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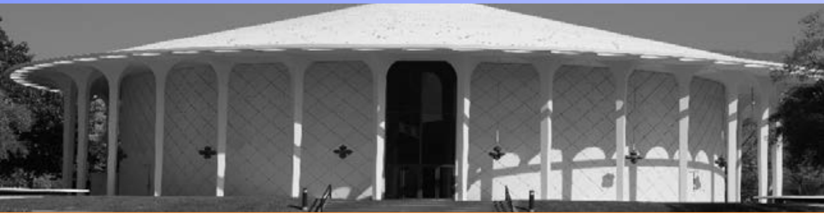


universität bonn

Fractional Strain-Gradient Plasticity

M. Ortiz

California Institute of Technology and
Rheinische Friedrich-Wilhelms Universität Bonn

with: Carl Dahlberg, KTH, Stockholm, Sweden

Century Fracture Mechanics Summit
Singapore, April 8, 2019

The genesis of this talk…

Technical University of Denmark

DTU

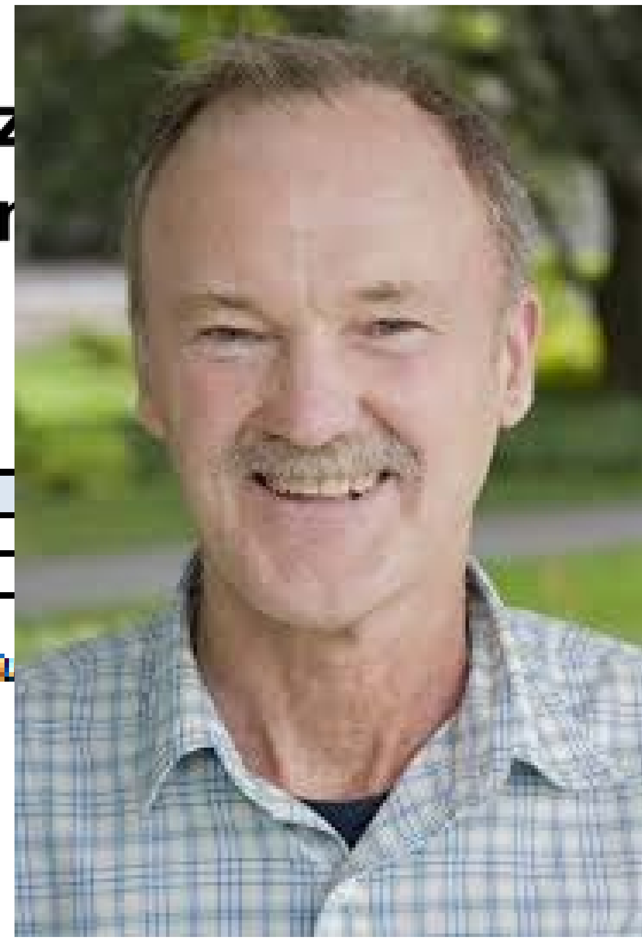
IUTAM Symposium on Siz
in Microstructure and Dar

May 27 - June 1, 2018

Thursday - 31 May 2018	
13.15 - 15.00	Room: M1

14.15 - 15.00 *Discussion Session on the Cu* *sticity*

J.W. Hutchinson



Michael Ortiz
CFMS 2019

The strength of confined Cu thin films

Extreme Mechanics Letters 1 (2014) 62–69

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journal homepage: www.elsevier.com/locate/eml

Micro-pillar measurements of plasticity in confined Cu thin films

Yang Mu [a], J.W. Hutchinson [b], W.J. Meng [a,*]

[a] Department of Mechanical and Industrial Engineering, Louisiana State University, Baton Rouge, LA 70803, United States

[b] School of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138, United States

Mu, Y., Chen, K., Meng, W.J., 2014. MRS Commun. Res. Lett. 4, 126–133.
Mu, Y., Zhang, X., Hutchinson, J.W., Meng, W.J., 2016. MRS Commun. Res. Lett.20, 1–6.
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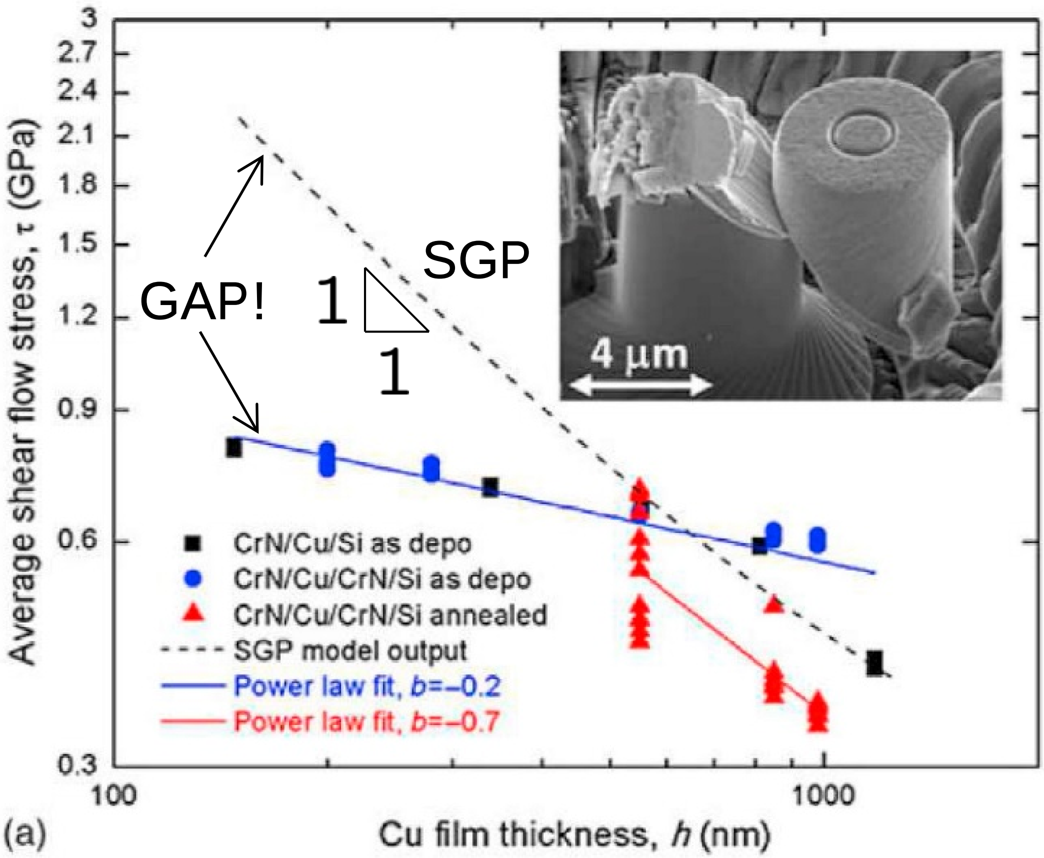
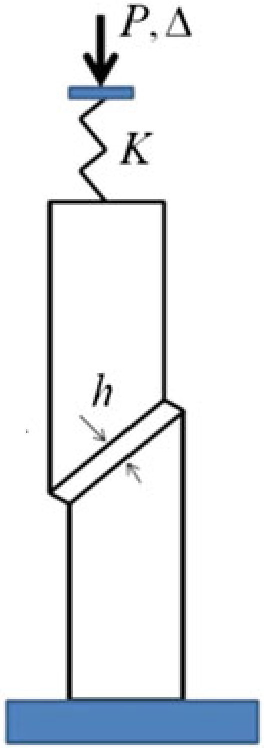
The strength of confined Cu think films





Shear flow stress as a function of thickness for Cu layers[1].
SGP model prediction shown as dashed line.
Insert shows SEM image of experimental setup.

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Classical strain-gradient plasticity

- Simplifying assumptions: Linearized kinematics, rigid-plastic deformations, proportional loading, deformation theory of plasticity
- Principle of minimum potential energy: Minimize

$$F(\boldsymbol{u}) = \int_\Omega \Big(\psi_{\mathrm{p}}(\varepsilon(\boldsymbol{u})) + \psi_{\mathrm{g}}(D\varepsilon(\boldsymbol{u})) \Big)\, dV + F.T.$$

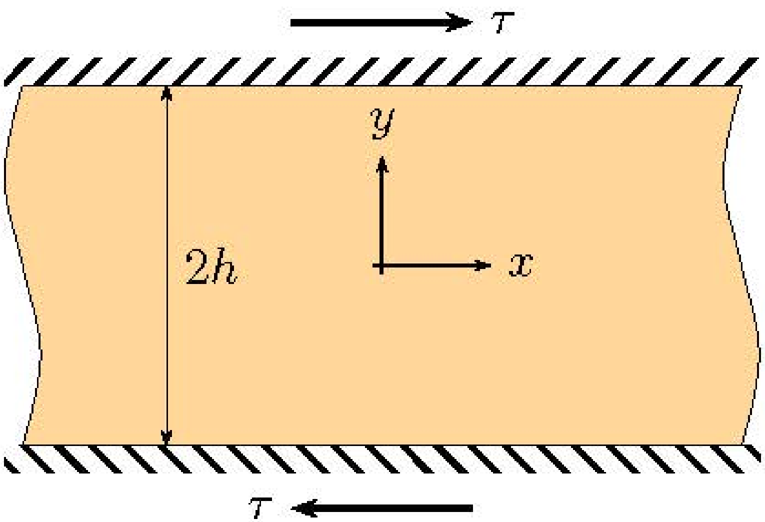
- Power law behavior (incompressible):

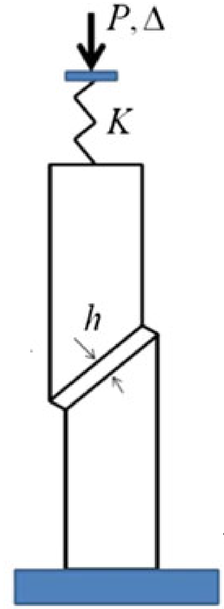
$$\psi_{\mathrm{p}} = \tau_0 \sqrt{2\varepsilon_{ij}\varepsilon_{ij}} + \frac{A\varepsilon_0^{-m}}{m+1} \Big(2\varepsilon_{ij}\varepsilon_{ij}\Big)^{\frac{m+1}{2}}$$

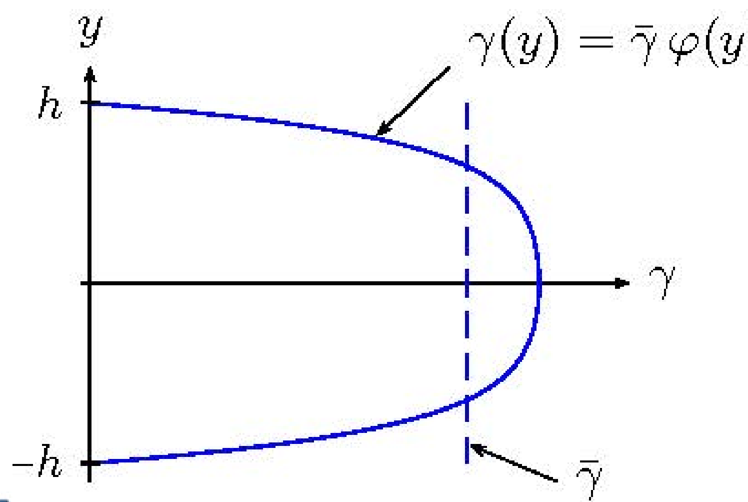
$$\psi_{\mathrm{g}} = \frac{B\varepsilon_0^{-n}\ell^{n+1}}{n+1} \Big(2\varepsilon_{ij,k}\varepsilon_{ij,k}\Big)^{\frac{n+1}{2}}$$

Aifantis, E.C., 1987. Int. J. Plast. 3, 211–247.
Fleck, N.A., Muller, G.M., Ashby, M.F., Hutchinson, J.W., 1994. Acta Metall. Mater. 42 (2), 475–487.

Metal interlayer deforming in shear







$$F(\gamma) = \int_{-h}^{+h} \Big(\psi_{\mathrm{p}}(\gamma(y)) + \psi_{\mathrm{g}}(\gamma_{,y}(y)) \Big)\, dy \to \text{min!}$$

- Power law: $$\psi_{\mathrm{p}}(\gamma) = \tau_0 |\gamma| + \frac{A}{m+1} \left|\frac{\gamma}{\gamma_0}\right|^{m+1}$$

$$\psi_{\mathrm{g}}(\gamma_{,y}) = \frac{B}{n+1} \left|\frac{\ell \gamma_{,y}}{\gamma_0}\right|^{n+1}$$

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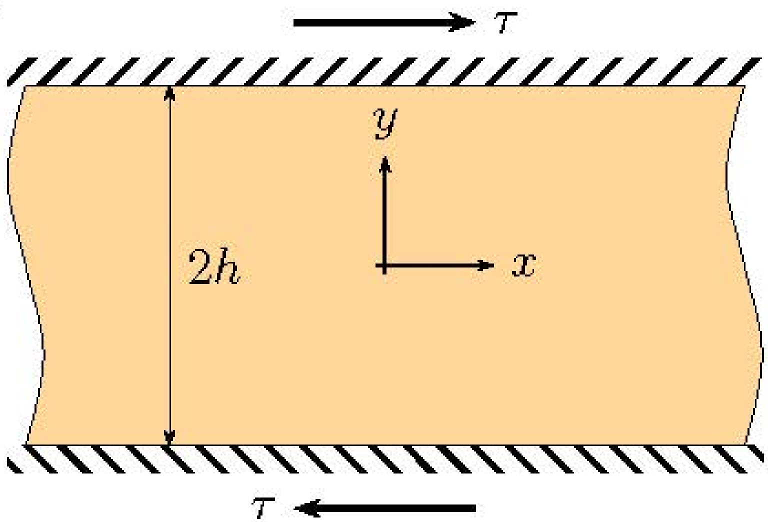
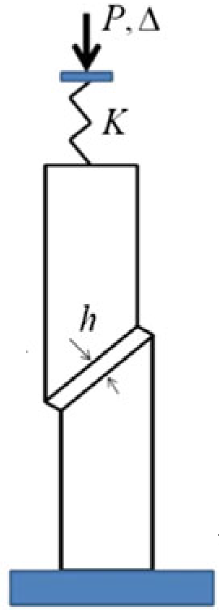
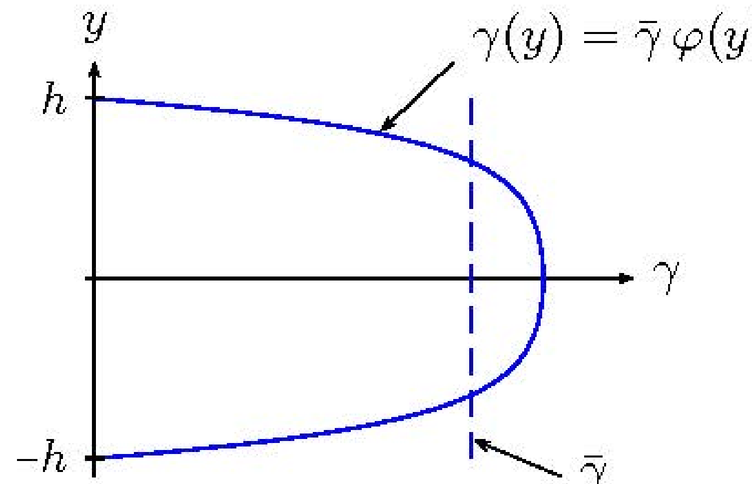
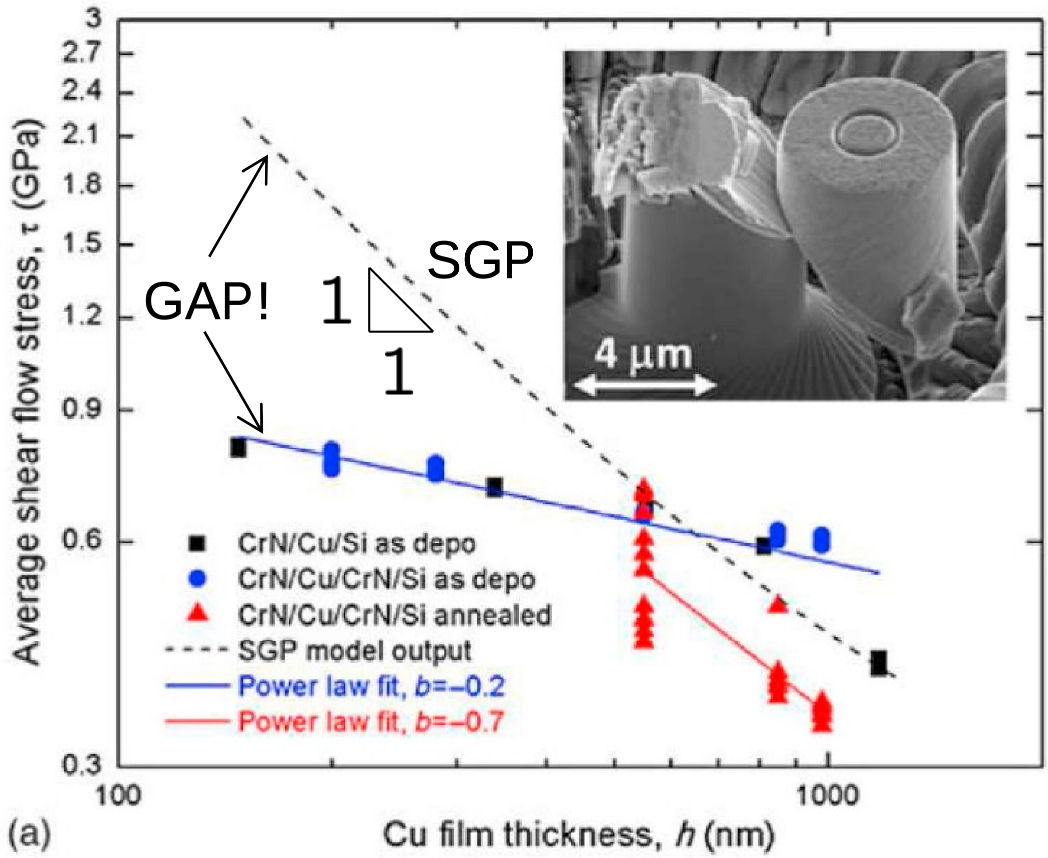


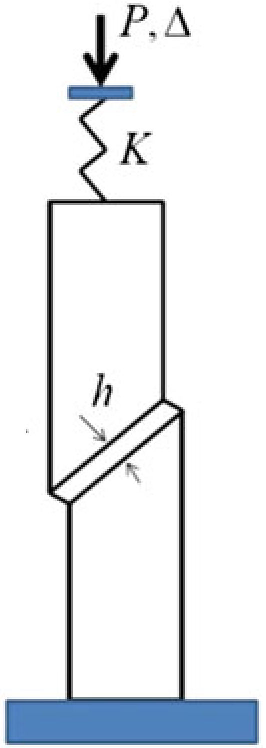




- Effective shear-yield stress: $\tau_\ell = \tau_0 \left(1 + \frac{\ell}{h}\right)$

The strength of confined Cu think films





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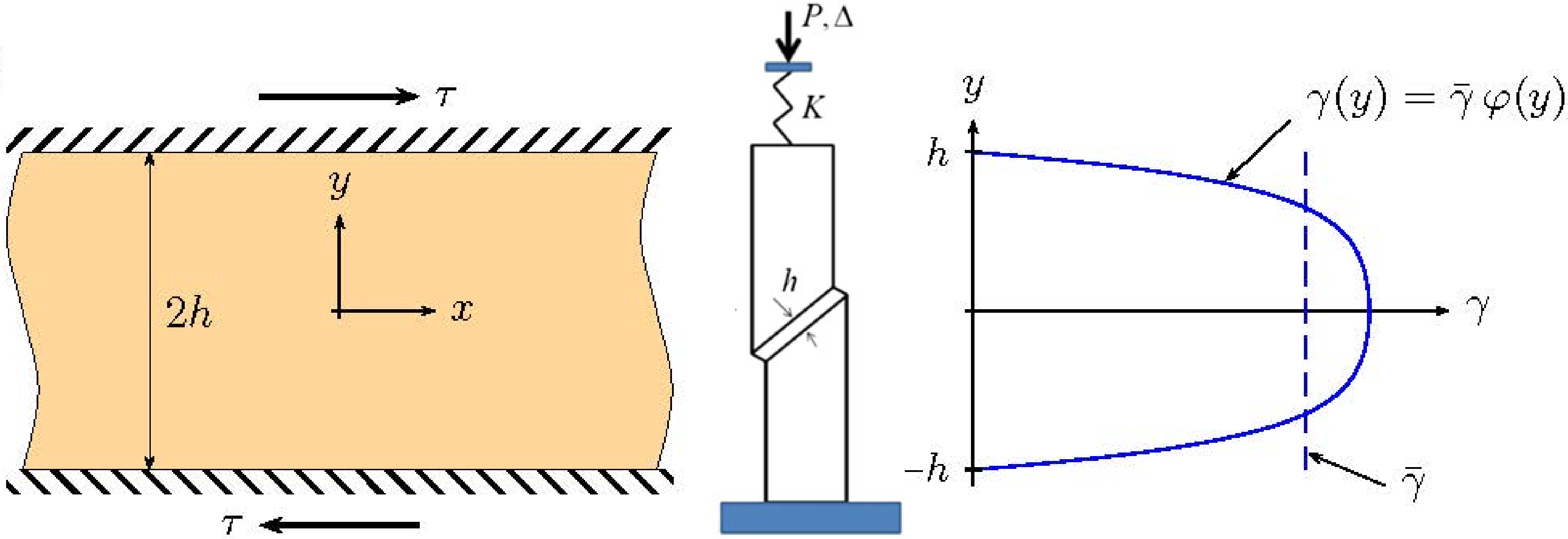

- Effective shear-yield stress: $\tau_\ell = \tau_0 \left(1 + \frac{\ell}{h}\right)$
- Linear scaling with inverse layer thickness → GAP!
- Linear scaling is a direct consequence of the differential structure of SGP (dependence on full strain gradient) → intrinsic limitation of SGP!

Fractional strain-gradient plasticity

- Extension: Allow for dependence of non-local energy on fractional derivatives of plastic strain

$$F(\boldsymbol{u}) = \int_\Omega \Big(\psi_{\mathrm{p}}(\varepsilon(\boldsymbol{u})) + \psi_{\mathrm{g}}(D^s \varepsilon(\boldsymbol{u})) \Big)\, dV + F.T.$$

- Recall: Fractional derivatives of order $0 \leq s \leq 1$ defined by means of Laplace transform (1D), Fourier transform, extension to higher dimensions...

- Representation by Gagliardo's formula:

$$\Psi_{\mathrm{g}}(\varepsilon) = \int_\Omega \int_\Omega \frac{B}{n+1} \frac{\ell^{s(n+1)}}{\varepsilon_0^{n+1}} \frac{|\varepsilon(\boldsymbol{x}') - \varepsilon(\boldsymbol{x}'')|^{n+1}}{|\boldsymbol{x}' - \boldsymbol{x}''|^{d+s(n+1)}}\, dV'\, dV''$$

- Energy non-local (double integral), fractional order of differentiation built directly into the kernel

C. Dahlberg & MO, European J. Mech. / A Solids 75 (2019) 348–354.

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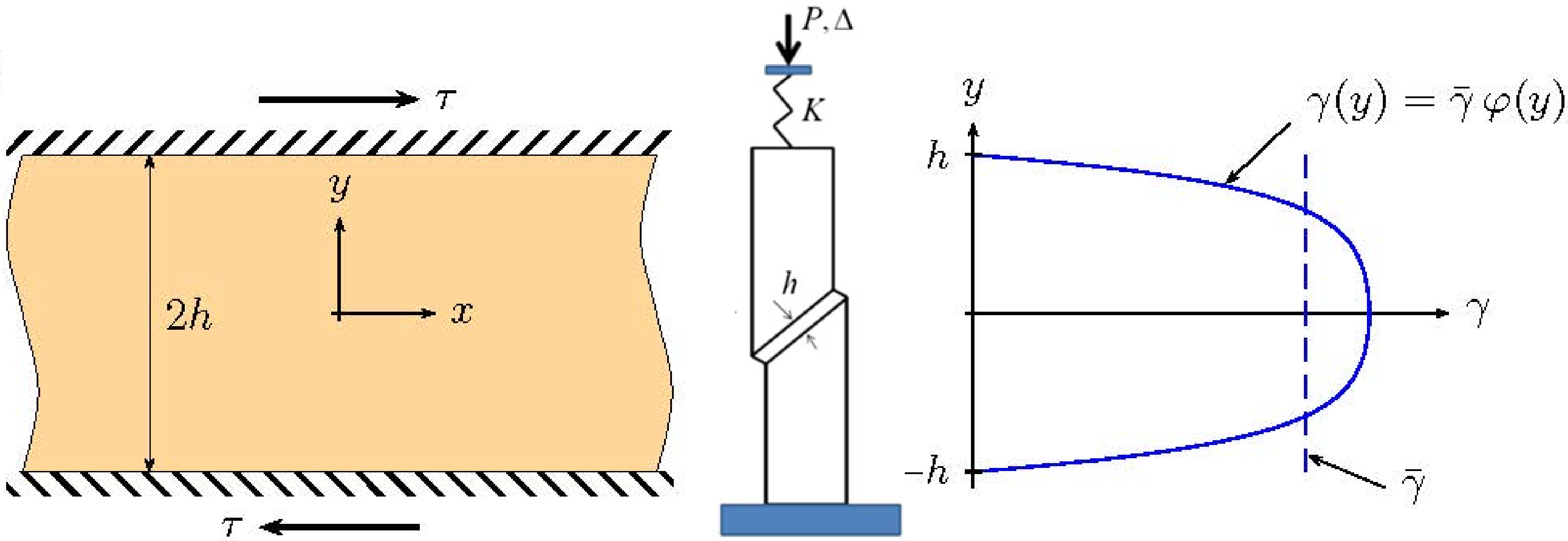
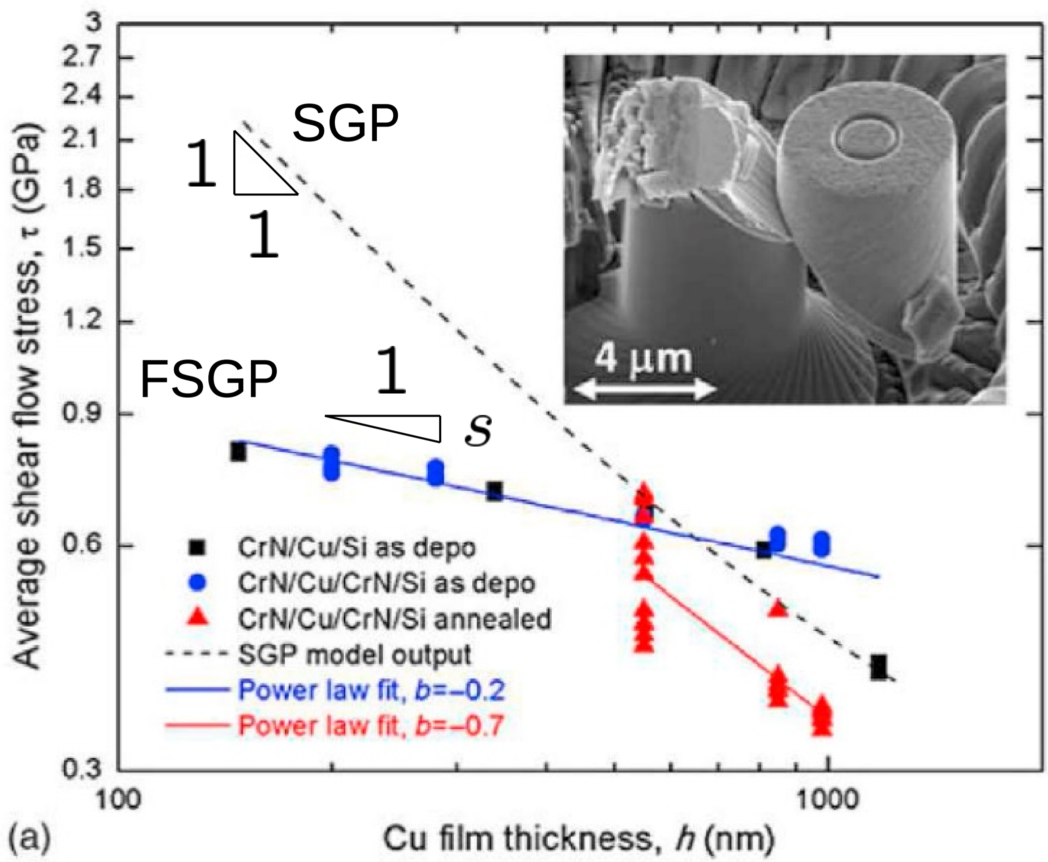
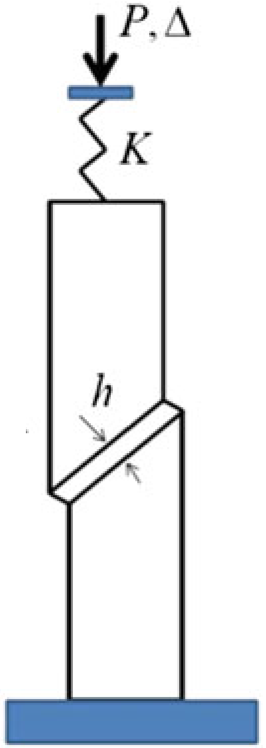


- Effective shear-yield stress: $\tau_\ell = \tau_0 \left(1 + \left(\frac{\ell}{h}\right)^s\right)$

The strength of confined Cu think films





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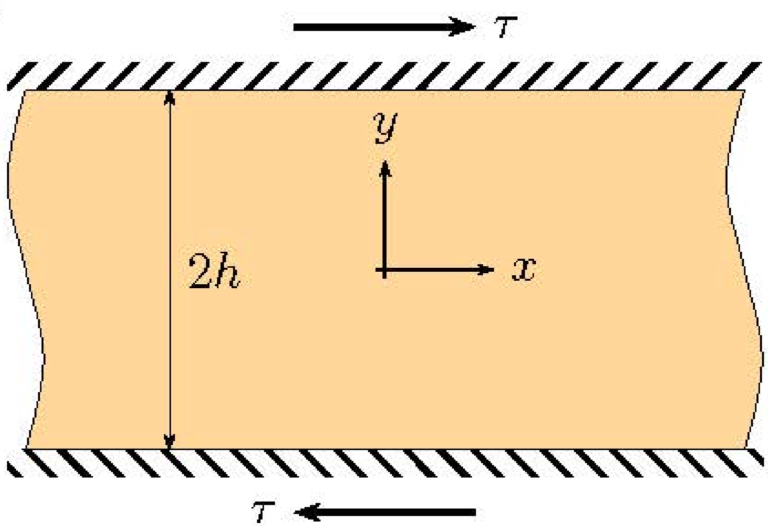
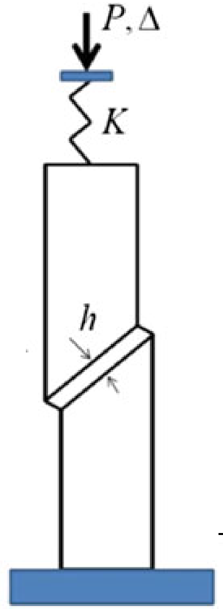
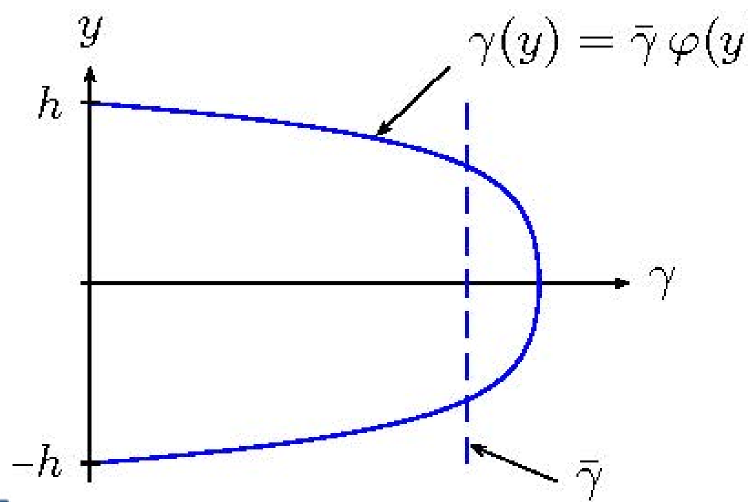
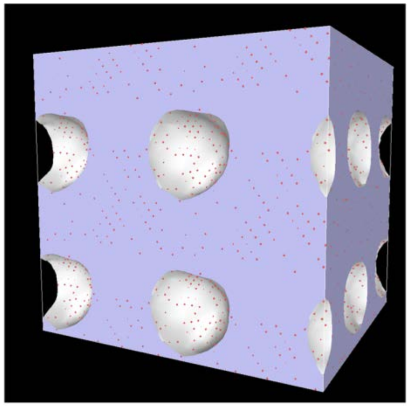






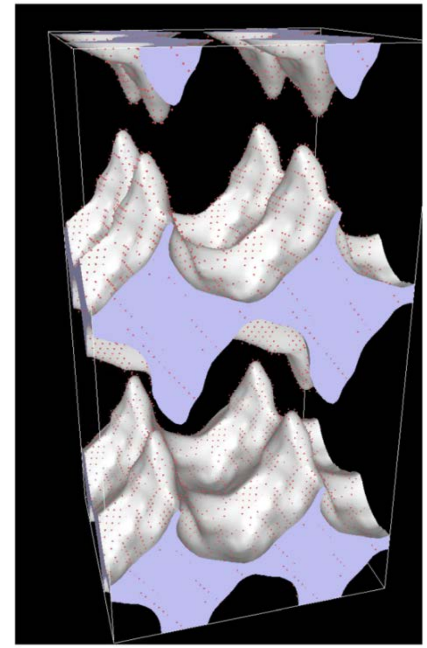
- Effective shear-yield stress: $\tau_\ell = \tau_0 \left(1 + \left(\frac{\ell}{h}\right)^s\right)$
- Can match the experimentally observed scaling of strength with layer thickness by choosing $s \sim 0.2$!
- Fractional strain-gradient plasticity closes the GAP between observed and predicted scaling!

FSGP *vs* SGP in ductile fracture



EAM Nickel, [111] loading, NPT 300K[1]

Ligament formation by void coalescence: Topological transition!



- Finite SGP energy: $\int_{\Omega} \psi_{\mathrm{g}}(D\varepsilon(\boldsymbol{u}))\, dV < +\infty \Rightarrow$

  continuous deformations, no topological transitions!
- Void coalescence precluded by SGP (it requires infinite energy), topological obstruction!

[1]M.I. Baskes and M. Ortiz, *JAM*, **82**: 071003-1-071003-5, 2015
Conti, S. & Ortiz, M. Arch Rational Mech Anal (2016) 219: 607

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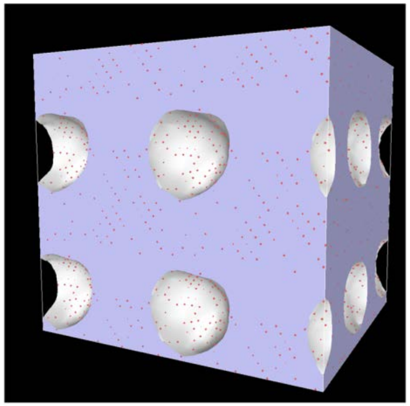

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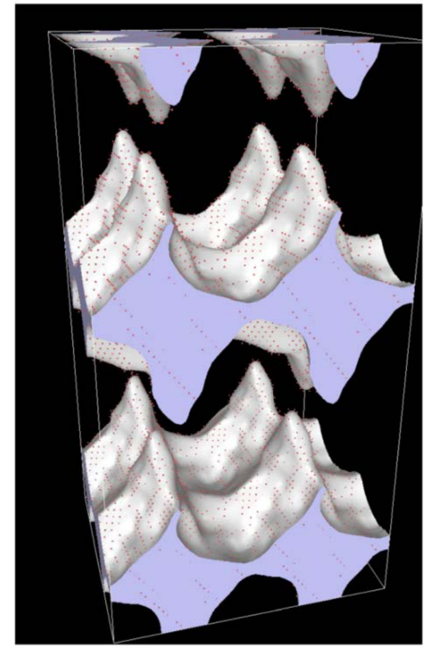

- Finite FSGP energy: $\int_\Omega \psi_{\rm g}(D^s\varepsilon(\boldsymbol{u}))\,dV < +\infty \Rightarrow$

  no continuity implied, topological transitions allowed!

- Void coalescence permitted under FSGP! (clears the topological obstruction of SGP)

[1]M.I. Baskes and M. Ortiz, *JAM*, **82**: 071003-1-071003-5, 2015
Conti, S. & Ortiz, M. Arch Rational Mech Anal (2016) 219: 607

Concluding remarks

- Fractional strain-gradient plasticity effectively closes the gap between observed and predicted size effect for yield strength
- Fractional strain-gradient plasticity permits topological transitions such as void coalescence, which are precluded by SGP
- Outlook, unresolved questions:
  - *Physical basis of FSGP? Stored energy?*
  - *Numerical implementation? Finite elements, phase-field models (coalescence)*
  - *Applications to ductile fracture, others...*

C. Dahlberg & MO, European J. Mech. / A Solids 75 (2019) 348–354.

Concluding remarks

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