

Problem Set 7

Due before class Thursday, 18 November 2010

Homework Problems:**1. Incandescent lights**

The filaments of incandescent light bulbs are made of refractory metals (e.g. Tungsten) so that when heated enough to radiate at optical wavelengths they don't sublimate too fast or melt.

- a) The resistance of a light bulb measured with a 3 V battery tester is about 10 times lower than it is when measured at 120 V line voltage. Why? Can you think of a consequence from your personal experience?
- b) Predict the length and thickness of the filament of a 100 W light bulb.

- 2. Fluorescent lights** A standard fluorescent light tube of the sort used in Caltech buildings is a cylinder about 1 m long, with an inside diameter of about 2 cm. Inside the tube is Argon at a pressure of 0.003 bar (1 bar is about standard atmospheric pressure), a 5 milligram drop of Mercury, and a cathode which emits electrons when heated.

First some facts about mercury: Mercury freezes at 234 K, boils at 630K, has a heat of vaporization of 65 cal/g, and has a vapor pressure at room temperature (293 K) of $p_{vap}(\text{Hg}, 293\text{K}) = 1.6 \times 10^{-6}\text{bar}$ ($= 1.2\text{mTorr}$). The first energy level of Mercury lies 4.89 eV above ground (corresponding to an ultraviolet photon of wavelength 253.7nm), and the ionization energy of Mercury is 10.44 eV. The Grotrian term diagram is shown in Figure 1. The oscillator strength of the 253.7nm transition is $f = 0.003$, and the Einstein A coefficient (inverse of the lifetime of the upper level) is $A = 8 \times 10^6\text{s}^{-1}$.

The lowest excited states of Argon are at 11.5 eV above ground, and Argon's ionization energy is 15.76 eV.

A consequence of all this is that electrons with kinetic energies $K_e < 4.89$ eV have purely elastic collisions with both Hg and Ar atoms. The elastic collision cross-section is of order the atom size, squared. Electrons of energies above the threshold of 4.89 eV can have inelastic collisions with neutral Hg atoms, in which they lose 4.89 eV and excite the Hg to its first excited state $\text{Hg} + e^- \rightarrow \text{Hg}^* + e^-$. The cross-section $\sigma_i(K_e)$ for these inelastic collisions is zero at threshold, but rises quickly to around $\sigma_i = 3\pi a_0^2$ (a_0 is the usual Bohr radius, 0.5 Angstrom) at electron energies $K_e = 6$ eV and then declines gradually with increasing electron energy, falling below πa_0^2 at $K_e > 15$ eV.

When the standard 110 Volts are applied, electrons from the cathode are accelerated until they ionize some of the mercury vapor creating more electrons. More numerous lower energy electrons accelerated in the electric field collide with Hg atoms, exciting them to the first excited state. These excited Hg atoms then decay by emitting a 4.89 eV UV photon, which tries to leave the tube. However, the walls of the tube are coated with crystals which absorb the UV photon and 86% of the time, fluorescently emit a photon of visible light, which is our ultimate goal.

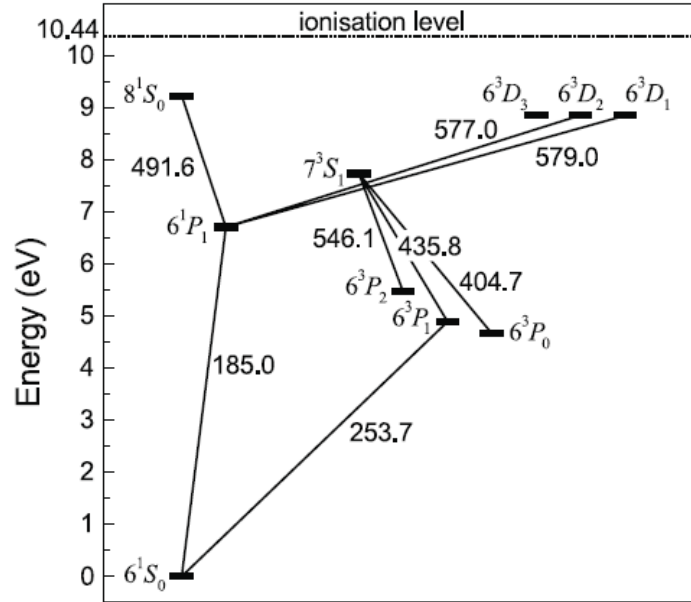


Figure 1: Term diagram of energy levels for Mercury. In this problem we will consider just the most important transition, the 253.7nm one to the lowest allowed level.

- Estimate the mean free path λ_e for elastic electron collisions with Argon.
- Show that in an elastic collision with an Argon atom, an electron on average loses a fraction $\sim m_e/M_{Ar} \sim 10^{-5}$ of its kinetic energy K_e . It also typically reverses its direction. Except for a centimeter near the cathode where the discharge is started, the electric field along the tube is about what you'd expect for a uniform gradient: $E = 110\text{V}/100\text{cm} \simeq 1\text{V cm}^{-1}$. If we neglected the loss, show that in the presence of the electric field, the electron would then diffuse in kinetic energy K , reaching an root-mean-square energy $K_e \sim eE\lambda_e\sqrt{n}$ after n collisions. Finally argue (or, if you know about them, solve a Fokker Planck equation to show) that if the elastic losses to Argon are included, the electrons will have an energy distribution with a typical value $K_e \sim eE\lambda_e(M_{Ar}/m_e)^{1/2} \sim 1.4\text{eV}$.
- The current through the fluorescent tube is measured to be 0.3 Amp. Estimate the electron density, assuming the characteristic electron energy from the previous part.
- If the tube operates at 40 C=313 K (note that the electron kinetic temperature, estimated above, is much higher than the atom kinetic temperature), estimate the density of gaseous Mercury (Hg) in the tube (hint: remember the argument given in class for the vapor pressure as a function of temperature), and check that there is plenty of liquid Hg left to maintain this.
- Another constraint on the electron temperature comes from the fact that the heating must be balanced by radiative cooling.

Compute the energy per unit volume per unit time transferred from electrons to Hg atoms in inelastic e^- -Hg collisions, keeping as free parameter the fraction f_e of the electrons with $K_e > 4.89\text{eV}$.

Show that even if $f_e = 1$, the rate of inelastic electron collisions an Hg atom experiences is small compared to the radiative decay rate A from the excited state. Thus (as long

as the radiation is not absorbed too much by other Hg atoms on the way out), the Hg atoms will spend most of their time in the ground state, and each collision will be quickly followed (long before any second collision) by a radiative decay, which will remove the energy from the plasma.

By equating the IV power per unit volume to the this cooling rate, calculate f_e . You should find that it is about what you would expect for a thermal electron distribution with the temperature you found in part (b).

- f) What would happen if the tube temperature were higher or lower (and hence the vapor-density set number density of Hg were much higher or much lower), so the “coincidence” of the last part did not happen?
 - g) The width of the 253.7nm line is approximately the sum of: the natural decay rate (A), the collision rate (with Argon atoms and with electrons) and the Doppler width set by the thermal motion of the Hg atoms. Which dominates?
 - h) Using your result from the previous part, use the relation derived in class for radiative absorption cross-sections $\int_0^\infty \sigma(\nu)d\nu = f\pi e^2/m_e c$ and the specified value of f to determine the cross-section for Hg in the ground state at 313 K to absorb the 253.7nm line (at line center). What is the optical depth of the tube (ratio of tube radius to the mean free path for the line absorption)? It is not desirable to have this be much larger than one. Can you figure out why?
 - i) As an upper limit to the operating temperature of the tube, estimate the tube temperature if it were in vacuum (with nearby walls at room temperature), and could cool only by radiation (tubes in normal rooms are more cooled by conduction and advection by the air around them).
3. **Nobel physics** Andrey Geim won the 2010 Nobel prize for isolating (using Scotch tape) and determining the remarkable electronic properties of graphene, a two-dimensional material composed of a single atomic layer of graphite.
- a) How many atoms thick is the graphite layer left by a pencil writing on a piece of paper?
 - i. Estimate this experimentally first
 - ii. Then, using equations given in class for absorption of electromagnetic radiation, estimate this theoretically.
 - b) How many words can a standard No. 2 pencil write (assume you don’t waste lead by over-sharpening)?
4. **Ig Nobel physics** Ten years before he won the Nobel prize, Andrey Geim won the 2000 Ig Nobel prize for magnetically levitating a small live frog (movies of this and various other levitations, including strawberries, waterdrops and hazelnuts can be viewed at <http://www.ru.nl/hfml/research/levitation/diamagnetic/>). JPL scientists last year levitated live mice in the guise of a microgravity experiment <http://dx.doi.org/10.1016/j.asr.2009.08.033> There are practical applications: growing perfect organic crystals for crystallography without the hassle and expense of going to a microgravity environment.

A Bitter magnet¹ replaces the wire coil of a solenoid with a helical staircase of vertically thin, radially wide copper plates separated by a similar staircase of insulator (see the exploded images at http://www.ru.nl/hfml/research/levitation/diamagnetic/bitter_solenoid/

¹I don’t know what they taste like, but they are named after Francis Bitter (1902-1967), a Caltech postdoc and later MIT Professor.

). As you should have found in the previous problem, cooling is the main limitation on the field strength, so holes are drilled through the plates, and (a really, really lot of) water is pumped through to advect the ohmic heat away. A hole along the axis of the staircase is left dry and empty for the test objects.

Geim's Bitter magnet had roughly the following dimensions. The copper plates had an inner radius of $R_1 \simeq 2\text{cm}$, an outer radius $R_2 \simeq 20\text{cm}$, and were stacked such that there are $n \simeq 10$ turns per cm. The length of the solenoid was $L \simeq 30\text{ cm}$. The magnet was oriented so that its axis was parallel to Earth's gravity.

- a) Where should you put the frog so it will levitate (top, middle, or bottom of solenoid)?
- b) Estimate the magnetic field required to levitate the frog. Give both an equation and a numerical estimate in Teslas or Gauss.
- c) How much power (in Watts) is dissipated in the electromagnet?
- d) How much cooling water (in liters or gallons per second) should be pumped through the magnet?
- e) Repeat (a) and (b) but for you instead of the frog. Is human levitation practical?