Equilibrium Effects of Superstition in the Housing Market

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In many durable goods markets, products are differentiated in terms of their quality, and furthermore, consumers are heterogeneous in terms of the value they place on product quality. An example is the housing market in some Asian communities with a superstitious culture. In those markets, some apartments are considered lucky because their address numbers contain the so-called “lucky numbers” (see, for example, Shum, Sun & Ye (2014)). Among the consumers, some are superstitious and value lucky apartments more highly than non-lucky ones, while the others are non-superstitious and value lucky and non-lucky apartments equally.

Another example is the car market, in which new cars have a higher quality than used cars, but consumers differ in how they value a car’s attribute of being new.1 Some consumers are “new car lovers” whose valuation of a new car is much higher than that of a used car, while some other consumers are “used car lovers” whose valuation does not differ significantly between new and used cars.

In this article, we report the progress in an on-going project in which we investigate the interaction of the product differentiation and consumer heterogeneity described above. One focus is the effects of secondary market liquidity and consumers’ preference shocks on the equilibrium prices.

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In particular, we want to understand how the equilibrium prices change (1) when the secondary market becomes less liquid as transaction cost in the secondary market is heightened, and (2) when consumer heterogeneity becomes more persistent as the magnitude of their per-period random preference shocks is reduced.

I Model

To investigate the above issues, we build a model of a durable goods market with product quality differentiation and consumer preference heterogeneity, using the apartments housing market as an example. Time is discrete. Consumers are infinitely-lived and forward-looking, and they incur transaction costs when selling the apartment that they own.

There are three goods indexed by \( j = 0, 1, 2 \), respectively, where \( j = 1 \) indicates a lucky apartment, \( j = 2 \) indicates a non-lucky apartment, and \( j = 0 \) indicates the outside good. There are two types of consumers indexed by \( l = 1, 2 \), respectively, where \( l = 1 \) indicates a superstitious consumer and \( l = 2 \) indicates a non-superstitious consumer.

For a non-superstitious consumer, the two types of apartments offer the same utility, whereas for a superstitious consumer, a lucky apartment offers a higher utility than a non-lucky apartment. Let \( \alpha_j^l \) denote the per-period utility of good \( j \) for a type \( l \) consumer. Furthermore, let \( \alpha_1 \equiv \alpha_1^1 \) denote the utility that a superstitious consumer obtains from owning a lucky apartment, and let \( \alpha_2 \equiv \alpha_2^1 = \alpha_2^2 = \alpha_2^3 \) denote the utility when the consumer is non-superstitious and/or when the apartment is non-lucky, with \( \alpha_1 > \alpha_2 \). The outside good’s utility \( \alpha_0 \equiv \alpha_0^1 = \alpha_0^2 \) is normalized to 0.

A Consumers’ problem

There is a continuum of infinitely-lived consumers with measure 1. Consumers are differentiated in two dimensions. First, consumers differ with respect to superstition...
measure $\lambda \in [0, 1]$ of consumers are superstitious ($l = 1$), and the remaining $1 - \lambda$ are non-superstitious ($l = 2$). A consumer’s superstition is unchanging across time periods and represents a persistent component of preference heterogeneity across consumers.

Second, consumers also experience preference shocks that vary period-by-period. Let $\epsilon_{it} \equiv (\epsilon_{ijt}, j = 0, 1, 2)$ be the vector of preference shocks of consumer $i$ in period $t$, where the shocks are i.i.d. across $(i, j, t)$. These preference shocks represent time-varying horizontal differentiation among consumers.

The apartments are infinitely-lived and do not depreciate, and there is no new supply. The stocks of lucky and non-lucky apartments are therefore constant across time periods. Let $\theta_1$ and $\theta_2$ denote the stocks of lucky and non-lucky apartments, respectively, with $\theta_1 + \theta_2 \leq 1$. The apartments are fully utilized, i.e., no apartments are unoccupied in any period. Each consumer owns a single good in each period.

Let $p_{jt}$ be the price of good $j$ in period $t$ and let $\bar{p}_t = (p_{0t}, p_{1t}, p_{2t})$ be the price vector. $p_{0t} = 0$ for all $t$. Let $k_j$ denote the transaction cost incurred when selling good $j$, with $k_1 = k_2 = k$ and $k_0 = 0$, i.e., the transaction cost $k$ is incurred if and only if the consumer sells an apartment. We assume each preference shock $\epsilon_{ijt}$ is distributed type I extreme-value. We then aggregate up the choices of all consumers to obtain the aggregate demand for each type of apartments in period $t$.

**B Equilibrium and steady state**

We compute the equilibrium in the model by solving a system of equations consisting of the market clearing conditions, the law of motion of the industry state, and consumers’ recursive expected value function derived from the Bellman equation. We then obtain the steady state of the model based on the equilibrium law of motion. The results we report below are steady state results.
II Preliminary results and next steps

In our baseline specification, we consider the following parameter values. We set $\alpha_1 = 2$ and $\alpha_2 = 1.6$, so that for superstitious consumers, a lucky apartment offers 25% higher utility than a non-lucky apartment. We set $\lambda = 0.1$, so that 10% of the consumer population is superstitious. We set $\theta_1 = 0.2$ and $\theta_2 = 0.6$, so that 80% ($=(0.2 + 0.6)/1$) of the consumers own apartments, and one quarter of the apartments are lucky apartments.

We then examine how the prices change as we vary the transaction cost $k$ and the variance of consumers’ preference shocks $Var(\epsilon)$. We find that as $k$ is increased so that the secondary market becomes less liquid, the difference between the lucky apartment price ($p_1$) and the non-lucky apartment price ($p_2$) becomes smaller. The price difference similarly becomes smaller as $Var(\epsilon)$ is decreased so that consumers’ preference heterogeneity becomes more persistent.

Table 1 reports the ratio $p_2/p_1$ for all combinations of the following $k$ and $Var(\epsilon)$ values: $k \in \{0,1,2,3,4\}$, and $Var(\epsilon) \in \{\pi^2/6, (7/8) \times \pi^2/6, (6/8) \times \pi^2/6, (5/8) \times \pi^2/6, (4/8) \times \pi^2/6\}$.\(^2\) With $Var(\epsilon) = \pi^2/6$, an increase of $k$ from 0 to 4 results in the price ratio increasing from 30.5% to 90.7%. Similarly, with $k = 0$, a reduction of $Var(\epsilon)$ from $\pi^2/6$ to $(4/8) \times \pi^2/6$ results in the price ratio increasing from 30.5% to 50.0%. In the table, the price ratio is the highest at 96.5% when $k$ is the highest at $k = 4$ and $Var(\epsilon)$ is the lowest at $Var(\epsilon) = (4/8) \times \pi^2/6$.

In ongoing research, we are exploring different specifications of the model (including alternative distributional assumptions on $\epsilon$) as well as calibration of the model using real-world data, in order to better understand the interesting dynamics resulting from the interaction of product quality differentiation and consumer preference heterogeneity in durable goods industries.

\(^2\)When the scale parameter of the type I extreme value distribution for $\epsilon$ is normalized to 1, $Var(\epsilon) = \pi^2/6$. 
References


Table 1. Ratio of $p_2/p_1$ for different combinations of $k$ and $Var(\varepsilon)$

$\alpha_1 = 2, \alpha_2 = 1.6, \lambda = 0.1, \theta_1 = 0.2, \theta_2 = 0.6$

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<th>2</th>
<th>3</th>
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