

TABLE IIa  
SUMMARY OF REPLACEMENT DATA  
(Subsample of buses for which at least 1 replacement occurred)

Bus Group	Mileage at Replacement				Elapsed Time (Months)				Number of Observations
	Max	Min	Mean	Standard Deviation	Max	Min	Mean	Standard Deviation	
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	273,400	124,800	199,733	37,459	74	38	59.1	10.9	27
4	387,300	121,300	257,336	65,477	116	28	73.7	23.3	33
5	322,500	118,000	245,291	60,258	127	31	85.4	29.7	11
6	237,200	82,400	150,786	61,007	127	49	74.7	35.2	7
7	331,800	121,000	208,963	48,981	104	41	68.3	16.9	27
8	297,500	132,000	186,700	43,956	104	36	58.4	22.2	19
Full Sample	387,400	83,400	216,354	60,475	127	28	68.1	22.4	124

are *right censored* since we do not observe the final age and mileage at which replacement occurs. We can see from Table IIb that despite the right censoring, both the mean elapsed age and mileage are significantly higher for this subsample. The data for bus groups 7 and 8 are also *left censored* since these buses were acquired in 1972 and my data begin in December, 1974. The presence of these biases makes it difficult to summarize the unconditional population distribution of the age and mileage at replacement. The empirical analysis in Section 5 implicitly accounts for censored spells through the use of a conditional likelihood function given the observed sample of data. I account for selection bias by allowing for heterogeneity in parameter estimates across bus groups.

The empirical analysis in Section 5 focuses on a subsample of the full data set, bus groups 1-4. The buses in these groups were the most recent acquisitions

TABLE IIb  
CENSORED DATA  
(Subsample of buses for which no replacements occurred)

Bus Group	Mileage at May 1, 1985				Elapsed Time (months)				Number of Observations
	Max	Min	Mean	Standard Deviation	Max	Min	Mean	Standard Deviation	
1	120,151	65,643	100,117	12,929	25	25	25	0	15
2	161,748	142,009	151,183	8,530	49	49	49	0	4
3	280,802	199,626	250,766	21,325	75	75	75	0	21
4	352,450	310,910	337,222	17,802	118	117	117.8	0.45	5
5	326,843	326,843	326,843	0	130	130	130	0	1
6	299,040	232,395	265,264	33,332	130	128	129.3	1.15	3
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0
Full Sample	352,450	65,643	207,782	85,208	130	25	66.4	34.6	49

TABLE X  
 STRUCTURAL ESTIMATES FOR COST FUNCTION  $c(x, \theta_1) = .001\theta_1x$   
 FIXED POINT DIMENSION = 175  
 (Standard errors in parentheses)

Parameter		Data Sample			Heterogeneity Test	
Discount Factor	Estimates Log-Likelihood	Groups 1, 2, 3 3864 Observations	Group 4 4292 Observations	Groups 1, 2, 3, 4 8156 Observations	LR Statistic (df = 6)	Marginal Significance Level
$\beta = .9999$	<i>RC</i>	11.7257 (2.597)	10.896 (1.581)	9.7687 (1.226)	237.53	1.89E - 48
	$\theta_{11}$	2.4569 (.9122)	1.1732 (0.327)	1.3428 (0.315)		
	$\theta_{30}$	.0937 (.0047)	.1191 (.0050)	.1071 (.0034)		
	$\theta_{31}$	.4475 (.0080)	.5762 (.0075)	.5152 (.0055)		
	$\theta_{32}$	.4459 (.0080)	.2868 (.0069)	.3621 (.0053)		
	$\theta_{33}$	.0127 (.0018)	.0158 (.0019)	.0143 (.0013)		
	<i>LL</i>	-3993.991	-4495.135	-8607.889		
$\beta = 0$	<i>RC</i>	8.2969 (1.0477)	7.6423 (.7204)	7.3113 (0.5073)	241.78	2.34E - 49
	$\theta_{11}$	56.1656 (13.4205)	36.6692 (7.0675)	36.0175 (5.5145)		
	$\theta_{30}$	.0937 (.0047)	.1191 (.0050)	.1070 (.0034)		
	$\theta_{31}$	.4475 (.0080)	.5762 (.0075)	.5152 (.0055)		
	$\theta_{32}$	.4459 (.0080)	.2868 (.0069)	.3622 (.0053)		
	$\theta_{33}$	.0127 (.0018)	.0158 (.0019)	.0143 (.0143)		
	<i>LL</i>	-3996.353	-4496.997	-8614.238		
Myopia tests:	LR Statistic (df = 1)	4.724	3.724	12.698		
$\beta = 0$ vs. $\beta = .9999$	Marginal Significance Level	0.0297	0.0536	.00037		

TABLE VIII  
SUMMARY OF SPECIFICATION SEARCH<sup>a</sup>

Cost Function	Bus Group		
	1, 2, 3	4	1, 2, 3, 4
<b>Cubic</b> $c(x, \theta_1) = \theta_{11}x + \theta_{12}x^2 + \theta_{13}x^3$	Model 1 -131.063 -131.177	Model 9 -162.885 -162.988	Model 17 -296.515 -296.411
<b>quadratic</b> $c(x, \theta_1) = \theta_{11}x + \theta_{12}x^2$	Model 2 -131.326 -131.534	Model 10 -163.402 -163.771	Model 18 -297.939 -299.328
<b>linear</b> $c(x, \theta_1) = \theta_{11}x$	Model 3 -132.389 -134.747	Model 11 -163.584 -165.458	Model 19 -300.250 -306.641
<b>square root</b> $c(x, \theta_1) = \theta_{11}\sqrt{x}$	Model 4 -132.104 -133.472	Model 12 -163.395 -164.143	Model 20 -299.314 -302.703
<b>power</b> $c(x, \theta_1) = \theta_{11}x^{\theta_{12}}$	Model 5 <sup>b</sup> N.C. N.C.	Model 13 <sup>b</sup> N.C. N.C.	Model 21 <sup>b</sup> N.C. N.C.
<b>hyperbolic</b> $c(x, \theta_1) = \theta_{11}/(91 - x)$	Model 6 -133.408 -138.894	Model 14 -165.423 -174.023	Model 22 -305.605 -325.700
<b>mixed</b> $c(x, \theta_1) = \theta_{11}/(91 - x) + \theta_{12}\sqrt{x}$	Model 7 -131.418 -131.612	Model 15 -163.375 -164.048	Model 23 -298.866 -301.064
<b>nonparametric</b> $c(x, \theta_1)$ any function	Model 8 -110.832 -110.832	Model 16 -138.556 -138.556	Model 24 -261.641 -261.641

<sup>a</sup> First entry in each box is (partial) log likelihood value  $\ell^2$  in equation (5.2) at  $\beta = .9999$ . Second entry is partial log likelihood value at  $\beta = 0$ .

<sup>b</sup> No convergence. Optimization algorithm attempted to drive  $\theta_{12} \rightarrow 0$  and  $\theta_{11} \rightarrow +\infty$ .

specification test cannot reject any of the particular parametric functional forms which I tried. As a result, I adopted more intuitive criteria in order to select a “best fit” model from the array of alternative functional forms. My decision was a compromise between the objectives of (i) choosing the functional form with the highest likelihood value, (ii) choosing a functional form which is parsimonious, yet consistent with my priors and other nonquantitative information about the bus replacement problem. These criteria lead me to choose the linear and square root functional forms as the “best fit” specifications.

Tables IX and X present the structural parameter estimates computed by maximizing the full likelihood function  $\ell^f$  using the nested fixed point algorithm. In Table IX I present structural estimates for the unknown parameters ( $RC, \theta_{11}$ ) of the linear specification for two alternative discount factors,  $\beta = 0$  and  $\beta = .9999$ . The estimation results for  $\beta = 0$  can be interpreted as a “myopic model” of bus engine replacement, under which a replacement occurs only when current operating costs  $c(x_t, \theta_1)$  exceed the current cost of replacement  $RC + c(0, \theta_1)$ . The

TABLE V  
 WITHIN GROUP ESTIMATES OF MILEAGE PROCESS  
 WITHIN GROUP HETEROGENEITY TESTS  
 (Standard errors in parentheses)

	Group 1 1983 Grumman	Group 2 1981 Chance	Group 3 1979 GMC	Group 4 1975 GMC	Group 5 1974 GMC (8V)	Group 6 1974 GMC (6V)	Group 7 1972 GMC (8V)	Group 8 1972 GMC (6V)
$\theta_{31}$	.197 (.021)	.391 (.035)	.307 (.008)	.392 (.007)	.489 (.013)	.618 (.014)	.600 (.010)	.722 (.009)
$\theta_{32}$	.789 (.021)	.599 (.035)	.683 (.008)	.595 (.007)	.507 (.013)	.382 (.014)	.397 (.010)	.278 (.009)
$\theta_{33}$	.014 (.006)	.010 (.007)	.010 (.002)	.013 (.002)	.005 (.002)	.000 (0)	.003 (.001)	.000 (0)
Restricted Log Likelihood	-203.99	-138.57	-2219.58	-3140.57	-1079.18	-831.05	-1550.32	-1330.35
Unrestricted Log Likelihood	-187.71	-136.77	-2167.04	-3094.38	-1068.45	-826.32	-1523.49	-1317.69
Likelihood ratio test statistic	32.56	3.62	105.08	92.39	21.46	9.46	53.67	25.31
Degrees of Freedom	42	9	141	108	33	18	51	34
Marginal Significance Level	.852	.935	.990	.858	.939	.948	.372	.859

TABLE VI  
 BETWEEN GROUP ESTIMATES OF MILEAGE PROCESS  
 BETWEEN GROUP HETEROGENEITY TESTS  
 (Standard errors in parentheses)

	1, 2, 3	1, 2, 3, 4	4, 5	6, 7	6, 7, 8	5, 6, 7, 8	Full Sample
$\theta_{31}$	.301 (.007)	.348 (.005)	.417 (.006)	.607 (.008)	.652 (.006)	.618 (.006)	.475 (.004)
$\theta_{32}$	.688 (.007)	.639 (.005)	.572 (.007)	.392 (.008)	.347 (.006)	.380 (.006)	.517 (.004)
$\theta_{33}$	.011 (.002)	.012 (.001)	.011 (.001)	.002 (.001)	.001 (.004)	.002 (.001)	.007 (.000)
Restricted Log Likelihood	-2575.98	-5755.00	-4243.73	-2384.50	-3757.76	-4904.41	-11,237.68
Unrestricted Log Likelihood	-2491.51	-5585.89	-4162.83	-2349.81	-3668.50	-4735.95	-10,321.84
Likelihood ratio test statistic	168.93	338.21	161.80	69.39	180.52	336.93	1,831.67
Degrees of Freedom	198	309	144	81	135	171	483
Marginal Significance Level	.934	.121	.147	.818	.005	1.5E-17	7.7E-10

the estimated value function for the linear case, model 11 (note that the value function is shown in terms of the original unscaled coefficient estimates). Figure 3 displays the estimated hazard function for model 11, including the nonparametric and myopic ( $\beta = 0$ ) hazard functions for comparison. We can see that the linear specification leads to a gently rising hazard function that appears to flatten out at a hazard rate of about 7 per cent at 450,000 miles. These estimates stand in marked contrast to the myopic model which implies a rapidly rising hazard function, with a hazard rate of over 20 per cent at 450,000 miles. It is unwise to use the nonparametric hazard estimate to try to decide whether or not the tail behavior of the dynamic model is more realistic than the tail behavior of the myopic model. Almost all of the observations are concentrated in bus mileages less than 100,000 and in fact we have very few observations for mileages beyond 300,000. As a result, the upper tail of the nonparametric hazard is estimated very erratically, leading ultimately to hazard rate estimates of 0 or 1 depending upon whether a single bus in a high mileage cell did or did not experience a replacement. This erratic “Dirac” behavior of the nonparametric hazard makes it unwise to try to infer anything about the precise nature of the tail behavior of the true underlying hazard function. Although the problem can be alleviated somewhat by choosing wider “windows” over which the nonparametric hazard is calculated, the basic problem is due to lack of observations in the upper tail and can only be addressed by increasing the size of the sample.

The lack of observations is reflected in the estimated value and hazard functions for the cubic and quadratic specifications. A positive estimated coefficient  $\theta_{13}$  on the  $x^3$  term in the cubic model leads to a sharply rising hazard function beyond 300,000 miles. A negative estimated coefficient  $\theta_{12}$  for the quadratic model leads to the opposite behavior, leading to a hazard rate which actually *decreases* after 350,000 miles. The wide divergence in the tail behavior of these two specifications was not accompanied by a significant change in the value of the log-likelihood function. Although the hazard function is precisely estimated until about 300,000 miles, the tail is essentially an artifact of the particular functional form chosen for  $c(x, \theta_1)$ . My prior belief that the hazard function should never decrease leads me to reject the quadratic specification, and conversations with Harold Zurcher lead me to reject the cubic model with its sharply rising hazard function. When asked to choose the hazard function which best represents his engine replacement behavior, Zurcher chose the hazards derived from the linear and square root specifications which flatten out at about 7 or 8 per cent after 350,000 miles. According to Zurcher, monthly maintenance costs increase very slowly as a function of accumulated mileage. If the mechanical reliability of a bus deteriorates only very gradually with accumulated mileage, then it makes sense that the hazard would flatten out instead of abruptly increasing after 400,000 miles as it does in the myopic and cubic models. Remember that the alternative to not replacing a bus engine is to replace individual components at time of failure. Eventually such a “replace on failure” strategy yields bus engines with a significant fraction of new components, even though some components may have significant accumulated mileage. Thus, even though a given bus may have gone 400,000 miles since

Table 3: Simulated steady-state in the dynamic model  
 Equilibrium values of endogenous variables evaluated at the steady-state of the dynamic model, using the parameter values in Table 1 above.<sup>a</sup>

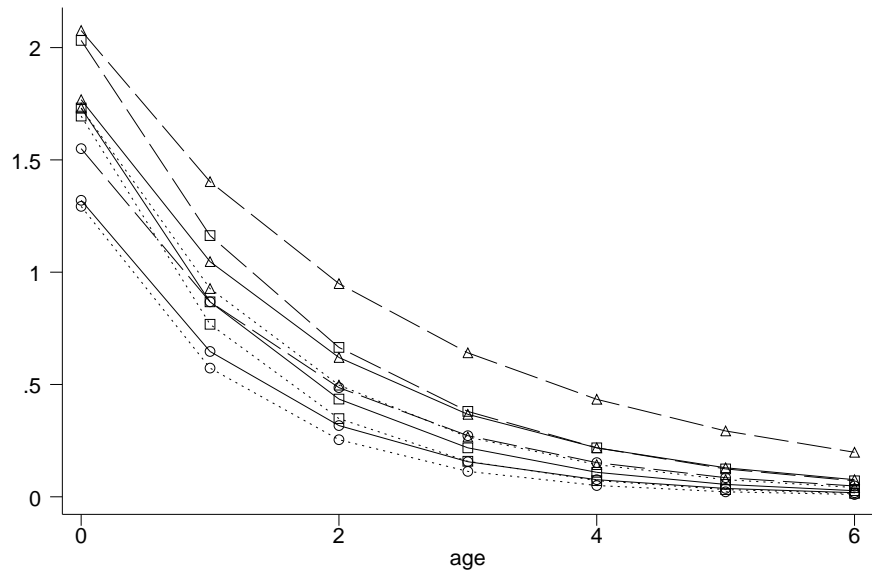
Manufacturer	Segment	Actual values		Simulated values		
		Quantity (millions)	Price ('000s)	Quantity ('000s)	Price ('000s)	Marginal Cost ('000s)
Chrysler	SC	0.176	5.344	0.158	6.254	6.137
Chrysler	C	0.379	7.923	0.400	6.798	6.667
Chrysler	MF	0.170	11.822	0.170	8.837	8.692
Ford	SC	0.322	6.098	0.326	9.258	8.956
Ford	C	0.267	7.457	0.298	10.547	10.197
Ford	MF	0.628	9.958	0.637	14.609	14.093
GM	SC	0.427	6.725	0.376	7.956	7.535
GM	C	0.526	8.530	0.475	9.051	8.572
GM	MF	1.430	10.253	1.263	12.054	11.416

<sup>a</sup>In the calculations, we used the  $\alpha$ 's for the import cars set at their 1987-1990 values (cf. the results in Table 1). Market shares are computed for the primary market and exclude imports, which are exogenous. We fixed the production of imports at 1.10 million cars, reflecting the average import volume during the sample period.

Table 4: Simulated markups when consumer population size is reduced

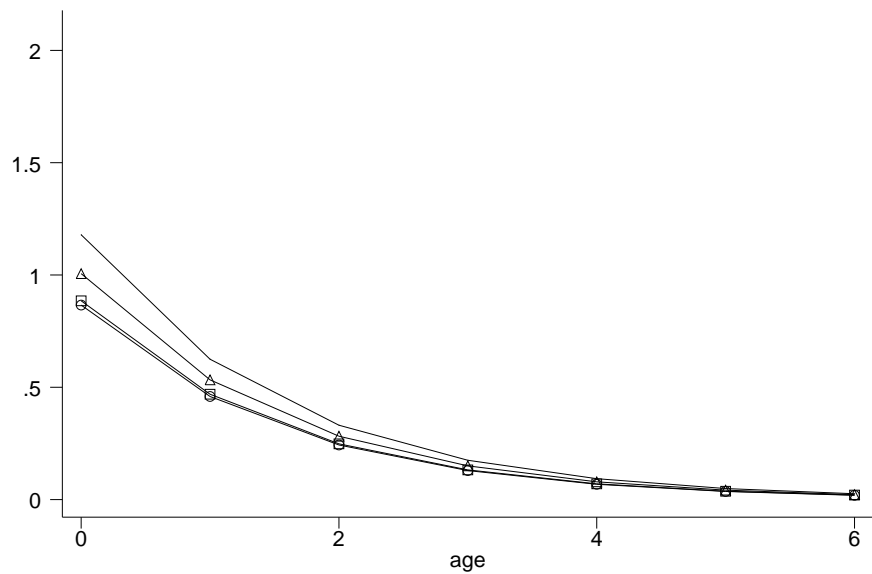
Manufacturer	Segment	Baseline <sup>a</sup>	75% pop'n	50% pop'n	25% pop'n
Chrysler	SC	1.860	2.270	2.952	4.2634
Chrysler	C	1.926	2.312	2.931	3.9894
Chrysler	MF	1.641	1.830	2.020	1.6065
Ford	SC	3.263	3.935	5.032	7.1601
Ford	C	3.314	3.905	4.801	6.1524
Ford	MF	3.533	4.029	4.697	5.2537
GM	SC	5.301	6.639	8.977	14.291
GM	C	5.294	6.558	8.709	13.281
GM	MF	5.296	6.410	8.188	11.313

<sup>a</sup>Computed using the marginal costs reported in Table 3. Baseline population value is 72.95 million households. We fixed the production of imports at 1.10 million cars, reflecting the average import volume during the sample period.

Figure 3: Graphs of the estimated car quality  $\alpha$  parameters, as reported in Table 1

Estimated  $\alpha$ 's for domestic car models:

Circle for Chrysler models, Triangle for Ford models, Square for GM models  
 Solid line for subcompacts, dotted line for compacts and dashed line for midsize



Estimated  $\alpha$ 's for imports:

Circle for 1971-78, Triangle for 1979-82, Square for 1983-86, Solid line for 1987-90

Note: age "0" denotes new car