Oligopoly: practice problems

Return to the 2-firm case. Assume each firm produces with $C(q) = cq$, and market demand curve is $p = a - bQ$.

1. Cournot:
   - (***) Solve for the Cournot Nash equilibrium quantities, prices, and profits for the two firms. Call these $q*, p*, \pi^*$.
   - (***) What if these two firms formed a cartel and maximized joint profits? Solve for the resulting quantities, prices, and profits; call these q^j, p^j, π^j.
   - What if firm 2 cheats when firm 1 sets $q_1 = q^j$? What are the resulting quantities, prices, and profits?
   - What does this have to do with the prisoner's dilemma?
   - (Think about) What if firms play the Cournot game for two consecutive periods? What are the chances that a cartel could survive, and how could this happen? What if they play for ten periods? What if they play forever? (Hint: how do we solve multi-period games?)
2. (***) Bertrand: derive the Bertrand nash equilibrium prices, quantities, and profits. Call these q^b, p^b, π^b.
3. (***) Stackelberg: If firm 1 is the Stackelberg leader, what are the resulting quantities, prices, and profits $(q_1^s, q_s^2), (p_1^s, p_2^s), (\pi_1^s, \pi_2^s)$.
4. Rank the quantities, prices, and profits computed in the problems marked (***).

5. Consider the following game tree (see figure 1)

(a) List all of player 1's strategies
(b) List all of player 2's strategies
(c) What are the Nash equilibria of this game? Show why.
(d) What are the subgame perfect equilibria of this game? Show why.

6. Construct a "Nash reversion"-type subgame-perfect equilibrium to the infinitely repeated Bertrand (price-setting) game. Assume there are two identical firms, each producing at constant marginal cost c. The market demand curve is $p = a - bQ$.

Figure 1: Game tree for question 5