As a matter of fact, the proof of the later statement does not follow from standard argument because $Q_{L,I}(\theta)$ is not continuous in I. However, because $Q_{L,I}(\theta)$ is continuous in θ for every I, and because of the above uniform convergence, we have $Q^*_{S,L,I_o}(\hat{\theta}_{I_o}) - Q_{L,I_o}(\theta_o) \xrightarrow{\text{a.s.}} 0$ and $\min_{I \in \mathscr{I}-\{I_o\}} Q^*_{S,L,I}(\hat{\theta}_I) - \min_{I \in \mathscr{I}-\{I_o\}} \min_\theta Q_{L,I}(\theta) \xrightarrow{\text{a.s.}} 0$. Because (I_o, θ_o) is identifiably unique by assumption, then $Q_{L,I_o}(\theta_o) < \min_{I \in \mathscr{I}-\{I_o\}} \min_\theta Q_{L,I}(\theta)$ for L sufficiently large a.s. Hence, $\hat{I} = I_o$ and $\hat{\theta}_{\hat{I}} = \hat{\theta}_{I_o}$ for L sufficiently large a.s. The desired result follows.

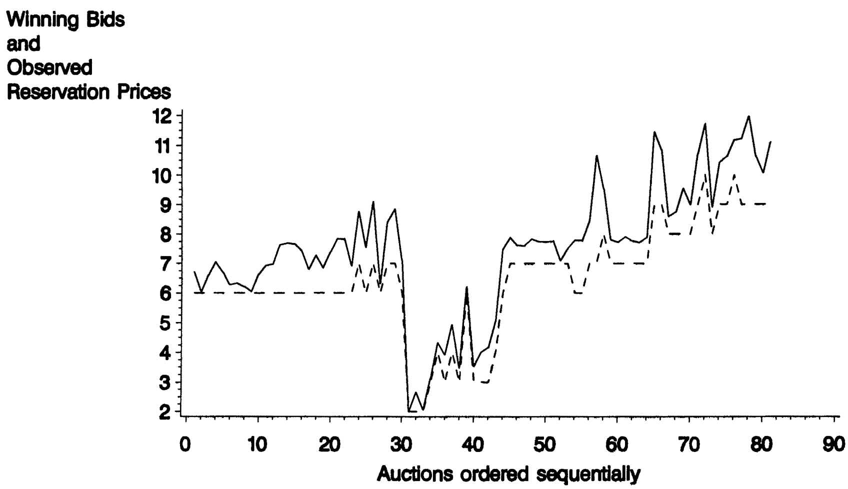

FIGURE 3.—Winning bids (continuous line) and reservation prices (dotted line).

APPENDIX B

Summary Statistics

Bids and reservation prices are measured in Francs per Kilo. The "supply" variable is measured in tons. The "date" variable and the qualitative variables are defined in Section 4.3.

NUMBER OF AUCTIONS: 81

Quantitative Variables

Variables	Mean	Standard Deviation	Maximum	Minimum
Bid	7.52	2.21	11.98	2.00
Reservation Price	6.43	1.81	10.00	2.00
Date	2.40	0.41	3.00	1.76
Supply	1.39	0.67	3.49	0.13

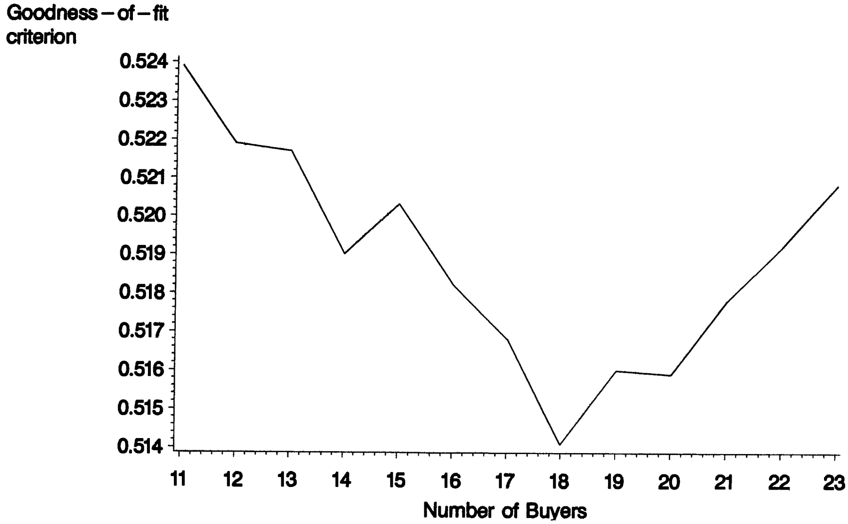

FIGURE 1.—Size of the market.

attended the auction. Thus, between $I = 11$ and $I = 23$, we search for the best value of I in terms of our lack-of-fit criterion $(1/L)Q^*_{S,L,I}(\hat{\theta}_I)$ (see Proposition 4). Figure 1 suggests that this value is obtained for $I = 18$.[21] This result lends support to our second hypothesis that the large buyer is an agent of a number of traders. Specifically $I = 18$ implies that this buyer represents 8 such traders, which is consistent with his share of the market (slightly less than one half). As long as this buyer is just an intermediary who bids according to the 8 traders' instructions determined independently, this does not affect the strategies of the other bidders. See also the conclusion.

Table I displays our empirical results for $I = 11$ and 18. Student statistics computed from Proposition 3 are shown in parentheses and the criterion value is the quantity $(1/L)Q^*_{S,L,I}(\hat{\theta}_I)$. Given the choice of the log-normal distribution and the parameterization (26), each parameter estimate of Table I can be interpreted as the percentage change of the expected value of the auctioned lot because $\theta_k = (1/E_l[V]\ \partial E_l[V]/\partial x_{kl}$. For instance, when $I = 18$, the midsize eggplants and the large size eggplants are respectively 24% and 12% more valuable then the small size eggplants. This agrees with the conventional wisdom that midsize eggplants are the highest quality.

Table I shows some slight differences between the coefficient estimates for $I = 11$ and 18. All parameters have the expected signs. The seller parameter identifies correctly the seller who is known to be of higher quality. The signs for the size dummies agree with conventional wisdom, as noted previously. The significantly positive coefficient for the period dummy agrees with a demand

[21] In principle, the significance of the difference between $I = 11$ and $I = 18$ can be tested but this requires the development of nonnested tests (following Vuong (1989)), which is outside the scope of this paper.

TABLE I

Variables	Parameters	
	First Model	Second Model
Number of Buyers (I)	11	18
Number of Simulations (S)	20	20
Number of Auctions (L)	81	81
Constant	0.1297	0.0286
	(0.02)	(0.06)
Seller	−0.0107	−0.0240
	(−0.17)	(−0.51)
Size 1	0.2402	0.2402
	(3.57)	(4.39)
Size 2	0.1373	0.1213
	(1.39)	(1.60)
Period	1.2404	1.1998
	(2.16)	(2.90)
Date	0.3115	0.3202
	(3.04)	(4.03)
Supply	−0.0340	−0.0357
	(−0.59)	(−0.81)
Criterion Value	0.52395	0.51401

shock due to the supply of a substitute good during the period coded zero. The trend coefficient is significantly positive. The supply coefficient is negative, although not significant. The lack of significance of the supply variable might be explained as follows. This variable is the local supply and not the global supply on this market. The final buyers buy on several markets so that the idiosyncratic local shocks in this market do not affect their willingness to pay. The global movement of supply is already taken into account by the period and date variables.

Concerning the goodness-of-fit of our estimated model for $I = 18$, our explanatory variables enable us to track closely the winning bids b_l^w by the simulated winning bids $\overline{X}_l(\hat{\theta})$ (see Figure 2). In view of Proposition 1, an R^2 measure can be computed as $1 - Q^*_{S,L}(\hat{\theta})/\hat{\operatorname{var}}\, b^w$. This gives $R^2 = .895$.[22]

5. CONCLUSION

The major contribution of this paper is to describe a new research strategy for analyzing auction data sets. Using a simulated NLLS estimation method we have shown that a traditional structural econometric approach can be employed.

[22] In principle, the log-normality of the private value distribution can also be tested. For instance, we can nest the log-normal family in $\{f^*(\cdot|z,\theta,\mu) = (1-\mu)f(\cdot|z,\theta) + \mu h(\cdot|z)\colon\ \mu \in \boldsymbol{R}\}$, where $h(\cdot|z)$ is a fixed distribution. The condition $\int v h(v|z)\,dv = 0$ may be imposed so that the conditional mean of v given z remains as specified in (26). Then standard classical tests of $\mu = 0$ can be used.

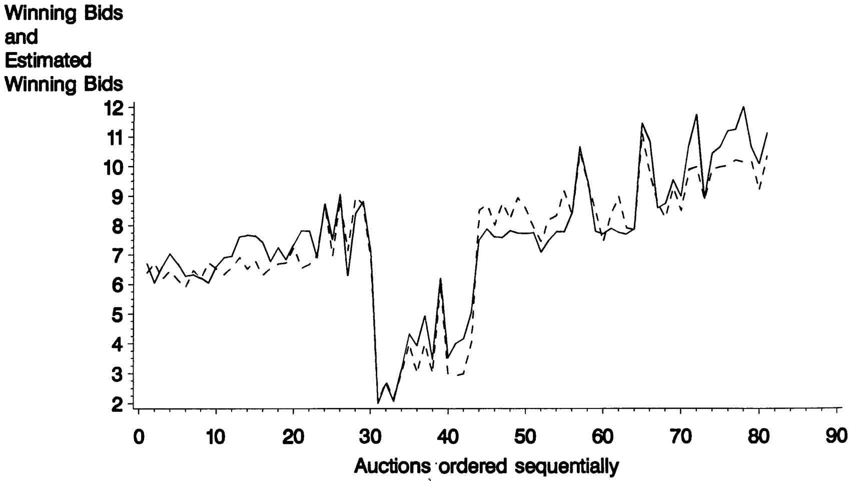

FIGURE 2.—Winning bids (continuous line) and estimated winning bids (dotted line).

We have developed the statistical tools necessary for the analysis of first-price or Dutch auctions with independent private values. Clearly similar methods can be developed for other types of auctions such as the English auction and the common value paradigm.

Many directions of research are possible with our statistical approach. They include:

—An analysis of the optimality of the observed reservation prices so as to provide a pricing strategy based on (4) that would enable the sellers to increase their expected income.[23]

—Tests of the gains provided by the game theoretic approach to auctions as opposed to nonstrategic approaches. For example the game theoretic approach can be confronted with a bounded rationality approach, where bid functions are restricted to be linear or quadratic. Such tests have been carried out in experimental economics where the distribution of characteristics is known.

—Tests of the strategic behavior of the large trader. This is explored in Laffont and Vuong (1994). In our case there is a natural potential coalition of traders. This is a modest step towards the more ambitious program of testing the existence of collusive behavior among buyers in general. In our view, this program still suffers from the weakness of economic theory in describing coalitional behavior under incomplete information. The step we propose raises a whole set of difficult issues associated with asymmetric auctions.[24]

[23] This is an important practical issue. Because our estimation method does not require sellers to behave according to (4), a Hausman test of the optimality of observed reservation prices can be derived by comparing our parameter estimate of θ with its parameter estimate obtained by simulation-based joint estimation of (2) and (4) (see footnote 1).

[24] These issues are not trivial because the asymmetry created by the large trader prevents the analytic solution for the optimal bidding strategy. Numerical integration of differential equations has to be part of the estimation procedure.

Table 3: Structural Demand Estimation

	(1)	(2)	(3)	(4)	(5)
μ					
Age	-0.2107	-0.2094	-0.2091	-0.2088	-0.2095
	(0.0023)	(0.0023)	(0.0023)	(0.0023)	(0.0024)
Age Squared	0.0043	0.0043	0.0043	0.0043	0.0043
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Log of Miles	-0.1258	-0.1248	-0.1246	-0.1242	-0.1256
	(0.0059)	(0.0058)	(0.0058)	(0.0058)	(0.0060)
Log of Text Size	0.0537	0.0471	0.0592	0.0610	0.0785
	(0.0065)	(0.0065)	(0.0068)	(0.0070)	(0.0079)
Number of Photos	0.0185	0.0179	0.0179	0.0176	0.0207
	(0.0009)	(0.0009)	(0.0009)	(0.0009)	(0.0010)
Manual Transmission	0.0404	0.0404	0.0385	0.0378	0.0341
	(0.0111)	(0.0111)	(0.0110)	(0.0110)	(0.0110)
Featured Listing		0.1915	0.1897	0.1925	0.1983
		(0.0159)	(0.0159)	(0.0160)	(0.0159)
Log Feedback			-0.0195	-0.0178	-0.0167
			(0.0029)	(0.0029)	(0.0029)
Percentage Negative Feedback			-0.0027	-0.0025	-0.0027
			(0.0013)	(0.0013)	(0.0013)
Total Number of Listings				-0.0040	-0.0044
				(0.0011)	(0.0011)
Phone Number Provided				0.0605	0.0641
				(0.0106)	(0.0105)
Address Provided				-0.0361	-0.0352
				(0.0154)	(0.0154)
Warranty					0.4761
					(0.1175)
Warranty*Logtext					-0.0503
					(0.0182)
Warranty*Photos					-0.0139
					(0.0022)
Inspection					0.4388
					(0.0937)
Inspection*Logtext					-0.0433
					(0.0142)
Inspection*Photos					-0.0052
					(0.0018)
Constant	10.6771	10.6952	10.6726	10.6485	10.4939
	(0.0764)	(0.0759)	(0.0759)	(0.0767)	(0.0853)

Estimated standard errors are given in parentheses. The table gives the estimated relationship between the prior mean valuation μ and auction covariates. The nested specifications (2)-(4) include controls for marketing effects, seller feedback and dealer status. Specification (5) includes warranty and inspection dummies and their interactions with the information measures, showing that the value of photos and text is lower when the car has been inspected or is under warranty.

Table 3: Structural Demand Estimation (continued)

	(1)	(2)	(3)	(4)	(5)
σ					
Age	0.0336	0.0337	0.0337	0.0335	0.0335
	(0.0022)	(0.0022)	(0.0022)	(0.0022)	(0.0022)
Log of Text Size	-0.0185	-0.0206	-0.0215	-0.0221	-0.0220
	(0.0357)	(0.0360)	(0.0360)	(0.0360)	(0.0363)
Photos	-0.0178	-0.0182	-0.0182	-0.0182	-0.0183
	(0.0053)	(0.0054)	(0.0054)	(0.0054)	(0.0055)
Constant	-1.1490	-1.1384	-1.1344	-1.1287	-1.1345
	(0.2234)	(0.2241)	(0.2239)	(0.2237)	(0.2254)
r_{INDEX}	-0.0630	-0.0648	-0.0637	-0.0619	-0.0642
	(0.0373)	(0.0364)	(0.0366)	(0.0374)	(0.0363)
Marginal Effects for σ					
Age	0.0127	0.0126	0.0126	0.0125	0.0124
Log of Text Size	-0.0070	-0.0077	-0.0081	-0.0083	-0.0082
Photos	-0.0067	-0.0068	-0.0068	-0.0068	-0.0068
Mean σ	0.8472	0.8448	0.8442	0.8433	0.8402
r	1.2099	1.2088	1.2095	1.2106	1.2092

Estimated standard errors are given in parentheses. The parameters in this table show the estimated relationship between the prior standard deviation σ, which is a measure of prior uncertainty, and auction covariates z. Since the relationship between σ and z is modeled as being non-linear, the marginal effects are easiest to interpret. An estimate of r, the standard deviation of bidders private signals, is also provided.

Figure 1. Empirical Distributions of Pseudo-values
From One Monte Carlo Sample

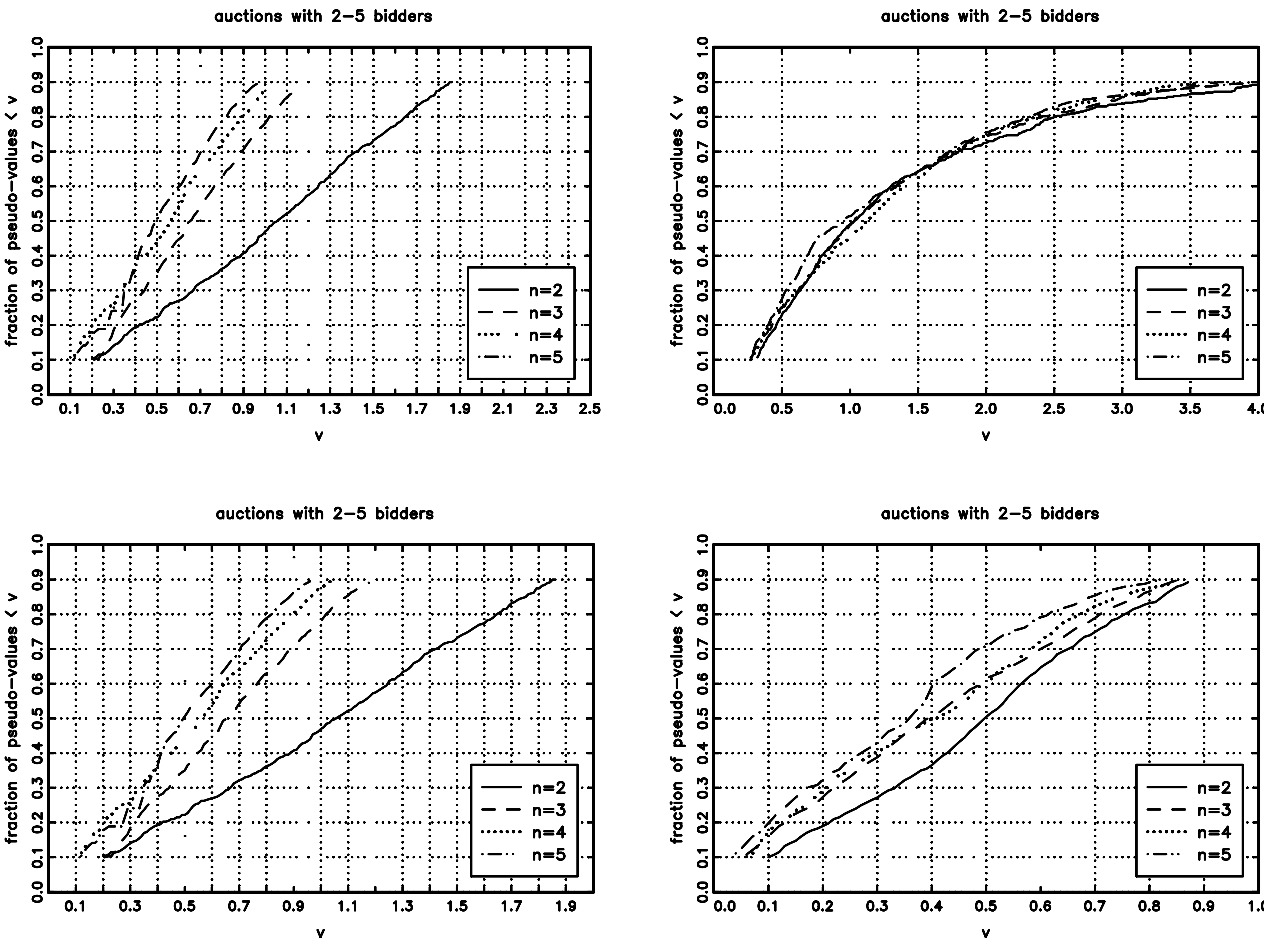

Table 1: Monte Carlo Results
500 replications of each experiment.

	PV1			CV1		
Range of n:	2–3	2–4	2–5	2–3	2–4	2–5
$n \cdot T^a$	300	300	300	300	300	300
%(reject at 5%)	0.064	0.086	0.164	0.532	0.850	0.898
%(reject at 10%)	0.112	0.146	0.214	0.678	0.904	0.960

	PV2			CV2		
Range of n:	2–3	2–4	2–5	2–3	2–4	2–5
$n \cdot T$	300	300	300	300	300	300
%(reject at 5%)	0.056	0.122	0.192	0.754	0.916	0.974
%(reject at 10%)	0.112	0.186	0.260	0.826	0.944	0.980

[a] T denotes number of auctions, and n denotes the number of bidders in each auction.

Table 2: Monte Carlo Results
Bootstrap Estimation of Σ
500 replications of each experiment.

	PV1			CV1		
Range of n:	2–3	2–4	2–5	2–3	2–4	2–5
$n \cdot T^a$	300	300	300	300	300	300
%(reject at 5%)	0.024	0.028	0.144	0.424	0.600	0.788
%(reject at 10%)	0.092	0.066	0.190	0.628	0.758	0.894

	PV2			CV2		
Range of n:	2–3	2–4	2–5	2–3	2–4	2–5
$n \cdot T$	300	300	300	300	300	300
%(reject at 5%)	0.024	0.030	0.050	0.530	0.740	0.852
%(reject at 10%)	0.062	0.070	0.084	0.680	0.852	0.906

[a] T denotes number of auctions, and n denotes the number of bidders in each auction.