# (Largely) Review: Cost concepts
# The Cost Function

- Cost function $C(q)$: **minimum** cost of producing a given quantity $q$
- $C(q) = F + VC(q)$, where
  - Fixed costs $F$: cost incurred regardless of output amount. Avoidable vs. sunk: crucial for determining shut-down decisions for the firm.
  - Variable costs $VC(q)$; vary with the amount produced.
  - Average cost $AC(q) = \frac{C(q)}{q}$
  - Marginal cost $MC(q) = \frac{\partial C(q)}{\partial q}$
  - $AVC(q) = \frac{VC(q)}{q}$; $AFC(q) = \frac{F}{q}$; $AC(q) = AVC(q) + AFC(q)$.

**Example**

- $C(q) = 125 + 5q + 5q^2$
- $AC(q) =$
- $MC(q) =$
- $AFC(q) = 125/q$
- $AVC(q) = 5 + 5q$
- 

| q | AC(q) | MC(q) |
|---|---|---|
| 1 | 135 | 15 |
| 3 | 61.67 | 35 |
| 5 | 55 | 55 |
| 7 | 57.86 | 75 |
| 9 | 63.89 | 95 |

- AC rises if MC exceeds it, and falls if MC is below it. Implies that MC intersects AC at the minimum of AC.

## Short-run vs. long-run costs:

- Short run: production technology given
- Long run: can adapt production technology to market conditions
- Long-run AC curve cannot exceed short-run AC curve: its the *lower envelope*

**Example:** "The division of labor is limited by the extent of the market" (Adam Smith)

- Division of labor requires high fixed costs (for example, assembly line requires high setup costs).
- Firm adopts division of labor only when scale of production (market demand) is high enough.
- Graph: Price-taking firm has "choice" between two production technologies.

## Opportunity cost

The *opportunity cost* of a product is the value of the best forgone alternative use of the resources employed in making it.

**Normal profit** of a product is its selling price minus opportunity cost. Quit when normal profit $< 0$.

Example:
Car factory: a worker would make \$5 an hour
Two brothers, who make one lamp each hour, with materials costing \$7
What is opportunity cost of lamp?
Normal profit when market price of lamp is \$11? \$10? \$9?

## Economies of scale: slope of AC curve

- $AC'(q) < 0$: increasing returns to scale
- $AC'(q) > 0$: decreasing returns to scale
- $AC'(q) = 0$: constant returns to scale

Example: U-shaped AC curve

- Factors affecting EOS:
  - Fixed costs
  - Congestion
  - specialization
- Measure of scale economies: $AC(q)/MC(q)$
  - Increasing RS
  - Decreasing RS
- Minimum efficient scale of production: smallest output which minimizes *long-run* AC.

## Economies of scope 1

- $C(q_1, q_2) < C_1(q_1) + C_2(q_2)$
- Example (common fixed costs):
  $C_1(q_1) = 10 + 2q_1$
  $C_2(q_2) = 10 + 3q_2$
  $C(q_1, q_2) = 10 + 2q_1 + 3q_2$
- Depends on levels of $q_1$, $q_2$: $C_1(q_1) = 5 + 2q_1$
  $C_2(q_2) = 5 + 3q_2$
  $C(q_1, q_2) = 10 + 3q_1 + 2q_2$
  Consider two output levels: 1 and 2

| $(q_1, q_2)$ | $C_1(q_1)$ | $C_2(q_2)$ | $C(q_1, q_2)$ |
|---|---|---|---|
| (1,1) | | | |
| (1,2) | | | |
| (2,1) | | | |
| (2,2) | | | |

## Economies of Scope 2: Ray Average Costs

How to measure economics of scale for multiproduct firms? Need to define appropriate notion of "average costs" for this firm.

- What is AC for a multiproduct firm?
- Assume production of the different products $i = 1, \ldots, N$ in **fixed proportions**, and let these proportions be $\lambda_1, \ldots, \lambda_N$, with $\sum_i \lambda_i = 1$.
- Let $q_1, \ldots, q_N$ denote production of the different products, and $q = q_1 + q_2 + ....$
- Then define $\lambda_i = q_i/q$, the "proportion" of component $i$ in the total production. Note that $q_i = \lambda_i q$.
- Example: Shoe factory
  $q_1$ is number of right shoes
  $q_2$ is number of left shoes
  $\lambda_1 = 0.5,\ \lambda_2 = 0.5$
- Example: auto (one body plus four wheels)

## Economies of Scope 3: Ray Average Costs (contd)

- Define: **Ray Average Costs (RAC(q))** for the composite quantity $q$: $\frac{C(\lambda_1 q,\ldots,\lambda_N q)}{q}$

  Graph: RAC concept only considers production combinations along "rays".

- Example: Shoe factory $C(q_1, q_2) = 100 + 5q_1 + 5q_2$, then RAC(q) is $\frac{1}{q} * [100 + 5\lambda_1 q + 5\lambda_2 q] = \frac{100+5q}{q}$.

- Slope of RAC(q) curve determines economies of scale for a multiproduct firm.

## Economies of Scope 4

- Reasons for Economies of Scope
  1. Common inputs (cow yields beef and leather)
  2. Marketing channels ("umbrella branding": Regular, Honey Nut, and Apple-Cinnamon Cheerios)

## Summary

- Define: cost function
- Returns to scale
- Economies of Scope