

Problem Set #6

Total points: 100. Each question weighted equally.

1. Consider $\mathbf{X}_n \equiv X_1, \dots, X_n \sim i.i.d. U[0, \theta]$. Consider the likelihood ratio statistic $\lambda(\mathbf{X}_n)$ for testing

$$H_0 : \theta = 3$$

$$H_1 : \theta \neq 3.$$

- (a) Consider a test of the form: $\mathbf{1}(\lambda(\mathbf{X}_n) < c)$, for some $c \in [0, 1]$.

(a) Derive the power function of this test.

(b) Is this test unbiased?

(c) What c should be chosen, for a size α test (where $\alpha \in [0, 1]$)?

- (b) Consider an alternative test statistic $Z(\mathbf{X}_n) \equiv 3 - \max(X_1, \dots, X_n)$, and a test

$$\mathbf{1}(Z_n < 0, \text{ or } Z_n > c), \quad c > 0.$$

(a) Derive the power function of this test.

(b) Is this test unbiased?

(c) What c should be chosen, for a size α test (where $\alpha \in [0, 1]$)?

2. CB, #8.5, parts (a) and (b)

3. CB, #8.6 (see pg. 624 for definition of exponential distribution)

4. CB, #8.13, parts (a)-(c)

5. CB, #8.15

6. CB, #10.34, part (a)