

Problem Set 2

Due in class Wednesday 15 April 2009

Homework Problems:

1. **Viscosity.** The dynamic viscosity η appears in the Navier-Stokes equations for the incompressible motion of a fluid:

$$\rho \frac{\partial \vec{u}}{\partial t} + \rho(\vec{u} \cdot \nabla)\vec{u} + \nabla p - \eta \nabla^2 \vec{u} = \rho \vec{g} \quad (1)$$

where $\rho(t, \vec{x})$ is the mass density (mass per unit volume), $\vec{u}(t, \vec{x})$ is the fluid velocity and $p(t, \vec{x})$ is the pressure.

- a) Determine the dimensions of η .
- b) The fluid with dynamic viscosity η is composed of hard spherical atoms or molecules of radius a , mass m , temperature T and number density n (number per unit volume $=\rho/m$). Adding Boltzmann's constant k to the mix, use Buckingham's Π theorem to determine the number of independent dimensionless quantities that can be formed from these variables, and give expressions for one possible choice for these dimensionless quantities.
- c) For dilute gases ($na^3 \ll 1$) η becomes independent of n . Show that in this limit, Buckingham's Π theorem determines η up to a purely numerical constant (which turns out to be $5\sqrt{\pi}/(64\pi)$).
- d) Estimate the viscosity of air at sea level, and compare to the value given in the Purcell sheet.
- e) Repeat part (b) if the gas is not composed of hard spheres but instead is a plasma composed of point-like charges $\pm e$ (in Gaussian units where the force between a proton and electron at distance r is $-e^2/r^2$), so that η depends on e, m, T, n, k .
- f) Repeat part (c) for the low density limit of the plasma, when η becomes independent of n . Estimate the Reynolds number of a galaxy-sized sphere (20,000 parsecs in diameter -see Purcell sheet if you don't know what a parsec is) moving at 400km/s through the ionized hydrogen plasma of a cluster of galaxies ($n_e = n_p = 0.01\text{cm}^{-3}$, $T = 5 \times 10^7\text{K}$).
- g) In part (e), what is the dimensionless Π analogous to na^3 in part (c), which must be less than 1 for the low density limit to apply? Give a physical interpretation of this number.

2. Gravitational radiation

- a) By direct analogy to the class discussion of quadrupole electromagnetic radiation, find an estimate for the power in quadrupole gravitational radiation radiated by a quadrupole μa^2 rotating at angular frequency ω ¹ [Comment: In order to do this problem, you

¹For two masses m_1 and m_2 separated by distance a in circular orbit about their center of mass, the mass quadrupole moment is μa^2 , where $\mu = m_1 m_2 / M$ and $\omega = \sqrt{GM/a^3}$, where $M = m_1 + m_2$ is the total mass, but you don't actually need this more detailed information to do the problem.

do not need to know anything about general relativity, except that the gravitational wave propagation is at the speed of light and that because of mass conservation and momentum conservation, the lowest permitted order of radiation is quadrupolar —i.e. there is no monopole or dipole radiation.]

- b) As a check on your answer, verify that your formula, applied to two equal-mass black holes orbiting each other at their horizon radius (the only length you can construct from M , G and c) predicts (correctly) that they radiate a fraction of order unity of their entire mass energy in gravitational radiation in a light-crossing time (the only time you can construct from M , G and c). Which is larger: the gravitational wave luminosity of this single pair of merging black holes, or the electromagnetic luminosity of the entire visible universe, which contains $\sim 10^{10}$ galaxies each composed of $\sim 10^{10}$ sun-like stars (see Purcell sheet for L_{\odot} , the luminosity of the sun)?
- c) In the case of the Earth-Sun system, the quadrupole can be estimated to be the Earth mass times one astronomical unit squared (Why is the Sun's mass not important?). Calculate the power radiated in gravitational radiation by the Earth-Sun binary and compare it with the average power you radiate as heat (burning the food you eat).
- d) How long would it take for Earth to spiral into the Sun because of this gravitational radiation?

3. **Blowing things up more thoroughly** In the Sedov-Taylor calculation of the radius $R(t)$ of an explosion of energy E in a medium of density ρ given in class, we neglected the mass in the bomb fragments and the pressure in the ambient air.

- a) For the Trinity test ($M_{bomb} = 5\text{tons}$, $E \simeq 25\text{kton TNT} \simeq 10^{21}\text{erg}$), calculate the minimum radius R_{min} for which it is appropriate to neglect the bomb's mass.
- b) Calculate the maximum radius R_{max} of the Trinity test for which it is appropriate to neglect the air pressure.
- c) What approximately is the Mach number ($dR/dt/c_s$, where c_s is the sound speed of the undisturbed air ahead of the blast) of the blast at R_{max} ?
- d) For $R > R_{max}$, the blast continues to propagate, but now as a weak shock wave, whose peak overpressure and postshock wind speed behaves (except for a logarithmic factor which can be neglected, since as Fermi pointed out, all logarithms appearing in physics are 10 to order of magnitude) like a sound wave, $\Delta p \propto \Delta v \propto 1/R$. Given what you know about hurricanes and tornadoes, out to what R do you think a bomb like the one in the Trinity test would destroy buildings?

4. **Heat transport** A thin, flat sheet of material, thickness d , is heated on one surface so that it absorbs power per unit area (energy flux) F . In steady state, it reradiates part of this energy as a black body at temperature T_1 and the remainder is conducted through to the other side in accordance with Fourier's law of heat conduction (heat flux is thermal conductivity κ times temperature gradient). This second surface is not heated but radiates as a black body at temperature T_2 . In accordance with the Buckingham Pi theorem construct a general solution for T_1/T_2 in terms of the other stated variables and the Stefan-Boltzmann constant σ . What are the expected limits of the formula you obtain for d very small and d very large? For the particular case where F is the luminosity of the Sun at Earth orbit and the sheet has the thermal conductivity of (a) copper, (b) a typical insulator (all these numbers are on your Purcell sheet), find the approximate thickness of the sheet for which there is a

transition between these limiting behaviors. (Note: The relevant equations can be written down explicitly but the intent here is that you use dimensional analysis to solve the general form of the solution and then physical reasoning to recognize the value of d for which there is a transition in behavior).

5. **Interstellar exploration** Would the energy in all the calories you have consumed as food be enough to eject you from the solar system?
6. **Invent your own** Invent a problem of your own (you don't have to know the answer). The most interesting problems submitted will be done in class, or assigned as homework in subsequent problem sets. Your problem can be like those above, or more general.