Lecture 1: Costs

EC 105. Industrial Organization.

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Technology and Costs

(Largely) Review: Cost concepts

- Two main components of a firm's costs:
  1. Fixed costs F: cost incurred regardless of output amount.
  2. Variable costs VC; vary with the amount produced.
- Examples:
  - Airline: fixed cost is cost of airplane; variable cost are costs of incremental customers
  - Computer factory: fixed cost is cost of setting up factory; variable cost include input costs for each PC produced
  - Starbucks: fixed cost is rent for space; VC are costs of making each cup of coffee (almost zero!)
- Magnitude of fixed vs. variable cost determine the *efficient size* of a firm.

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The Cost Function

- Cost function $C(q)$: **minimum** cost of producing a given quantity q
- $C(q) = F + VC(q)$, where
  - Fixed costs F: cost incurred regardless of output amount.
    - Avoidable vs. sunk: crucial for determining shut-down decisions for the firm.
  - Variable costs $VC(q)$; vary with the amount produced.
  - Average cost $AC(q) = \frac{C(q)}{q}$
  - Marginal cost $MC(q) = \frac{\partial C(q)}{\partial q}$
  - $AVC(q) = \frac{VC(q)}{q}$; $AFC(q) = \frac{F}{q}$; $AC(q) = AVC(q) + AFC(q)$.

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Example

- $C(q) = 125 + 5q + 5q^2$
- $AC(q) =$
- $MC(q) =$
- $AFC(q) = 125/q; \quad AVC(q) = 5 + 5q$

q	AC(q)	MC(q)
1	135	15
3	61.67	35
5	55	55
7	57.86	75
9	63.89	95

- AC rises if MC exceeds it, and falls if MC is below it. Implies that MC intersects AC at the minimum of AC.

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Traditional vs. internet firms

- Traditional companies vs. internet companies can have very different cost structures:
- Compare to traditional products, some internet versions have *lower fixed costs* but *higher variable costs*
- Ex: Restaurants
  - Traditional restaurant: FC (costs of renting space; hiring workers) >> VC (cost of ingredients for each dish)
  - Food delivery service: FC (no need for physical restaurant) <<(?) VC (costs for delivery)
- Ex: Internet retailers (books, electronics, clothing, etc.)
  - Lower overhead costs, but *high shipping costs*
- As a result, internet companies actively try to reduce VC
  - automation based on machine learning in warehouses;
  - using drones for delivery

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Traditional products vs. internet: cont'd

- Other internet ventures dominate traditional in both FC and VC
- Ex: movies
  - Traditional movie theater: FC>> VC
  - Online movie streaming: very low *FC* and *VC*
- Ex: music streaming, eBooks, etc.
- In these cases, however, traditional versions still surviving!
  - Traditional and internet versions are becoming *differentiated products*
    - Watching movie at home (with friends) vs. in theater ("date")
  - can profitably *coexist*

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Economies of scale

- Magnitude of FC and VC determine the best size for a firm.
- When FC high relative to VC, there are *Economies of scale:*
- Larger firms are more *efficient*, bc they *produce at lower avg costs*:
  - $AC'(q) < 0$: increasing returns to scale
  - $AC'(q) > 0$: decreasing returns to scale
  - $AC'(q) = 0$: constant returns to scale
  - Example: U-shaped AC curve
- **Minimum Efficient Scale (MES):** level of production where AC is minimized. At minimum of AC curve (more later).
- Factors affecting scale economies:
  - Fixed costs
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  - specialization: "division of labor limited by the extent of the market"

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Multiproduct firms: Economies of Scope

- *Subadditive* cost function:

$$C(q_1, q_2) < C_1(q_1) + C_2(q_2)$$

- Example 1: common fixed costs
  - Ricardo: rancher produces beef & leather
  - "joint production"
  - Leads to *global* EOS

$$C_1(q_1) = 10 + 2q_1; \quad C_2(q_2) = 10 + 3q_2; \quad C(q_1, q_2) = 10 + 2q_1 + 3q_2$$

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Economies of Scope, cont'd

- More often, EOS is local & depends on levels of q_1, q_2:

$$C_1(q_1) = 5 + 2q_1; \quad C_2(q_2) = 5 + 3q_2; C(q_1, q_2) = 10 + 3q_1 + 2q_2$$

(q_1, q_2)	$C_1(q_1)$	$C_2(q_2)$	$C(q_1, q_2)$
(1,1)			
(1,2)			
(2,1)			
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Economies of Scope, cont'd

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Economies of Scope: Ray Average Costs

How to measure economics of scale for multiproduct firms? Need to define appropriate notion of "average costs" for this firm.

- What is AC for a multiproduct firm?
- Not straightforward to answer, except in special cases.
- Assume production of the different products $i = 1, \ldots, N$ in **fixed proportions**, and let these proportions be $\lambda_1, \ldots, \lambda_N$, with $\sum_i \lambda_i = 1$.
- Let $q_1, \ldots, q_N$ denote production of the different products, and $q = q_1 + q_2 + \ldots$.
- Then define $\lambda_i = q_i / q$, the "proportion" of component i in the total production. Note that $q_i = \lambda_i q$.

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Economies of Scope: Ray Average Costs (contd)

- Strict eample: Shoe factory
  q_1 is number of right shoes
  q_2 is number of left shoes
  $\lambda_1 = 0.5, \ \lambda_2 = 0.5$
- More general: take λ as quantity shares of production:
  - schools, restaurants, factory, etc.
- Define: **Ray Average Costs**

$$RAC(q) = \frac{C(\lambda_1 q, \ldots, \lambda_N q)}{q}$$

  RAC considers production combinations along "rays".
- Example: Shoe factory $C(q_1, q_2) = 100 + 5q_1 + 5q_2$,
  - $RAC(q) = \frac{1}{q} * [100 + 5\lambda_1 q + 5\lambda_2 q] = \frac{100+5q}{q}$.
- $RAC'(q)$ determines economies of scale for a multiproduct firm.
- Weakness: fixed proportions only *approximate*

Economies of Scope: Ray Average Costs (contd)

- Strict eample: Shoe factory
  q_1 is number of right shoes
  q_2 is number of left shoes
  $\lambda_1 = 0.5, \; \lambda_2 = 0.5$
- More general: take λ as quantity shares of production:
  - schools, restaurants, factory, etc.
- Define: **Ray Average Costs**

$$RAC(q) = \frac{C(\lambda_1 q, \ldots, \lambda_N q)}{q}$$

  RAC considers production combinations along "rays".
- Example: Shoe factory $C(q_1, q_2) = 100 + 5q_1 + 5q_2$,
  - $RAC(q) = \frac{1}{q} * [100 + 5\lambda_1 q + 5\lambda_2 q] = \frac{100+5q}{q}$.
- $RAC'(q)$ determines economies of scale for a multiproduct firm.
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- Classic example: common inputs (cow yields beef and leather)
- Marketing channels ("umbrella branding": Regular, Honey Nut, and Apple-Cinnamon Cheerios)
- Japanese *keiretsu*; Korean *chaebol*
  - Toyota, Sony, Sumitomo, Mitsubishi
  - Samsung, Hyundai, Kia
- Most major internet companies are involved in many products
  - Are Amazon, Google, Alibaba, Tencent, the "new keiretsu"??
    - retail, transportation, banking and finance, health insurance
  - Economies of scope arise from *information* on consumers?
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- Short run: production technology given
- Long run: can adapt production technology to market conditions
- Long-run AC curve cannot exceed short-run AC curve: its the *lower envelope*
- **Minimum efficient scale** of a firm: smallest output which minimizes *long-run* AC.

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"The division of labor is limited by the extent of the market" (Adam Smith)

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Define: Opportunity cost

- The *opportunity cost* of a product is the value of the best forgone alternative use of the resources employed in making it.
- **Normal profit** of a product is its selling price minus opportunity cost. Quit when normal profit < 0.
- (vs. **accounting profits**: quit when revenue minus *production cost* < 0)
- Example:
  - Car factory: a worker would make \$5 an hour
  - Two brothers, who make one lamp each hour, with \$7 prod cost
  - What is opportunity cost of lamp?
  - Normal profit when market price of lamp is \$11? \$10? \$9?
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Summary

- Define: cost function
- Economies of Scale
- Economies of Scope
- Long-run vs. short-run
- Accounting vs. opportunity cost