

**Problem Set 3**

Due before class Thursday, 21 October 2010

**Homework Problems:**

1. **“The sun was in my eyes.”** This Monday afternoon, your instructor’s not-quite-so-beloved-as-before-Monday spouse drove her car up the driveway and into their wrought-iron gate, which had the misfortune to be solidly closed across the driveway at the time. For the purposes of this problem, the relevant structural elements of the gate are three rectangular iron bars, one above the other, each  $2\text{cm} \times 2\text{cm}$  in cross-section and about  $L = 200\text{cm}$  long, which were firmly anchored at the two sides of the driveway.

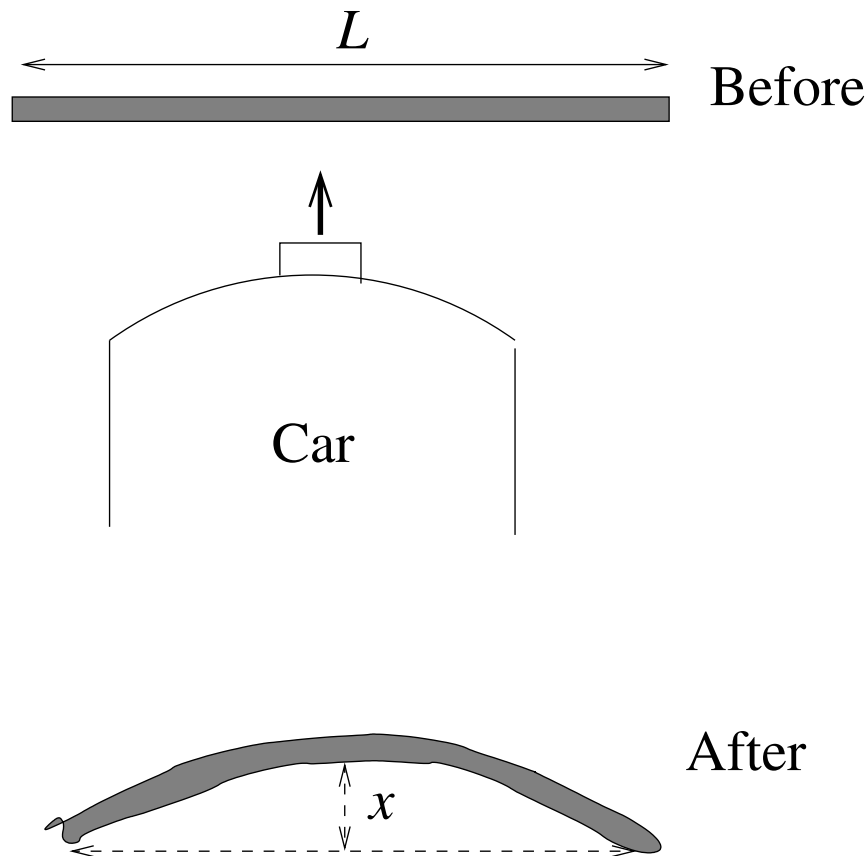


Figure 1: Top view of the gate, before (top) and after (bottom) the collision (problem 1). The 3 iron bars are arranged vertically above one another, so appear as a single shaded object.

- a) Estimate the kinetic energy of the car shortly before the license-plate holder at the front of the bumper contacted the iron bars barring its way (Figure 1, Before).

- b) If the iron bars are bent into a semi-circular arc, with central displacement  $x$  (Figure 1, After), estimate the dimensionless strain  $\epsilon$  in each bar in terms of  $x$ ,  $L$  and the bar thickness  $t = 2\text{cm}$ . Try to get an expression for  $\epsilon$  good to within a factor of 2.
- c) Estimate how large  $x$  must have been if all the kinetic energy of the car went into strain energy of the three bars.
- d) Given the value of the strain  $\epsilon$  for this  $x$ , did you successfully predict the observational result that the deformation of the bars was inelastic?
- e) Your instructor's spouse proposed that we fix the gate by closing it and leaning hard on the middle to push it back into shape. Would you expect this effort to succeed? Be quantitative.
- f) Did the airbag of the car deploy when it ran into the gate?

## 2. Bombs away!

- a) How large (give both size and mass) must a bomb be to fall supersonically through the air? [A problem with the WWII Norden bomb sight was that as bombs got bigger, no one knew how to calculate the trajectories of such supersonic bombs.]
- b) A cubical bomb of a type which is known to fall at about Mach 1 (i.e at the speed of sound in air) rests on the floor of the hanger before being loaded into the bomber. What pressure does it exert on the floor of the hanger, in atmospheres?

## 3. Cooling powerplants

At what rate is water used to cool a 1000 MW coal-fired powerplant? Consider two popular methods:

- a) "once-through", where river water is diverted into the plant, used to cool the steam, and released back into the river, and
- b) "evaporative cooling tower", in which water is evaporated from cooling fins in a large cooling tower (the city of Pasadena's are visible at the entrance to the 110 Freeway at Arroyo Blvd.).
- c) Which method would be preferred in dry climates (say 10cm annual precipitation, as in much of the Southwestern US)? In such a climate, what land area's runoff is required to supply the plants' needs?

## 4. Thermal Expansion

- a) Complete the calculation of the linear coefficient of thermal expansion,  $\alpha$ , sketched in class: i.e. for an anharmonic interatomic potential, where the energy  $\Delta E$  above the minimum  $-E_b$  is given by the displacement  $\Delta a \equiv xa$ , where  $a$  is the equilibrium separation, by

$$\frac{\Delta E}{E_b} \simeq \frac{x^2}{2} + f_T x^3 \quad (1)$$

(where for typical potentials  $f_T \sim 1/6 - 1/3$ ) show that for  $\Delta E = kT$ ,

$$\langle x \rangle \simeq \frac{kT}{E_b} f_T \quad (2)$$

and thus that the coefficient of thermal expansion is

$$\alpha \simeq \frac{k}{E_b} f_T \simeq 3 \times 10^{-5} \text{K}^{-1} . \quad (3)$$

- b) Cooking dishes are composed of special glasses that have low coefficients of expansion. For example, the linear coefficient of thermal expansion  $\alpha$ , in units of inverse centigrade degree, is  $1 \times 10^{-5}$  for ordinary glass,  $3 \times 10^{-6}$  for pyrex, and  $8 \times 10^{-7}$  for vycor. Why is ordinary glass inappropriate for cooking vessels? Be quantitative.

5. **Quantum liquids** The binding energies of atoms in the liquid phase are given by the heat of vaporisation (the binding energy in the solid state is given by the heat of sublimation, which is typically  $\sim 1.1$  times larger).

Substance	He	H <sub>2</sub>	Ne	Ar	N <sub>2</sub>	Kr	H <sub>2</sub> O	Hg	Pt
Heat of vaporisation (kJ/mole)	0.09	0.9	1.8	6	6	9	44	59	509

Note that  $1 \text{ kJ/mole} = 10^{-2} \text{ eV}/(\text{atom or molecule}) = k(120\text{K})$ .

- a) Estimate the melting temperatures of the substances in the table.
- b) For each substance in the table, calculate the de Broglie wavelength of the atom or molecule at its melting temperature. Use this to explain why, of all elements and molecules, only He (and to a lesser extent H<sub>2</sub>) show interesting quantum effects in their liquid state.