27 April 2009 **Ph 101** E.S. Phinney & D.J. Stevenson

Order of Magnitude Physics

Problem Set 5

Due **in class** Monday 4 May 2009

Homework Problems:

1. **Piano physics** The upper 3/4 of piano strings are bare steel wires, stretched to the yield point of steel.

   a) Estimate the speed of transverse waves on such a piano string, and compare to the speed of sound in air.

   b) Estimate (using only the properties of steel) the length of a piano string whose fundamental frequency ν_1 is middle C (262Hz).

   c) By what distance does the A string (tuned to 440 Hz) on a grand piano sag in the middle owing to its own weight?

   d) Is the vibrating string a monopole, dipole or quadrupole source of radiation? What is the purpose of the piano soundboard?

   e) A pianist plays a fast and loud piano piece. Estimate the ratio of the mechanical energy she puts into the keys to the acoustic power radiated by the piano.

2. **Waves on Titan** Saturn's moon Titan ($g_T = 160\text{cm s}^{-2}$) is larger than the planet Mercury, and has many lakes, some hundreds of km across, and presumed to be filled with a methane-dominated liquid (density $\rho = 0.5\text{g cm}^{-3}$, surface tension $\gamma = 17\text{erg cm}^{-2}$). The depths of these lakes are not known but are inferred to be as much as hundreds of meters in some places; in other places they may be relatively shallow. Near surface winds on Titan are variable (just as on Earth) and are often only 1m/s or less (they were measured by the Huygens probe a few years ago), but may occasionally be much larger because of storms.

   a) The boiling point of methane at 1 bar pressure is 112K. Why is the surface tension of methane about a factor of three less than that of water?

   b) At approximately what wavelength and phase speed do the waves transition from ripples (surface tension dominance) to being dominated by gravity?

   c) Lakes will usually be deepest far from the shore and shallow as you approach the shore. Assume that observations can be made both when the winds are 1m/s and on the infrequent occasions when the winds are about 10m/s and assume that waves are excited when the wind speed is the phase velocity of the wave. Under what circumstances could you determine something about lake depth(s)? How?

   d) Titan is in an eccentric orbit about Saturn and as a consequence experiences tides (tidal period=orbital period $\sim$ 15 Earth days.) Is there any realistic situation in which a resonance might arise between the tidal forcing and waves? Hint: Titan, like Earth's moon and unlike Earth, has a rotation period equal to its orbital period, so the same hemisphere always faces Saturn. Thus you should think of the forcing as being approximately fixed in position in the sky; i.e., potentially capable of exciting a standing wave.

3. **Water bug propulsion and navigation**

a) The marine water strider *Halobates* has a mass of about 10mg and is about 5mm long (with legs of 2mm in length). It can not only walk on water but is able to jump several cm into the air. It does so by using both its non-wettable legs and non-wettable special hairs on its legs. (These hairs are very long and can support some stress.) What is the length of contact (between parts of its anatomy including hairs, and the water surface) required at take-off to perform these jumps?

b) The similar-sized beetle *Stenus* can float on water (i.e., like a boat, about one third submerged) and will, when necessary, propel itself at high speed by squirting a fluid out of its backside that lowers the surface tension dramatically. How does this provide propulsion? Estimate the peak velocity at which it can move.

c) What is the smallest amount of material it needs to excrete per unit time to maintain this velocity? Compare the thrust with what would be possible if the same mass flux of material were ejected as a gas of density similar to air ("jet" propulsion) assuming the overpressure is of order atmospheric pressure and the gas has a sound speed not very different from air.

d) Whirligig beetles (length 10mm) are able to echolocate through reflection of their ripples from obstacles nearby. (They create these waves as they bounce their abdomens on the surface of the water.) They have been observed to move as fast as 0.4m/sec on water (with their abdomens remaining partly in contact with the water). Roughly what is the peak speed at which their echolocation system will function? [Note: Some of these bugs also use echolocation to communicate their sex to other bugs. The male adds a high frequency flourish to the wave train of the ripples it sends. But this won't work if they're fast movers!]

4. **Exciting waves**

a) Why is it difficult to walk carrying a bowl of soup without spilling any? Be quantitative.

b) Estimate the energy per unit area stored in ocean waves. Express your answer in terms of seconds of solar insolation.

5. **Ship Wakes**

a) Prove that surface gravity waves generated by a ship in steady motion are confined to a wedge with opening angle $2\theta = 2\sin^{-1}(1/3)$.

b) How does the wavelength of forward propagating waves depend upon the speed of the ship?