I. Noncooperative Oligopoly

- Oligopoly: interaction among small number of firms
- Conflict of interest:
 - Each firm maximizes its own profits, but...
 - Firm j's actions affect firm i's profits
 - Example: price war
 - PC: firms are small, so no single firm's actions affect other firms' profits
 - Monopoly: only one firm
- Game theory: mathematical tools to analyze situations involving conflicts of interest
- Three game-theoretic models of oligopolistic behavior in homogeneous good markets
 - 1. quantity-setting Cournot model
 - 2. price-setting Bertrand model
 - 3. sequential quantity-setting *Stackelberg* models

Conflict of interest: Prisoner's dilemma Introduce some terminology and the prototypical example (prisoner's dilemma):

player $2 \rightarrow$	Confess	Don't confess
player $1 \downarrow$		
confess	2,2	5,1
don't confess	1,5	$4{,}4$

What would you do??

Game theory: terminology

- Game: model of interaction between a group of players (prisoners, firms, people)
- Each player (or firm) attempts to maximize its own payoffs
- Strategy: an action that a player can take
- A player's **best response strategy** specifies the payoff-maximizing (or *optimal*) move that should be taken in response to a set of strategies played by the other players.
- A Nash equilibrium is a set of strategies, one for each player, from which, holding the strategies of all other players constant, no player can obtain a higher payoff.
- Each player's NE strategy is a best-response strategy to his opponents' NE strategies
- Example: prisoner's dilemma
 - What are strategies?
 - What is $BR_1(confess)$? $BR_1(don't confess)$?
 - Why isn't (don't confess, don't confess) a NE?

First we focus on games in which each player only moves once — static games.

Cournot quantity-setting model 1

- Players: 2 identical firms
- Strategies: firm 1 set q_1 , firm 2 sets q_2
- Inverse market demand curve: $p = a bQ = a b(q_1 + q_2)$.
- Constant marginal costs: C(q) = cq
- Payoffs are profits, as a function of strategies:

$$\pi_1 = q_1(a - b(q_1 + q_2)) - cq_1 = q_1(a - b(q_1 + q_2) - c).$$

$$\pi_2 = q_2(a - b(q_1 + q_2)) - cq_2 = q_2(a - b(q_1 + q_2) - c).$$

Cournot quantity-setting model 2

- Firm 1: $\max_{q_1} \pi_1 = q_1(a b(q_1 + q_2) c)$.
- FOC: $a - 2bq_1 - bq_2 - c = 0 \rightarrow q_1 = \frac{a-c}{2b} - \frac{q_2}{2} \equiv BR_1(q_2).$
- Similarly, $BR_2(q_1) = \frac{a-c}{2b} \frac{q_1}{2}$.
- Symmetric, so in a Nash equilibrium firms will produce same amount so that $q_1 = q_2 \equiv q^*$.
- Symmetric NE quantity q^* satisfies $q^* = BR_1(q^*) = BR_2(q^*) \Longrightarrow q^* = \frac{a-c}{3b}$.
- Graph: NE at intersection of two firms' BR functions.
- Equilibrium price: $p^* = p(q^*) = \frac{1}{3}a + \frac{2}{3}c$
- Each firm's profit: $\pi^* = \pi_1 = \pi_2 = \frac{(a-c)^2}{9b}$

Cournot quantity-setting model 3

Prisoner's dilemma flavor in Nash Equilibrium of Cournot game

- If firms cooperate: $\max_{q} = 2q(a b(2q) c) \rightarrow q^{j} = \frac{(a-c)}{4b}$
- $p^j = \frac{1}{2}(a+c)$, higher than p^* .
- $\pi^j = \frac{(a-c)^2}{8b}$, higher than π^* .
- But why can't each firm do this? Because NE condition is not satisfied: $q^j \neq BR_1(q^j)$, and $q^j \neq BR_2(q^j)$. Analogue of (don't confess, don't confess) in prisoner's dilemma.
- What if we repeat the game? Possibility of punishment for cheating (next class).

EXTRA: Cournot quantity-setting model 4

For general N-firm symmetric case:

- Industry inverse demand curve: $p = a b(q_1 + q_2 + \cdots + q_N)$
- Firm i's profit: $\pi_i = q_i(a b(q_1 + q_2 + \dots + q_N) c)$
- Firm *i*'s best-response function: $BR_i(q_{-i}) = \frac{a-c}{2b} \frac{q_1 + \dots + q_N q_1}{2}$
- Symmetric NE quantities: $q^{N*} = \frac{(a-c)}{(N+1)b}$
- Market price: $p^{N*} = \frac{1}{(N+1)}a + \frac{N}{(N+1)}c$.
- Per-firm profits: $\pi^{N*} = \frac{(a-c)^2}{(N+1)^2b}$.
- Note: as N grows large, $p^{N*} \to c$ and $\pi^{N*} \to 0$, as in PC.

Bertrand price-setting model

- Players: 2 identical firms
- Firm 1 sets p_1 , firm 2 sets p_2
- Market demand is $q = \frac{a}{b} \frac{1}{b}p$. C(q) = cq.
- Recall: products are homogeneous, or identical. This implies that all the consumers will go to the firm with the lower price:

$$\pi_1 = \begin{cases} (p_1 - c)(\frac{a}{b} - \frac{1}{b}p_1) & \text{if } p_1 < p_2\\ \frac{1}{2}(p_1 - c)(\frac{a}{b} - \frac{1}{b}p_1) & \text{if } p_1 = p_2\\ 0 & \text{if } p_1 > p_2 \end{cases}$$
(1)

• Firm 1's best response:

$$BR_1(p_2) = \begin{cases} p_2 - \epsilon & \text{if } p_2 - \epsilon > c \\ c & \text{otherwise} \end{cases}$$
 (2)

- NE: $p^* = BR_1(p^*) = BR_2(p^*)$ Unique $p^* = c!$ The "Bertrand paradox".
- Recall: with homogeneous products, firms are "price takers". Bertrand outcome is game-theoretic retelling of PC outcome.
- Contrast with Cournot results. Some resolutions:
 - Capacity constraints: one firm can't supply the whole market
 - Differentiated products

Stackelberg leader-follower quantity-setting model 1

- Example of multi-period game.
- Players: two identical firms
- Sequential game: firm 1 moves before firm 2. Insight into "first-mover" or incumbent advantage in markets. Graph: game tree.
- Strategies: firm 1 sets q_1 , firm 2 sets q_2
- Inverse market demand curve: $p = a bQ = a b(q_1 + q_2)$.
- constant marginal costs: C(q) = cq
- Profits:

$$\pi_1 = q_1(a - b(q_1 + q_2) - c)$$

$$\pi_2 = q_2(a - b(q_1 + q_2) - c)$$

• Firm 2's best response:

$$BR_2(q_1) = \frac{a-c}{2b} - \frac{q_1}{2}$$

• Firm 1's best response: Since firm 1 moves first, it takes firm 2's best-response function as given. In other words, firm 1 "picks its most preferred point off of firm 2's best-response function".

Stackelberg model 2

- Firm 1: $\max_{q_1} q_1(a b(q_1 + BR_2(q_1)) c) = q_1(a b(\frac{q_1}{2} + \frac{(a-c)}{2b}) c).$
- FOC: $q_1(\frac{-b}{2}) + a b(\frac{q_1}{2} + \frac{(a-c)}{2b}) c) = 0 \longrightarrow q_1^{S*} = \frac{a-c}{2b}$
- Note: Firm 1 has no "best-response" function in this case, since its profit function is not a function of q_2
- $q_1^{S*} = \frac{a-c}{2b} \ (>q^* = \frac{a-c}{3b})$ $q_2^{S*} = \frac{a-c}{4b} \ (<q^*).$
- Backward induction: how you solve multi-period games
- Issues:
 - Entry situation: incumbents can put entrant at a disadvantage (capacity overinvestment?)
 - Uncertainty: about market environment, about entrant's characteristics, can erode first-mover advantage

Summary

- Nash equilibrium: a set of strategies for each player; each player's NE strategy is a best-response to opponents' best-response strategies 2-player case: $s_1 = BR_1(s_2)$, $s_2 = BR_2(s_1)$
- Cournot: noncooperative quantity-choice game.
- Bertrand: noncooperative price-setting game.
 - Bertrand paradox: when goods are homogeneous, firms are price-takers!
- Stackelberg: leader-follower quantity-setting game. Solve by **backward induction**.

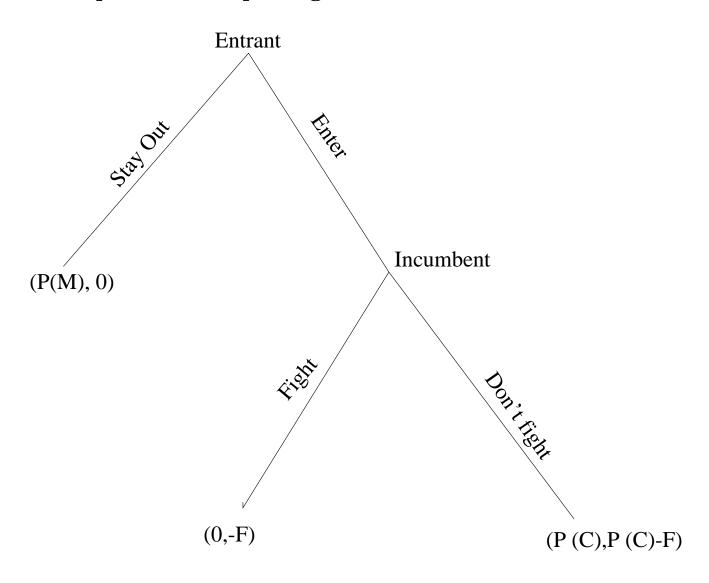
II. Cartels and collusion in oligopoly

- Single-period non-cooperative Cournot game: unique NE when firms produce higher-output, receive lower profits than if they cooperated (prisoners' dilemma)
- Can cooperation occur in multi-period games?
- Equilibrium concept for multi-period games: **Subgame**Perfect Equilibrium
- Finitely-repeated Cournot game
- Infinitely-repeated Cournot game

Subgame-Perfect Equilibrium 1

- Game: model of interaction between a group of players (prisoners, firms, people)
- Each player (or firm) attempts to maximize its own payoffs
- Multi-period game most conveniently represented by a **game tree**. **Nodes** are junctures of game tree, at which a player moves.
- A **Strategy** for each player specifies an action to be taken *at each of her nodes*.
- A *subgame* is the part of the multi-period game that starts from any given node onwards.
- Example 1: Limit pricing
 - Subgames?
 - Strategies for each player?
 - What are NE?
 - But what if entrant enters?
 - SPE: removes all non-credible threats, including incumbent's threat to counter entry with "limit pricing"

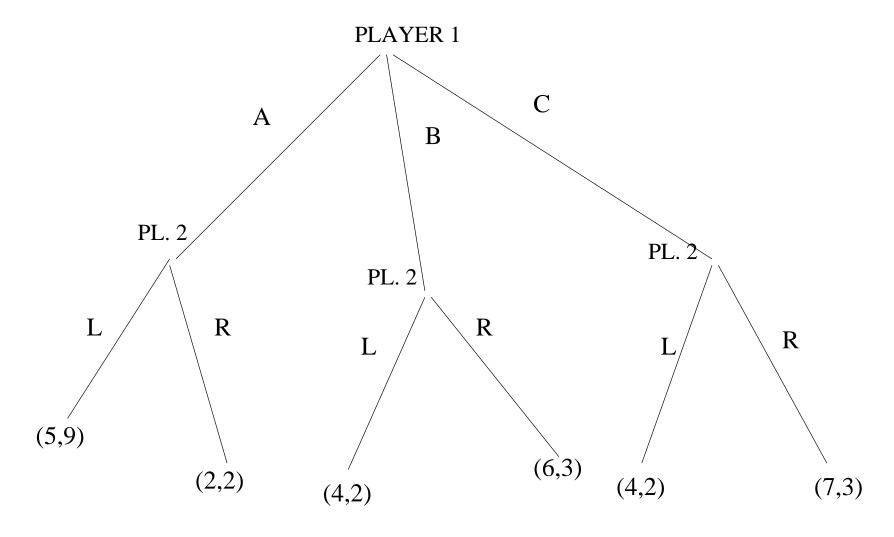
Example 1: Limit pricing



Subgame Perfect Equilibrium 2

- A subgame perfect equilibrium is a set of strategies, one for each player, from which, no player can receive a higher payoff *in any subgame*, i.e., each player's SPE strategy must be a best-response in any subgame
- All SPE are NE, not all NE are SPE
- Solve by backwards induction

Example 2



Subgame Perfect Equilibrium 3

Example 2: Are these equilibria, in any sense?

- (A; (L, L, L)): P2 "threatens" P1
- (B; (L, R, R))
- (C;(L,R,R))
- Backwards induction: eliminate all non-BR actions from player 2's subgame

2-firm Cournot quantity-setting game. Relevant quantities are:

- NE profits $\pi^* = \frac{(a-c)^2}{9b}$
- Cartel profits $\pi^j = \frac{(a-c)^2}{8b}$
- Firm 1 cheats on firm 2: $\pi^x = \pi_1(BR_1(q_2^j)) = \frac{9(a-c)^2}{64b}$
- Prisoners' dilemma analogy:

Firm $2 \rightarrow$	cheat	cartel
Firm 1 ↓		
cheat	$\frac{(a-c)^2}{9b}, \frac{(a-c)^2}{9b}$	$\frac{9(a-c)^2}{64b}$, $\frac{3(a-c)^2}{32b}$
cartel	$\frac{3(a-c)^2}{32b}, \frac{9(a-c)^2}{64b}$	$\frac{(a-c)^2}{8b}, \frac{(a-c)^2}{8b}$

2-period Cournot game

- Firm 1 chooses quantities (q_{11}, q_{12}) Firm 2 chooses quantities (q_{21}, q_{22})
- What are the subgames?
- What are SPE: solve backwards
- Second period: unique NE is (cheat, cheat)
- First period: (cheat,cheat) unique SPE is ((cheat,cheat), (cheat,cheat))
- What about ((cartel, cartel), (cartel, cartel))?
- What about ((cartel, cheat), (cartel, cheat))?
- What about
 Firm 1 plays (cartel; cheat if cheat, cartel if cartel)
 Firm 2 plays (cartel; cheat if cheat, cartel if cartel) ???
- What about 3 periods? N periods?

What if the 2-firm Cournot game is repeated forever? Are there SPE of this game in which both firms play q^{J*} each period?

- Discount rate $\delta \in [0, 1]$, which measures how "patient" a firm is.
- The "discounted present value" of receiving \$10 both today and tomorrow is $10 + \delta 10$.
- If $\delta = 1$, then there is no difference between receiving \$10 today and \$10 tomorrow.
- Property: $x + \delta x + \delta^2 x + \dots + \delta^n x + \dots = \frac{x}{1-\delta}$.

Define a "punishment period" (P) as one following a period where either firm has cheated (ie. produced an amount other than q^j . A "cartel period" (C) is one which follows a period in which both firms produced q^j .

Proposition: If the discount rate is "high enough", then these strategies constitute a SPE of the infinitely-repeated Cournot game:

- 1. Play q^j in a C-period
- 2. Play q^* in a P-period.

According to this strategy, firms cooperate as along as they observe their rival to be cooperating. Once a firm deviates, firms produce at the Cournot-Nash output forever: **Nash reversion** (or "grim strategy").

Show that this strategy constitute a SPE by finding conditions such that they prescribe best-response behavior for firm 1 given that firm 2 is following this strategy also in each subgame.

Consider firm 1 (symmetric for firm 2). There are two relevant subgames for firm 1.

1. In a P-period: proposed strategy prescribes playing q^* forever, given that firm 2 also does this. This is NE of the subgame: playing q^* is a best-response to firm 2 playing q^* . This satisfies SPE conditions.

2. In a C-period:

- Proposed strategy prescribes cooperating and playing q^j , with discounted PV of payoffs = $\pi^j/(1-\delta)$.
- The best other possible strategy is to play $BR_1(q_2^j) \equiv q_1^x$ this period, but then be faced with $q_2 = q^*$ forever. This yields discounted PV = $\pi^x + \delta(\pi^*/(1-\delta))$.
- In order for q_j to be NE of this subgame, require $\pi^j/(1-\delta) > \pi^x + \delta(\pi^*/(1-\delta))$ (profits from cooperating exceed profits from deviating). This is satisfied if $\delta > 9/17$.
- 3. Therefore, the Nash reversion specifies a best response in both of these subgames if $\delta > 9/17$ ("high enough"). In this case, Nash reversion constitutes a SPE.

Note: Firms will never cheat under Nash reversion.

Nash reversion is but one example of strategies which yield cooperative outcome in an infinitely-repeated Cournot game.

In general, firm 1 need not punish firm 2 forever to induce it to cooperate; after firm 2 deviates, just produce q^* for long enough so that it never pays for firm 2 to ever deviate. These "carrot and stick" strategies used to interpret price wars.

In general, the *Folk Theorem* says that, if the discount rate is "high enough", an infinite number of SPE exist for infinite-horizon repeated games, which involve higher payoffs than in the single-period Nash outcome.

 δ "high enough": punishments must be severe. If δ too low, firm 2 prefers higher profits from cheating now, undeterred from lower future profits from firm 1's punishment.

Generally: threats of punishment must be *credible* — it must be a firm's best-response to punish when it detects cheating

Industry and/or firm characteristics can make punishments more credible:

- 1. Flexible capacity: punishment may involve a large hike in quantity, this must be relatively costless
- 2. Good monitoring technology: cheating must be detected rather quickly. Trade journals facilitate collusion? Fewer number of firms?
- 3. Demand uncertainty foils detection of cheating: is low profits due to lower demand or cheating?
- 4. Homogeneous products: so firm 1 can hurt firm 2 by producing more

Porter (1983)

- Study of pricing in a "legal" railroad cartel: Joint Executive Commmittee
- Price series: two "regimes" of pricing. Can periods of low pricing be explained as "price wars"?
- Repeated games theory: view observed price series as realization of equilibrium price process.
- Standard repeated games (e.g. repeated Cournot game) with unchanging economic environment: equilibrium price path is *constant*!

Note: Need to specify punishment strategies which support collusive equilibrium, but punishment is never "observed" on the equilibrium path.

Need for model with nonconstant equilibrium price process.

Two important models

1. Rotemberg and Saloner (1986): with *i.i.d.* demand fluctuations, fixed discount rate, and constant marginal production costs, collusive prices will be lower in periods of above-average demand ("Price wars during booms").

Intuitively: cheat when (current) gains exceed (future) losses. Current gains highest during "boom" periods; reduce incentives to cheat by lowering collusive price. Demand shocks are observed by firms.

2. Green and Porter (1983): same framework as R-S, but introduce *imperfect information* — firms cannot observe the output choices of their competitors, only observed realized market price. Market price can be low due to either (i) cheating; or (ii) adverse demand shocks. Firms cannot distinguish.

Result: Prices can be lower during periods of low demand ("Price wars during recessions").

Intuitively: equilibrium "trigger" strategies involve "low price" regime when prices are low.

Note: these two models generate periods of low and high pricing on the equilibrium path. But low prices not caused by cheating; rather they are manifestation of collusive behavior!

Green-Porter Model

- Price-setting duopoly game
- Random demand: with probability α , demand is zero, while demand is high with probability 1α
- Firm's profits:
 - equals $\Pi^m/2$ if both firms collude, and demand high
 - equals zero if demand low
 - if one firm cheats: it earns Π^m , while rival makes zero
- Demand is unobserved; all firms observe are their profits. If firms observe that they make zero profits, could be due to either zero demand, or cheating rival.
- Consider following strategy:
 - Start in collusive phase. Set collusive price $p = p^m$.
 - Once profits are zero, enter punishment phase, and set price p = c.
 - Punishment phase lasts T periods, then revert back to collusive phase.
- Note: low prices when demand is low

Check for equilibrium:

 \bullet V^+ : discounted future profits in collusive phase

$$V^{+} = (1 - \alpha)(\Pi^{m}/2 + \delta V^{+}) + \alpha(\delta V^{-})$$

ullet V^- : discounted future profits in first period of punishment phase

$$V^- = \delta^T V^+$$

• These can be solved as:

$$-V^{+} = \frac{(1-\alpha)\Pi^{m}/2}{1-(1-\alpha)\delta - \alpha\delta^{T+1}}$$

$$-V^{-} = \frac{(1-\alpha)\delta^{T}\Pi^{m}/2}{1-(1-\alpha)\delta-\alpha\delta^{T+1}}$$

- In punishment phase, it is Nash equilibrium to play p = c, since your rival is playing p = c.
- In collusive phase, need conditions such that

$$V^{+} \ge (1 - \alpha)(\Pi^{m} + \delta V^{-}) + \alpha(\delta V^{-})$$

which is equivalent to

$$\delta(V^+ - V^-) \ge \Pi^m/2.$$

• By substituting in the exptressions for V^+ and V^- , we obtain

$$1 \le 2(1-\alpha)\delta + (2\alpha - 1)\delta^{T+1}.$$

- Fails for T=1
- As $T \to \infty$, condition becomes $\frac{1}{2(1-\alpha)\delta} \le 1$
- If $\alpha = 1/2$, then condition is $\delta \geq 1$.

Porter (1983) 1

- Test the Green-Porter model: two regimes of behavior ("cooperative" vs. "noncooperative/price war"). Subtle: noncooperative regime arises due to low demand, *not* due to cheating.
- Empirical problem: don't (or only imperfectly) observe when a "price war" is occurring. How can you estimate this model then?
- *Prices* should be lower in price war periods, holding the demand function constant. Price war triggered by change in firm behavior
- Observed weekly data (1880-1886):
 - Market-level output (Q_t) and price (p_t)
 - Cost shifters S_t : dummies DM1, DM2, DM3, DM4 for entry by additional rail companies
 - Demand shifter L_t : =1 if Great Lakes open to navigation (availability of substitute to rail transport)
 - $-PO_t$: =1 if price war reported in week t

Porter (1983): Model

- N firms (railroads), each producing a homogeneous product (grain shipments). Firm i chooses q_{it} in period t.
- Market demand:

$$logQ_t = \alpha_0 + \alpha_1 \log p_t + \alpha_2 L_t + U_{1t},$$

where $Q_t = \sum_i q_{it}$.

- Firm i's cost fxn: $C_i(q_{it}) = a_i q_{it}^{\delta} + F_i$
- Firm i's pricing equation: $p_t(1 + \frac{\theta_{it}}{\alpha_1}) = MC_i(q_{it})$, where:

 $\theta_{it} = 0$: Bertrand pricing

 $\theta_{it} = 1$: Monopoly pricing

 $\theta_{it} = s_{it}$: Cournot outcome

- Aggregate: $p_t(1 + \frac{\theta_t}{\alpha}) = DQ^{\delta 1}$. $D \equiv \delta \left(\sum_i a_i^{1/(1 - \delta)}\right)^{1 - \delta}$
- Markup: $= -\theta_t/\alpha$ (varies across weeks depending on competition)
- Define $I_t = \begin{cases} 0 & \text{if in "punishment" week} \\ 1 & \text{if in "collusive" week} \end{cases}$
- Allow θ_t to vary, depending on I_t :

$$\theta_t = I_t \cdot \theta_1 + (1 - I_t) \cdot \theta_0.$$

According to G-P model, $\theta_1 > \theta_0$.

Porter (1983) Model

• After some manipulation, aggregate supply relation is:

$$\log p_t = \log D - (\delta - 1) \log Q_t - \log(1 + \theta_t/\alpha_1)$$

with empirical version

$$\log p_t = \beta_0 + \beta_1 \log Q_t + \beta_2 S_t + \beta_3 I_t + U_{2t}$$

• Since θ_t varies depending on I_t , we see that

$$\beta_3 = \log(1 + \theta_0/\alpha) - \log(1 + \theta_1/\alpha).$$

• Porter estimates this model in two ways.

First estimation approach: 2SLS

- Take $I_t = PO_t$
- Estimate demand and supply functions separately using 2SLS.
- Use S_t as instruments for price in demand equation
- Use L_t as instrument for quantity in supply equation
- Results:
 - price coefficient $\alpha = -0.742$
 - If assume $\theta_0 = 0$, then $\beta_3 = -\log(1 + \theta_1/\alpha) = 0.345$
 - Markup in punish phase=0, in collusive phase=46.5%

Second estimation approach: ML

• $(U_{1t}, U_{2t})' \sim N(0, \Sigma)$, estimate using ML as in previous example:

Structural model is $\Gamma Y = BX + CI + U$, with $Y = (\log Q_t, \log p_t)'$.

$$L(Y_t|X_t, I_t) = |\Sigma|^{-1/2} |\Gamma|$$

$$\exp\left\{\frac{1}{2}(\Gamma Y - BX + CI)'\Sigma^{-1}(\Gamma Y - BX + CI)\right\}$$

• Allow I_t to be unobserved. Consider integrating it out of the likelihood

$$L(Y_t|X_t) = \int L(Y_t \mid X_t, I_t)g(I_t)dI_t$$

• I_t follows a discrete, two-point distribution (from structure of GP equilibrium):

$$I_t = \begin{cases} 1 & \text{with prob } \lambda \\ 0 & \text{with prob } 1 - \lambda \end{cases}$$

• So likelihood function is:

$$L(Y_t|X_t) = \lambda L(Y_t \mid X_t, I_t = 1) + (1 - \lambda)L(Y_t \mid X_t, I_t = 0)$$

Porter (1983) 4

Results: Largely unchanged from 2SLS results

- Price coefficient $\alpha = -0.800$
- Estimate of β_3 is 0.545: prices higher when firms are in "cooperative" regime.
- If we assume that $\theta = 0$ in non-cooperative periods, then this implies $\theta = 0.336$ in cooperative periods. Low? (Recall $\theta = 1$ under cartel maximization)
- Markup in collusive periods: -0.336/0.8 = 0.42
- What if we assume θ_1 and θ_2 in the two different regimes?
- Cartel earns \$11,000 more in weeks when they are cooperating

Ellison (1994)

Ellison (1994, RAND Journal of Economics) extends the analysis on several fronts:

- Allows for variables to enter λ : theory gives guidance that price wars precipitated by "triggers": low demand (in GP model), large shifts in market share. Allows I_t to be an endogenous in this fashion.
- Allows unobserved I_t 's to be serially correlated
- Also tests Rotemberg and Saloner (1986) model: alternative explanation for price wars (where serial correlation in demand, not unobserved firm actions, drives price fluctuations). In equilibrium, prices lower in periods of relatively high demand, when demand is expected to fall in the future. Finds little evidence in favor of this hypothesis.
- Test for explicit cheating on the part of firms, which doesn't happen in the equilibrium of *any* of these models. Introduces additional regimes to the model.

Other work on collusion

- Bresnahan's (1987, Journal of Industrial Economics) work on price wars in the automobile industry. Focuses on static pricing models, among differentiated products. Evidence that manufacturers colluding in 1955, but not in surrounding years.
- Borenstein and Shepard (1996, RAND Journal of Economics) find support for Rotemberg and Saloner theory in retail gasoline markets.
- Chevalier, Kashyap, Rossi (2005, American Economic Review): tests R-S models versus other explanations of "price wars during booms".