Lecture 5: Collusion and Cartels in Oligopoly

EC 105. Industrial Organization

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HSS, California Institute of Technology

Outline

1. Introduction

2. Dynamic Games: introduction

Cartels and collusion in oligopoly

- Recall: in static games from last lecture:
  - firms produce "too much"
  - relative to joint profit maximization
  - as in Prisoner's dilemma
- Can cooperation occur in multi-period ("dynamic") games?
  - main idea: repeated interactions allow for threats/rewards
  - examples: roommates? restaurant? bank?
- In order to study dynamic games, we need to introduce a new concept of equilibrium
  - Nash equilibrium not enough
- Introduce: **Subgame Perfect Equilibrium**
- Finitely-repeated Cournot game
- Infinitely-repeated Cournot game

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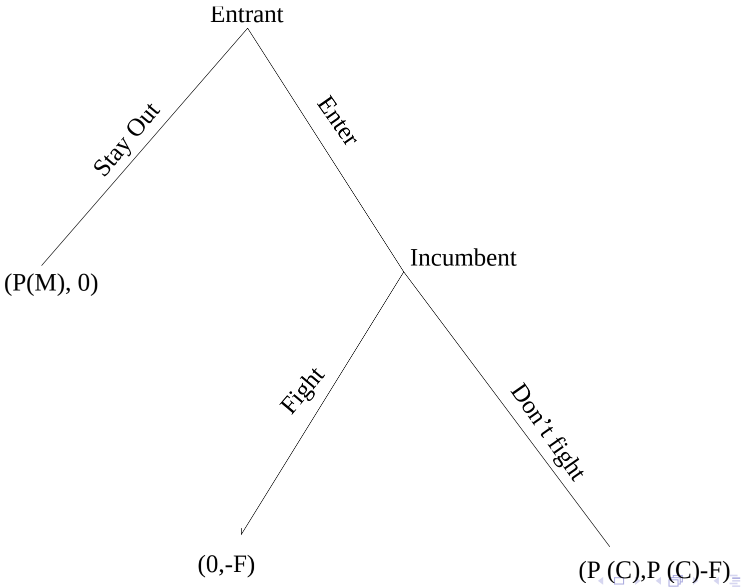
Simple model of threat: Limit pricing



Extensive Form ("Tree") Representation of a Game

- Extensive Form Representation specifies:

1. players in the game.
2. when each player has the move.
3. what each player can do at each of his or her opportunities to move.
4. what each player knows at each of his or her opportunities to move.
5. the payoff received by each player for each combination of moves that could be chosen by the players.

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Strategy in dynamic games

Definition

A strategy for a player is a complete plan of action.
It specifies a feasible action for the player in every contingency in which the player might be called to act

What are strategies in limit pricing game?

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Limit pricing analysis

- One way to analyze this game, is to "flatten" the tree into a game matrix.

Incumbent:	Fight	Don't fight
Entrant:		
Stay Out	0, Monop	0, Monop
Enter	-F, 0	Prof-F, Prof

- What are NE?
- But what if entrant enters?
- Some Nash equilibria seem unpalatable, bc they involve *noncredible threats*

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Subgame-Perfect Equilibrium

- A *subgame* is the part of the multi-period game that starts from any given node onwards.
- A **subgame perfect equilibrium (SPE)** is a strategy profile, from which, no player can receive a higher payoff *in any subgame*. That is, each player's SPE strategy must be a best-response in any subgame
- Find SPE by doing **backwards induction** on the game tree
  - this eliminates all non-BR actions in any subgame
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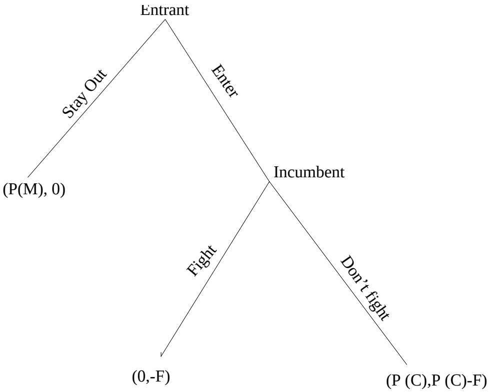

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- What are SPE?

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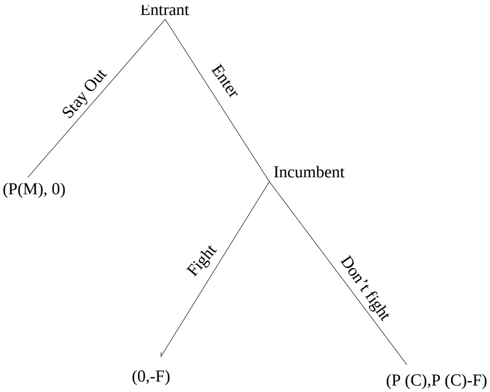

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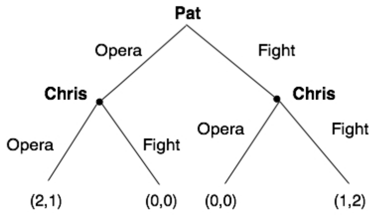
- What are SPE?

Sequential Version of BoS: Pat moves first

Strategic / Normal Form Representation

		Chris			
		Op-Op	*Op-Fi*	*Fi-Op*	*Fi-Fi*
Pat	*Opera*	(2,1)	(2,1)	(0,0)	(0,0)
	Fight	(0,0)	(1,2)	(0,0)	(1,2)

Extensive Form / Game Tree



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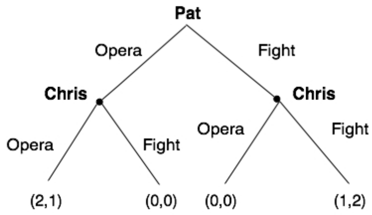

Now let's consider repeating the Cournot Game

2-firm Cournot quantity-setting game. Relevant quantities are:

- NE profits $\pi^* = \frac{(a-c)^2}{9b}$
- Cartel profits $\pi^j = \frac{(a-c)^2}{8b}$
- Firm 1 cheats on firm 2: $\pi^x = \pi_1(BR_1(q_2^j)) = \frac{9(a-c)^2}{64b}$
- Prisoners' dilemma analogy:

Firm 2 → Firm 1 ↓	cheat	cartel
cheat	$\frac{(a-c)^2}{9b}, \frac{(a-c)^2}{9b}$	$\frac{9(a-c)^2}{64b}, \frac{3(a-c)^2}{32b}$
cartel	$\frac{3(a-c)^2}{32b}, \frac{9(a-c)^2}{64b}$	$\frac{(a-c)^2}{8b}, \frac{(a-c)^2}{8b}$

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Let's repear the Cournot Game twice

2-period Cournot game

- Firm 1 chooses quantities (q_{11}, q_{12})
  Firm 2 chooses quantities (q_{21}, q_{22})

What are SPE: solve backwards

- Second period: unique NE is (cheat,cheat)
- First period: (cheat,cheat) $\longrightarrow$ unique SPE is ((cheat,cheat), (cheat,cheat))
- What about ((cartel,cartel), (cartel,cartel))?
- What about ((cartel, cheat), (cartel, cheat))?
- What about
  Firm 1 plays (cartel; cheat if cheat, cartel if cartel)
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What about 3 periods? N periods?

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Infinitely-repeated Cournot Game

What if the 2-firm Cournot game is repeated forever? Are there SPE of this game in which both firms play "cartel" each period?

Need to introduce the concept of *discounting*:

- Discount rate $\delta \in [0, 1]$, which measures how "patient" a firm is.
- The "discounted present value" of receiving \$10 both today and tomorrow is $10 + \delta 10$.
- If $\delta = 1$, then there is no difference between receiving \$10 today and \$10 tomorrow.
- Geometric series property: $x + \delta x + \delta^2 x + \cdots + \delta^n x + \cdots = \frac{x}{1-\delta}$.

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Infinitely-repeated Cournot Game (part 2)

- Let q^j denote the cartel (joint profit-maximizing) quantity.
- **Proposition**: If the discount rate is "high enough", then these strategies constitute a SPE of the infinitely-repeated Cournot game:
  1. In period t, firm 1 plays $q_{1t} = q^j$ if $q_{i,t-1} = q^j$ for both $i = 1, 2$.
  2. Play q^* if $q_{i,t-1} \neq q^j$ for either $i = 1, 2$.
- Firm 1 cooperates as long as it observes firm 2 to be cooperating. Once firm 2 cheats firm 1 produces the Cournot-Nash quantity every period hereafter: **Nash reversion**
  - "Grim strategy": no second chances.
- Show that these strategies constitute a SPE by finding conditions such that they prescribe best-response behavior for firm 1 given that firm 2 is following this strategy also in each subgame.

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Infinitely-repeated Cournot Game (part 3)

Consider firm 1 (symmetric for firm 2).
There are two relevant (types of) subgames for firm 1. Consider each in turn.

Subgame type #1: After a period in which cheating (either by himself or the other firm) has occurred.

Proposed strategy prescribes playing q^* forever, given that firm 2 also does this. This is NE of the subgame: playing "q^* forever" is a best-response to firm 2 playing "q^* forever". This satisfies SPE conditions.

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One-Shot Deviation Principle

It is enough to check one-shot deviations to check for SPNE

- No need to consider deviations now followed by other deviations later

1. **If there is no profitable one-shot deviation, then there is no profitable finite sequence of deviations**
   - Last deviation can't be profitable . . .

2. **If there is no profitable finite deviation, then there is no profitable infinite sequence of deviations**
   - Suppose there are no one-shot profitable deviations from a strategy p_i, but there is a stage t and a history h_t where player i could improve his payoff using a different strategy $\hat{p}_i$ in the subgame starting at h_t.
   - Since payoff is a discounted sum and feasible profits are bounded, distant payoffs can't matter much. Thus, there is a $t' > t$ such that the strategy p_i' that coincides with $\hat{p}_i$ at all stages before t' and agrees with p_i at all stages from t' on must improve on p_i in the subgame starting at h_t. But this contradicts the fact that no finite sequence of deviations can make no improvement at all.

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Remarks

- Nash reversion is but one example of strategies which yield cooperative outcome in an infinitely-repeated Cournot game.
- In general, firm 1 need not punish firm 2 forever to induce it to cooperate; after firm 2 deviates, just produce q^* for long enough so that it never pays for firm 2 to ever deviate.
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- In general, the *Folk Theorem* says that, if the discount rate is "high enough", an infinite number of SPE exist for infinite-horizon repeated games, which involve higher payoffs than in the single-period Nash outcome.
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Credible threats

Generally: threats of punishment must be *credible* — it must be a firm's best-response to punish when it detects cheating

Industry and/or firm characteristics can make punishments more credible:

1. Flexible capacity: punishment may involve a large hike in quantity, this must be relatively costless
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Same arguments for infinitely-repeated Bertrand

- Consider Nash reversion strategy:
  - Let π^m denote monopoly profits each period.
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