Online Appendix to “A Hybrid Incentive Scheme: Promotion beyond Pay for Performance”

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November 3, 2016

This online appendix contains all proof of the paper, “A Hybrid Incentive Scheme: Promotion beyond Pay for Performance” and details of the firm’s problem in Section 7.1, 7.2 and 9.

Appendix A  Proofs

A.1  Derivation of $P(e_{Mi})$

Note that

$$P(e_{Mi}) = \sum_{j=0}^{N-1} \frac{1}{j+1} \binom{N-1}{j} s(e_{Mi})^j (1 - s(e_{Mi}))^{N-1-j}$$

$$= \frac{1}{Ns(e_{Mi})} \sum_{j=0}^{N-1} \binom{N}{j+1} s(e_{Mi})^{j+1} (1 - s(e_{Mi}))^{N-1-j}$$

$$= \frac{1}{Ns(e_{Mi})} \left( \sum_{s=0}^{N} \binom{N}{s} s(e_{Mi})^s (1 - s(e_{Mi}))^{N-s} - (1 - s(e_{Mi}))^N \right)$$

$$= \frac{1 - (1 - s(e_{Mi}))^N}{Ns(e_{Mi})}.$$ 

A.2  Proof of Lemma 1

Denote the firm’s profit when the CEO’s IR condition binds as $\Pi$ Then, the firm’s optimal profit must be greater than or equal to $\Pi$.

First, for an action $(e_C^*, e_M^*) \in [0, 1] \times [0, 1]$, I show that there exists a solution $(W_G^C, W_B^C, W_G^M, W_B^M)$ to the firm’s problem. Note that when the CEO’s IR condition binds the firm’s problem is reduced to the case of Grossman and Hart (1983). Hence, a solution exists to this restricted problem. Now, I show that I can artificially bound the constraint set of $(W_G^C, W_B^C, W_G^M, W_B^M)$. They are bounded below by two IR
conditions. Moreover, they are also bounded above since the firm’s optimal profit is lower than $\Pi$ and the firm’s profit is a strictly decreasing function in all four components in $(W^G, W^B, W_g, W_b)$ without a lower bound. Also, the constraint set is closed according to two IC and two IR conditions. Hence, there exists a solution by the Extreme value theorem. The remaining proof exactly follows the proof in Grossman and Hart (1983).

### A.3 Proof of Lemma 2

Suppose this is not true. That is,

$$s(e_M)u(W^G_M) + (1 - s(e_M))u(W^B_M) - g(e_M) + s(e_M)P(e-M)VC > \underline{U}_M.$$  

Then, choosing new wage scheme $(\tilde{W}^G_M, \tilde{W}^B_M) = (W^G_M - \epsilon_1, W^B_M - \epsilon_2)$, where $\epsilon_1 > 0$ and $\epsilon_2 > 0$, satisfying

$$u(W^G_M) - u(W^B_M) = u(\tilde{W}^G_M) - u(\tilde{W}^B_M)$$ and

$$s(e_M)u(\tilde{W}^G_M) + (1 - s(e_M))u(\tilde{W}^B_M) - g(e_M) + P(e-M)VC \geq \underline{U}_M$$

gives a higher profit to the firm without affecting other constraints. Hence, the wage scheme $(W^G_M, W^B_M)$ is not optimal.

### A.4 Proof of Lemma 3

Note that

$$\frac{\partial F(V)}{\partial V} = -\frac{1}{u'(W_C)} + \frac{1}{u'(W_M)}$$

using the envelope theorem.
Differentiating this with respect to $\mathcal{V}$ gives

$$\frac{\partial^2 F(\mathcal{V})}{\partial \mathcal{V}^2} = \frac{u''(W_C)}{u'(W_C)^3} + \frac{1}{N} \frac{u''(W_M)}{u'(W_M)^3} + \frac{u''(W_C)}{u'(W_C)^3} g'(e_C^*) \frac{\partial e_C^*}{\partial \mathcal{V}} - \frac{u''(W_M)}{u'(W_M)^3} g'(e_M^*) \frac{\partial e_M^*}{\partial \mathcal{V}}$$

$$= \frac{u''(W_C)}{u'(W_C)^3} - \frac{u''(W_C)}{u'(W_C)^3} g'(e_C^*) \frac{u''(W_C)}{u'(W_C)^2} g'(e_C^*)^2 - g''(e_C^*)$$

$$+ \frac{1}{N} \frac{u''(W_M)}{u'(W_M)^3} \frac{1}{u'(W_M)^2} g'(e_M^*) \frac{1}{u'(W_M)^2} g'(e_M^*)^2 - g''(e_M^*)$$

$$= \frac{u''(W_C)}{u'(W_C)^3} \left( 1 - \frac{u''(W_C)}{u'(W_C)^2} g'(e_C^*)^2 - g''(e_C^*) \right)$$

$$+ \frac{1}{N} \frac{u''(W_M)}{u'(W_M)^3} \left( 1 - \frac{u''(W_M)}{u'(W_M)^2} g'(e_M^*)^2 - g''(e_M^*) \right)$$

$$< 0.$$

That is, $F(\mathcal{V})$ is a strictly concave function.

### A.5 Proof of Proposition 1

It is enough to show that

$$\frac{\partial^2 F(\mathcal{V})}{\partial N \partial \mathcal{V}} > 0.$$

Notice that

$$\frac{\partial^2 F(\mathcal{V})}{\partial N \partial \mathcal{V}} = -\frac{1}{N^2 u'(W_M)^3} \frac{u''(W_M)}{u'(W_M)^3} g'(e_M^*) \left( -\frac{\beta(G_M - B_M)}{\beta^2(G_M - B_M)^2 u''(W_M)} \right) \mathcal{V}$$

$$= -\frac{1}{N^2 u'(W_M)^3} \left( 1 - \frac{\beta(G_M - B_M)}{\beta^2(G_M - B_M)^2 u''(W_M)} g'(e_M^*) \right) \mathcal{V}$$

$$= -\frac{1}{N^2 u'(W_M)^3} \left( 1 - \frac{\beta^2(G_M - B_M)^2 u''(W_M)}{\beta^2(G_M - B_M)^2 u''(W_M)} g'(e_M^*) \right) \mathcal{V}$$

$$> 0.$$
A.6 Proof of Corollary 2

Suppose this is not the case. From the first order condition

\[ \beta (G_C - B_C) = \frac{g'(e_C)}{u'(W_C)}, \]

it can be shown that \( e_C^* \) and \( W_C^* \) move in the opposite direction since the left hand side is a constant. Therefore, \( e_C^* \) should increase if \( W_C^* \) decreases. Since \( \psi^* \) increases as \( N \) increases, \( W_C^* \) must increase according to

\[ u(W_C^*) = \psi^* + g(e_C^*). \]

This contradicts the premise that \( W_C^* \) decreases. Hence,

\[ \frac{\partial W_C^*}{\partial N} > 0. \]

A.7 Proof of Proposition 2

Recall that the first order condition is

\[ -\frac{1}{u'(W_C^*)} + \frac{1}{u'(W_M^*)} = 0. \]

Hence, \( W_C^* \) should be the same as \( W_M^* \).

If \( G_C - B_C = G_M - B_M \), using the previous result and two first order conditions, it can be shown that

\[ g'(e_C^*) = g'(e_M^*). \]

That is, \( e_C^* = e_M^* \).

Also, this result and the two individual rationality conditions imply that

\[ u(W_M) - g(e_M) + \frac{1}{N} \psi^* = \psi^* + \frac{1}{N} \psi^* = U_M. \]

Hence,

\[ \psi^* = \frac{N}{N+1} U_M. \]
A.8 Proof of Proposition 3

Note that when agents are risk-neutral
\[
\frac{\partial F(V)}{\partial V} = -s(e_C) - (1 - s(e_C)) + N \cdot s(e_M)P(e_{-M})
\]
\[
= -1 + (1 - (1 - s(e_M))^N)
\]
\[
< 0
\]
using the envelope theorem.\textsuperscript{54} Hence, the firm’s profit decreases as the level of \(V\) increases.

A.9 Proof of Lemma 4

Using the envelope theorem,
\[
\frac{\partial F(V)}{\partial V} = \frac{s(e_C)}{u'(W_G^C)} - \frac{1 - s(e_C)}{u'(W_B^C)} + Ns(e_M)P(e_{-M})\frac{1}{u'(W_M^G)}.
\]

Differentiating this with respect to \(V\) gives
\[
\frac{\partial^2 F(V)}{\partial V^2} = s(e_C)\frac{u''(W_G^C)}{u'(W_G^C)^2} + (1 - s(e_C))\frac{u''(W_B^C)}{u'(W_B^C)^3} + Ns(e_M)P(e_{-M})\frac{2}{u'(W_M^G)}\frac{u''(W_M^G)}{u'(W_M^G)^3} < 0.
\]

A.10 Proof of Proposition 4

It is enough to show that
\[
\frac{\partial^2 F(V)}{\partial N \partial V} > 0.
\]

Note that
\[
\frac{\partial^2 F(V)}{\partial N \partial V} = -(1 - s(e_M))^N \log(1 - s(e_M)) \frac{1}{u'(W_M^G)}
\]
\[
+ [1 - (1 - s(e_M))^N] \frac{\partial P(e_{-M})}{\partial N}\frac{u''(W_M^G)}{u'(W_M^G)^3} > 0,
\]

where
\[
\frac{\partial P(e_{-M})}{\partial N} = \frac{1}{s(e_M)^N} \left[- \log(1 - s(e_M))(1 - s(e_M))^N - 1 + (1 - s(e_M))^N\right]
\]
\[
< 0
\]
since \(k(s) \equiv - \log(1 - s)(1 - s)^N - 1 + (1 - s)^N\) is equal to zero when \(s = 0\) and \(k'(s) < 0\). Here, I use the condition that \(V \geq 0\).

\textsuperscript{54}In equilibrium, \(s(e_M)\) is equal to \(s(e_{-M})\) since I am considering a symmetric equilibrium.
A.11 Proof of Corollary 4

Denote the expected compensation to CEO by $E[W_C]$, 

$$E[W_C] = s(e_C)W_C^G + (1 - s(e_C))W_C^B.$$ 

Since $\frac{\partial V^*}{\partial N} > 0$, it is enough to show that 

$$\frac{\partial E[W_C]}{\partial V} = \frac{s(e_C)}{u'(W_C^G)} + \frac{1 - s(e_C)}{u'(W_C^B)} > 0.$$ 

A.12 Proof of Corollary 5

I need to show that the wage gap 

$$[s(e_C)(W_C^G)^* + (1 - s(e_C))(W_C^B)^*] - [s(e_M)(W_M^G)^* + (1 - s(e_M))(W_M^B)^*]$$ 

widens as $N$ increases. Since $(1 - s(e_M))(W_M^B)^*$ has a fixed value regardless of the number of managers, it is enough to show that 

$$[s(e_C)(W_C^G)^* + (1 - s(e_C))(W_C^B)^*] - s(e_M)(W_M^G)^*$$ 

is an increasing function in $N$. When agents have the log utility function, the first order condition with respect to $V$ is 

$$s(e_C)(W_C^G)^* + (1 - s(e_C))(W_C^B)^* = (1 - (1 - s(e_M))^N)(W_M^G)^*.$$ 

Since the left hand side of the equation is a strictly increasing function in $V$ and $\frac{\partial V^*}{\partial N} > 0$, this side strictly increases as $N$ increases. Hence, 

$$\frac{\partial}{\partial N}(1 - (1 - s(e_M))^N)(W_M^G)^* = -\log(1 - s(e_M))(1 - s(e_M))^N(W_M^G)^*$$ 

$$+ (1 - (1 - s(e_M))^N)\frac{\partial (W_M^G)^*}{\partial N} > 0.$$ 

Since 

$$[s(e_C)(W_C^G)^* + (1 - s(e_C))(W_C^B)^*] - s(e_M)(W_M^G)^* = (1 - s(e_M) - (1 - s(e_M))^N)(W_M^G)^* \quad (10)$$
\[
\frac{\partial}{\partial N} \left\{ \left[ s(e_C)(W^G_C)^* + (1 - s(e_C))(W^B_C)^* \right] - s(e_M)(W^G_M)^* \right\} = \\
- \log(1 - s(e_M))(1 - s(e_M))^N(W^G_M)^* + (1 - s(e_M) - (1 - s(e_M))^N) \frac{\partial (W^G_M)^*}{\partial N} \\
> - \log(1 - s(e_M))(1 - s(e_M))^N(W^G_M)^* + (1 - s(e_M) - (1 - s(e_M))^N). \\
\log(1 - s(e_M))(1 - s(e_M))^N(W^G_M)^* \\
= - \frac{s(e_M)}{1 - (1 - s(e_M))^N} \log(1 - s(e_M))(1 - s(e_M))^N(W^G_M)^* \\
> 0
\]
when \(N > 1\). Also, when \(N = 1\), the wage gap is equal to zero according to 10. On the other hand, the gap has a positive value when \(N = 2\) since \((W^G_M)^* > 0\). Therefore, the expected compensation gap is a strictly increasing function in \(N\).

**A.13 Proof of Proposition 5**

First, I consider a case when \(e_C = e_M\).

Note that

\[ F(U_M|N) = \frac{s(e_C)}{u'(W^G_C)} - \frac{1 - s(e_C)}{u'(W^B_C)} + \frac{1 - (1 - s(e_M))^N}{u'(W^G_M)}, \]

where

\[ u(W^G_C) = U_M + g(e_C) + (1 - s(e_C)) \frac{g'(e_C)}{h'(e_C)}, \]

\[ u(W^B_C) = U_M + g(e_C) - s(e_C) \frac{g'(e_C)}{h'(e_C)}, \]

and

\[ u(W^G_M) = U_M + g(e_M) + (1 - s(e_M)) \frac{g'(e_M)}{h'(e_M)} - P(e_M)U_M. \]

Denote the difference between \(1/u'(W^G_C)\) and \(1/u'(W^B_C)\) by \(D\).

For given \((1 - s(e_C))D > \epsilon > 0\), there is \(\tilde{N}\) such that

\[ \frac{1}{u'(W^G_C)} - \frac{1 - (1 - s(e_M))^N}{u'(W^G_M)} < \epsilon \]
when $N \geq \tilde{N}$ since $P(e_{-M}) \to 0$ and $(1 - s(e_M))^N \to 0$ as $N \to \infty$. Therefore, when $N \geq \tilde{N}$,

$$F(U_M|N) = \frac{s(e_C)}{u'(W_C^G)} - \frac{1 - s(e_C)}{u'(W_M^B)} + \frac{1 - (1 - s(e_M))^N}{u'(W_g)}$$

$$> \frac{s(e_C)}{u'(W_C^G)} - \frac{1 - s(e_C)}{u'(W_M^B)} + \frac{1}{u'(W_C^B)} - \epsilon$$

$$= (1 - s(e_C)) \left( \frac{1}{u'(W_C^G)} - \frac{1}{u'(W_M^B)} \right) - \epsilon$$

$$> 0.$$  

Since $F(\mathcal{V}|N)$ is a strictly concave in $\mathcal{V}$, $\mathcal{V}^* > U_M$ when $N \geq \tilde{N}$.

Second, I show that there is $N^*$ such that $\mathcal{V}^* > U_M$ if $N > N^*$ when $0 < e_C < e_M < 1$.

There are two possibilities:

$$\frac{s(e_M)}{u'(W_M^G)} \geq \frac{1}{u'(W_C^G)} \quad \text{or} \quad \frac{s(e_M)}{u'(W_M^G)} < \frac{1}{u'(W_C^G)}$$

when $\mathcal{V} = U_M$ and $N = 1$.

1. \( \left( \frac{s(e_M)}{u'(W_M^G)} \geq \frac{1}{u'(W_C^G)} \right) \)

   This condition implies that

   $$F(U_M|1) = \frac{s(e_C)}{u'(W_C^G)} - \frac{1 - s(e_C)}{u'(W_M^B)} + \frac{s(e_M)}{u'(W_M^G)}$$

   $$\geq (1 - s(e_C)) \left( \frac{1}{u'(W_C^G)} - \frac{1}{u'(W_M^G)} \right)$$

   $$> 0.$$  

Since \( \frac{\partial \mathcal{V}^*}{\partial N} > 0 \), $\mathcal{V}^* > U_M$ for every $N$.

2. \( \left( \frac{s(e_M)}{u'(W_M^G)} < \frac{1}{u'(W_C^G)} \right) \)

   Again, denote the difference between $1/u'(W_C^G)$ and $1/u'(W_M^B)$ by $\mathcal{D}$. Then, for given $(1 - s(e_C))\mathcal{D} > \epsilon$, there exists $\tilde{N}$ such that

   $$0 \leq \frac{1}{u'(W_C^G)} - \frac{1 - (1 - s(e_M))^N}{u'(W_M^G)} < \epsilon.$$  

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Therefore, when \( N \geq \bar{N} \),

\[
F(\underline{U}_M|N) = -\frac{s(e_C)}{u'(W^G_C)} - \frac{1 - s(e_C)}{u'(W^B_C)} + \frac{1 - (1 - s(e_M))^N}{u'(W^G_M)} \\
> -\frac{s(e_C)}{u'(W^G_C)} - \frac{1 - s(e_C)}{u'(W^B_C)} + \frac{1}{u'(W^G_C)} - \epsilon \\
= (1 - s(e_C)) \left( \frac{1}{u'(W^G_C)} - \frac{1}{u'(W^B_C)} \right) - \epsilon \\
> 0.
\]

Since \( F(\mathcal{V}|N) \) is a strictly concave in \( \mathcal{V} \), \( \mathcal{V}^* > \underline{U}_M \) when \( N \geq \bar{N} \).

### A.14 Proof of Proposition 6

Under the given assumption, it can be shown that

\[
\frac{\partial^2 F(\mathcal{V})}{\partial \mathcal{V} \partial e_C} = -\beta \left( \frac{1}{u'(W^G_C)} - \frac{1}{u'(W^B_C)} \right) + s(e_C)(1 - s(e_C)) \frac{g''(e_C)}{\beta} \left( \frac{u''(W^G_C)}{u'(W^G_C)^3} - \frac{u''(W^B_C)}{u'(W^B_C)^3} \right) < 0,
\]

\[
\frac{\partial^2 F(\mathcal{V})}{\partial \mathcal{V} \partial e_M} = \bar{N} \beta \frac{(1 - s(e_M))^{N-1}}{u'(e_M)} \\
- [1 - (1 - s(e_M))^N] \frac{u''(W^G_M)}{u'(W^G_M)^3} \left( 1 - s(e_M) \right) \frac{g''(e_M)}{\beta} - \left( \frac{\partial P(e^{-M})}{\partial e_M} \right) \mathcal{V} > 0.
\]

### A.15 Proof of Proposition 7

First, I show that \( \mathcal{V}^* \) increases as \( \underline{U}_M \) increases. From the first order condition with respect to \( \mathcal{V}^* \):

\[
-\frac{s(e_C)}{u'(W^G_C)} - \frac{1 - s(e_C)}{u'(W^B_C)} + \frac{1 - (1 - s(e_M))^N}{u'(W^G_M)} = 0,
\]

\[
\frac{\partial \mathcal{V}^*}{\partial \underline{U}_M} = \frac{[1 - (1 - s(e_M))^N] \frac{u''(W^G_M)}{u'(W^G_M)^3}}{s(e_C) \frac{u''(W^G_C)}{u'(W^G_C)^3} + (1 - s(e_C)) \frac{u''(W^B_C)}{u'(W^B_C)^3} + \bar{N} s(e_M) P(e_M)^2 \frac{u''(W^G_M)}{u'(W^G_M)^3}} > 0.
\]

Note that for a given \((N, e_M), (W^G_M)^* = (W^B_M)^* \) if

\[
\mathcal{V}^* = \frac{g'(e_M)}{\beta P(e_M)}
\]
since \( u(W^G_M) - u(W^B_M) = \frac{g'(e_M)}{\beta} - P(e_-)V \). For given \((N, e_C, e_M)\), denote \( V \) satisfying \((W^G_M)^* = (W^B_M)^*\) by \( \hat{V} \). That is,
\[
\hat{V} = \frac{g'(e_M)}{\beta P(e_-)} > 0.
\]
Then,
\[
\frac{\partial F(V|U_M)}{\partial V} \bigg|_{V=\hat{V}} = -\frac{s(e_C) - (1 - s(e_M))^N}{u'(W^G_M)} + \frac{1}{u'(W^B_C)}.
\]
There are two possible cases, \( \frac{\partial F(V|U_M)}{\partial V} \bigg|_{V=\hat{V}} \geq 0 \) and \( \frac{\partial F(V|U_M)}{\partial V} \bigg|_{V=\hat{V}} < 0 \). If \( \frac{\partial F(V|U_M)}{\partial V} \bigg|_{V=\hat{V}} \geq 0 \), then \( V^* > \hat{V} \). This implies that \((W^G_M)^* \leq (W^B_M)^*\). Suppose that \( \frac{\partial F(V|U_M)}{\partial V} \bigg|_{V=\hat{V}} < 0 \). Since \( V \) is fixed at \( \hat{V} \),
\[
\lim_{U_M \to \infty} \frac{1 - (1 - s(e_M))^N}{u'(W^G_M)} = \infty.
\]
Hence, there is \( 0 < U^*_M < \infty \) such that
\[
\frac{\partial F(V|U_M)}{\partial V} \bigg|_{V=\hat{V}} = 0.
\]
Since \( \frac{\partial V^*}{\partial U_M} > 0 \), \((W^G_M)^* \leq (W^B_M)^*\) if \( U_M \geq U^*_M \).

### A.16 Proof of Corollary 6

It is enough to show that there is \( U_M \) such that \( W^B_C \geq W^G_M \) when \( V = 0 \) since this implies that
\[
\frac{\partial F(V)}{\partial V} \bigg|_{V=0} < 0.
\]
Note that
\[
u(W^B_C) - u(W^G_M) = g(e_C) - s(e_C) \frac{g'(e_C)}{\beta} - U_M - g(e_M) - (1 - s(e_M)) \frac{g'(e_M)}{\beta}
\]
when \( V = 0 \). Hence, if
\[
U_M \leq g(e_C) - s(e_C) \frac{g'(e_C)}{\beta} - g(e_M) - (1 - s(e_M)) \frac{g'(e_M)}{\beta},
\]
\( W^B_C \geq W^G_M \).

Since \( \frac{\partial V^*}{\partial U_M} > 0 \) and there is \( U_M \) such that \( V^* > 0 \) according to **Proposition 7**, there exists \( U_M \) such that \( V^* = 0 \) with \( \frac{\partial F(V)}{\partial V} \bigg|_{V=V^*} = 0 \). Also, if \( U_M \) is less than \( U_M^* \), the solution is \( V^* = 0 \) with \( \frac{\partial F(V)}{\partial V} \bigg|_{V=V^*} < 0 \).
A.17 Proof of Proposition 8

Before I prove the proposition, I show that $\mathcal{V}^*$ is bounded for every $N$. The first order condition with respect to $\mathcal{V}$ implies that $(W_M^G)^* > (W_C^B)^*$ for every $N$. Therefore,

$$U_M + g(e_M) + (1 - s(e_M)) \frac{g'(e_M)}{\beta} - P(e_{-M}) \mathcal{V}^* \geq \mathcal{V}^* + g(e_C) - s(e_C) \frac{g'(e_C)}{\beta}.$$ 

Since $0 \leq P(e_{-M}) \leq 1$,

$$U_M + g(e_M) + (1 - s(e_M)) \frac{g'(e_M)}{\beta} - g(e_C) + s(e_C) \frac{g'(e_C)}{\beta} > \mathcal{V}^*,$$

where the left hand side does not depend on $N$.

First, I denote the optimal compensations by $(W_G^C(N), W_B^C(N), W_G^M(N), W_B^M)$ for a given $N$.\(^{55}\)

Then, the difference between the two average profit is

$$2(N + 1)((N + 1)(\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)) = (N - 1)(s(e_M) - s(e_C))(G - B)
- 2W_G(N) + (N + 1)W_C(1) - 2NW_M(N) + (N + 1)W_M(1),$$

where

$$W_G(N) = s(e_C)W_G^C(N) + (1 - s(e_C))W_C^B(N)$$
$$W_M(N) = s(e_M)W_M^G(N) + (1 - s(e_M))W_M^B(N).$$

Since $\mathcal{V}^*$ is bounded, optimal compensations $(W_G(N), W_M(N), W_C(1), W_M(1))$ are also bounded.

Also, they are not depend on $G$ and $B$. Hence, there is $G^* - B^*$ such that the difference has a positive value for a given $N$ since $s(e_M) > s(e_C)$.

For the second part, I impose a restriction on $e_M$. Namely, for a given $e_C$, $e_M$ satisfies the condition that

$$\mathcal{V}^*(e_C, e_M|N = 1) + \frac{g'(e_C)}{\beta} - \frac{g'(e_M)}{\beta} \geq 0,$$

where $\mathcal{V}^*(e_C, e_M|N = 1)$ is the optimal $\mathcal{V}$ when $N = 1$ for a given $(e_C, e_M)$. Since $\mathcal{V}^*(e_C, e_M|N = 1) + \frac{g'(e_C)}{\beta} - \frac{g'(e_M)}{\beta} > 0$ if $e_M = e_C$, there is $\bar{e}_M$ such that $\mathcal{V}^*(e_C, e_M|N = 1) + \frac{g'(e_C)}{\beta} - \frac{g'(e_M)}{\beta} \geq 0$ if $e_M \in (e_C, \bar{e}_M]$.\(^{55}\)

\(^{55}\)Note that $W_M^B$ does not depend on the number of candidates.
Note that
\[
\Pi(\mathcal{Y}^*|N) - \Pi(\mathcal{Y}^*|1) = \frac{N - 1}{2(N + 1)} (s(e_M) - s(e_C))(\mathcal{G} - B)
- \frac{1}{2} s(e_M)(W^G_M(N) - W^G_M(1))
+ \frac{1}{2(N + 1)} [-(N - 1)W_M(N) - 2W_C(N) + (N + 1)W_C(1)]
< \frac{N - 1}{2(N + 1)} (s(e_M) - s(e_C))(\mathcal{G} - B)
- \frac{1}{2} s(e_M)(W^G_M(N) - W^G_M(1))
+ \frac{1}{2(N + 1)} [-(N - 1)W_M(N) - 2W_C(N) + (N - 1)W_M(1) + 2W_C(N)]
= \frac{N - 1}{2(N + 1)} (s(e_M) - s(e_C))(\mathcal{G} - B)
- \frac{N}{N + 1} s(e_M)(W^G_M(N) - W^G_M(1)).
\]

The inequality holds since \(W_C(N) > W_C(1)\) and \(W_M(1) > W_C(1)\). Note that \(W_C(N) > W_C(1)\) is true because \(\frac{\partial V^*}{\partial N} > 0\). On the other hand, the condition imposed on \(e_M\) guarantees that \(W_M(1) > W_C(1)\). Here, I show that \(W_M(1) > W_C(1)\) if \(e_M \in (e_C, \bar{e}_M]\). The first order condition with respect to \(\mathcal{V}\) when \(N = 1\) is
\[
\frac{s(e_C)}{u'(W^G_M(1))} + \frac{1 - s(e_C)}{u'(W^G_M(1))} = \frac{s(e_M)}{u'(W^G_M(1))},
\]
which implies that \(u(W^G_M(1)) > u(W^G_C(1))\). Therefore,
\[
\mathcal{U}_M + g(e_M) + (1 - s(e_M)) \frac{g'(e_M)}{\beta} > 2\mathcal{V}^* + g(e_C) + (1 - s(e_C)) \frac{g'(e_C)}{\beta}.
\]
This inequality and the condition on \(e_M\) imply that
\[
u(W^B_M) = \mathcal{U}_M + g(e_M) - s(e_M) \frac{g'(e_M)}{\beta}
> \mathcal{V}^* + g(e_C) - s(e_C) \frac{g'(e_C)}{\beta} + \mathcal{V}^* + \frac{g'(e_C)}{\beta} - \frac{g'(e_M)}{\beta}
\geq \mathcal{V}^* + g(e_C) - s(e_C) \frac{g'(e_C)}{\beta}
= u(W^B_C(1)).
\]

Therefore, \(W_M(1) > W_C(1)\).

Hence, if
\[
\mathcal{G} - B < \frac{2N}{N - 1} s(e_M) - s(e_C) \left[ W^G_M(N) - W^G_M(1) \right],
\]
\({}^{56}\) if agents have the log utility function, the condition is not needed.
\[ \Pi(V^*|N) < \Pi(V^*|1). \]

Notice that there is \( \tilde{N} \) such that \( W_M^G(N) > W_M^G(1) \) if \( N > \tilde{N} \) since \( P(\text{e}_M) \) converges to zero as \( N \) approaches infinity and \( V^* \) is bounded. Let \( \bar{\mathcal{O}} \) denote

\[
\inf_{N \in [\mathbb{N},\infty)} \frac{2N}{N-1} \frac{s(e_M)}{s(e_M) - s(e_C)} [W_M^G(N) - W_M^G(1)].
\]

Then, \( \Pi(V^*|N) < \Pi(V^*|1) \) if \( N > \tilde{N} \) and \( G - B \leq \bar{\mathcal{O}}. \)

### A.18 Proof of Proposition 9

**Proof.** First, note that

\[
\Pi(V^*|N) - \Pi(V^*|1) = \frac{(N+1)W_C(1) - 2W_C(N) + (N+1)W_M(1) - 2NW_M(N)}{2(N+1)},
\]

where \( W_C(k) \) and \( W_M(k) \) represent CEO’s and managers’ expected compensation when the firm hires \( k \) managers\(^{57} \), respectively. When agents have the log utility function, the first order condition with respect to \( V \) is

\[
s(e)W_C^G(N) + (1-s(e))W_C^B(N) = (1-(1-s(e))^N)W_M^G(N).
\]

Using this condition, it can be shown that

\[
\Pi(V^*|N) - \Pi(V^*|1) = s(e)[W_M^G(1) - W_M^G(N)]
\]

\[
\quad - \frac{(1-s(e))(1-(1-s(e))^{N-1})W_M^G(N)}{N+1} - \frac{N-1}{2(N+1)}(1-s(e))W_M^B.
\]

This indicates that \( \Pi(V^*|N) - \Pi(V^*|1) < 0 \) if \( W_M^G(N) > W_M^G(1). \)

\(^{57}\)These are defined in A.17.
Also, when \( u(x) = \log(x) \) and \( e_C = e_M = e \), \( \mathcal{V}(N) \) is\(^{58}\)

\[
\mathcal{V}^*(N) = -\frac{1}{1 + P(e_M)} \log \left[ \frac{s(e) \exp \left[ g(e) + (1 - s(e)) \frac{g'(e)}{\beta} \right] + (1 - s(e)) \exp \left[ g(e) - s(e) \frac{g'(e)}{\beta} \right]}{(1 - (1 - s(e))^N) \exp \left[ U_M + g(eM) + (1 - s(e)) \frac{g'(e)}{\beta} \right]} \right] = -\frac{1}{1 + P(e_M)} \log \left[ \frac{s(e) + (1 - s(e)) \exp \left[ -\frac{g'(e)}{\beta} \right]}{(1 - (1 - s(e))^N) \exp [U_M]} \right] = \frac{1}{1 + P(e_M)} U_M + \frac{1}{1 + P(e_M)} \log \left[ \frac{(1 - (1 - s(e))^N)}{s(e) + (1 - s(e)) \exp \left[ -\frac{g'(e)}{\beta} \right]} \right].
\]

Note that when \( U_M = -\log \left[ \frac{s(e)}{s(e) + (1 - s(e)) \exp \left[ -\frac{g'(e)}{\beta} \right]} \right] \), \( \mathcal{V}^*(1) = 0 \), and

\[
\mathcal{V}^*(N) = \frac{1}{1 + P(e_M)} \log \left[ \frac{(1 - (1 - s(a))^N)}{s(a)} \right].
\]

Therefore, \( \Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1) \) is greater than 0 if

\[
s(e) - \left[ \frac{Ns(e) + 1 - (1 - s(a))^N}{N + 1} \right] \exp [-P(e_M|N)\mathcal{V}^*(N)] - \frac{N - 1}{2(N + 1)} (1 - s(e)) \exp \left( -\frac{g'(e)}{\beta} \right) > 0.
\]

This is equivalent to

\[
1 > \frac{N}{N + 1} \frac{1 + P(e_M|N)}{[NP(e_M|N)]^{1+P(e_M|N)}} + \frac{N - 1}{2(N + 1)} \frac{1 - s(e)}{s(e)} \exp \left( -\frac{g'(a)}{\beta} \right).
\]

For a fixed \( N \), the first term on the right hand side is increasing in the agents’ effort level \( e \) since

\[
\frac{d}{de} \left( \frac{1 + P(e_M|N)}{[NP(e_M|N)]^{1+P(e_M|N)}} \right) = -\frac{\log [NP(e_M|N)]}{(1 + P(e_M|N))[NP(e_M|N)]^{1+P(e_M|N)}} \frac{dP(e_M|N)}{de} > 0.
\]

Also, the first term on the right hand side is bounded above by 1. On the other hand, the second term on the right hand side is decreasing in \( a \) and converges to zero as \( a \) goes to 1.\(^{59}\) Therefore, there is

\(^{58}\)In this proof, I explicitly indicate the dependency of the variable on \( N \).

\(^{59}\)This result relies on the condition that \( \lim_{e \to 1} g'(e) = \infty \).
Suppose that $V > N$ such that $V$ is greater than zero. That is, $\Pi(\nu^*|N) - \Pi(\nu^*|1) > 0$ if $U_N > U_M^\nu$. Since $P(e_{-M}|N)$ is strictly less than 1 when $N \geq 2$, there is $U_M^\nu(N)$ such that $W_M^G(1) < W_M^G(N)$ if $U_M > U_M^\nu(N)$. This implies that $\Pi(\nu^*|N) - \Pi(\nu^*|1) < 0$ when $U_M > U_M^\nu(N)$.

Now, I show that $\frac{\partial^2}{\partial U_M^2} [\Pi(\nu^*|N) - \Pi(\nu^*|1)] < 0$ if $\frac{\partial}{\partial U_M} [\Pi(\nu^*|N) - \Pi(\nu^*|1)] \leq 0$.

Suppose that

$$
\frac{\partial}{\partial U_M} [\Pi(\nu^*|N) - \Pi(\nu^*|1)] = \frac{s(e)}{2} W_M^G(1) - \frac{N}{N + 1} s(e) W_M^G(N) - \frac{N - 1}{2(N + 1)} (1 - s(e)) W_M^B
$$

has a negative value. Then,

$$
\frac{\partial^2}{\partial U_M^2} [\Pi(\nu^*|N) - \Pi(\nu^*|1)] = \frac{s(e)}{4} W_M^G(1) - \frac{N}{N + 1} \frac{s(e)}{1 + P(e_{-M}|N)} W_M^G(N)
$$

$$
- \frac{N - 1}{2(N + 1)} (1 - s) W_M^B
$$

$$
< \frac{s(e)}{4} W_M^G(1) - \frac{N}{N + 1} \frac{s(e)}{2} W_M^G(N)
$$

$$
- \frac{N - 1}{4(N + 1)} (1 - s) W_M^B
$$

$$
= \frac{1}{2} \left[ \frac{s(e)}{2} W_M^G(1) - \frac{N}{N + 1} s(e) W_M^G(N) - \frac{N - 1}{2(N + 1)} (1 - s(e)) W_M^B \right]
$$

$$
= \frac{1}{2} \frac{\partial}{\partial U_M} [\Pi(\nu^*|N) - \Pi(\nu^*|1)] \leq 0.
$$

When $\Pi(\nu^*|N) - \Pi(\nu^*|1) > 0$ as $U_M = -\log \left[ \frac{s(a)}{s(a) + (1-s(a)) \exp \left[ -\frac{2(a)}{\beta} \right]} \right] = U_M^0$, there are two possible cases.
1. \( \left( \frac{\partial}{\partial U_M} [\Pi(Y^*|N) - \Pi(Y^*|1)] \leq 0 \right) \)

Since there is \( U^*_M(N) \) such that \( \Pi(Y^*|N) - \Pi(Y^*|1) < 0 \) when \( U_M > U^*_M(N) \), there is a unique \( \tilde{U}_M(N) \) such that \( \Pi(Y^*|N) - \Pi(Y^*|1) > 0 \) if \( U_M \in [U^*_M, \tilde{U}_M(N)] \).

2. \( \left( \frac{\partial}{\partial U_M} [\Pi(Y^*|N) - \Pi(Y^*|1)] > 0 \right) \).

The condition that \( \frac{\partial^2}{\partial(U_M)^2} [\Pi(Y^*|N) - \Pi(Y^*|1)] < 0 \) if \( \frac{\partial}{\partial U_M} [\Pi(Y^*|N) - \Pi(Y^*|1)] = 0 \) implies that there is a unique \( \tilde{U}_M \) such that \( \frac{\partial}{\partial U_M} [\Pi(Y^*|N) - \Pi(Y^*|1)] = 0 \). Hence, there is a unique \( \tilde{U}_M(N) \) such that \( \Pi(Y^*|N) - \Pi(Y^*|1) > 0 \) if \( U_M \in [U^*_M, \tilde{U}_M(N)] \).

Lastly, I show that there is \( N^* \) such that \( W^G_M(1) < W^G_M(N) \) if \( N \geq N^* \) for a given \( (U_M, e) \). This implies that \( \Pi(Y^*|N) < \Pi(Y^*|1) > 0 \). In the equation (11), \( R(N, e) \) is equal to

\[
R(N, e) = \frac{1}{2} \log \left( \frac{s(e)}{s(e) + (1-s(e)) \exp \left[ -\frac{g'(a)}{\beta} \right]} \right) - \frac{P(e-M|N)}{1 + P(e-M|N)} \log \left( \frac{1 - (1-s(e))^N}{s(e) + (1-s(e)) \exp \left[ -\frac{g'(e)}{\beta} \right]} \right).
\]

Since \( \left( \frac{1}{2} - \frac{1}{1 + P(e-M|N)} \right) U_M > 0 \), it is enough to show that \( R(N, e) > 0 \) if \( N \geq N^* \). The first term of \( R(N, e) \) does not depend on \( N \) and has a strictly positive number. Denote this number by \( C \). On the other hand, the second term is always less than

\[
\frac{P(e-M|N)}{1 + P(e-M|N)} \log \left( \frac{1}{s(e) + (1-s(e)) \exp \left[ -\frac{g'(e)}{\beta} \right]} \right), \tag{12}
\]

which is strictly decreasing function in \( N \) and converges to zero. Hence, there is \( N^* \) such that (12) is less than \( C \) if \( N \geq N^* \). This implies that \( R(N, e) > 0 \).
A.19 Proof of Proposition 10

Note that
\[
\frac{\partial^2 F}{\partial V \partial N} = -(1 - s(e_M))^N \log(1 - s(e_M)) \frac{1}{u'(W_g)} > 0,
\]
\[
\frac{\partial^2 F}{\partial (-e_C) \partial N} = 0, \text{ and}
\]
\[
\frac{\partial^2 F}{\partial V \partial (-e_C)} = s(e_C) \frac{\partial^2 W^e_C}{\partial V \partial e_C} + (1 - s(e_C)) \frac{\partial^2 W^B_C}{\partial V \partial e_C} + \beta \left( \frac{\partial W^G_C}{\partial V} - \frac{\partial W^B_C}{\partial V} \right)
\]
\[
= -s(e_C)(1 - s(e_C)) \frac{g''(e_C)}{\beta} \left( \frac{u''(W^G_C)}{u'(W^G_C)^3} - \frac{u''(W^B_C)}{u'(W^B_C)^3} \right) + \beta \left( \frac{1}{u'(W^G_C)} - \frac{1}{u'(W^B_C)} \right).
\]

This result shows that \( \frac{\partial y^*}{\partial N} \geq 0 \) and \( \frac{\partial e^*_C}{\partial N} \leq 0 \) according to Milgrom and Shannon (1994). First, consider \( e^*_C \) as a function of \( V \). Then, under the condition that \( u''(x)/u'(x)^3 \) is a decreasing function in \( x \),
\[
\frac{\partial^2 F}{\partial V \partial N} = \left[ -\beta \left( \frac{1}{u'(W^G_C)} - \frac{1}{u'(W^B_C)} \right) + s(e^*_C) \frac{u''(W^G_C)}{u'(W^G_C)^2} \frac{\partial W^G_C}{\partial e_C} + (1 - s(e^*_C)) \frac{u''(W^B_C)}{u'(W^B_C)^2} \frac{\partial W^B_C}{\partial e_C} \right] \frac{\partial e^*_C}{\partial N}
\]
\[
- (1 - s(e_M))^N \log(1 - s(e_M)) \frac{1}{u'(W_g)} > 0
\]
since \( \frac{\partial e^*_C}{\partial N} \leq 0 \). This implies that \( \frac{\partial y^*}{\partial N} > 0 \). Now, consider the first order condition with respect to \( e_C \):
\[
\beta(G - B) = \beta(W^G_C - W^B_C) + s(e^*_C) \frac{\partial W^G_C}{\partial e_C} + (1 - s(e_C)) \frac{\partial W^B_C}{\partial e_C}
\]
\[
= \beta(W^G_C - W^B_C) + s(e^*_C)(1 - s(e^*_C)) \frac{g''(e_C)}{\beta} \left( \frac{1}{u'(W^G_C)} - \frac{1}{u'(W^B_C)} \right).
\]
The right hand side of the equation is a strictly increasing function in \( e^*_C \) if \( g''(e_C) \geq 0 \) and a strictly decreasing function in \( V \). Hence, \( \frac{\partial y^*}{\partial N} > 0 \) indicates that \( \frac{\partial e^*_C}{\partial N} < 0 \).

A.20 Proof of Corollary 7

Note that
\[
\frac{\partial}{\partial N} [(W^e_C)^* - (W^B_C)^*] = \frac{\partial V}{\partial N} \left[ \frac{1}{u'(W^G_C)^*} - \frac{1}{u'(W^B_C)^*} + \left( \frac{1 - s(e^*_C)}{u''((W^G_C)^*)} + \frac{s(e^*_C)}{u''((W^B_C)^*)} \right) \frac{g''(e^*_C)}{\beta} \frac{\partial e^*_C}{\partial V} \right]
\]
\[
= \frac{\partial V}{\partial N} \left[ (W^e_C)^* - (W^B_C)^* + (1 - s(e^*_C))(W^G_C)^* + s(e^*_C)(W^B_C)^* \right] \frac{g''(e^*_C)}{\beta} \frac{\partial e^*_C}{\partial V}.
\]
when agents have the log utility function. Also, it can be shown that

\[
\frac{\partial e^*_C}{\partial \nu} = -\left(\beta + s(e^*_C)(1 - s(e^*_C))\frac{g''(e^*_C)}{\beta}\right) \frac{(W^G_C)^* - (W^B_C)^*}{D_1 + D_2},
\]

where

\[
D_1 = \left(\beta(1 - 2s(e^*_C))\frac{g''(e^*_C)}{\beta} + s(e^*_C)(1 - s(e^*_C))\frac{g'''(e^*_C)}{\beta}\right)((W^G_C)^* - (W^B_C)^*)
\]

\[
D_2 = \left(\beta + s(e^*_C)(1 - s(e^*_C))\frac{g''(e^*_C)}{\beta}\right)\frac{g''(e^*_C)}{\beta}((1 - s(e^*_C))(W^G_C)^* + s(e^*_C)(W^B_C)^*)
\]

Hence,

\[
\frac{\partial}{\partial N} [(W^G_C)^* - (W^B_C)^*] = \frac{(W^G_C)^* - (W^B_C)^*}{D_1 + D_2} D_1 > 0.
\]

**A.21 Proof of Proposition 11**

Note that when \(\nu = 0\), \(e^*_C = e^*_M\) by two first order conditions. Also, this implies that \(W^G_C = W^G_M\).

Therefore,

\[
\frac{\partial F(\nu)}{\partial \nu} \bigg|_{\nu=0} = -\frac{s(e^*_C)}{u'(W^G_C)} - \frac{1 - s(e^*_C)}{u'(W^B_C)} + \frac{1 - (1 - s(e^*_M))^N}{u'(W^G_M)} - \frac{1}{u'(W^B_M)}.
\]

Since \(W^G_C > W^B_C\) and \((1 - s(e^*_C))^N \rightarrow 0\) as \(N \rightarrow \infty\), there is \(\tilde{N}\) such that \(\frac{\partial F(\nu)}{\partial \nu} \bigg|_{\nu=0} > 0\) if \(N > \tilde{N}\). This implies that \(\nu^* > U_M = 0\) when \(N > \tilde{N}\). As a next step, I show that the promotion incentive is bounded regardless of the firm size.

**Claim 1.** The optimal promotion incentive \(\nu^*\) is bounded for any \(N\).

**Proof.** First, fix \((e_M) \in (0, 1)\) for a given \(N\). Denote the optimal \(\nu\) by \(\nu^*(e_C)\) for a given \(e_C\). Recall that \(\frac{\partial \nu^*(e_C)}{\partial e_C} < 0\). Therefore,

\[
\nu^*(e_C) \leq \nu^*(0).
\]

When \(e_C = 0\), the first order condition with respect to \(\nu\) is

\[
-\frac{1}{u'(W^G_C)} + \frac{1 - (1 - s(e))^N}{u'(W^G_M)} = 0.
\]
where \( u(W_C^F) = \mathcal{V}^*(0) \). It can be easily shown that
\[
\mathcal{V}^*(0) = u(W_C^F) < u(W_M^G) = U_M + g(e_M) + (1 - s(e_M)) \frac{g'(e_M)}{\beta} - P(e_{-M})\mathcal{V}^*(0)
\]
implies that
\[
\mathcal{V}^*(0) < \frac{1}{1 + P(e_{-M})} \left[ U_M + g(e_M) + (1 - s(e_M)) \frac{g'(e_M)}{\beta} \right].
\]
Notice that this bound does not depend on \( e_C \). Now, suppose that
\[
\mathcal{V} = \frac{1}{1 + P(e_{-M})} \left[ U_M + g(e_M) + (1 - s(e_M)) \frac{g'(e_M)}{\beta} \right].
\]
Then, the manager’s wage for good performance satisfies
\[
u(W_G^M) = \frac{1}{1 + P(e_{-M})} \left[ U_M + g(e_M) + (1 - s(e_M)) \frac{g'(e_M)}{\beta} \right] \geq \frac{1}{2} \left[ U_M + g(e_M) + (1 - s(e_M)) \frac{g'(e_M)}{\beta} \right].
\]
The bound for \( u(W_G^M) \) does not depend on \( N \). This result means that the firm has to pay the wage satisfying the lower bound if it requires an effort level \( e_M \) from its managers. Since the wage approaches infinity as \( e_M \) converges to one, there is \( \bar{e}_M < 1 \) such that \( e_C^* \leq \bar{e}_M \) regardless of \( N \) and \( e_C \). Hence, \( \mathcal{V}^* \) is bounded by \( \frac{1}{1 + P(e_{-M})} \left[ U_M + g(\bar{e}_M) + (1 - s(\bar{e}_M)) \frac{g'(\bar{e}_M)}{\beta} \right] \), which is less than \( \left[ U_M + g(\bar{e}_M) + (1 - s(\bar{e}_M)) \frac{g'(\bar{e}_M)}{\beta} \right] \). I denote this bound by \( \bar{\mathcal{V}} \). This upper bound does not depend on \( N \).

Based on this result, I show the following result.

**Claim 2.** There is \( \bar{N} \) such that \( (W_M^G)^* > (W_M^B)^* \) if \( N > \bar{N} \).

**Proof.** Recall that
\[
(W_M^G)^* \leq (W_M^B)^*
\]
if and only if
\[
\frac{g'(e_M^*)}{\beta} \leq P(e_{-M})\mathcal{V}^*.
\]
Since \( e_M^* \geq e_M \), where \( e_M \) is the firm’s optimal effort choice when \( \mathcal{V} = 0 \), and \( P(e_{-M}) \to 0 \) as \( N \to \infty \), there exists \( \bar{N} \) such that
\[
\frac{g'(e_M^*)}{\beta} \geq \frac{g'(e_M)}{\beta} > P(e_{-M})\bar{\mathcal{V}} \geq P(e_{-M})\bar{\mathcal{V}}
\]
if \( N > \bar{N} \). Therefore, \( (W_M^G)^* > (W_M^B)^* \) if \( N > \bar{N} \). 

\(\square\)
Now, I show that $e^*_M > e^*_C$ if $N > N^* \equiv \max\{\bar{N}, N\}$.

Note that two optimal effort levels $e^*_C$ and $e^*_M$ are decided by two first order conditions for a given $\mathcal{V}$:

$$
\beta(\mathcal{G} - \mathcal{B}) = \beta(W^G_C - W^B_C) + s(e^*_C) \frac{\partial W^G_C}{\partial e_C} + (1 - s(e^*_C)) \frac{\partial W^B_C}{\partial e_C}
$$

$$
= \beta(W^G_C - W^B_C) + s(e^*_C)(1 - s(e^*_C)) \frac{g''(e^*_C)}{\beta} \left( \frac{1}{u'(W^G_C)} - \frac{1}{u'(W^B_C)} \right), \quad \text{and} \quad (13)
$$

$$
\beta(\mathcal{G} - \mathcal{B}) = \beta(W^G_M - W^B_M) + s(e^*_M) \frac{\partial W^G_M}{\partial e_M} + (1 - s(e^*_M)) \frac{\partial W^B_M}{\partial e_M}
$$

$$
= \beta(W^G_M - W^B_M) + s(e^*_M)(1 - s(e^*_M)) \frac{g''(e^*_M)}{\beta} \left( \frac{1}{u'(W^G_M)} - \frac{1}{u'(W^B_M)} \right)
$$

$$
- s(e^*_M) \frac{\partial P(e^*_M)}{\partial e_M} \frac{1}{u'(W^G_M)} \mathcal{V}, \quad \text{(14)}
$$

If $\mathcal{V}$ is equal to $U_M = 0$, two conditions yield $e^*_C = e^*_M$. The right hand side of (13) is a strictly increasing function in $\mathcal{V}$ while that of (14) is a strictly decreasing function in $\mathcal{V}$ since

$$
\frac{\partial J_C(\mathcal{V}, e_C)}{\partial \mathcal{V}} = \beta \left( \frac{1}{u'(W^G_C)} - \frac{1}{u'(W^B_C)} \right) - s(e_C)(1 - s(e_C)) \frac{g''(e_C)}{\beta} \left( \frac{u''(W^G_C)}{u'(W^G_C)^3} - \frac{u''(W^B_C)}{u'(W^B_C)^3} \right) > 0,
$$

$$
\frac{\partial J_M(\mathcal{V}, e_M)}{\partial \mathcal{V}} = - \beta P(e^-_M) \frac{1}{u'(W^G_M)} + s(e_M)(1 - s(e_M)) \frac{g''(e_M)}{\beta} \frac{u''(W^G_M)}{u'(W^G_M)^3} P(e^-_M) - s(e_M) \frac{\partial P(e^-_M)}{\partial e_M} \frac{1}{u'(W^G_M)} \mathcal{V}
$$

$$
= s(e_M)(1 - s(e_M)) \frac{g''(e_M)}{\beta} P(e^-_M) \frac{u''(W^G_M)}{u'(W^G_M)^3} - (1 - s(e_M))^{N-1} \frac{1}{u'(W^G_M)} \mathcal{V} < 0,
$$

where

$$
J_C(\mathcal{V}, e_C) = \beta(W^G_C - W^B_C) + s(e_C)(1 - s(e_C)) \frac{g''(e_C)}{\beta} \left( \frac{1}{u'(W^G_C)} - \frac{1}{u'(W^B_C)} \right), \quad \text{and}
$$

$$
J_M(\mathcal{V}, e_M) = \beta(W^G_M - W^B_M) + s(e_M)(1 - s(e_M)) \frac{g''(e_M)}{\beta} \left( \frac{1}{u'(W^G_M)} - \frac{1}{u'(W^B_M)} \right)
$$

$$
- s(e_M) \frac{\partial P(e^-_M)}{\partial e_M} \frac{1}{u'(W^G_M)} \mathcal{V}, \quad \text{and}
$$

$$
\frac{\partial P(e^-_M)}{\partial e_M} = \frac{\beta}{s(e_M)} [(1 - s(e_M))^{N-1} - P(e^-_M)] < 0.
$$
In addition, the following results

\[ \frac{\partial J_C(V, e_C)}{\partial e_C} = 2g''(e_C) \left( \frac{1 - s(e_C)}{u'(W_G^C)} + \frac{s(e_C)}{w'(W_B^C)} \right) - g''(e_C) \left( \frac{s(e_C)}{u'(W_G^C)} + \frac{1 - s(e_C)}{u'(W_B^C)} \right) \]

\[ - s(e_C)(1 - s(e_C)) \frac{g''(e_C)}{\beta^2} \left[ (1 - s(e_C)) \frac{u''(W_G^C)}{u'(W_B^C)} + s(e_C) \frac{u''(W_B^C)}{u'(W_B^C)^3} \right] \]

\[ + s(e_C)(1 - s(e_C)) \frac{g''(e_C)}{\beta} \left( \frac{1}{u'(W_G^C)} - \frac{1}{u'(W_B^C)} \right) > 0, \]

\[ \frac{\partial J_M(V, e_M)}{\partial e_M} = \frac{\partial J_C(V, e_C)}{\partial e_C} \bigg|_{e_C = e_M} - 2\beta \frac{\partial P(e^{-}_M)}{e_M} \frac{1}{u'(W_M^G)} \] \[ + \left[ 2s(e_M)(1 - s(e_M)) \frac{g''(e_M)}{\beta} \frac{\partial P(e^{-}_M)}{e_M} \right] - s(e_M) \left( \frac{\partial P(e^{-}_M)}{e_M} \right)^2 \frac{u''(W_M^G)}{u'(W_M^G)^3} \]

\[ > 0, \]

where

\[ \frac{\partial^2 P(e^{-}_M)}{\partial(e_M)^2} = \frac{\beta^2}{Ns(e_M)^3} \left[ -N(N - 1)s(e_M)^2(1 - s(e_M))^{N-2} \right. \]

\[ + 2 - 2(1 - s(e_M))^{N-1} - 2(N - 1)s(e_M)(1 - s(e_M))^{N-1} \]

\[ - \frac{\beta^2}{s(e_M)} (N - 1)(1 - s(e_M))^{N-2} - 2\frac{\beta}{s(e_M)} \frac{\partial P(e^{-}_M)}{e_M} \geq 0, \]

validate the first order approach.

Hence, \( e^*_C < e^*_M \) since \( V^* > U^*_M = 0 \) when \( N > \tilde{N} \).

**A.22 Proof of Proposition 12**

For a given \((V, N)\), \( U^*_{M2} \) is determined by the equation (9). I denote this by \( U^*_{M2}(V, N) \) to explicitly express the dependency. The first order condition with respect to \( V \) and (9) imply that \( V^* \) and \( U^*_{M2}(V^*, N) \) satisfy

\[ \frac{s(e_C)}{u'((W_G^C)^*)} + \frac{1 - s(e_C)}{u'((W_B^C)^*)} + (1 - s(e_M^1))^N = \delta \left( \frac{s(e_M^2)}{u'((W_M^G)^*)} + \frac{1 - s(e_M^2)}{u'((W_M^B)^*)} \right) \],

...
where the first order condition with respect to \( V \) is
\[
\frac{\partial F(V)}{\partial V} = -\frac{s(e_C)}{u'(W^G_C)} - \frac{1 - s(e_C)}{u'(W^B_C)} + Ns(e_{M1}) \left[ \frac{1}{u'(W^G_M)} \left( P(e_{-M}) + (1 - P(e_{-M})) \frac{\partial U_{M2}(V)}{\partial V} \right) \right] - \delta \left( Ns(e_{M1}) - 1 + (1 - s(e_{M1}))^N \right) \left( \frac{s(e_{M2})}{u'(W^{GG}_M)} + \frac{1 - s(e_{M2})}{u'(W^{GB}_M)} \right) \frac{\partial U_{M2}(V)}{\partial V} \]

by (9).

Note that \( \frac{(1-s(e_{M1}))^N}{u'(W^M)} \) approaches zero as \( N \) goes to infinity. Also, when \( e_C = e_{M2}, V = U_{M2}, \) and \( \delta = 1, \) the condition is equal to
\[
\frac{s(e_C)}{u'(W^G_C)} + \frac{1 - s(e_C)}{u'(W^B_C)} = \frac{s(e_{M2})}{u'(W^{GG}_M)} + \frac{1 - s(e_{M2})}{u'(W^{GB}_M)}.
\]

Since the left hand side of this equation is a strictly increasing function in \( e_C \) and \( V, \) there is \( \hat{N}, \)
\( \hat{e}_C < e_{M2} \) such that
\[
\frac{s(\hat{e}_C)}{u'(W^G_C)} + \frac{1 - s(\hat{e}_C)}{u'(W^B_C)} + (1 - s(e_{M1}))^{\hat{N}} \leq \delta \left[ \frac{s(e_{M2})}{u'(W^{GG}_M)} + \frac{1 - s(e_{M2})}{u'(W^{GB}_M)} \right]
\]
when \( V = U_M \) for a sufficiently large \( \delta. \) This implies that for given (\( \delta, e_C, e_{M1}, e_{M2}, N, \)) where \( e_C \leq \hat{e}_C < e_{M2} \) and \( N \geq \hat{N}, \) \( V^* > U_{M2}(V^*, N). \)

Now, I show that \( V^* \) is an increasing function in \( N > \hat{N} \) when \( V^* \geq U_{M2}(V^*, \hat{N}). \)

First, note that for a given \( V \) and \( N, \)
\[
\frac{\partial U_{M2}(V, N)}{\partial N} = -\frac{u''(W^G_M)}{u'(W^G_M)^3} \frac{\partial P(e_{-M1})}{\partial N} (V - U_{M2}^*(V, N)) - \frac{u''(W^{GG}_M)}{u'(W^{GG}_M)^3} (1 - P(e_{-M1}))^2 - \frac{u''(W^{GB}_M)}{u'(W^{GB}_M)^3} (1 - s(e_{M2})) \frac{\partial U_{M2}(V, N)}{\partial N}.
\]

Therefore,
\[
\frac{\partial^2 F(V)}{\partial N^2} = -(1 - s(e_{M1}))^N \log(1 - s(e_{M1})) - \frac{1}{u'(W^G_M)} \left[ 1 - s(e_{M1})^N \right] \frac{u''(W^G_M)}{u'(W^G_M)^3}.
\]

\[
\left[ \frac{\partial P(e_{-M1})}{\partial N} (V^* - U_{M2}^*) + (1 - P(e_{-M1})) \frac{\partial U_{M2}(V, N)}{\partial N} \right] \]

\[
= -(1 - s(e_{M1}))^N \log(1 - s(e_{M1})) - \frac{1}{u'(W^G_M)} \left[ 1 - s(e_{M1})^N \right] \frac{u''(W^{GG}_M)}{u'(W^{GG}_M)^3} + (1 - s(e_{M2})) \frac{u''(W^{GB}_M)}{u'(W^{GB}_M)^3} \geq 0.
\]
Moreover,
\[
\frac{\partial^2 F(V)}{\partial V^2} = s(e_C) \frac{u''(W_{E}^G)}{u'(W_{E}^G)^3} + (1 - s(e_C)) \frac{u''(W_E)}{u'(W_E)^3} \\
+ (1 - (1 - s(e_{M1}))^N) \frac{u''(W_{E}^G)}{u'(W_{E}^G)^3} \left[ P(e_{-M1}) + (1 - P(e_{-M1})) \frac{\partial U_{M2}^*(V, N)}{\partial V} \right]
\]
\[
= s(e_C) \frac{u''(W_{E}^G)}{u'(W_{E}^G)^3} + (1 - s(e_C)) \frac{u''(W_E)}{u'(W_E)^3} \\
+ (1 - (1 - s(e_{M1}))^N) \frac{u''(W_{E}^G)}{u'(W_{E}^G)^3} \\
\cdot \left[ P(e_{-M1}) \frac{s(e_{M2})}{(1 - P(e_{-M1}))} \frac{u''(W_{M}^G)}{u'(W_{M}^G)^3} + (1 - s(e_{M2})) \frac{u''(W_{M}^G)}{u'(W_{M}^G)^3} \right]
\]
< 0,

where I exploit
\[
\frac{\partial U_{M2}^*(V, N)}{\partial V} = -\frac{P(e_{-M1}) \frac{u''(W_{M}^G)}{u'(W_{M}^G)^3}}{(1 - P(e_{-M1})) \frac{u''(W_{M}^G)}{u'(W_{M}^G)^3} + \delta \left[ s(e_{M2}) \frac{u''(W_{M}^G)}{u'(W_{M}^G)^3} + (1 - s(e_{M2})) \frac{u''(W_{M}^G)}{u'(W_{M}^G)^3} \right]}
\]
using the implicit function theorem.

### A.23 Derivation of the Firm’s Problem in Section 7.2

The firm’s problem can be written as
\[
\max_{C, M_1, M_2} E_0 \left[ \sum_{t=1}^{\infty} \delta^{t-1} P_t(C, M_1, M_2 | H_{t-1}) \right]
\]
subject to (IRc), (ICc), (IRM1), (ICM1), (IRM2), (ICM2),

where
\[
P_t(C, M_1, M_2 | H_{t-1}) = \begin{cases} 
  P_C(C) + NP_M(M_1) & \text{if } H_{t-1} \in S_1 \\
  NP_M(M_2) & \text{if } H_{t-1} \in S_2
\end{cases}
\]
with
\[ P_C(C) = s(e_{C1})(G_C - W_C^G) + (1 - s(e_{C1}))(B_C - W_C^B) \]
\[ + \delta s(e_{C1})[s(e_{C2})(G_C - W_C^{GG}) + (1 - s(e_{C2}))(B_C - W_C^{GB})] \]
\[ P_M(M_i) = s(e_{Mi})(G_M - W_M^G) + (1 - s(e_{Mi}))(B_M - W_M^B) \]
\[ C = (W_C^G, W_C^B, W_C^{GG}, W_C^{GB}) \]
\[ M_i = (W_{Mi}^G, W_{Mi}^B). \]

Also, \( H_t \) denotes the CEO’s seniority and outcome at time \( t \). Hence,
\[ H_t \in \{(C_1, G_C), (C_1, B_C), (C_2, G_C), (C_2, B_C)\}, \]
where \( C_i \) is equal to \( C_1 \) \((C_2)\) if the CEO is her first period \((\text{second period})\) in the position. For brevity, I use two terms, \( S_1 \) and \( S_2 \), in order to represent
\[ S_1 = \{(C_1, B_C), (C_2, G_C), (C_2, B_C)\} \]
\[ S_2 = \{(C_1, G_C)\}, \]
respectively.

Then,
\[ E_0 \left[ \sum_{t=1}^{\infty} \delta^{t-1} P_t(C, M_1, M_2|H_{t-1}) \right] = P_C(C) + NP_M(M_1) \]
\[ + \delta s(e_{C1}) \left\{ NP_M(M_2) + \delta E_0 \left[ \sum_{t=1}^{\infty} \delta^{t-1} P_t(C, M_1, M_2|H_{t-1}) \right] \right\} \]
\[ + \delta(1 - s(e_{C1}))E_0 \left[ \sum_{t=1}^{\infty} \delta^{t-1} P_t(C, M_1, M_2|H_{t-1}) \right], \]
where I exploit
\[ E_0 \left[ \sum_{t=1}^{\infty} \delta^{t-1} P_t(C, M_1, M_2|H_{t-1}) \right] = E_s \left[ \sum_{t=s+1}^{\infty} \delta^{t-(s+1)} P_t(C, M_1, M_2|H_{t-1}) \right] \]
if \( H_s \in S_1 \). Therefore,
\[ E_0 \left[ \sum_{t=1}^{\infty} \delta^{t-1} P_t(C, M_1, M_2|H_{t-1}) \right] = \frac{1}{1 - \delta(1 + \delta s(e_{C1}))} [P_C(C) + NP_M(M_1) + \delta s(e_{C1})NP_M(M_2)]. \]

Since I treat \( e_{C1} \) as an exogenous variable, the firm’s problem is to choose \((C, M_1, M_2)\) maximizing
\[ P_C(C) + NP_M(M_1) + \delta s(e_{C1})NP_M(M_2). \]
A.24 Proof of Proposition 13

The firm’s problem for guaranteed situation is to choose \( \tilde{\nu} \in [0, \infty) \) maximizing \( \tilde{F}(\tilde{\nu}) \) defined by

\[
\tilde{F}(\tilde{\nu}) \equiv \max_{\tilde{A}} s(e_C)(G_C - \tilde{W}^G_C) + (1 - s(e_C))(B_C - \tilde{W}^B_C)
\]

\[
+ \delta s(e_C)[s(e_C)(G_C - \tilde{W}^{GG}_C) + (1 - s(e_C))(B_C - \tilde{W}^{GB}_C)]
\]

\[
+ \delta(1 - s(e_C))[s(e_C)(G_C - \tilde{W}^{BG}_C) + (1 - s(e_C))(B_C - \tilde{W}^{BB}_C)]
\]

\[
+ N \left[ s(e_M)(G_M - \tilde{W}^G_{M1}) + (1 - s(e_M))(B_M - \tilde{W}^B_{M1}) \right]
\]

\[
+ \delta N \left[ s(e_M)(G_M - \tilde{W}^G_{M2}) + (1 - s(e_M))(B_M - \tilde{W}^B_{M2}) \right]
\]

subject to

\[
u(\tilde{W}^C) = \mathcal{V} + g(e_C) + (1 - s(e_C))\frac{g'(e_C)}{\beta} - V^G_2,\\
\]

\[
u(\tilde{W}^B) = \mathcal{V} + g(e_C) - s(e_C)\frac{g'(e_C)}{\beta} - V^B_2,\\
\]

\[
u(\tilde{W}^{GG}) = V^G_2 + g(e_C) + (1 - s(e_C))\frac{g'(e_C)}{\beta},\\
\]

\[
u(\tilde{W}^{GB}) = V^G_2 + g(e_C) - s(e_C)\frac{g'(e_C)}{\beta},\\
\]

\[
u(\tilde{W}^{BG}) = V^B_2 + g(e_C) + (1 - s(e_C))\frac{g'(e_C)}{\beta},\\
\]

\[
u(\tilde{W}^{BB}) = V^B_2 + g(e_C) - s(e_C)\frac{g'(e_C)}{\beta},\\
\]

\[
u(\tilde{W}^M_{M1}) = \mathcal{U}_M + g(e_M) + (1 - s(e_M))\frac{g'(e_M)}{\beta},\\
\]

\[
u(\tilde{W}^B_{M1}) = \mathcal{U}_M + g(e_M) - s(e_M)\frac{g'(e_M)}{\beta},\\
\]

\[
u(\tilde{W}^M_{M2}) = \mathcal{U}_M + g(e_M) + (1 - s((e_M))\frac{g'((e_M))}{\beta} - P(e_{-M})\mathcal{V}, \text{ and}\\
\]

\[
u(\tilde{W}^B_{M2}) = \mathcal{U}_M + g(e_M) - s(e_M)\frac{g'(e_M)}{\beta}.
\]

\[60^I \text{I use the hat notation to indicate guaranteed job security case.} \]
the optimal promotion incentive for a given CEO's effort level $e$

Also, this means that $\hat{\mathcal{A}} = \{\hat{W}_G^G, \hat{W}_M^G, \hat{W}_C^G, \hat{W}_C^{BG}, \hat{W}_C^{BB}, (\hat{W}_G^M, \hat{W}_M^B, (\hat{W}_G^M, \hat{W}_M^B)\}$.

$V_2^G = s(e)u(\hat{W}_C^{BG}) + (1 - s(e))u(\hat{W}_C^{BB}) - g(e)$, and

$V_2^B = s(e)u(\hat{W}_C^{BG}) + (1 - s(e))u(\hat{W}_C^{BB}) - g(e)$.

Then, the first order condition with respect to $\hat{\mathcal{V}}$ for unguaranteed situation is

$$-\frac{s(e)}{u'(\hat{W}_C^{G})} - \frac{1 - s(e)}{u'(\hat{W}_C^{B})} + \delta(1 - s(e))\frac{1 - (1 - s(e))}{u'(\hat{W}_C^{G})} + \frac{s(e)}{u'(\hat{W}_C^{G})} \frac{1 - (1 - s(e))}{u'(\hat{W}_C^{G})} = 0.$$

On the other hand, the condition for guaranteed case is

$$-\frac{s(e)}{u'(\hat{W}_C^{G})} - \frac{1 - s(e)}{u'(\hat{W}_C^{B})} + \delta \frac{1 - (1 - s(e))}{u'(\hat{W}_C^{G})} = 0.$$

First, I show that there is $\delta^* \in (0, 1)$ such that $(V_2^B)^* < \hat{\mathcal{V}}$ for a given $\hat{\mathcal{V}} \in [0, \infty)$. Note that $\hat{\mathcal{V}}$ and $(V_2^B)^*$ satisfy

$$\frac{1}{u'(\hat{W}_C^{B})} - \delta \left[ \frac{s(e)}{u'(\hat{W}_C^{G})} + \frac{1 - s(e)}{u'(\hat{W}_C^{G})} \right] = 0.$$

Suppose that $\delta = 1$. Then, the equation cannot hold if $\hat{\mathcal{V}} \leq (V_2^B)^*$ since this inequality implies that $(\hat{W}_C^{BG})^* > (\hat{W}_C^{BB})^* \geq (\hat{W}_C^G)^*$. Here, the first inequality holds since $e_C > 0$ and the last inequality holds as a strict inequality unless $\mathcal{V} = (V_2^B)^* = 0$. Since this is true for all $\hat{\mathcal{V}} \in [0, \infty)$, there is $\delta^* \in (0, 1)$ such that $(V_2^B)^* < \hat{\mathcal{V}}$.

There are two possible cases when $e_C = 0$: 1) $\hat{\mathcal{V}}^*(0) > 0$, and 2) $\hat{\mathcal{V}}^*(0) = 0$, where $\hat{\mathcal{V}}^*(e_C)$ is the optimal promotion incentive for a given CEO’s effort level $e_C$. First, I show that $\frac{\partial (V_2^B)^*}{\partial \hat{\mathcal{V}}} > 0$ and $\frac{\partial \hat{\mathcal{V}}^*(e_C)}{\partial e_C} < 0$ when $\hat{\mathcal{V}}^*(e_C) > 0$ for $e_C \in (0, 1)$. Notice that, by the implicit function theorem,

$$\frac{\partial (V_2^B)^*}{\partial \hat{\mathcal{V}}} = \frac{u''(\hat{W}_C^B)}{u'(\hat{W}_C^B)^3} + \frac{\delta s(e_C)u''(\hat{W}_C^{BG})}{u'(\hat{W}_C^{G})^3} + \frac{\delta(1 - s(e_C))u''(\hat{W}_C^{BB})}{u'(\hat{W}_C^{G})^3} > 0.$$

Also, this means that $\frac{\partial (V_2^B)^*}{\partial \hat{\mathcal{V}}}$ is less than 1. Likewise, $0 < \frac{\partial (V_2^G)^*}{\partial \hat{\mathcal{V}}} < 1$. Hence,

$$\frac{\partial \hat{\mathcal{V}}^*(e_C)}{\partial e_C} = -\frac{\partial^2 \hat{F}(\hat{\mathcal{V}}^*)}{\partial e_C \partial \hat{\mathcal{V}}} < 0,$$

26
where

\[
\frac{\partial^2 F(\hat{\varphi})}{\partial e_C \partial \hat{\varphi}} = -\beta \left[ \frac{1}{u'(\hat{W}_C^G)} - \frac{1}{u'(\hat{W}_B^G)} \right] + s(e_C) \frac{u''((\hat{W}_C^G)^*)}{u'(\hat{W}_C^G)^3} \left[ (1 - s(e_C)) \frac{g''(e_C)}{\beta} - \frac{\partial(V_2^G)^*}{\partial e_C} \right] \\
+ (1 - s(e_C)) \frac{u''((\hat{W}_B^G)^*)}{u'(\hat{W}_B^G)^3} \left[ -s(e_C) \frac{g''(e_C)}{\beta} - \frac{\partial(V_B^G)^*}{\partial e_C} \right] \\
= \beta \left[ \frac{1}{u'(\hat{W}_C^G)} - \frac{1}{u'(\hat{W}_B^G)} \right] \\
- s(e_C) \frac{u''((\hat{W}_C^G)^*)}{u'(\hat{W}_C^G)^3} \cdot \\
\left[ \frac{u''((\hat{W}_C^G)^*)}{u'(\hat{W}_C^G)^3} + \delta \left[ s(e_C) \frac{u''((\hat{W}_C^G)^*)}{u'(\hat{W}_C^G)^3} + (1 - s(e_C)) \frac{u''((\hat{W}_B^G)^*)}{u'(\hat{W}_B^G)^3} \right] \right] \\
- (1 - s(e_C)) \frac{u''((\hat{W}_B^G)^*)}{u'(\hat{W}_B^G)^3} \cdot \\
\left[ \frac{u''((\hat{W}_B^G)^*)}{u'(\hat{W}_B^G)^3} + \delta \left[ s(e_C) \frac{u''((\hat{W}_B^G)^*)}{u'(\hat{W}_B^G)^3} + (1 - s(e_C)) \frac{u''((\hat{W}_B^G)^*)}{u'(\hat{W}_B^G)^3} \right] \right] \\
+ \delta s(e_C)(1 - s(e_C)) \frac{g''(e_C)}{\beta},
\]

and

\[
\frac{\partial^2 F(\hat{\varphi})}{\partial \hat{\varphi}^2} = s(e_C) \frac{u''((\hat{W}_C^G)^*)}{u'(\hat{W}_C^G)^3} \left[ 1 - \frac{\partial(V_2^G)^*}{\partial \hat{\varphi}} \right] + (1 - s(e_C)) \frac{u''((\hat{W}_B^G)^*)}{u'(\hat{W}_B^G)^3} \left[ 1 - \frac{\partial(V_B^G)^*}{\partial \hat{\varphi}} \right] \\
+ \delta(1 - (1 - s(e_M))^N) P(e_M) \frac{u''((\hat{W}_M^G)^*)}{u'(\hat{W}_M^G)^3} < 0,
\]
where I use the following two results

$$
\frac{\partial (V_2^G)^*}{\partial e_C} = \frac{u''(\hat{W}_C^G)}{u'(\hat{W}_C^G)^3} + \delta \left[ s(e_C) \frac{u''(\hat{W}_C^{GG})}{u'(\hat{W}_C^{GG})^3} + (1 - s(e_C)) \frac{u''(\hat{W}_C^{GB})}{u'(\hat{W}_C^{GB})^3} \right] \left\{ (1 - s(e_C)) \frac{g''(e_C)}{\beta} \frac{u''(\hat{W}_C^G)}{u'(\hat{W}_C^G)^3} \right. \\
+ \delta \left[ \beta \left( \frac{1}{u'(\hat{W}_C^{GG})} - \frac{1}{u'(\hat{W}_C^{GB})} \right) - s(e_C)(1 - s(e_C)) \frac{g''(e_C)}{\beta} \left( \frac{u''(\hat{W}_C^{GG})}{u'(\hat{W}_C^{GG})^3} - \frac{u''(\hat{W}_C^{GB})}{u'(\hat{W}_C^{GB})^3} \right) \right\}, \text{ and}
$$

$$
\frac{\partial (V_2^B)^*}{\partial e_C} = \frac{u''(\hat{W}_C^B)}{u'(\hat{W}_C^B)^3} + \delta \left[ s(e_C) \frac{u''(\hat{W}_C^{BG})}{u'(\hat{W}_C^{BG})^3} + (1 - s(e_C)) \frac{u''(\hat{W}_C^{BB})}{u'(\hat{W}_C^{BB})^3} \right] \left\{ -s(e_C) \frac{g''(e_C)}{\beta} \frac{u''(\hat{W}_C^B)}{u'(\hat{W}_C^B)^3} \right. \\
+ \delta \left[ \beta \left( \frac{1}{u'(\hat{W}_C^{BG})} - \frac{1}{u'(\hat{W}_C^{BB})} \right) - s(e_C)(1 - s(e_C)) \frac{g''(e_C)}{\beta} \left( \frac{u''(\hat{W}_C^{BG})}{u'(\hat{W}_C^{BG})^3} - \frac{u''(\hat{W}_C^{BB})}{u'(\hat{W}_C^{BB})^3} \right) \right\}
$$

based on the implicit function theorem.

For the previous results, I exploit the condition \((\hat{W}_C^G)^* > (\hat{W}_C^B)^*, (\hat{W}_C^{GG})^* > (\hat{W}_C^{BG})^*, \) and \((\hat{W}_C^{GB})^*) > (\hat{W}_C^{BB})^*, \) which all hold since \((V_2^G)^* > (V_2^B)^*. \) These imply that

$$
\frac{1}{u'(\hat{W}_C^G)^*} = \delta \left[ \frac{s(e_C)}{u'(\hat{W}_C^{GG})^*} + \frac{1 - s(e_C)}{u'(\hat{W}_C^{GB})^*} \right] > \delta \left[ \frac{s(e_C)}{u'(\hat{W}_C^{GB})^*} + \frac{1 - s(e_C)}{u'(\hat{W}_C^{BB})^*} \right] = \frac{1}{u'(\hat{W}_C^B)^*}.
$$

Here, I show that why the condition, \((V_2^G)^* > (V_2^B)^*, \) holds. Suppose \((V_2^G)^* \leq (V_2^B)^*. \) Then \((\hat{W}_C^G)^* \leq (\hat{W}_C^B)^* \) according to the same logic above. Note that \(u((\hat{W}_C^G)^*) + (V_2^G)^* \) must be strictly greater than \(u((\hat{W}_C^B)^*) + (V_2^B)^* \) in order to induce managers to exert a positive effort. However, two conditions, \((V_2^G)^* \leq (V_2^B)^* \) and \((\hat{W}_C^G)^* \leq (\hat{W}_C^B)^*, \) yield \(u((\hat{W}_C^G)^*) + (V_2^G)^* \leq u((\hat{W}_C^B)^*) + (V_2^B)^*. \) Therefore, \((V_2^G)^* \) must be strictly greater than \((V_2^B)^*. \)

The next step is to show that there is \(\bar{e}_C \in (0, 1) \) such that \(\hat{V}^*(e_C) = 0 \) if \(e_C \in [\bar{e}_C, 1) \) and \(\hat{V}^*(e_C) > 0 \) if \(e_C \in [0, \bar{e}_C). \) Since \(\frac{\partial \hat{V}^*(e_C)}{\partial e_C} < 0 \) when \(\hat{V}^*(e_C) > 0, \) it is enough to show that there is \(\bar{e}_C \) such that \(\hat{V}^*(e_C) = 0. \) Recall that

$$
\frac{\partial \hat{F}(\hat{V})}{\partial \hat{V}} = -\frac{s(e_C)}{u'(\hat{W}_C^G)} - \frac{1 - s(e_C)}{u'(\hat{W}_C^B)} + \delta \frac{1 - (1 - s(e_M))^{N}}{u'(\hat{W}_M^{M2})}.
$$

When \(\hat{V} = 0, \) the last term is a positive constant regardless of the value of \(e_C. \) On the other hand, from
the condition

\[ \frac{1}{u'(W^G_C)} - \delta \left[ \frac{s(e_C)}{u'(W^G_G)} + \frac{1 - s(e_C)}{u'(W^G_B)} \right] = 0 \]

, it can be shown that the first term approaches negative infinity as \( e_C \) approaches one because \((V^G_2)^* \geq \frac{\hat{\nu}}{2} = 0 \) and \( u((W^G_C)^*) \) approaches positive infinity as \( e_C \) converges to one.

Hence, there is \( \bar{e}_C \) supporting the optimal choice of zero promotion incentive. This result yields that there is \( \bar{e}_C \in [0, \bar{e}_C) \) such that \((V^B_2)^* \leq 0 \) if \( e_C \in [\bar{e}_C, 1) \) since \((V^B_2)^* < \hat{\nu}^*(e_C)\).

The remaining proof is to show that \( \nu^*(e_C) \geq \hat{\nu}^*(e_C) \) when \( e_C \in [\bar{e}_C, 1) \). Notice that, when \( \nu = \hat{\nu} \in [0, \hat{\nu}^*(e_C)] \) for \( e_C \in [\bar{e}_C, 1) \),

\[ \frac{\partial F(\nu)}{\partial \nu} > \frac{\partial \hat{F}(\hat{\nu})}{\partial \hat{\nu}} \]

since \((W^G_C)^* = (\hat{W}^G_C)^*, (W^B_C)^* < (\hat{W}^G_C)^*, \) and \((W^G_M)^* > (\hat{W}^G_M)^*\). Hence, \( \nu^*(e_C) \geq \hat{\nu}^*(e_C) \) when \( e_C \in [\bar{e}_C, 1) \). Moreover, when \( e_C \in [\bar{e}_C, \bar{e}_C], \nu^*(e_C) > \hat{\nu}^*(e_C) \) since \( \frac{\partial \hat{F}(\hat{\nu})}{\partial \hat{\nu}} |_{\hat{\nu} = \hat{\nu}^*(e_C)} = 0 \).

Consider the second case, \( \hat{\nu}^*(0) = 0 \). In this case, \( \hat{\nu}^*(e_C) = 0 \) for every \( e_C \in (0, 1) \). Hence, \( \nu^*(e_C) \geq \hat{\nu}^*(e_C) \) regardless of the value of \( e_C \in (0, 1) \).

### A.25 Proof of Proposition 14

For brevity, I denote \( e_{M11} = e_{M21} \) by \( e_M \) and \( e_{M12} = e_{M22} \) by \( e_M \). Suppose that \((U^2_M)^* \leq (U^2_M)^*\). Note that, for a given \( \nu \), the expected utility for the second period, \( U^2_M \), is determined according to the equation

\[ \frac{1}{u'(W^G_{Mi})} = \delta \left[ \frac{s(e_{Mi})}{u'(W^G_G)} + \frac{1 - s(e_{Mi})}{u'(W^G_B)} \right], \]

\( i = 1, 2 \). This equation and the condition that \((U^2_M)^* \leq (U^2_M)^* \) imply that \((W^G_M)^* \leq (W^G_M)^* \) in order for the inequality to hold, the following must hold

\[ (1 - s(e_{C1}))P(e_{-M1})\nu + (1 - (1 - s(e_{C1}))P(e_{-M1}))(U^2_M)^* \geq P(e_{-M1})\nu + (1 - P(e_{-M1}))(U^2_M)^*, \]

which implies that \((1 - P(e_{-M1}))(U^2_M)^* < (U^2_M)^* \) + \( s(e_{C1})P(e_{-M1})(U^2_M)^* \geq s(e_{C1})P(e_{-M1})\nu \).

Then, \((U^2_M)^* \) must be greater than \( \nu \) since \((U^2_M)^* \leq (U^2_M)^* \). This contradicts the given condition. Hence, \((U^2_M)^* > (U^2_M)^* \) if \( \nu^* > (U^2_M)^* \). Moreover, the difference between \( u(W^G_M) \) and \( u(W^G_M) \)
is
\[u(W_{M1}^G) - u(W_{M2}^G) = s(e_C)P(e_-)V - [(1 - P(e_-))(U_{M1}^2 - U_{M2}^2) + s(e_C)P(e_-)U_{M1}^2],\]
which has a positive value when \(V > (U_{M1}^2) > (U_{M2}^2).\) That is, \((W_{M1}^G)^* > (W_{M2}^G)^*\).

**A.26 Proof of Proposition 15**

First, I show that there is a constant \(\tilde{N}\) such that \(V^*(N+1) - V^*(N) \leq 0\) if \(N > \tilde{N}\).

For a given \(V\) and \(N\), \(e_C(N)\) is determined by
\[
E \left[ f \left( \sum_{i=1}^{N} X_i \right) \right] \beta(G - B_C) = \beta(W_C^G(N) - W_B^B(N))
+ s(e_C(N))(1 - s(e_C(N))) \frac{g''(e_C(N))}{\beta} \left[ \frac{1}{u'(W_C^G(N))} - \frac{1}{u'(W_B^B(N))} \right]. \tag{15}
\]

**Claim 3.** There is \(\mathcal{M}\) such that
\[
E \left[ f \left( \sum_{i=1}^{N+1} X_i \right) \right] - E \left[ f \left( \sum_{i=1}^{N} X_i \right) \right] > \mathcal{M}
\]
for every \(N\).

**Proof.** For brevity, denote
\[
E \left[ f \left( \sum_{i=1}^{N} X_i \right) \right] = \sum_{i=0}^{N} \binom{N}{i} s(e_M)^i(1 - s(e_M))^{N-i} f(iG_M + (N - i)B_M)
\]
by \(I(N)\). Also, I denote
\[
\min_{i} \left[ f(iG_M + (N + 1 - i)B_M) - f(iG + (N - i)B_M) \right]
\]
by \(f\). Note that there is \(\mathcal{M}_f > 0\) such that \(f \geq \mathcal{M}_f\) for every \(N\) since \(f'(x) > 0\) for every \(x \geq 0\).

Then,
\[ I(N + 1) - I(N) = \sum_{i=0}^{N} \binom{N}{i} s(e_M)^i(1 - s(e_M))^{N-i}. \]

\[
\left[ \frac{N + 1}{N + 1 - i} (1 - s(e_M)) f(iG_M + (N + 1 - i)B_M) - f(iG_M + (N - i)B_M) \right] \\
+ s(e_M)^{N+1} f((N + 1)G_M) \\
\geq \sum_{i=0}^{N} \binom{N}{i} s(e_M)^i(1 - s(e_M))^{N-i}. \\
\left[ \frac{N + 1}{N + 1 - i} (1 - s(e_M)) - 1 \right] f(iG_M + (N - i)B_M) \\
+ s(e_M)^{N+1} f((N + 1)G_M) + f. 
\]

Denote \([ (N + 1)s(e_M) ]\) by \(\hat{s}\). Then,

\[
\left[ \frac{N + 1}{N + 1 - i} (1 - s(e_M)) - 1 \right] \begin{cases} > 0 & \text{if } i \geq \hat{s} \\ \leq 0 & \text{otherwise.} \end{cases}
\]

There are two possible cases.

1. \((\hat{s} = N + 1)\)

Then,

\[ I(N + h) - I(h) \geq \sum_{i=0}^{N} \binom{N}{i} s(e_M)^i(1 - s(e_M))^{N-i}. \]

\[
\left[ \frac{N + 1}{N + 1 - i} (1 - s(e_M)) - 1 \right] f(iG_M + (N - i)B_M) \\
+ s(e_M)^{N+1} f(NG_M) \\
+ s(e_M)^{N+1} [f((N + 1)G_M) - f(NG_M)] + f \\
> s(e_M)^{N+1} [f((N + h)G_M) - f(NG_M)] + f, 
\]

where the last inequality holds since

\[
\sum_{i=0}^{N} \binom{N}{i} s(e_M)^i(1 - s(e_M))^{N-i} \left[ \frac{(N + h)!(N - i)!}{N!(N + h - i)!} (1 - s(e_M))^h - 1 \right] = s(e_M)^{N+1}.
\]

2. \((\hat{s} < N + 1)\)
First, notice that if $\frac{N}{N+1} > s(e_M)$, then $\hat{s} < N+1$. That is, if $N$ is sufficiently large, $\hat{s} < N+1$. In this case,

$$I(N+1) - I(N) \geq \sum_{i=0}^{\hat{s}-1} \binom{N}{i} s(e_M)^i (1 - s(e_M))^{N-i} \cdot \left[ \frac{N+1}{N+1-i} (1 - s(e_M)) - 1 \right] f(iG_M + (N-i)B_M)$$

$$+ \sum_{i=\hat{s}}^{N} \binom{N}{i} s(e_M)^i (1 - s(e_M))^{N-i} \cdot \left[ \frac{N+1}{N+1-i} (1 - s(e_M)) - 1 \right] f(iG_M + (N-i)B_M)$$

$$+ s(e_M)^{N+1} f(NG_M) + f$$

$$> \sum_{i=\hat{s}}^{N} \binom{N}{i} s(e_M)^i (1 - s(e_M))^{N-i} \cdot \left[ \frac{N+1}{N+1-i} (1 - s(e_M)) - 1 \right] \cdot [f(\hat{s}G_M + (N-\hat{s})B_M) - f((\hat{s}-1)G_M + (N-\hat{s}+1)B_M)] + f$$

Hence

$$E \left[ f \left( \sum_{i=1}^{N+1} X_i \right) \right] - E \left[ f \left( \sum_{i=1}^{N} X_i \right) \right] > f \geq M_f > 0.$$  

This result means that $e_C(N+1) > e_C(N)$ if $\mathcal{V}$ is fixed.

Now, I show that there is $\tilde{N}$ such that $\frac{\partial F(\mathcal{V})}{\partial \mathcal{V}} < 0$ for every $\mathcal{V} \in [0, \tilde{\mathcal{V}}]$. Note that

$$\frac{\partial F(\mathcal{V}|N)}{\partial \mathcal{V}} = \frac{s(e_C(\mathcal{V}, N))}{u'(M_{C}^{\mathcal{V}}(\mathcal{V}, e_C(\mathcal{V}, N)))} - \frac{1 - s(e_C(\mathcal{V}, N))}{u'(M_{C}^{\mathcal{V}}(\mathcal{V}, e_C(\mathcal{V}, N)))} + \frac{1 - (1 - s(e_M))}{u'(M_{\tilde{w}M}^{\mathcal{V}}(\mathcal{V}, e_C(\mathcal{V}, N)))}$$

$$\leq \frac{-s(e_C(\tilde{\mathcal{V}}, N))}{u'(M_{C}^{\mathcal{V}}(0, e_C(\mathcal{V}, N)))} - \frac{1 - s(e_C(\tilde{\mathcal{V}}, N))}{u'(M_{C}^{\mathcal{V}}(0, e_C(\mathcal{V}, N)))} + \frac{1 - (1 - s(e_M))}{u'(M_{\tilde{w}M}^{\mathcal{V}}(0, e_C(\mathcal{V}, N)))},$$

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where
\[ W^G_C(V_1, e_C(V_2, N)) = V_1 + g(e_C(V_2, N)) + (1 - s(e_C(V_2, N))) \frac{g'(e_C(V_2, N))}{\beta} \]
\[ W^B_C(V_1, e_C(V_2, N)) = V_1 + g(e_C(V_2, N)) - s(e_C(V_2, N)) \frac{g'(e_C(V_2, N))}{\beta} \]
and \( e_C(V_2, N) \) satisfies
\[ E \left[ f \left( \sum_{i=1}^{N} X_i \right) \right] \beta(G_C - B_C) = \beta(W^G_C(V_2, e_C(V_2, N)) - W^B_C(V_2, e_C(V_2, N)) \]
\[ + s(e_C(V_2, N))(1 - s(e_C(V_2, N))) \frac{g''(e_C(V_2, N))}{\beta} \left[ \frac{1}{u'(W^G_C(V_2, e_C(V_2, N)))} - \frac{1}{u'(W^B_C(V_2, e_C(V_2, N)))} \right] \]
\[ \text{since} \]
\[ \frac{\partial}{\partial V_1} \left[ \frac{s(e_C(V_2, N))}{u'(W^G_C(V_1, e_C(V_2, N)))} + \frac{1 - s(e_C(V_2, N))}{u'(W^B_C(V_1, e_C(V_2, N)))} \right] > 0, \]
\[ \frac{\partial}{\partial V_2} \left[ \frac{s(e_C(V_2, N))}{u'(W^G_C(V_1, e_C(V_2, N)))} + \frac{1 - s(e_C(V_2, N))}{u'(W^B_C(V_1, e_C(V_2, N)))} \right] < 0. \]
\[ \text{Since } V \text{ is bounded } \frac{1 - (1 - s(e_M))^{N}}{u'(W^G_M)} \leq - \frac{1}{u'(W^B_M)}, \text{ where } W^G_M \text{ satisfies} \]
\[ u(W^G_M) = U_M + g(e_M) + (1 - s(e_M)) \frac{g'(e_M)}{\beta}. \]
Moreover, there is \( \hat{e}_C \in (0, 1) \) such that
\[ \frac{s(e_C)}{u'(W^G_C(0, e_C))} + \frac{1 - s(e_C)}{u'(W^B_C(0, e_C))} > \frac{1}{u'(W^G_M)} \]
if \( e_C \geq \hat{e}_C \) since \( \lim_{e_C \to 1} W^G_C(0, e_C) = \infty. \) Since there is \( N_1 \) such that \( e_C(\bar{V}, N) \geq \hat{e}_C \text{ if } N \geq N_1, \)
\[ \frac{\partial F(V|N)}{\partial V} \leq - \frac{s(e_C(\bar{V}, N))}{u'(W^G_C(0, e_C(\bar{V}, N)))} - \frac{1 - s(e_C(\bar{V}, N))}{u'(W^B_C(0, e_C(\bar{V}, N)))} + \frac{1 - (1 - s(e_M))^{N}}{u'(W^G_M)} \]
\[ < - \frac{s(e_C(\bar{V}, N))}{u'(W^G_C(0, e_C(\bar{V}, N)))} - \frac{1 - s(e_C(\bar{V}, N))}{u'(W^B_C(0, e_C(\bar{V}, N)))} + \frac{1}{u'(W^G_M)} \]
\[ < 0 \]
when \( N \geq N_1. \)

Hence, \( V^* = 0 \text{ if } N \geq N_1. \) Also, this implies that there is \( N_2 < N_1 \) such that \( V^*(N+1) - V^*(N) < 0 \) when \( N \in [N_2, N_1 - 1] \) if there is \( N^* < N_1 \) such that \( V^*(N^*) > 0. \)
Moreover, $e_C^*(N + 1) - e_C^*(N) > 0$ when $N \geq N_2$ since
\[
\frac{\partial E(V, e_C)}{\partial V} > 0 \text{ and } \frac{\partial E(V, e_C)}{\partial e_C} > 0,
\]
where
\[
E(V, e_C) \equiv \beta(W_C^G(V, e_C) - W_C^B(V, e_C))
\]
\[+ s(e_C)(1 - s(e_C)) \frac{g''(e_C)}{\beta} \left[ \frac{1}{u'(W_C^G(V, e_C))} - \frac{1}{u'(W_C^B(V, e_C))} \right]
\]
comes from the right hand side of (15).

A.27 Proof of Proposition 16

First, note that constraints regarding managers give the following results

\[
u(W_C) = N
\]
\[
u(W_M^G) = U_M + g(\epsilon_L) + (1 - s_L(\epsilon_L)) \frac{g'(\epsilon_L)}{\beta} - \beta(\epsilon_M)N,
\]
\[
u(W_M^B) = U_M + g(\epsilon_L) - s_L(\epsilon_L) \frac{g'(\epsilon_L)}{\beta} - \beta(\epsilon_M)N, \text{ and}
\]
\[
\epsilon_H = \epsilon_L \frac{\beta}{\beta}
\]
for a given $(\epsilon_L, V)$. From now on, I use subscript 1 for promotion rule 1 and subscript 2 for promotion 2 in order to distinguish two problems. For a given $\epsilon_L^1$ and $\epsilon_L^2$, denote the optimal $V$ by $V_C^1(\epsilon_L^1)$ under promotion rule 1 and $V_C^2(\epsilon_L^1)$ under promotion rule 2. Also, denote the firm’s objective function under promotion rule 1 and promotion rule 2 by $F_1(\epsilon_L^1|\gamma)$ and $F_2(\epsilon_L^2|\gamma)$, respectively, for a given $\gamma$. That is,

\[
F_1(\epsilon_L^1|\gamma) = \gamma \left[ (\beta - \beta)H_1(\epsilon_L) + \beta \right] - W_C1 + 2[q_sH(\epsilon_H) + (1 - q)s_L(\epsilon_L)](G - W_M^G)
\]
\[+ 2[1 - q_sH(\epsilon_H) - (1 - q)s_L(\epsilon_L)](B - W_M^B), \text{ and}
\]
\[
F_2(\epsilon_L^2|\gamma) = \gamma \left[ (\beta - \beta)H_2(\epsilon_L) + \beta \right] - W_C2 + 2[q_sH(\epsilon_H) + (1 - q)s_L(\epsilon_L)](G - W_M^G)
\]
\[+ 2[1 - q_sH(\epsilon_H) - (1 - q)s_L(\epsilon_L)](B - W_M^B),
\]
where

\[
H_1(\epsilon_L^1) = \frac{q_sH(\epsilon_H^1)}{q_sH(\epsilon_H^1) + (1 - q)s_L(\epsilon_L^1)} \left[ 2(q_sH(\epsilon_H^1) + (1 - q)s_L(\epsilon_L^1)) - (q_sH(\epsilon_H^1) + (1 - q)s_L(\epsilon_L^1))^2 \right]
\]
\[+ q \left[ 2(q_sH(\epsilon_H^1) + (1 - q)s_L(\epsilon_L^1)) + (q_sH(\epsilon_H^1) + (1 - q)s_L(\epsilon_L^1))^2 \right], \text{ and}
\]
\[
H_2(\epsilon_L^2) = q + q(1 - q)(s_H(\epsilon_H^2) - s_L(\epsilon_L^2)).
\]
Then, the firm’s problem is to choose $e_L$ in order to maximize its objective function. Notice that, for a given $e_{Lj}$, $V_j^e(e_{Lj})$, $j = 1$ and $2$, satisfies

$$\frac{\partial F_1(e_{L1}|\gamma)}{\partial V_1} = -\frac{1}{u'(W_{C1})} + \frac{2(qs_H(e_{H1}) + (1-q)s_L(e_{L1}))}{u'(W_{M1}^G)} P(e_{-M1}) = 0,$$

and

$$\frac{\partial F_2(e_{L2}|\gamma)}{\partial V_2} = -\frac{1}{u'(W_{C2})} + \frac{2(qs_H(e_{H2}) + (1-q)s_L(e_{L2}))}{u'(W_{M2}^G)} P(e_{-M2}) + \frac{2(1 - qs_H(e_{H2}) - (1-q)s_L(e_{L2}))}{u'(W_{L2}^B)} R(e_{-M2}) = 0.$$ 

Also, the first order conditions with respect to $e_{Lj}$ are

$$\frac{\partial F_1(e_{L1}|\gamma)}{\partial e_{L1}} = \gamma(\beta - \beta)\frac{\partial H_1(e_{L1})}{\partial e_{L1}} + 2 \left( q\frac{\beta^2}{\beta} + (1-q)\beta \right) \left[ G - B - (W_{M1}^G - W_{M1}^B) \right]$$

$$- 2\frac{\kappa}{\beta} \left[ (1 - s_L(e_{L1})) \frac{qs_H(e_{H1}) + (1-q)s_L(e_{L1})}{u'(W_{M1}^G)} - s_L(e_{L1}) \frac{1 - qs_H(e_{H1}) - (1-q)s_L(e_{L1})}{u'(W_{M1}^B)} \right]$$

$$+ 2 \frac{qs_H(e_{H1}) + (1-q)s_L(e_{L1})}{u'(W_{M1}^G)} \frac{\partial P(e_{-M1})}{\partial e_{L1}} V_1^e(e_{L1}),$$

and

$$\frac{\partial F_2(e_{L2}|\gamma)}{\partial e_{L2}} = \gamma(\beta - \beta)\frac{\partial H_2(e_{L2})}{\partial e_{L2}} + 2 \left( q\frac{\beta^2}{\beta} + (1-q)\beta \right) \left[ G - B - (W_{M2}^G - W_{M2}^B) \right]$$

$$- 2\frac{\kappa}{\beta} \left[ (1 - s_L(e_{L2})) \frac{qs_H(e_{H2}) + (1-q)s_L(e_{L2})}{u'(W_{M2}^G)} - s_L(e_{L2}) \frac{1 - qs_H(e_{H2}) - (1-q)s_L(e_{L2})}{u'(W_{M2}^B)} \right]$$

$$+ 2 \frac{qs_H(e_{H2}) + (1-q)s_L(e_{L2})}{u'(W_{M2}^G)} \frac{\partial P(e_{-M2})}{\partial e_{L2}} V_2^e(e_{L2})$$

$$+ 2 \frac{1 - qs_H(e_{H2}) - (1-q)s_L(e_{L2})}{u'(W_{M2}^B)} \frac{\partial R(e_{-M2})}{\partial e_{L2}} V_2^e(e_{L2}).$$

According to Milgrom and Shannon (1994), $\frac{\partial e_{L1}}{\partial \gamma} \geq 0$ and $\frac{\partial e_{L1}}{\partial \gamma} \geq 0$ since

$$\frac{\partial^2 F_1(e_{L1}|\gamma)}{\partial e_{L1}\partial \gamma} = (\beta - \beta)\frac{\partial H_1(e_{L1})}{\partial e_{L1}}$$

$$= 2(\beta - \beta)q(1-q) \left( \frac{\beta^2}{\beta} - \beta \right) \left[ 1 - qs_H(e_{H1}) - (1-q)s_L(e_{L1}) \right] > 0,$$

and

$$\frac{\partial^2 F_2(e_{L2}|\gamma)}{\partial e_{L2}\partial \gamma} = (\beta - \beta)\frac{\partial H_2(e_{L2})}{\partial e_{L2}}$$

$$= (\beta - \beta)q(1-q) \left( \frac{\beta^2}{\beta} - \beta \right) > 0.$$ 

Moreover, these inequalities imply that $\frac{\partial e_{L1}}{\partial \gamma} > 0$ and $\frac{\partial e_{L1}}{\partial \gamma} > 0$ if $e^*_L \in \left( 0, \frac{\beta}{\beta} \right)$ and $e^*_L \in \left( 0, \frac{\beta}{\beta} \right)$, respectively, according to Edlin and Shannon (1998).
Also, there is $\gamma_1^*$ such that $e_{L1}^* = \frac{\beta}{\bar{\beta}}$, which means that $e_{H1} = 1$, if $\gamma \geq \gamma_1^*$ since $\left. \frac{\partial F_1(e_{L1}|\gamma)}{\partial e_{L1}} \right|_{e_{L1}=1}$ is a strictly increasing function in $\gamma$ and $\lim_{\gamma \to \infty} \left. \frac{\partial F_1(e_{L1}|\gamma)}{\partial e_{L1}} \right|_{e_{L1}=1} = \infty$.

Moreover, by the envelope theorem,

\[
\frac{\partial F_1(e_{L1}^*|\gamma)}{\partial \gamma} = (\bar{\beta} - \beta)H_1(e_{L1}^*) + \beta, \quad \text{and} \quad \frac{\partial F_2(e_{L2}^*|\gamma)}{\partial \gamma} = (\bar{\beta} - \beta)H_2(e_{L2}^*) + \beta.
\]

Since $H_1(e_{L1}) > H_2(e_{L1})$ when $e_{L1} = e_{L2}$, there is $\hat{e}_{L1} \in \left(0, \frac{\beta}{\bar{\beta}}\right)$ such that $H_1(\hat{e}_{L1}) > H_2 \left(\frac{\beta}{\bar{\beta}}\right)$. Hence, there is $\hat{\gamma}$ such that $F_1(e_{L1}^*|\gamma) > F_2(e_{L2}^*|\gamma)$ if $\gamma \geq \hat{\gamma}$.

Now, I show that $F_1(e_{L1}^*|\gamma) < F_2(e_{L2}^*|\gamma)$ when $\gamma = 0$. This is true since

\[
F_2(e_{L2}^*|\gamma) \geq F_2(e_{L1}^*|\gamma) > F_1(e_{L1}^*|\gamma).
\]

The second inequality holds since $(W_{M1}^G)^* = W_{M2}^G$ and $(W_{M1}^B)^* > W_{M2}^B$ if $(e_{L2}, \nu_2) = (e_{L1}^*, \nu_1^*)$. Hence, $\hat{\gamma} > 0$.

**Appendix B  Firm’s Problems in Detail**

**B.1 The Firm’s Problem in Section 7.1**

Under this extension, the firm’s problem is to choose $\nu \in [0, \infty)$ maximizing $F(\nu)$ defined by

\[
F(\nu) = \max_{\{(W_{M1}^G, W_{M1}^B), (W_{M2}^G, W_{M2}^B), (W_{G1}^G, W_{G1}^B), \}} \left\{ s(e_{C})(G_C - W_{C}^G) + (1 - s(e_{C}))(B_C - W_{C}^B) \right. \\
+ N \left[ s(e_{M1})(G_M - W_{M}^G) + (1 - s(e_{M1}))(B_M - W_{M}^B) \right] \\
+ \delta(Ns(e_{M1}) - 1 + (1 - s(e_{M1}))^N)\left[ s(e_{M2})(G_M - W_{M}^{GG}) + (1 - s(e_{M2}))(B_M - W_{M}^{GB}) \right] \\
\left. \right\}
\]

subject to

\[
E[\mathcal{U}(W_{C}^G, W_{C}^B, e_{C})] = \nu \quad (IC_C), \\
E[\mathcal{U}(W_{M1}^G, W_{M1}^B, e_{M1})] + s(e_{M1})\left\{ P(e_{M1})\nu + (1 - P(e_{M1}))E[\mathcal{U}(W_{M1}^{GG}, W_{M1}^{GB}, e_{M2})] \right\} = U_{M1} \quad (IR_{M1}), \\
\]

\[
e_{C} \in \arg \max_{e} \quad E[\mathcal{U}(W_{C}^G, W_{C}^B, e)] \quad (IC), \\
e_{M1} \in \arg \max_{\hat{e}} \quad E[\mathcal{U}(W_{M1}^G, W_{M1}^B, \hat{e})] + s(\hat{e})\left\{ P(e_{M1})\nu + (1 - P(e_{M1}))E[\mathcal{U}(W_{M1}^{GG}, W_{M1}^{GB}, e_{M2})] \right\} \quad (IC_{M1}), \\
\]

\[
e_{M2} \in \arg \max_{\hat{e}} \quad E[\mathcal{U}(W_{M2}^{GG}, W_{M2}^{GB}, \hat{e})] \quad (IC_{M2}),
\]

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where

\[ E[\mathcal{U}(W^G, W^B, e)] = s(e)u(W^G) + (1 - s(e))u(W^B) - g(e). \]

Note that the expected number of senior managers in the second period is

\[ \sum_{k=0}^{N} \binom{N}{k} s(e_{M1})^k (1 - s(e_{M1}))^{N-k} (k - 1) = \sum_{k=0}^{N} \binom{N}{k} ks(e_{M1})^k (1 - s(e_{M1}))^{N-k} - \sum_{k=0}^{N} \binom{N}{k} s(e_{M1})^k (1 - s(e_{M1}))^{N-k} + (1 - s(e_{M1}))^N = Ns(e_{M1}) - 1 + (1 - s(e_{M1}))^N. \]

### B.2 The Firm’s Problem in Section 7.2

The objective of the firm is to choose \( V \in [0, \infty) \) maximizing \( F(V) \) defined by

\[
F(V) \equiv \max_A s(e_1)(G_C - W^G) + (1 - s(e_1))(B_C - W^B) \\
+ \delta s(e_1)[s(e_2)(G_C - W^{GG}) + (1 - s(e_2))(B_C - W^{GB})] \\
+ N [s(e_{M1})(G_M - W^G_{M1}) + (1 - s(e_{M1}))(B_M - W^B_{M1})] \\
+ \delta Ns(e_1) [s(e_{M2})(G_M - W^G_{M2}) + (1 - s(e_{M2}))(B_M - W^B_{M2})]
\]

subject to

\[
E[\mathcal{U}(W^G_C, W^B_C, e_1)] + s(e_1)E[\mathcal{U}(W^{GG}_C, W^{GB}_C, e_2)] = V \ (IR) \\
E[\mathcal{U}(W^G_{M1}, W^B_{M1}, e_{M1})] + s(e_{M1})(1 - s(e_{M1}))P(e_{-M1})V = U_{M1} \ (IRM1), \\
E[\mathcal{U}(W^G_{M2}, W^B_{M2}, e_{M2})] + s(e_{M2})P(e_{-M2})V = U_{M2} \ (IRM2) \\
e_{C1} \in \arg \max_{e} E[\mathcal{U}(W^G_C, W^B_C, e)] + s(\hat{e})E[\mathcal{U}(W^{GG}_C, W^{GB}_C, e_2)] \ (IC1) \\
e_{C2} \in \arg \max_{e} E[\mathcal{U}(W^{GG}_C, W^{GB}_C, \hat{e})] \ (IC2) \\
e_{M1} \in \arg \max_{e} E[\mathcal{U}(W^G_{M1}, W^B_{M1}, \hat{e})] + s(\hat{e})(1 - s(e_{C1}))P(e_{-M1})V \ (ICM1), \\
e_{M2} \in \arg \max_{e} E[\mathcal{U}(W^G_{M2}, W^B_{M2}, \hat{e})] + s(\hat{e})P(e_{-M2})V \ (ICM2),
\]
where
\[ \mathcal{A} = \{(W_C^G, W_C^B, W_C^{GG}, W_C^{GB}), (W_M^G, W_M^B), (W_M^G, W_M^B)\} \]
and
\[ E[U(W^G, W_B, e)] = s(e)u(W^G) + (1 - s(e))u(W^B) - g(e). \]

When the CEO’s individual rationality condition binds at \( \mathcal{V} \), the compensation scheme \((W_C^G, W_C^B, W_C^{GG}, W_C^{GB})\) for the CEO satisfies
\[
\begin{align*}
u(W_C^G) &= \mathcal{V} + g(e_{C1}) + (1 - s(e_{C1}))\frac{g'(e_{C1})}{\beta} - V_2, \\
u(W_C^B) &= \mathcal{V} + g(e_{C1}) - s(e_{C1})\frac{g'(e_{C1})}{\beta}, \\
u(W_C^{GG}) &= V_2 + g(e_{C2}) + (1 - s(e_{C2}))\frac{g'(e_{C2})}{\beta}, \\
u(W_C^{GB}) &= V_2 + g(e_{C2}) - s(e_{C2})\frac{g'(e_{C2})}{\beta},
\end{align*}
\]
where
\[ V_2 = s(e_{C2})u(W_C^{GG}) + (1 - s(e_{C2}))u(W_C^{GB}) - g(e_{C2}) \]
is the successful CEO’s expected utility in the second period.

On the other hand, the compensation schemes for managers are characterized by
\[
\begin{align*}
u(W_M^G) &= U_M + g(e_{M1}) + (1 - s(e_{M1}))\frac{g'(e_{M1})}{\beta} - (1 - s(e_{C1}))P(e_{-M1})\mathcal{V}, \\
u(W_M^B) &= U_M + g(e_{M1}) - s(e_{M1})\frac{g'(e_{M1})}{\beta}, \\
u(W_M^{G}) &= U_M + g(e_{M2}) + (1 - s(e_{M2}))\frac{g'(e_{M2})}{\beta} - P(e_{-M2})\mathcal{V}, \text{ and} \\
u(W_M^{B}) &= U_M + g(e_{M2}) - s(e_{M2})\frac{g'(e_{M2})}{\beta}.
\end{align*}
\]

\[ \text{B.3 The Firm’s Problem in Section 9} \]

The expected profit from two managers is
\[
E[\Pi_M(e_H, e_L, W_M^G, W_M^B)] = q^2[2s_H(e_H)(G_M - W_M^G) + (1 - s_H(e_H))(B_M - W_M^B)] \\
+ 2q(1 - q)[s_H(e_H)(G_M - W_M^G) + (1 - s_H(e_H))(B_M - W_M^B)] \\
+ [s_L(e_L)(G_M - W_M^G) + (1 - s_L(e_L))(B_M - W_M^B)] \\
+ (1 - q)^2[2s_L(e_L)(G_M - W_M^G) + (1 - s_L(e_L))(B_M - W_M^B)].
\]

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Also, the choice of promotion rule determines the expected $\beta$ when the firm requires $e_H$ and $e_L$ from high-type and low-type managers according to: 1) when the firm uses promotion rule 1, $E[\beta|e_H, e_L]$ is equal to

$$E^{P1}[\beta|e_H, e_L] = \frac{qs_H(e_H)}{qs_H(e_H) + (1 - q)s_L(e_L)} \left[2(qs_H(e_H) + (1 - q)s_L(e_L)) - (qs_H(e_H) + (1 - q)s_L(e_L))^2\right]$$

$$+ q \left[1 - 2(qs_H(e_H) + (1 - q)s_L(e_L)) + (qs_H(e_H) + (1 - q)s_L(e_L))^2\right],$$

2) while the expectation has the following value

$$E^{P2}[\beta|e_H, e_L] = q + q(1 - q)(s_H(e_H) - s_L(e_L))$$

if the firm adopts promotion rule 2.