

Consider the variational integral for the shortest light time between two points in a generalized 2D space:

$$T = \frac{1}{c} \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{d}{dx} y(x) \right)^2} dx$$

If we consider a function which distorts normal Euclidean space in the following way:

$$P(x, y(x)) = 1 - \frac{\alpha \cdot mG}{v^2 \sqrt{x^2 + y(x)^2}}$$

where to make physical sense this distortion must be a dimensionless function, and it should distort space with a $1/r$ dependence. It is formed like $1-A$ so that at large 'r', the distortion of space is unity. If we consider a photon of speed 'c' scattering through a field created by a mass 'm' then the Buckingham- π Theorem gives:

$$P(x, y(x)) = 1 - \frac{2mG}{c^2 \sqrt{x^2 + y(x)^2}}$$

Then the function has the form:

$$T = \frac{1}{c} \int_{x_1}^{x_2} \frac{\sqrt{1 + \left(\frac{d}{dx} y(x) \right)^2}}{\left(1 - \frac{2mG}{c^2 \sqrt{x^2 + y(x)^2}} \right)} dx \rightarrow L = \frac{\sqrt{1 + \left(\frac{d}{dx} y(x) \right)^2}}{\left(1 - \frac{2mG}{c^2 \sqrt{x^2 + y(x)^2}} \right)}$$

then...

$$\frac{\partial L}{\partial y} - \frac{d}{dx} \left(\frac{\partial L}{\partial y_x} \right) = 0$$

where this gives the following DE:

$$0 = \frac{2mG(y_x \cdot x - y)}{(x^2 + y^2)^{3/2} c^2 - 2mG(x^2 + y^2)} + \frac{y_x^2 y_{xx}}{(1 + y_x^2)^2} - y_{xx}$$

For realistic values of 'm', 'x₁' and 'x₂', DE yields the following (multiple masses, photonic pathes)

