This test has a total of 80 points. Please submit your answers to the questions in the same order as they appear in the exam. Indicate clearly which question is being worked upon, and please highlight your answers with boxes. Good luck!

Total allowed time: 90 minutes. Closed book, closed notes.

1. **15 pts**

   Consider a monopolist facing a demand curve of $q = 1/p^\alpha$, where $\alpha > 0$. The cost function is $C(q) = q$.

   (a) Using the inverse elasticity principle, derive an expression for the markup under monopoly pricing, as a function of p and α.

   (b) Based on this, what is the monopolist's optimal production and price when $\alpha = 2$?

   (c) Based on this, what is the monopolist's optimal production and price when $\alpha = 1/2$?

2. **15 pts**

   Short answer questions:

   (a) The short-run supply of cotton is perfectly inelastic, because it has already been harvested and no more of it can be grown until the next year. In order to raise the price received by farmers, the government orders the destruction of a percentage of the harvest. Does this action result in a deadweight loss? Why or why not?

   (b) Daily newspaper publishers routinely order newsstands to destroy unsold copies of newspapers at the end of each day, thus preventing them from selling the previous day's newspaper (potentially at a discounted price). Does this result in a deadweight loss? Why or why not?

   (c) Assume that the demand for gasoline is completely inelastic. The gasoline monopolist raises the price of gasoline from $3 per gallon to $4 per gallon. Does this result in a deadweight loss? Why or why not?

3. **(30 points)**

   The demand for Brazilian BBQ in the rural town of Eggnog, Kansas is given by the demand function $Q = 95 - p$, where q measures the quantity of meals. The costs of providing a meal are given by the cost function $C(Q) = 5Q$.

   The Rio Grill is the incumbent monopolist in the opera market in Eggnog.

   (a) What is the optimal price which the Rio Grill will charge? (5 pts)

   Now assume that a second Brazilian BBQ restaurant, Fogo de Chao (FDC), is contemplating opening in Eggnog. Since FDC is a nationwide chain, its production costs are lower than that of the Rio Grill; the cost function for FDC is $C(Q) = 4Q$. Furthermore, if FDC enters, the two restaurants would compete in Bertrand fashion.

   (b) If FDC enters the market, what will the profits for both firms be? (5 pts)

   Now assume that, before FDC decides whether to enter, the Rio Grill has the opportunity to invest in a "turbogriller" which allows it to pre-grill a specified quantity Q^* (which can be chosen by the Rio Grill) of meat during the night, and serve it to diners the next day. If the Rio Grill buys this machine, its cost function would change to:

   $$C(Q) = \begin{cases} 0 & \text{if } Q \leq Q^* \\ 5(Q - Q^*) & \text{otherwise.} \end{cases}$$

   (c) Assume that the Rio Grill owns this machine. What is the subgame perfect equiilbrium of the game, and how does it change as Q^* varies? What value of Q^* would the Rio Grill choose? (15 pts)

   (d) If the turbogriller costs $300 to operate (regardless of what level of Q^* is chosen), would the Rio Grill be willing to purchase it? (5 pts)

4. **(30 points)**

   Consider a market with 2 firms. Assume that firms produce at zero cost, and the
   market demand curve is:

   $$Q = \begin{cases} 0 & \text{with probability 0.25; "low demand"} \\ 1 - p & \text{with probability 0.75; "high demand"} \end{cases}$$

   That is, each period could be a "high" or "low" demand period with demand curves
   as above. Also, we assume that each period, each firm observe whether it is a high or
   low demand period, before choosing its price.

   (a) Derive the joint profit maximizing quantities and profits in each type of period.
       Let q_H^* and q_L^* denote these optimizing quantities in, respectively, high and low
       demand periods. (5 points)

   (b) Now assume that the two firms repeat the Bertrand game forever. Derive con-
       ditions under which "Bertrand reversion" strategies constitute subgame perfect
       equilibrium of this game, in which full cooperation between the two firms in both
       types of periods – that is, production at q_H^* in high-demand periods, and q_L^* in
       low-period periods – can be sustained. (25 pts)