If LaGrange Could Drive

Tristan S. Ursell & Joseph A. Schreier

Discussion:
-Assume a one-wheeled vehicle, i.e. no suspension or weight transfer
-Assume the vehicle path is described by two orthogonal functions x(t) and y(t)
-The most effective driving occurs when the cars is *always* at the traction limit, from both turning and linear acceleration then….

Nomenclature and Equations:
M = traction limit (m/s^2)

$\vec{v}(t) = (\dot{x}, \dot{y})$ (m/s)

$\vec{a}(t) = (\ddot{x}, \ddot{y})$ (m/s^2)

$v(t) = \sqrt{\dot{x}^2 + \dot{y}^2}$ (m/s)

$a(t) = \sqrt{\ddot{x}^2 + \ddot{y}^2}$ (m/s^2)

$$R(t) = \frac{(\dot{x}^2 + \dot{y}^2)^{3/2}}{\dot{x}\ddot{y} - \dot{y}\ddot{x}} = \frac{\left\|\vec{v}(t)\right\|^3}{\vec{v}(t) \times \vec{a}(t)} = \frac{v(t)^3}{\dot{x}\ddot{y} - \dot{y}\ddot{x}} \quad \text{(m)}$$

$$a_c(t) = \frac{v(t)^2}{R} = \frac{\vec{v}(t) \times \vec{a}(t)}{\left\|\vec{v}(t)\right\|} = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{v(t)} \quad \text{(m/s}^2\text{)}$$

The traction limit equation:

$$\mathrm{M}^2 = a_c(t)^2 + a(t)^2$$

The U-Gauge equation is:

$$U = P_{track}(x(t), y(t)) \cdot P_{acc}(v(t), a(t)) \cdot P_{traction}(a_c(t)^2 + a(t)^2), x(t), y(t)) \quad \text{(unit-less)}$$

The governing equation: (presumed)

$$T = \int_{t_i}^{t_f} \frac{ds}{v} = \int_{t_i}^{t_f} \frac{\sqrt{\frac{dx^2}{dt^2} + \frac{dy^2}{dt^2}}\,dt}{\sqrt{\dot{x}^2 + \dot{y}^2} \cdot U(x, y, \dot{x}, \dot{y}, \ddot{x}, \ddot{y})} = \int_{t_i}^{t_f} \frac{\sqrt{\dot{x}^2 + \dot{y}^2}}{\sqrt{\dot{x}^2 + \dot{y}^2} \cdot U(x, y, \dot{x}, \dot{y}, \ddot{x}, \ddot{y})} dt$$

$$T = \int_{t_i}^{t_f} \frac{dt}{U(x, y, \dot{x}, \dot{y}, \ddot{x}, \ddot{y})}$$

$$L(t, x_i, \dot{x}_i, \ddot{x}_i) = U(x, y, \dot{x}, \dot{y}, \ddot{x}, \ddot{y})^{-1}$$

$$\frac{\partial}{\partial x_i} L - \frac{d}{dt}\left(\frac{\partial}{\partial \dot{x}_i} L\right) + \frac{d^2}{dt^2}\left(\frac{\partial}{\partial \ddot{x}_i} L\right) = 0 \quad \rightarrow \quad \frac{1}{U^2}\frac{\partial}{\partial x_i} U - \frac{d}{dt}\left(\frac{1}{U^2}\frac{\partial}{\partial \dot{x}_i} U\right) + \frac{d^2}{dt^2}\left(\frac{1}{U^2}\frac{\partial}{\partial \ddot{x}_i} U\right) = 0$$