frac{\boldsymbol{\Delta} - \boldsymbol{\Delta}_n}{\Delta t}; \mathbf{F}, q, \boldsymbol{\Delta}\right)\right\}$$

subject to: $\boldsymbol{\Delta}\cdot\mathrm{N} \geq 0 \, , \qquad q \geq q_n$

- PK stresses and tangent moduli follow as

$$\mathbf{P} = \frac{\partial W_n(\mathbf{F})}{\partial \mathbf{F}}, \qquad D\mathbf{P} = \frac{\partial^2 W_n(\mathbf{F})}{\partial \mathbf{F}\,\partial \mathbf{F}}$$

Fault inception and optimal orientation

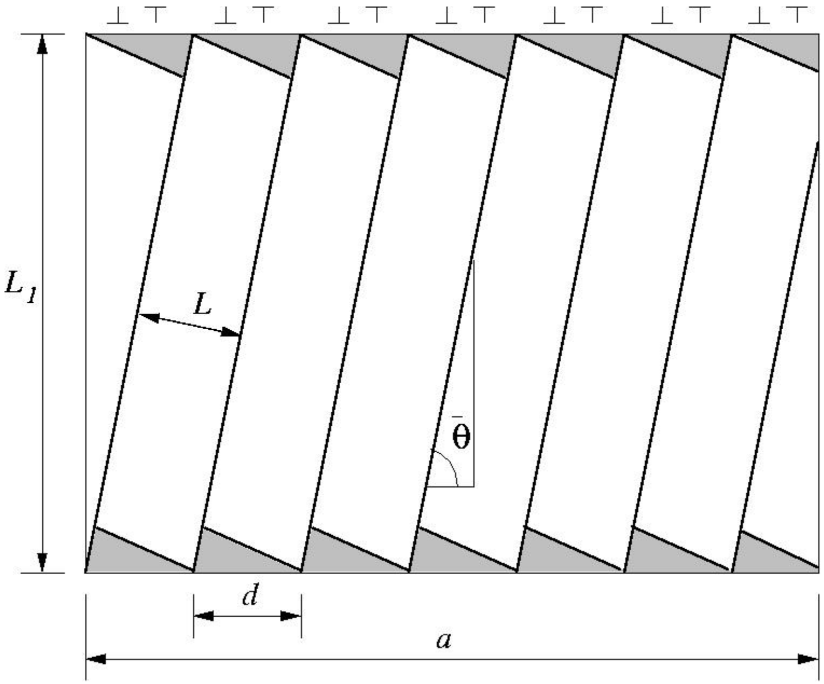
- Is the insertion of faults energetically favorable?
  - Test two end states of the material, without and with faults.
  - Chose the one resulting in the lowest incremental energy W_n.
- There is an energetically optimal orientation $\mathbf{N}$? The optimal orientation is given by the solution of the extended constrained minimum problem

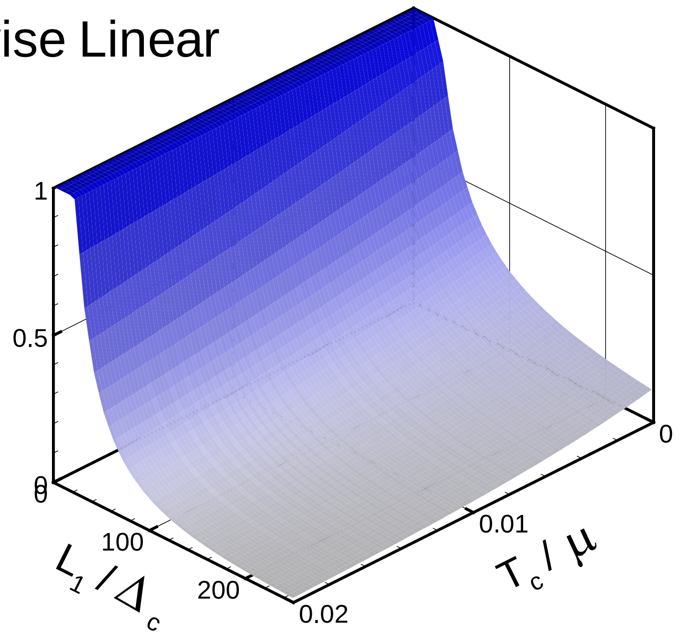
$$W_n(\mathbf{F}) = \inf_{\mathbf{\Delta}, q, \mathbf{N}} \left\{ W_{\text{mat}}(\mathbf{F}_{\text{mat}}) + \frac{1}{L} \Phi(\mathbf{\Delta}, q) + \frac{\Delta t}{L} \Psi^* \left(\frac{\mathbf{\Delta} - \mathbf{\Delta}_n}{\Delta t}; \mathbf{F}, q, \mathbf{\Delta} \right) \right\}$$

subject to: $\mathbf{\Delta} \cdot \mathbf{N} \geq 0 \qquad q \geq q_n \qquad |\mathbf{N}|^2 = 1$

- The optimum $\mathbf{N}$ is given by:
  - If faults open (tensile state): direction of maximum tensile stress
  - If faults slide (compressive state): normal to the plane of maximum shear, inclined of $\theta = \pi/4 - \phi/2$ with respect to the maximum principal stress direction (Mohr-Coulomb failure criterion)

Misfit energy and optimal fault separation





- Misfit energy: $E^{\text{mis}}(L_{n+1}) = \frac{C|\mathbf{\Delta}|^2}{L_1} \frac{1}{L_{n+1}} \log \frac{L_{n+1}}{L_0}$
- Total energy:

$$W_n(\mathbf{F}) = \inf_{\mathbf{\Delta}, q, L} \left\{ W^{\text{m}}(\mathbf{F}^{\text{m}}) + \frac{1}{L} \Phi(\mathbf{\Delta}, q) + \frac{\Delta t}{L} \Psi^* \left(\frac{\mathbf{\Delta} - \mathbf{\Delta}_n}{\Delta t}; \mathbf{F}, q, \mathbf{\Delta} \right) + E^{\text{mis}}(L) \right\}$$

- Optimize: $L_{n+1} = \Delta_c \exp \left[1 - \frac{L_1}{\Delta_c} \frac{T_c}{2\,\mu} \right]$

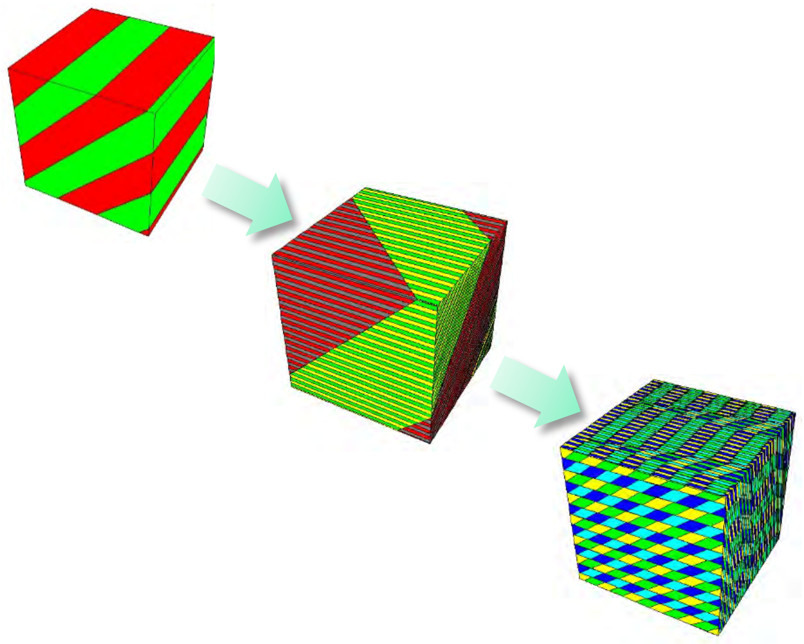
Recursive faulting construction

- Once the first fault family has developed, the matrix between faults may experience a tensile/shear state resulting in further faulting on a sublevel.
- The matrix deformation gradient $\mathbf{F}_{\text{mat}}$ at the first level can be in turn decomposed into further matrix and cohesive components governed by the single-family model
- Nested faulting of any depth can be implemented simply by calling the single-family model recursively (supported in C and C++ languages)

$$\mathbf{F} = \mathbf{F}^{(1)}_{\text{mat}} \mathbf{F}^{(1)}_{\text{coh}}$$ Rank-1

$$\mathbf{F}^{(1)}_{\text{mat}} = \mathbf{F}^{(2)}_{\text{mat}} \mathbf{F}^{(2)}_{\text{coh}}$$ Rank-2

$$\mathbf{F}^{(2)}_{\text{mat}} = \mathbf{F}^{(3)}_{\text{mat}} \mathbf{F}^{(3)}_{\text{coh}}$$ Rank-3



Optimality of sequential faulting

- Is sequential optimal faulting optimal? Are there other microstructures (fracture patterns) that are more efficient at relaxing the energy of a brittle solid under geostatic (triaxial) stresses?
- Recall: Quasiconvex envelop $W\mathrm{qc}(\boldsymbol{F})$ = smallest energy density/volume obtained by considering all possible microstructures (fracture patterns) consistent with macroscopic deformation $\boldsymbol{F}$
- $W\mathrm{qc}(\boldsymbol{F})$ describes the optimal (softest) effective macroscopic behavior of a brittle solid undergoing fracture under triaxial compression

Theorem [PCO'06] Assume no friction, and

$$W_{\mathsf{mat}}(\boldsymbol{F}) = W_{\mathsf{dev}}(\boldsymbol{F}_{\mathsf{dev}}) + W_{\mathsf{vol}}(J).$$

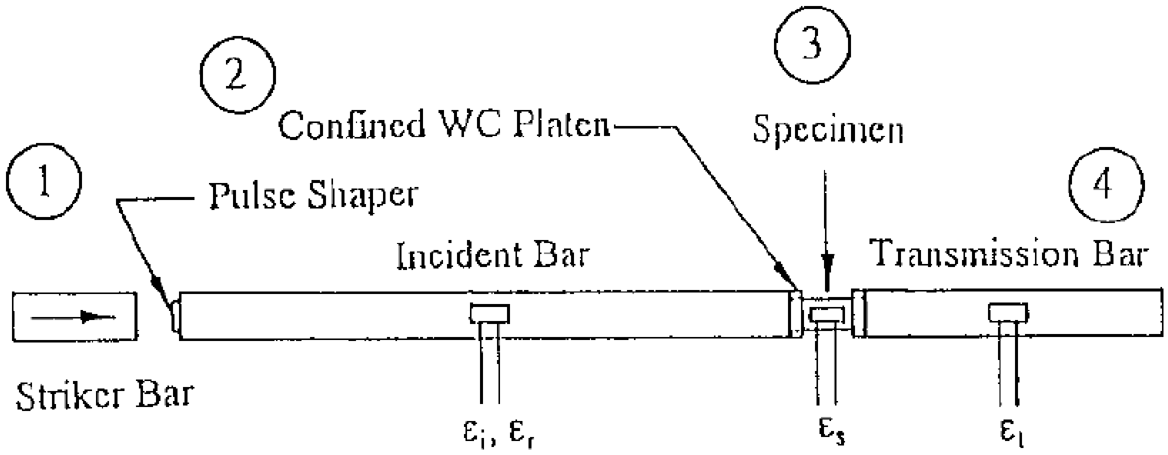
Then: $$W_{\mathsf{qc}}(\boldsymbol{F}) = \begin{cases} W_{\mathsf{vol}}(J), & \text{if } J \leq 1, \\ 0, & \text{if } J > 1, \end{cases}$$

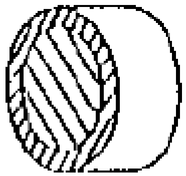
attained by sequential-faulting construction.

- Sequential faulting fully relaxes all geostatic deviatoric stresses!

Experimental validation

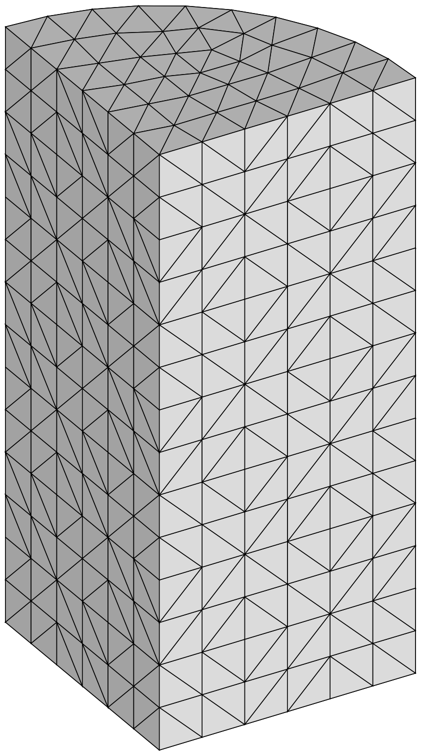
- Refer to the experiments by Chen and Ravichandran on Sintered Aluminum Nitride (AIN) [Chen and Ravichandran, JAS, 1996].
- Special experimental technique to impose lateral confinement on AIN cylinder, by using a shrink-fit metal sleeve.
- Confinement increases the resistance and the ductility of the specimen, both in static and in dynamic case.

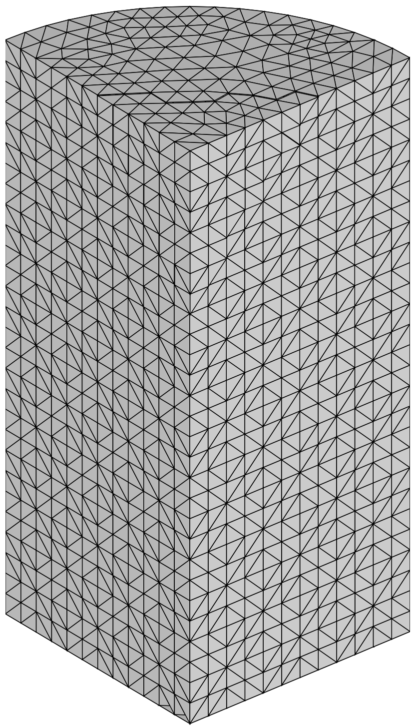




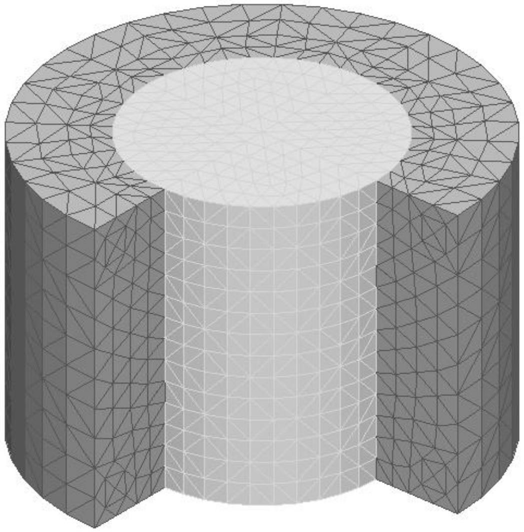
(Courtesy of G. Ravichandran)

Experimental validation



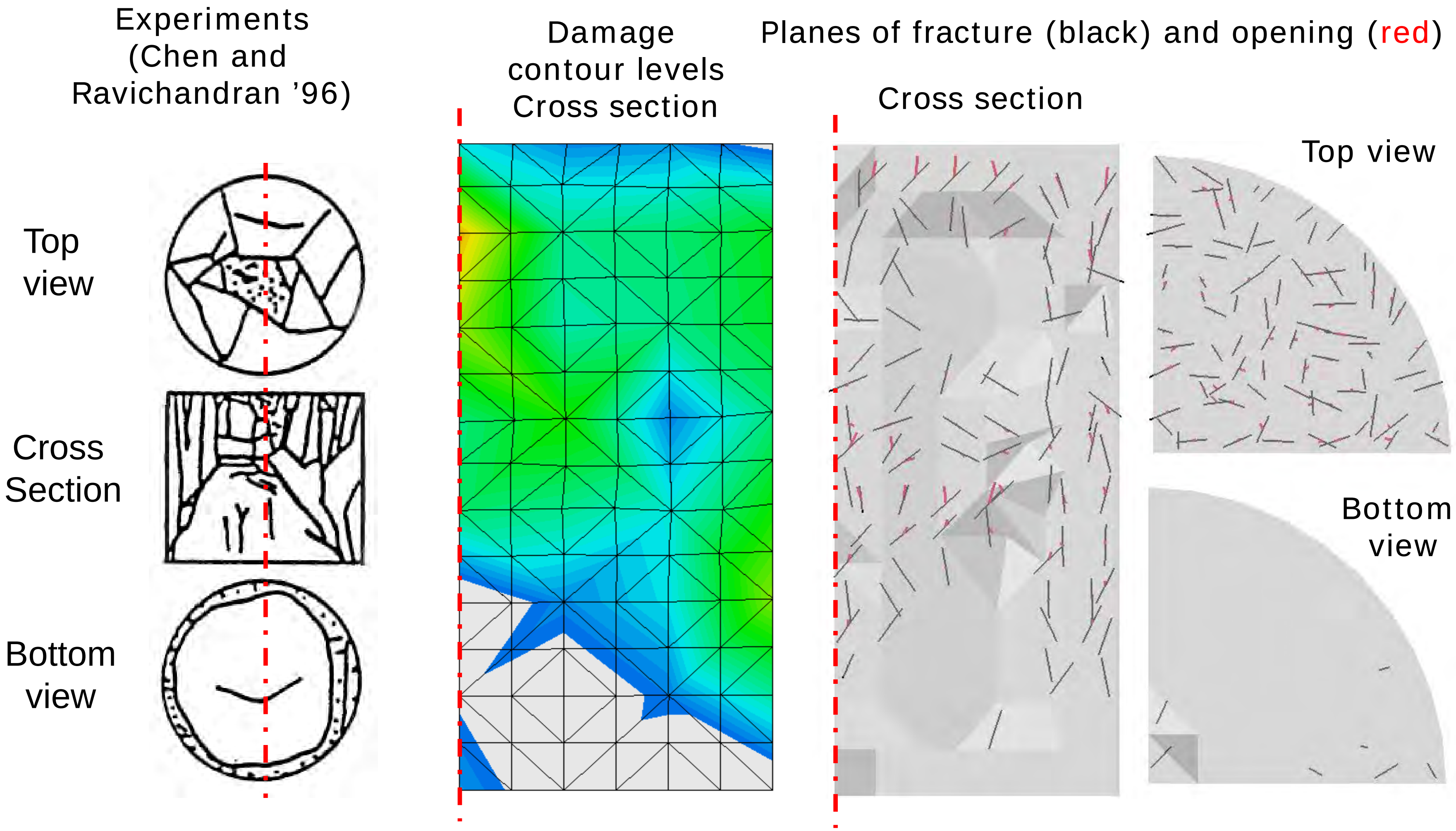


$$E = 310\,\text{GPa}$$
$$\nu = 0.237$$
$$T_c = 180\,\text{MPa}$$
$$G_c = 162\,N/\text{m}$$
$$\beta^2 = 12$$
$$L_0 / L_c = 100$$
$$\rho = 3200\,\text{kg/m}^3$$

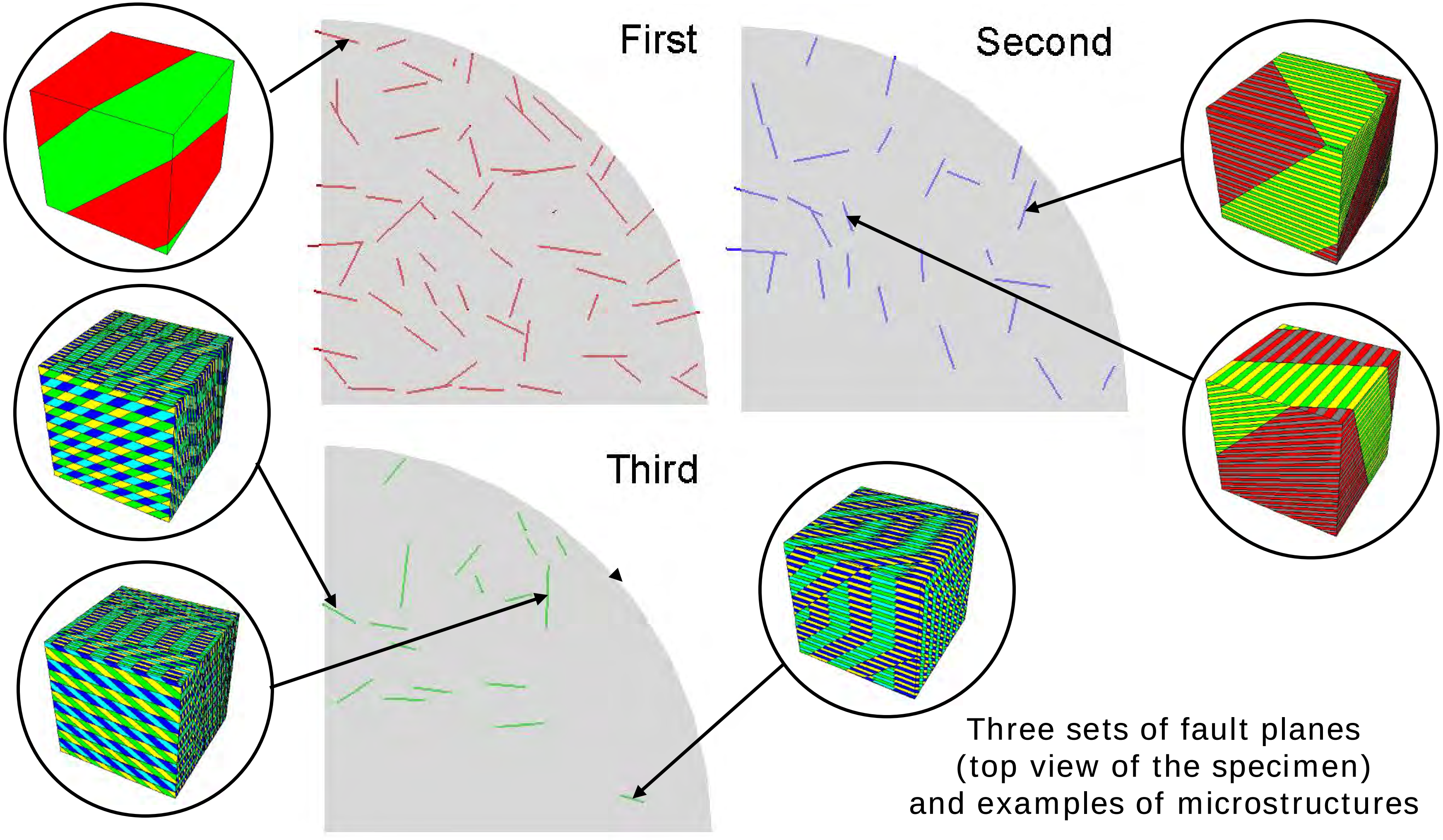


- Tetrahedral FE meshes of AlN specimen and steel sleeve
- Two mesh sizes to verify mesh-size insensitivity of calculations

Experimental validation



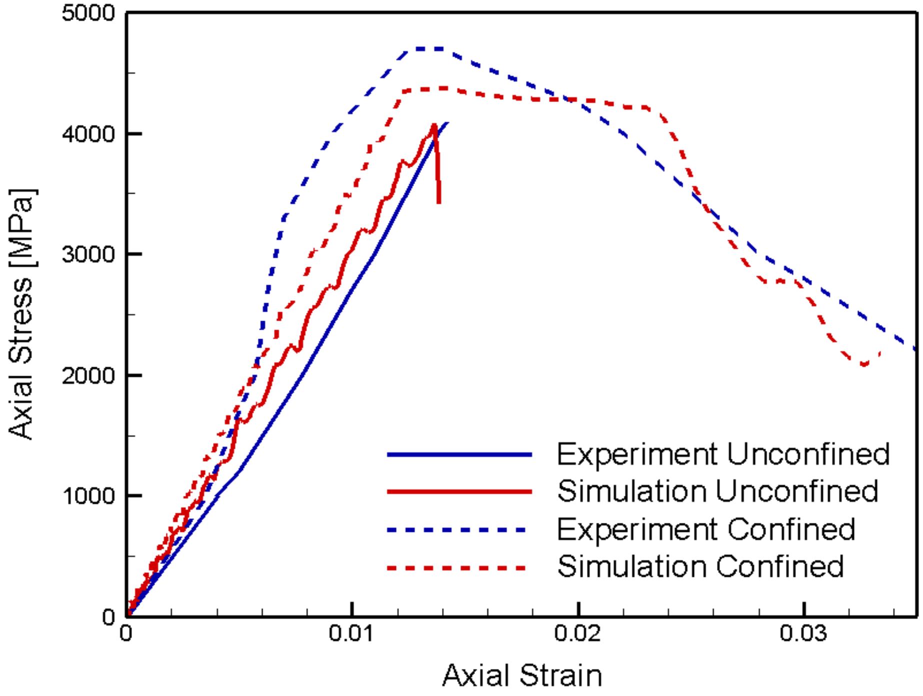
Experimental validation



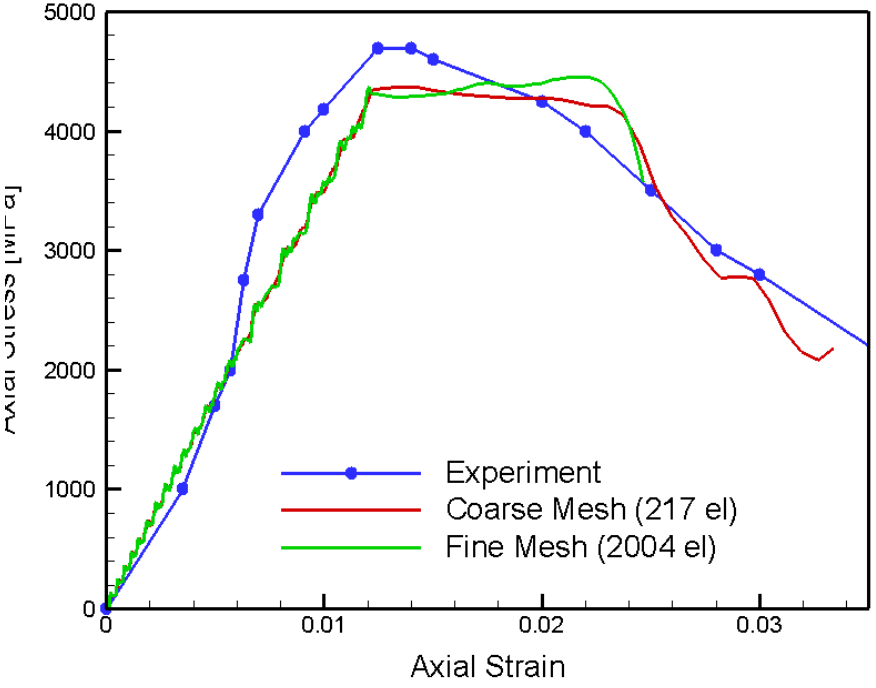
Three sets of fault planes
(top view of the specimen)
and examples of microstructures

Experimental validation

Comparison with experiment
[Chen and Ravichandran, 1996]



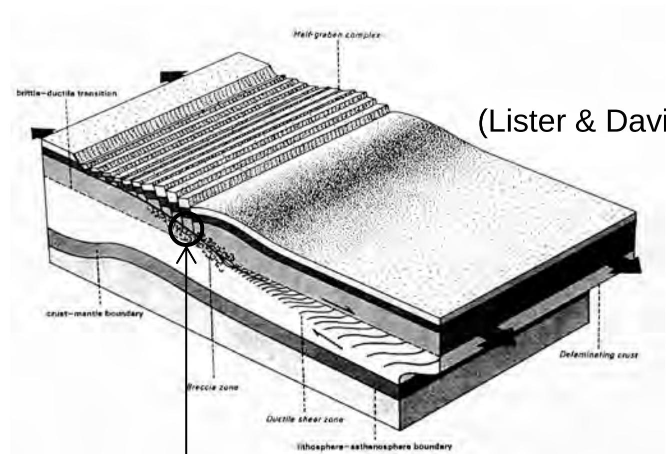
Mesh-size sensitivity



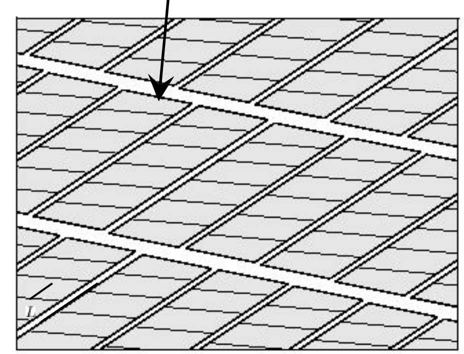
- Model captures brittle-to-ductile transition with increasing confinement
- Model results are mesh-size insensitive

Damage-enhanced permeability

- Explicit **recursive-faulting** construction characterizes both **average macroscopic behavior** and **fine structure** of the fracture pattern at the microscale
- Can couple the predicted microscopic fracture geometry and evolution to **permeability model**



(Lister & Davis, 89)



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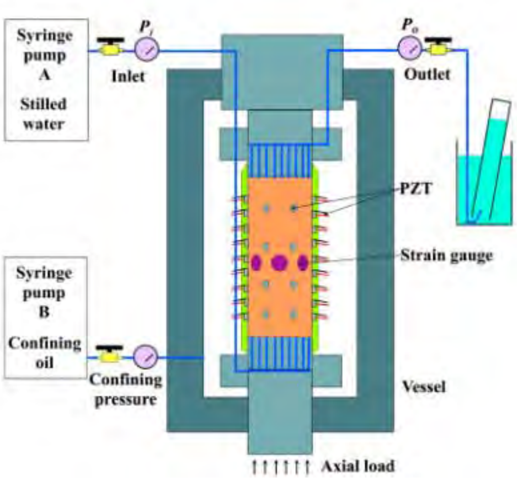


Lei et al., *Geophys. Res. Let.*, **38**, L24310, 2011

Porous media equations in finite kinematics

- Terzaghi's effective stress principle, p pore pressure: $\boldsymbol{P} = \boldsymbol{P}' + pJ\,\boldsymbol{F}^{-T}$
- Total porosity and permeability: $n = n_{\mathrm{m}} + n_{\mathrm{f}}$ $\qquad \kappa = \kappa_{\mathrm{m}} + \kappa_{\mathrm{f}}$
- Matrix porosity and permeability (Kozeni-Carman model)

$$n_{\mathrm{m}} = 1 + \frac{1}{J}(n_0 - 1), \qquad \kappa_{\mathrm{m}} = C_{KC}\frac{(n_{\mathrm{m}})^3}{(1 - n_{\mathrm{m}})^2}$$

- Permeability due to fracture (laminar flow between parallel planes):

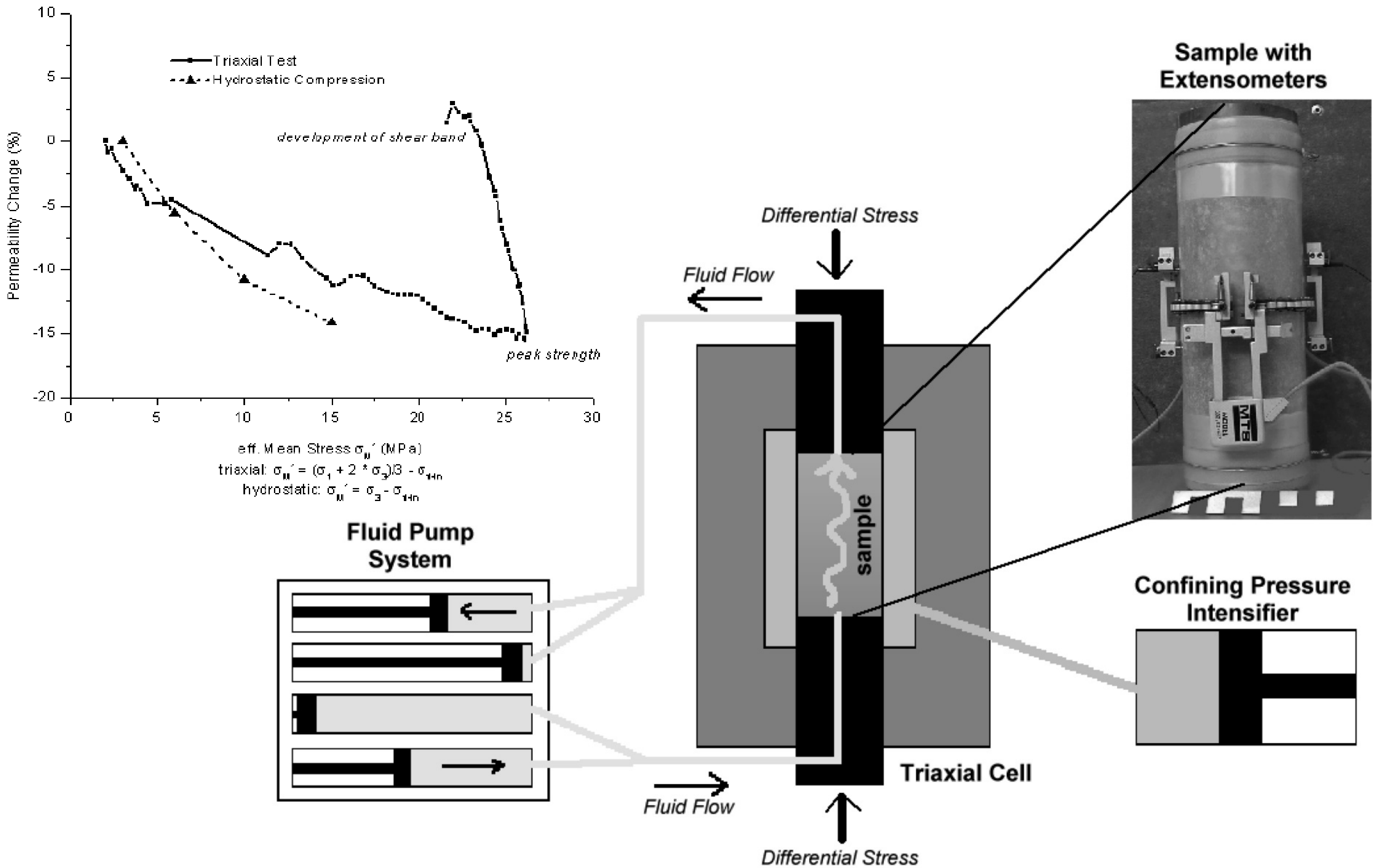
$$\kappa^{\mathrm{f}} = \sum_{k=1}^{Q} \frac{\Delta_N^k}{L^k + \Delta_N^k}\,\frac{{\Delta_N^k}^2}{12}\left(\boldsymbol{I} - \boldsymbol{N}^k \otimes \boldsymbol{N}^k\right)$$

fault-orientation dependence
(anisotropic permeability…)

fault-opening dependence
(effect of stress, pore pressure, friction…)

fault-density dependence
(specific fracture energy, size effect…)

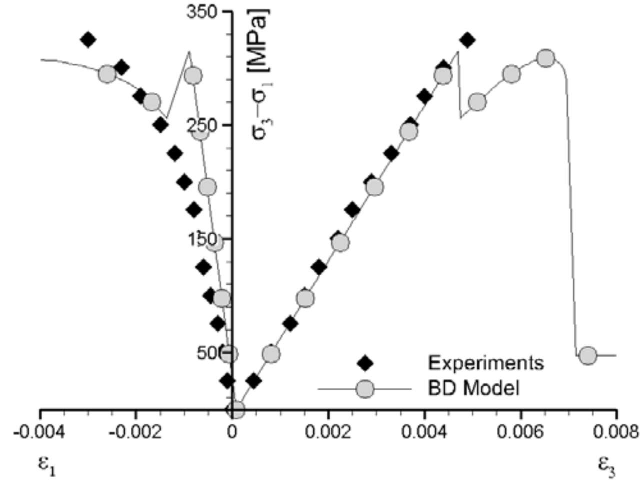
Typical post-peak permeability behavior in rocks

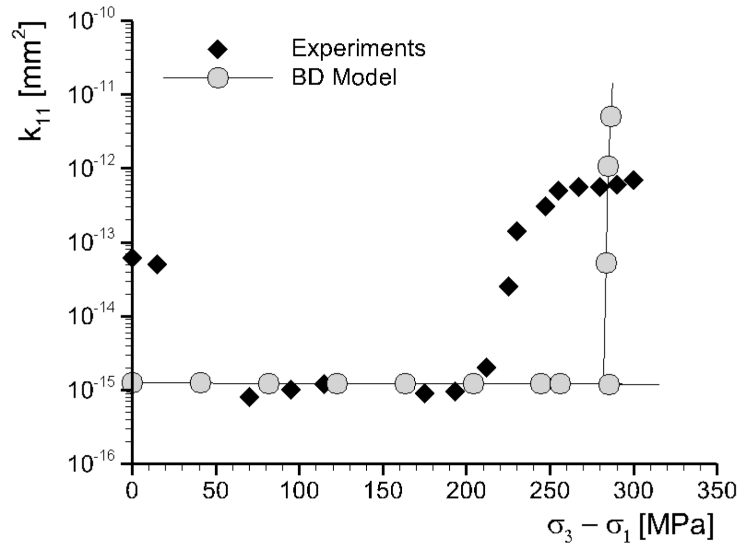


J. Heiland, *Hydrogeology Journal*, **11**:122-141, 2003.

Material-point validation: Lac du Bonnet granite

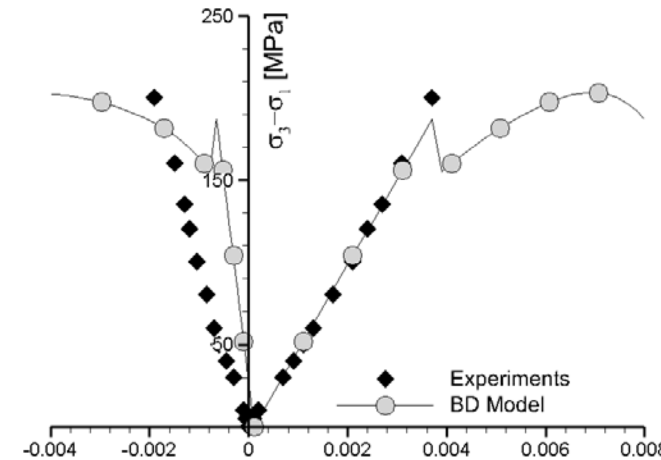
- Experimental data from [Souley et al., 2001]
- Confinement 10 MPa
- Triaxial test up to the peak, with measurements of the permeability
- Missing material data on strength and fracture energy
- Satisfactory deviatoric stress versus axial and lateral strain
- Prediction of permeability shows sharp upturn after peak, corresponding to usual experimental trend

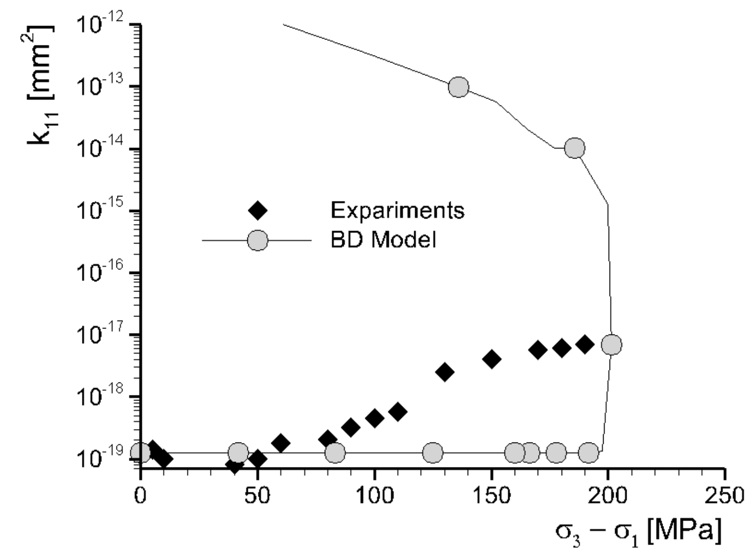




Material-point validation: Beishan granite

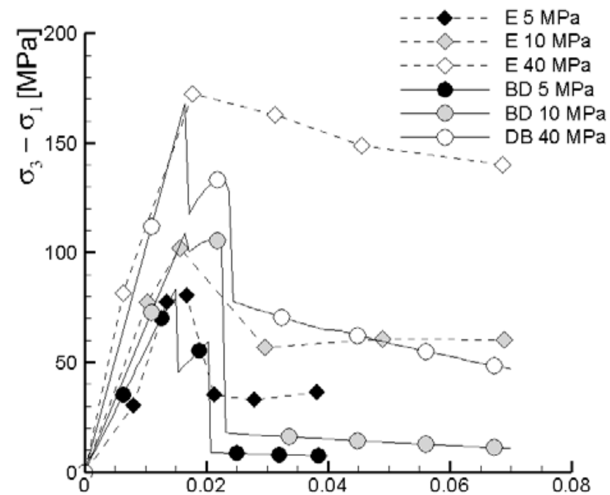
- Experimental data from [Ma et al., 2012]
- Confinement 10 MPa
- Triaxial test up to the peak, with measurements of the permeability
- Missing material data on resistence and fracture energy
- Satisfactory deviatoric stress versus axial and lateral strain
- Prediction of permeability shows sharp upturn followed by recovery after peak, corresponding to usual experimental trend

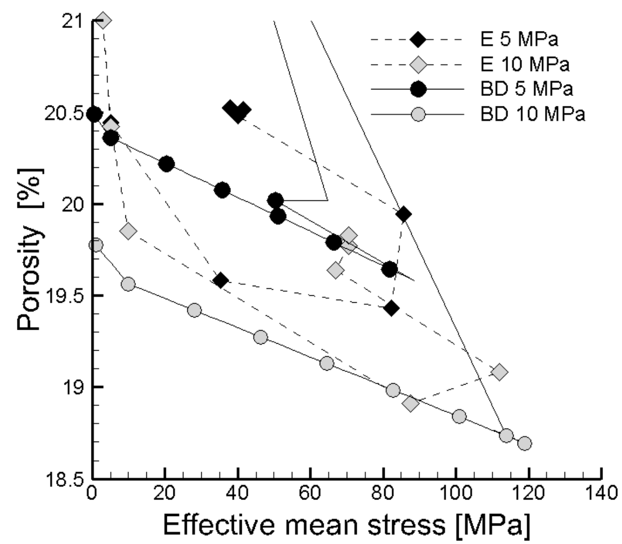




Material-point validation: Berea sandstone

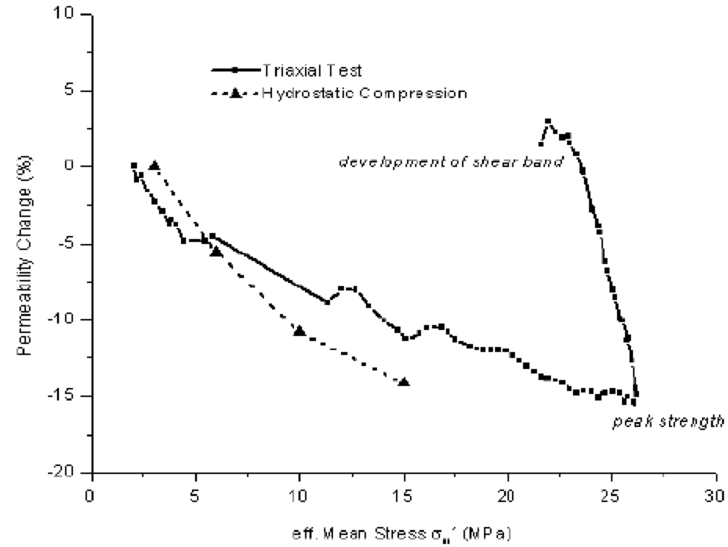
- Experiments from [Zhu & Wong, 1997]
- Triaxial tests with post-peak measurements of porosity and permeability
- Missing the data on strength and fracture energy
- Confinement 5, 10 and 40 MPa
- Good agreement on the stress-strain curves for all levels of confinement
- Good agreement on the porosity curves, especially for the lower confinement, showing decrease during loading and recovery during unloading

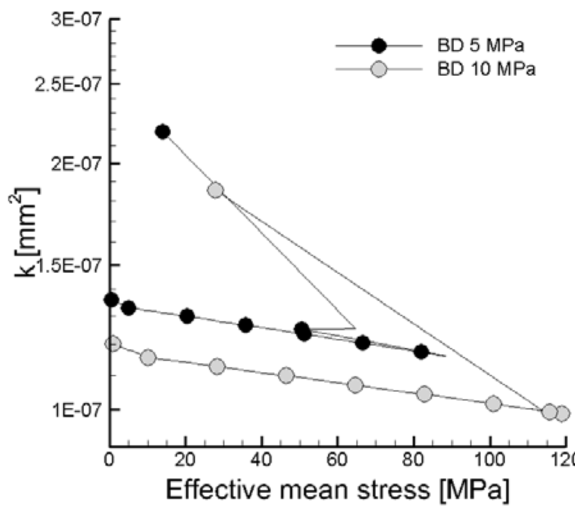




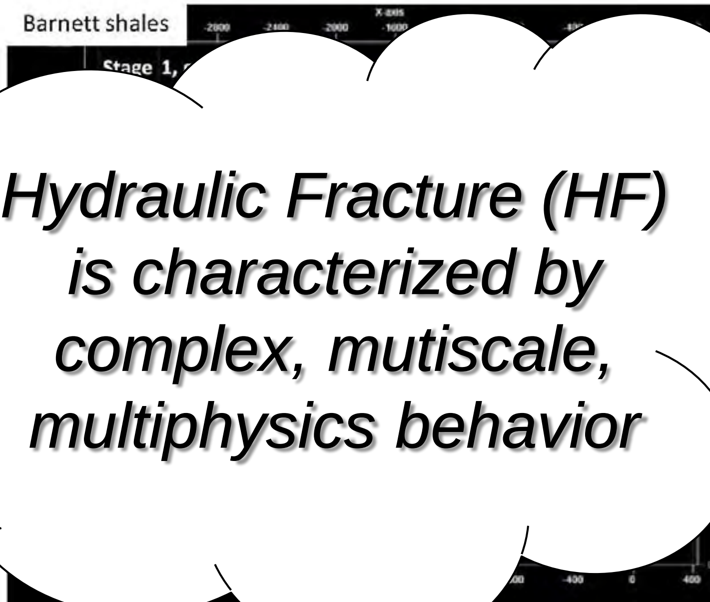
Material-point validation: Berea sandstone

- Experiments from [Zhu & Wong, 1997]
- Triaxial tests with post-peak measurements of porosity and permeability
- Missing the data on strength and fracture energy
- Confinement 5, 10 and 40 MPa
- Good agreement on the stress-strain curves for all levels of confinement
- Predicted permeability shows decrease during loading and recovery during unloading, in agreement with usual experimental trend



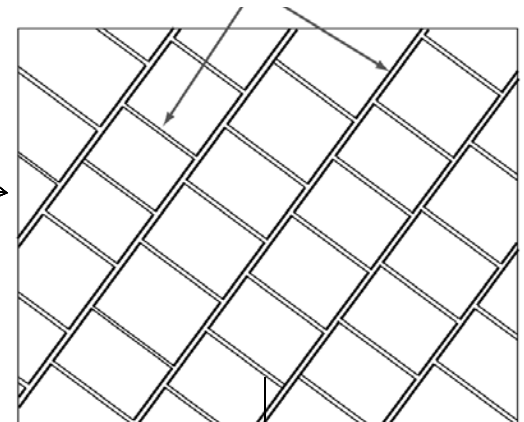


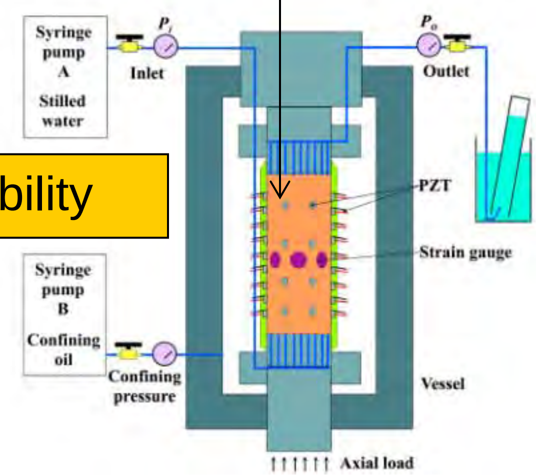
Concluding remarks



Hydraulic Fracture (HF) is characterized by complex, mutiscale, multiphysics behavior

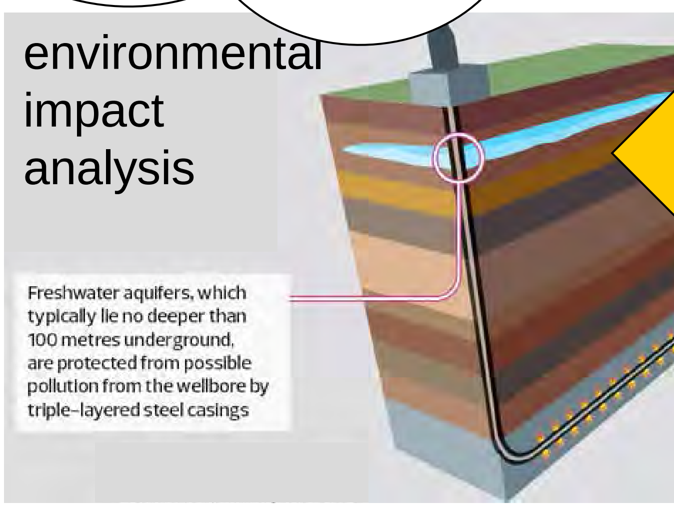
subgrid model





permeability

environmental impact analysis



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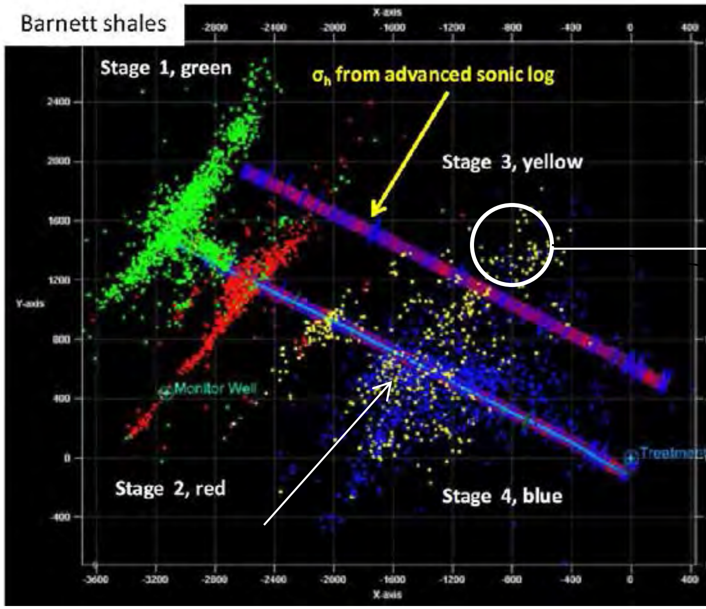
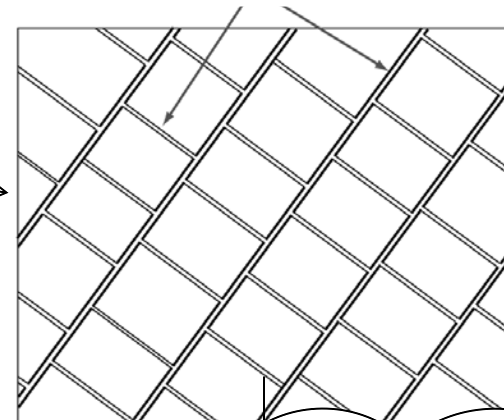
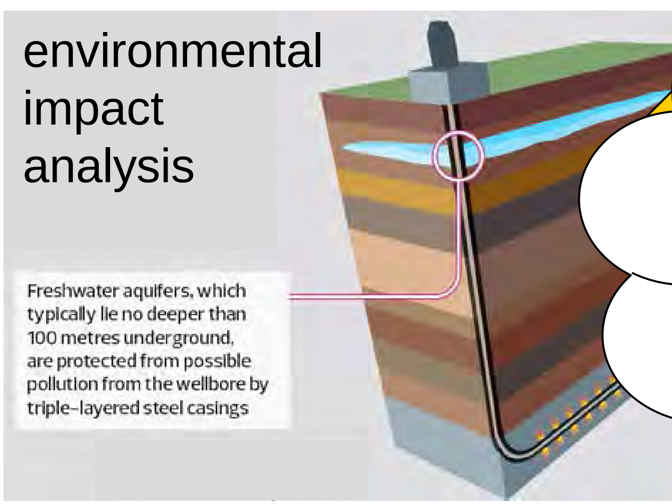
Concluding remarks



subgrid model





Permeability depends on fine structure of fracture patterns (no only averages)

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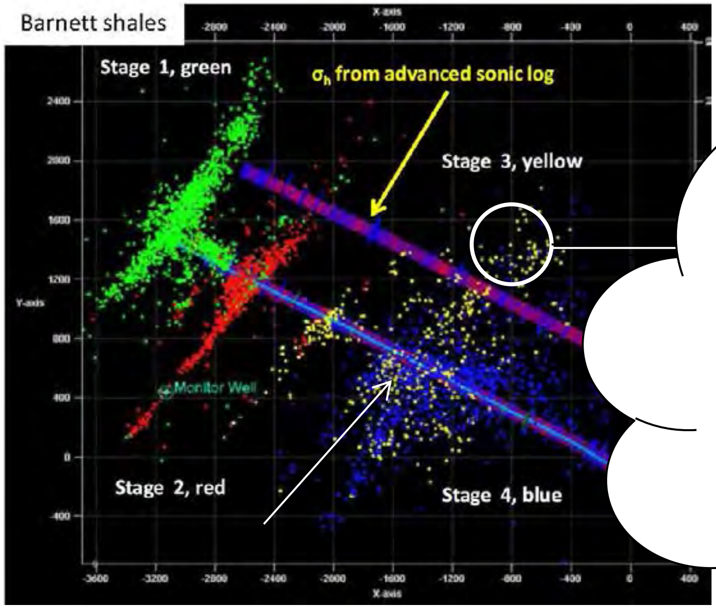
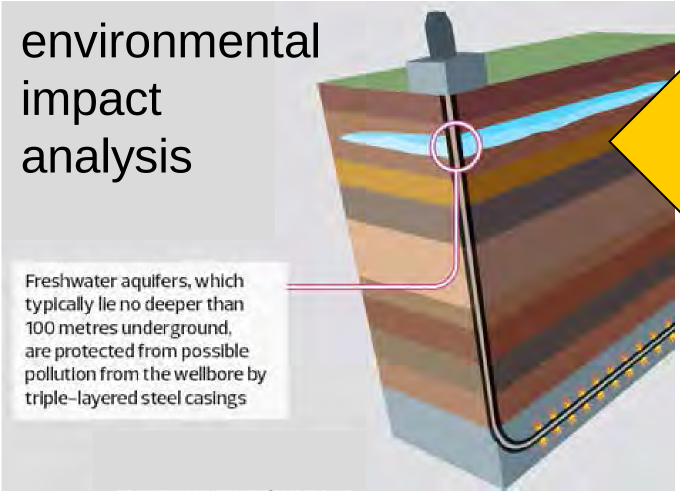
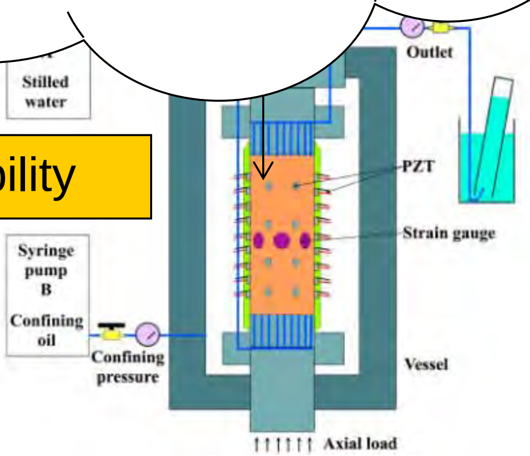
Concluding remarks



Sequential faulting is optimal at relaxing geostatic stresses



permeability



Lei et al., *Geophys. Res. Let.*, **38**, L24310, 2011

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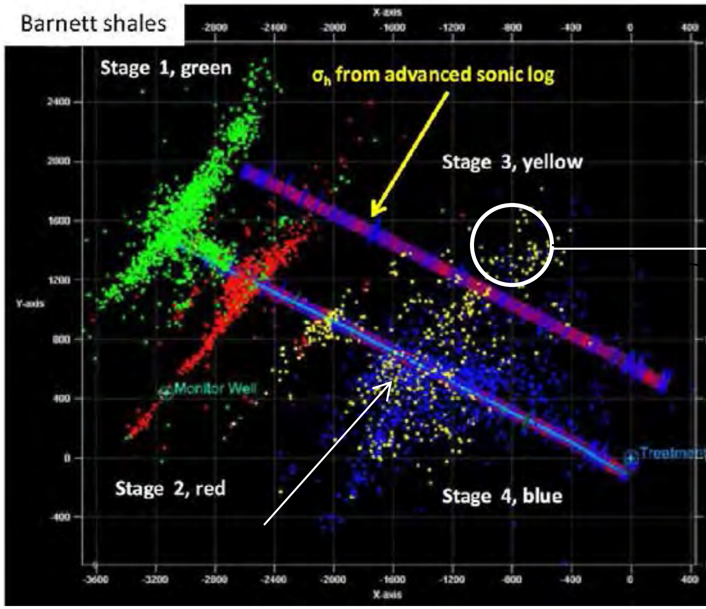
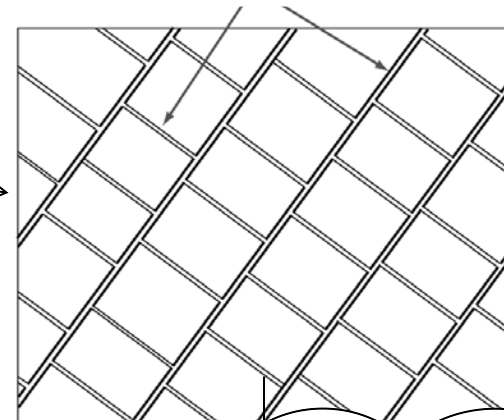
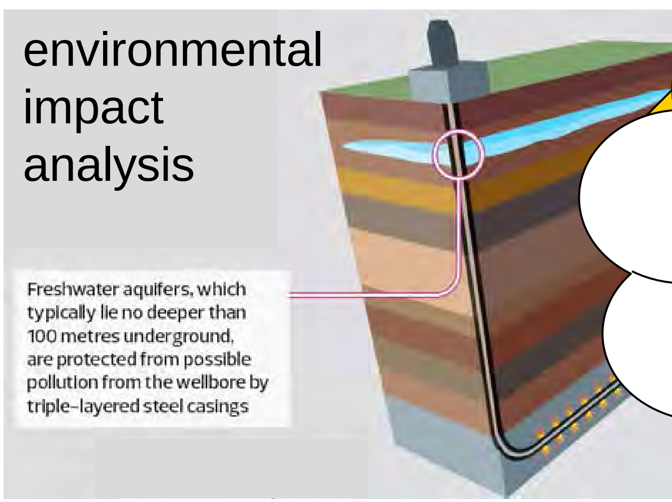


subgrid model





Sequential faulting introduces channels for fluid flow, resulting in enhanced permeability

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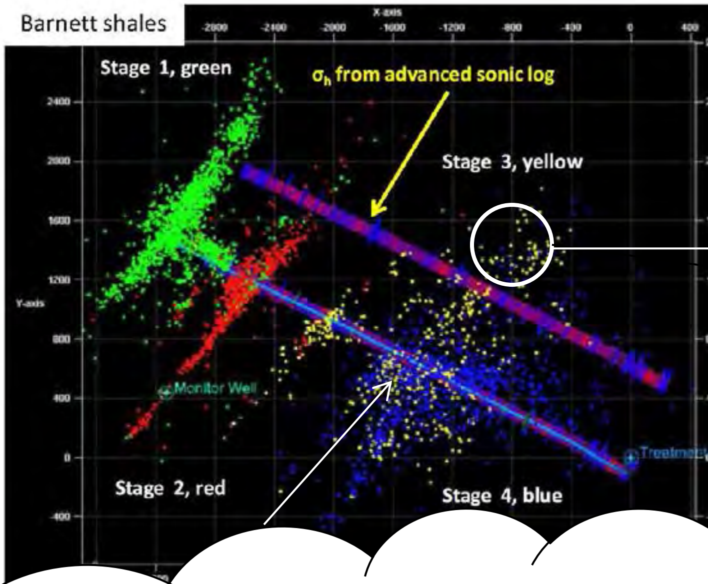
Concluding remarks



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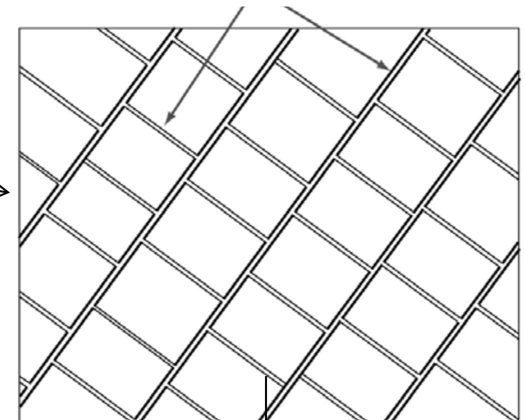

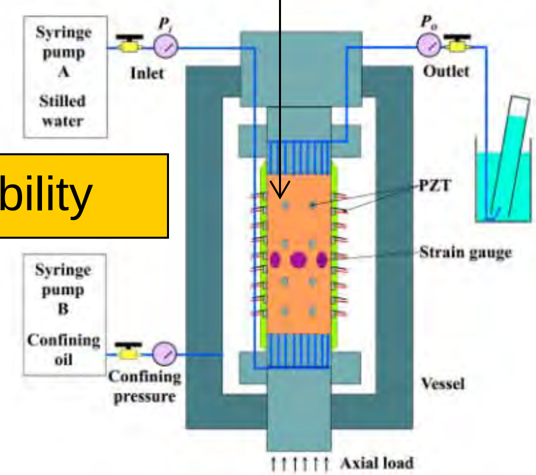

ability

Potential payoff: HF process optimization, environmental impact (microseismicity, waste fluid disposal…)

Lei et al., *Geophys. Res. Let.*, **38**, L24310, 2011

Concluding remarks

Thank you!