Product Differentiation

- In homogeneous goods markets, price competition leads to perfectly competitive outcome, even with two firms
- Price competition with differentiated products
- Models where differentiation is modeled as spatial location:
  1. Linear (Hotelling) model
  2. Circular (Salop) model
- Compare prices and variety in competitive equilibrium versus "social" optimum.

Types of product differentiation

Two important types of product differentiation.

1. **Horizontal differentiation**: people differ in their rankings of a group of products, even if they were all priced identically (cereals, cars, etc.)
2. **Vertical differentiation**: products differ in quality, which is equally perceived by all people. So if all products have the same price, people would only buy the one with highest quality.
3. Examples: cereals, medical care

Today we focus only on horizontal differentiation.

Bertrand competition with differentiated products

- 2 firms, producing *imperfect substitutes.* Firms' demand curves are:

$$q_1 = a - b_1 p_1 + b_2 p_2$$
$$q_2 = a - b_1 p_2 + b_2 p_1$$

  Assume: $b_1 > b_2$ (so demand more responsive to own-price)

- Firms' costs are $C(q) = cq$
- Firm 1 maximizes:

$$\max_{p_1} \pi_1 = (p_1 - c)(a - b_1 p_1 + b_2 p_2)$$

  FOC is:

$$a - 2b_1 p_1 + b_2 p_2 + cb_1 = 0$$

- Firms' (symmetric) best-response functions are:

$$BR_1(p_2) = \frac{a + cb_1 + b_2 p_2}{2b_1}$$
$$BR_2(p_1) = \frac{a + cb_1 + b_2 p_1}{2b_1}$$

Graph.

Bertrand with differentiated products 2

- NE: $p_1 = p_2 \equiv p^* = BR_1(p^*) = BR_2(p^*)$
  $p^* = \frac{a+cb_1}{2b_1-b_2}$.
- $q_1 = q_2 \equiv q^* = \frac{b_1(a+c(b_2-b_1))}{2b_1-b_2}$.
- Assume: $a > c(b_1 - b_2)$, so that q^* is positive.
- Given this assumption, $p^* > c$.
- What happens with N firms? As $N \to \infty$?
- Clearly, firms have an incentive to differentiate their products, to avoid toughness of price competition

Location models of product differentiation

- Differences between products modeled as differences in a product's location in "product space", or the spectrum of characteristics.
- Differences in consumer preferences modeled by their location on that same product space.
- Models differ in how product space is modeled. Study models with *linear* and *circular* product spaces.

Linear Hotelling model 1

- Town with just one street of length 1, along which all reside.
- Consumers located on the street with *uniform density*, i.e., there are 0.25 "consumers" living between 0 and 0.25.
- Two pizza places located at a and $1-b$
  Cost function $c(q) = cq$.
- Each consumer eats only one piece of pizza (i.e., buys from only one of the stores), and incur transportation costs which are quadratic in the distance needed to travel to the store.
  Ex: consumer located at x and buying from store 1 gets utility $u - p_1 - t(a-x)^2$.
- *Two-stage* game where
  1. Location chosen in first stage
  2. Prices chosen in second stage.

Linear Hotelling model 2

Second stage: firms choose prices, taking locations as given

Derive each firm's demand function.

1. Given locations $(a, 1-b)$, solve for location of the consumer who is just indifferent between buying at the two stores. Then all consumers to the left buy from store 1 and all consumers to the right buy from store 2.

2. Consumer x is indifferent if:

$$u - p_1 - t(a-x)^2 = u - p_2 - t(1-b-x)^2 \Leftrightarrow$$

$$x = a + \frac{1-a-b}{2} + \frac{p_2 - p_1}{2t(1-a-b)}$$

3. Demand for the two firms is:

$$q_1(p_1, p_2) = a + \frac{1-a-b}{2} + \frac{p_2 - p_1}{2t(1-a-b)}$$

$$q_2(p_1, p_2) = b + \frac{1-a-b}{2} + \frac{p_1 - p_2}{2t(1-a-b)}.$$

Hotelling linear model 3

- Next, each firms chooses price to maximize profits, given demand functions derived earlier.

.... (Practice problem #1)

- Best response functions:

$$BR_1(p_2) = at(1-a-b) + \frac{t(1-a-b)^2}{2} + \frac{p_2+c}{2}$$
$$BR_2(p_1) = bt(1-a-b) + \frac{t(1-a-b)^2}{2} + \frac{p_1+c}{2}$$

- Conditional price functions:

$$p_1^c(a,b) = c + t(1-a-b)(1+\frac{a-b}{3})$$
$$p_2^c(a,b) = c + t(1-a-b)(1+\frac{b-a}{3}).$$

Hotelling linear model 4

First stage: firms choose locations.

For simplicity's sake, focus on **symmetric case**: $a = b$

- $p_1 = p_2 \equiv p = c + t(1 - 2a)$.
- $q_1 = q_2 = q = 1/2$, independently of a
- Profits, given a, are therefore: $\Pi(a) = \frac{t(1-2a)}{2}$.
- Firms just choose location (a) so as to get the highest profit:

$$\max_a \Pi(a).$$

- NOTE $\frac{\partial \Pi(a)}{\partial a} = -t < 0$: profits are always falling the more towards the center a firm locates. Therefore **firms will opt to locate at the ends of the street** (a=0).
- **Maximal differentiation principle**: firms avoid the "toughness of price competition" by differentiating themselves as much relative to each other as possible.

Hotelling linear model 5

What if you eliminate the toughness of price competition? Suppose government sets $p = \hat{p} \geq c$. Where do firms locate to maximize profits?

- Eliminate symmetry, otherwise profits don't depend on a at all. Profits are:

$$\Pi_1(\hat{p}, a, b) = (\hat{p} - c)(a + \frac{1 - a - b}{2})$$
$$\Pi_2(\hat{p}, a, b) = (\hat{p} - c)(b + \frac{1 - a - b}{2}).$$

- Now $\frac{\partial \Pi_1}{\partial a} = (\hat{p} - c)(1/2) > 0$, so profits are *increasing* as firm moves towards center of town. Therefore firms will locate right next to each other!

- **Principle of minimal differentiation**: in the absence of price competition, firms will “go where the demand is”, which in this example is in the middle of town.
- Question: Why are so many businesses clustered together?
  - Going where the demand is?
  - Possible collusion story? Being next to the competitor facilitates monitoring?

Hotelling linear model 6

- How optimal are these configurations, either at the ends or the middle of the street?
- In both cases, total transportation costs incurred by consumers is $2\int_0^{1/2} tx^2 dx = 2t/24 = t/12$.
- Optimal (a, b), which minimizes total transportation costs? Assume symmetry again, so $a = b$.

$$\min_a 2\int_0^{1/2} t(a-x)^2 dx \iff \min_a \frac{-at}{4} + \frac{a^2 t}{2} \implies a = \frac{1}{4}$$

- $p = c + (1/2)t$ and $\Pi = (1/4)t$, which is lower than maximal differentiation, where $\Pi = (1/2)t$.
- For this example, "too much differentiation": consumers are willing to sacrifice some variety (have the products "closer together" in product space) for lower price.
- Interpret linear model metaphorically: differences in pizza offerings, political competition

Salop circular model 1

1. Use model to look at optimal diversity.
2. Consumers uniformly located along a circle with circumference 1.
3. Consumer demand one unit, and receive utility equal to $u - p - t|d|$, where d is distance traveled (*linear* transportation costs).
4. Cost function $C(q) = F + cq$. Once they are in the market, they can costlessly change locations.
5. Market equilibrium determined by **free entry** condition: firms enter the market until profits go to zero.

Salop circular model 2

Focus on two stage game:

1. Firm chooses whether to enter, and where to locate. Focus on entry process to determine whether free-entry equilibrium has too much, or too little variety, relative to "social optimum" where sum of production and transportation costs are minimized.

2. Price competition in second stage.

To simplify the first stage, appeal to the **maximal differentiation principle**: If N firms decide to enter, they will be located *equidistant* from each other along circle, i.e., there is distance $1/N$ between each of them.

This is only matter in which circular assumption is important.

Salop circular model 3

Second stage: price competition

- Symmetric equilibrium where $p_1 = \cdots = p_N = p^c$.
- Firm i has only two competitors, those located to the right and left of it, both of whom are charging price p. Now if firm i charges p_i, what is its demand?
- A consumer located at a distance x between firm i and firm $i+1$ is indifferent between the two firms if:

$$u - p_i - tx = u - p - t(1/N - x) \Leftrightarrow x = \frac{(p - p_i + t/N)}{2t}$$

so that demand for firm i is

$$d_i = 2x = \frac{p - p_i + t/N}{t}.$$

- Therefore, firm i maximizes

$$(p_i - c)\left(\frac{p - p_i + t/N}{t}\right) - F$$

and set symmetric price

$$p = c + t/N.$$

Salop circular model 4

First stage: how many firms enter?

- In symmetric equilibrium, all firms charge $p = c + t/N$.
- Demand for each firm is $1/N$ and profits $(p - c)(1/N) = t/N^2 - F$.
- Number of firms in the market is N^c which satisfy the free-entry/zero-profit condition:

$$t/(N^c)^2 = F \Leftrightarrow N^c = \sqrt{\frac{t}{F}}.$$

- At N^c, price is $c + \sqrt{tF} \equiv p^c$.

Salop circular model 5

Social optimum: choose N which minimizes total production and transportation costs

$$\min_N N\left(\left[2\int_0^{\frac{1}{2N}} txdx\right] + \left[F + \frac{c}{N}\right]\right) = \min_N \frac{t}{4N} + NF + c.$$

- $N^s = \frac{1}{2}\sqrt{\frac{t}{F}} = \frac{1}{2}N^c$.
- Profits are positive, and equal to $3F$.
- Excessive entry (or excessive variety) in the competitive market equilibrium, relative to the social optimum.

- Note: total transportation costs are $\frac{t}{4N}$, which *decrease* in N. Since $N^s < N^c$, this implies that total transportation costs are *higher* at the social optimum.

  If goal were just to minimize consumer transportation costs, there would be an infinite number of firms in the market, so that entire circle is covered.

- Therefore, entry is excessive because firms incur too much fixed costs in entering market.

Conclusions

Models:

- Differentiated Product Bertrand Model
- Hotelling linear model
- Salop circular model

Three important principles:

- Maximal differentiation to counteract price competition.
- Minimum differentiation when there is no price competition.
- Free entry can lead to excessive variety. Implication: from a social point of view, firms' costly attempts to differentiate themselves are wasteful.