Problem Set #3: Misc.

1. Let $X \sim U[1,2]$, and define $Y = \log X$.

   (a) What is $\text{Prob}\,(Y \geq \epsilon)$, for any $\epsilon > 0$?
   (b) Use Chebyshev's inequality to bound $\text{Prob}\,(Y \geq \epsilon)$.

2. Suppose U and V are independent with *exponential distribution* with parameter λ. (A random variable T is exponentially distributed with parameter λ if its density is given by $f(t) = \lambda \exp(-\lambda t)$ with support $T > 0$.) Define $X = U + V$ and $Y = UV$. (Source: Amemiya, p. 86, #18)

   (a) Derive the joint density of (X, Y). *For part (a) only*, consider the truncated distribution of (U, V), truncated to the region $U > V$.

   (b) Find the best linear predictor of Y given X.
   (c) Find the best predictor of Y given X.

3. Find the best linear predictor and best predictor of Y given X if (X, Y) are bivariate normal $N(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$.

4. Let $x \sim \chi_k^2$. Derive the distribution of $(x - k)/\sqrt{2k}$, as $k \to \infty$.