Auctions: problems

1. Which random variables are the expectations taken over in (all from Milgrom and Weber (1982)):

- The proof on the top of p. 1101?
- The series of equations on p. 1106?

2. Implement the Guerre, Perrigne, and Vuong (2000) procedure for an IPV auction model:

- Generate 1000 valuations $x \sim U[0,1]$. Recall (as derived in lecture notes) the equilibrium bid function in this case is
$$b(x) = \frac{N-1}{N} \cdot x.$$
- For 500 of the valuations, split them into 125 4-bidder auctions. For each of these valuations, calculate the corresponding equilibrium bid.
- For the other 500 valuations, split them into 100 5-bidder auctions. For each of these valuations, calculate the corresponding equilibrium bid.
- For each b_i, compute the estimated valuation $\tilde{x}_i$ using the GPV equation:
$$\frac{1}{g(b_i)} = (N_i - 1)\frac{x_i - b_i}{G(b_i)}$$
$$\Leftrightarrow x_i = b_i + \frac{G(b_i)}{(N_i - 1)g(b_i)}$$
(where N_i denotes the number of bidders in the auction that the bid b_i is from).

  In computing the G and g functions, try

  1. Epanechnikov kernel ($\mathcal{K}(u) = \frac{3}{4}(1-u^2)\mathbf{1}(|u| \leq 1)$)
  2. Uniform kernel ($\mathcal{K}(u) = \frac{1}{2}\mathbf{1}(|u| \leq 1)$).

  Also, try different bandwidths $h \in \{0.5, 0.1, 0.05, 0.01\}$.

  For each case, plot x vs. $\tilde{x}$. Can you comment on performance of the procedure for different bandwidth values?
- Compute and plot the empirical CDF's for the estimated valuations $\tilde{x}_i$, separately for $N = 4$ and $N = 5$.

3. Consider an example of a common-value model with conditionally independent signals, drawn from Matthews (1984). Namely

⇒ Pareto-distributed common values: $v \sim g(v) = \alpha v^{-(\alpha+1)}$, with support $v \in [1, +\infty)$. Note that corresponding CDF is $G(v) = 1 - v^{-\alpha}$.

⇒ Conditionally independent signals: $x|v \sim U[0, v]$.

⇒ Equilibrium bidding strategy:

$$b(x) = \left[\frac{N - 1 + \max(1, x)^{-N}}{N}\right] \cdot \left(\frac{N + \alpha}{N + \alpha - 1}\right) \cdot \max(1, x) \tag{1}$$

So do the following:

- Simulate the common values v_t i.i.d. from $G(v)$,[1] for $t = 1, 225$ (225 auctions).
- For each auction $t = 1, 125$, generate 4 signals each, where $x_{it} \sim U[0, v_t]$, for $i = 1, \ldots, 4$, and $t = 1, \ldots, 125$.

  Then for each signal x_{it}, generate the corresponding equilibrium bid b_{it} for a 4-bidder auction, using Eq. (1).

  For each bid b_{it}, pick out the maximum among rivals' bids in auction t: $b*_{it} \equiv \max_{j \neq i} b_{jt}$.

  For each bid in the simulated 4-bidder auctions, recover the corresponding pseudovalue $\xi(b_{it}, N_t)$, using Eq. (10) from auction lecture notes.
- For each auction $t = 126, 225$, generate 5 signals each, where $x_{it} \sim U[0, v_t]$, for $i = 1, \ldots, 5$, and $t = 126, \ldots, 225$.

  As above, generate the corresponding b_{it}, $b*_{it}$ for each signal.

  Then, for each bid in these 5-bidder auctions, recover the pseudovalue $\xi(b_{it}, N_t)$.
- Compute and plot the empirical CDF's for the estimated pseudovalues $\xi(b_{it}, N_t)$, separately for $N_t = 4$ and $N_t = 5$.

References

Guerre, E., I. Perrigne, and Q. Vuong (2000): "Optimal Nonparametric Estimation of First-Price Auctions," *Econometrica*, 68, 525–74.

[1] To simulate from any non-uniform CDF, use the "inverse-quantile" procedure. Generate $w \sim U[0, 1]$, then transform $v = G^{-1}(x)$. The random variable $v \sim G(v)$.

MATTHEWS, S. (1984): "Information Acquisition in Discriminatory Auctions," in *Bayesian Models in Economic Theory*, ed. by M. Boyer, and R. Kihlstrom. North-Holland.

MILGROM, P., AND R. WEBER (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1089–1122.