Oligopoly

Return to the 2-firm case. Assume each firm produces with $C(q) = cq$, and market demand curve is $p = a - bQ$.

1. Cournot:
   - (***) Solve for the Cournot Nash equilibrium quantities, prices, and profits for the two firms. Call these $q*, p*, \pi^*$.
   - (***) What if these two firms formed a cartel and maximized joint profits? Solve for the resulting quantities, prices, and profits; call these q^j, p^j, π^j.
   - What if firm 2 cheats when firm 1 sets $q_1 = q^j$? What are the resulting quantities, prices, and profits?
   - What does this have to do with the prisoner's dilemma?
2. (***) Bertrand: derive the Bertrand nash equilibrium prices, quantities, and profits. Call these q^b, p^b, π^b.
3. Rank the quantities, prices, and profits computed in the problems marked (***).

5. Consider the following game tree (see figure 1)

(a) List all of player 1's strategies
(b) List all of player 2's strategies
(c) What are the Nash equilibria of this game? Show why.
(d) What are the subgame perfect equilibria of this game? Show why.

6. Construct a "Nash reversion"-type subgame-perfect equilibrium to the infinitely repeated Bertrand (price-setting) game. Assume there are two identical firms, each producing at constant marginal cost c. The market demand curve is $p = a - bQ$.

7. Consider two firms interacting in two identical and independent markets. The markets differ in that in market 1 a firm's price at time t is observed at $t + 1$, whereas in market 2 it is learned only at $t + 2$. Thus, although each of the markets meets every period, market 2 has longer information lags.

(i) Derive conditions on the discount rate δ for sustainable collusion *only in market 2*.

(ii) Derive conditions on the discount rate δ under which firms and collude *in both markets simultaneously*, allowing for punishments potentially across markets.

Figure 1: Game tree for question 5