
Problem Set 7

Due in class Monday 18 May 2009

Homework Problems:

1. **Life on the Mississippi** In *Life on the Mississippi* (1883) Mark Twain famously wrote that “There is something fascinating about science. One gets such wholesale returns of conjecture out of such a trifling investment of fact.” This quote comes from the end of a typically outrageous (but less frequently quoted) description of scientific information about how the length of the river is changing over time because of erosion. In the spirit of Twain, here are some facts from which you are expected to obtain wholesale returns that are not entirely conjectural: As we estimated in class: the outflow of the river is about $2 \times 10^{10} \text{cm}^3 \text{s}^{-1}$, and it took Twain the better part of a year to float down the river. The sediment load is about 300 parts per million by volume of this flow
 - a) What size particles will stay in suspension during passage down the river? Ignore re-suspension (the plucking of previously deposited particles from the river bottom). To do this problem, you need to think through the consequences of turbulence. Contrary to what you might at first suppose, turbulence (in the form of random vertical motions of fluid eddies) does not change the order of magnitude of the timescale for the settling of most of the particles because the mass flux (per unit area) of settling must be about mnv in the laminar bottom boundary layer, where m is the particle mass, n is the number density of particles and v is the Stokes velocity (speed particles fall at low Re). There cannot be vertical motion of the fluid right next to the bottom. The main effect of the turbulence is to make n nearly uniform in the vertical column; it cannot keep the particles suspended forever (re-suspension is inefficient for particles as small as those implied by the answer to this question).
 - b) Would you expect a glass beaker of river water to appear clear or cloudy to the naked eye? How cloudy? (This turbidity can be expressed as optical depth. The standard method of estimating turbidity is in fact the one implied by this question, though one might use a light source other than the Sun of course). Assume that most of the sediment mass is in the largest particles that can remain suspended (i.e., the size given by the answer to part (i)).
 - c) Suppose this sediment load is supplied by reducing the altitude of the catchment area (the upstream source of the river). How long does it take to lower the altitude by 1 km (and thereby shut off the river, if nothing else were happening)?
 - d) Suppose that the method of making the particles in the sediment load requires doing work equivalent to the new surface energy created when converting a very large rock into a pile of small particles. What fraction of the gravitational energy flux of the river does this correspond to? (In reality, the particles are mostly clay, water chemistry is involved, and gravitational energy is not necessarily used to make the particles, but this approach is not wrong by orders of magnitude in respect of the energy required to make the particles.)

2. **Photoacoustics** Photoacoustics was discovered by Alexander Graham Bell. His device was somewhat more complicated than the following set-up for this problem, but the basic physics is the same. A square of material of area A is illuminated by a light beam of intensity $F_o(1 + \epsilon \sin(\omega t))$, with $\epsilon \ll 1$ (this assumption merely simplifies the calculation; it is not essential to the idea). It absorbs all this radiation and reradiates as a black body. Beneath this surface is material of thermal conductivity k , linear thermal expansion coefficient α , density ρ and specific heat C_p .
- What is the approximate thickness of the region that is heated by the oscillatory part of the heating (assuming this thickness is small compared to the total thickness of the medium.)?
 - What is the amplitude of the surface modulation resulting from this oscillatory heating?
 - What is the resulting acoustic radiation?
 - Assume $\omega = 2\pi(400\text{Hz})$, A is $10\text{cm} \times 10\text{cm}$, $\epsilon = 0.1$, and $F_o = 10^7\text{erg cm}^{-2}\text{s}^{-1}$ (corresponding to 100 watts of energy input on average). Would you be able to hear the output? (Assume the usual numbers for ambient air in the region near the heated surface). What is the efficiency in this case (i.e., the conversion of the energy input to acoustic output)?
3. **Boiling Water and Whistling Tea Kettles** It takes about 5 minutes to bring a liter of water to a boil on a kitchen stove. Many tea kettles come with whistles. The basic whistle is a hole of radius $\approx 0.15\text{cm}$ through which water vapor can exit the kettle.
- How much *power* is being absorbed by the water?
 - At what rate does the boiling water evaporate?
 - At what velocity does water vapor exit the hole when water is boiling inside the kettle?
 - What is the Reynolds number of the flow near the hole?
 - Why does the kettle whistle and what determines its frequency?
 - Which multipole dominates the acoustic radiation? Estimate the acoustic power.
4. **CO₂ Production in the classroom** CO₂ comprises about 350 ppmv (parts per million by volume) of the earth's atmosphere.
- What is the doubling time for the CO₂ concentration in our original classroom, 103 Downs? How does it compare to the time for the temperature to rise by 5 K? Assume that the classroom is sealed and every seat is occupied.
 - Now suppose that we unseal the vents, and a ventilation system steadily blows air at temperature 18 C (65 F) into 103 Downs.
 - If there are enough people in the room to raise the temperature to a steady state temperature of 24 C (75 F), by what fraction is the CO₂ concentration increased?
 - On what timescale should the ventilation system of 103 Downs completely replace the air, in order to prevent the temperature of the occupied room (50 students) from rising above 24 C (75 F)?
 - The atmospheric mixing ratio of CO₂ in the northern hemisphere has been measured for several decades. It displays an annual oscillation with peak to peak amplitude of about 5 ppmv. Provide a quantitative explanation for this effect.

5. **Watch physics** The quartz crystals in electronic wrist watches are 1 mm thick. They are excited (and monitored) piezoelectrically in their fundamental mode.
- a) Estimate the frequency at which they oscillate.
 - b) If you spend a lot of time outdoors, and your watch kept good time in summer, estimate its daily drift (seconds per day) in winter, if there were no compensation for changes in the quartz. Do you think the watch does have such compensation?
 - c) Predict the sign of the drift (ie does the watch run fast or slow in winter?). Be sure to consider all effects changing the crystal oscillation frequency.