Lecture 1: Costs

EC 105. Industrial Organization.

Matt Shum
HSS, California Institute of Technology
Technology and Costs
(Largely) Review: Cost concepts

- Two main components of a firm’s costs:
  1. Fixed costs $F$: cost incurred regardless of output amount.
  2. Variable costs $VC$: vary with the amount produced.

- Examples:
  - Airline: fixed cost is cost of airplane; variable cost are costs of incremental customers
  - Computer factory: fixed cost is cost of setting up factory; variable cost include input costs for each PC produced
  - Starbucks: fixed cost is rent for space; $VC$ are costs of making each cup of coffee (almost zero!)

- Magnitude of fixed vs. variable cost determine the efficient size of a firm.
Two main components of a firm’s costs:

1. Fixed costs $F$: cost incurred regardless of output amount.
2. Variable costs $VC$: vary with the amount produced.

Examples:

- Airline: fixed cost is cost of airplane; variable cost are costs of incremental customers
- Computer factory: fixed cost is cost of setting up factory; variable cost include input costs for each PC produced
- Starbucks: fixed cost is rent for space; VC are costs of making each cup of coffee (almost zero!)

Magnitude of fixed vs. variable cost determine the efficient size of a firm.
Two main components of a firm’s costs:
1. Fixed costs $F$: cost incurred regardless of output amount.
2. Variable costs $VC$: vary with the amount produced.

Examples:
- Airline: fixed cost is cost of airplane; variable cost are costs of incremental customers
- Computer factory: fixed cost is cost of setting up factory; variable cost include input costs for each PC produced
- Starbucks: fixed cost is rent for space; VC are costs of making each cup of coffee (almost zero!)

Magnitude of fixed vs. variable cost determine the efficient size of a firm.
Two main components of a firm’s costs:
1. Fixed costs $F$: cost incurred regardless of output amount.
2. Variable costs $VC$: vary with the amount produced.

Examples:
- Airline: fixed cost is cost of airplane; variable cost are costs of incremental customers
- Computer factory: fixed cost is cost of setting up factory; variable cost include input costs for each PC produced
- Starbucks: fixed cost is rent for space; VC are costs of making each cup of coffee (almost zero!)

Magnitude of fixed vs. variable cost determine the efficient size of a firm.
The Cost Function

- Cost function $C(q)$: **minimum** cost of producing a given quantity $q$
- $C(q) = F + VC(q)$, where
  - Fixed costs $F$: cost incurred regardless of output amount.
    - Avoidable vs. sunk: crucial for determining shut-down decisions for the firm.
  - Variable costs $VC(q)$; vary with the amount produced.
- Average cost $AC(q) = \frac{C(q)}{q}$
- Marginal cost $MC(q) = \frac{\partial C(q)}{\partial q}$
- $AVC(q) = \frac{VC(q)}{q}; AFC(q) = \frac{F}{q}; AC(q) = AVC(q) + AFC(q)$. 
The Cost Function

- Cost function \( C(q) \): **minimum** cost of producing a given quantity \( q \)
- \( C(q) = F + VC(q) \), where
  - Fixed costs \( F \): cost incurred regardless of output amount.
    - Avoidable vs. sunk: crucial for determining shut-down decisions for the firm.
  - Variable costs \( VC(q) \); vary with the amount produced.
  - Average cost \( AC(q) = \frac{C(q)}{q} \)
  - Marginal cost \( MC(q) = \frac{\partial C(q)}{\partial q} \)
  - \( AVC(q) = \frac{VC(q)}{q} \); \( AFC(q) = \frac{F}{q} \); \( AC(q) = AVC(q) + AFC(q) \).
The Cost Function

- **Cost function** \( C(q) \): **minimum** cost of producing a given quantity \( q \)

\[ C(q) = F + VC(q), \text{ where} \]
  - **Fixed costs** \( F \): cost incurred regardless of output amount.
    - Avoidable vs. sunk: crucial for determining shut-down decisions for the firm.
  - **Variable costs** \( VC(q) \); vary with the amount produced.

- **Average cost** \( AC(q) = \frac{C(q)}{q} \)

- **Marginal cost** \( MC(q) = \frac{\partial C(q)}{\partial q} \)

- **AVC(q) = \frac{VC(q)}{q}; AFC(q) = \frac{F}{q}; AC(q) = AVC(q) + AFC(q).**
The Cost Function

- **Cost function** $C(q)$: **minimum** cost of producing a given quantity $q$
- $C(q) = F + VC(q)$, where
  - Fixed costs $F$: cost incurred regardless of output amount.
    - Avoidable vs. sunk: crucial for determining shut-down decisions for the firm.
  - Variable costs $VC(q)$; vary with the amount produced.
- Average cost $AC(q) = \frac{C(q)}{q}$
- Marginal cost $MC(q) = \frac{\partial C(q)}{\partial q}$
- $AVC(q) = \frac{VC(q)}{q}$; $AFC(q) = \frac{F}{q}$; $AC(q) = AVC(q) + AFC(q)$. 
The Cost Function

- Cost function $C(q)$: **minimum** cost of producing a given quantity $q$
- $C(q) = F + VC(q)$, where
  - Fixed costs $F$: cost incurred regardless of output amount.
    - Avoidable vs. sunk: crucial for determining shut-down decisions for the firm.
  - Variable costs $VC(q)$; vary with the amount produced.
- Average cost $AC(q) = \frac{C(q)}{q}$
- Marginal cost $MC(q) = \frac{\partial C(q)}{\partial q}$
- $AVC(q) = \frac{VC(q)}{q}$; $AFC(q) = \frac{F}{q}$; $AC(q) = AVC(q) + AFC(q)$. 
Cost function $C(q)$: minimum cost of producing a given quantity $q$

$C(q) = F + VC(q)$, where

- Fixed costs $F$: cost incurred regardless of output amount. Avoidable vs. sunk: crucial for determining shut-down decisions for the firm.
- Variable costs $VC(q)$; vary with the amount produced.
- Average cost $AC(q) = \frac{C(q)}{q}$
- Marginal cost $MC(q) = \frac{\partial C(q)}{\partial q}$

$AVC(q) = \frac{VC(q)}{q}; AFC(q) = \frac{F}{q}; AC(q) = AVC(q) + AFC(q)$. 


The Cost Function

- Cost function \( C(q) \): **minimum** cost of producing a given quantity \( q \)
- \( C(q) = F + VC(q) \), where
  - Fixed costs \( F \): cost incurred regardless of output amount.
    - Avoidable vs. sunk: crucial for determining shut-down decisions for the firm.
  - Variable costs \( VC(q) \); vary with the amount produced.
- Average cost \( AC(q) = \frac{C(q)}{q} \)
- Marginal cost \( MC(q) = \frac{\partial C(q)}{\partial q} \)
- \( AVC(q) = \frac{VC(q)}{q} \); \( AFC(q) = \frac{F}{q} \); \( AC(q) = AVC(q) + AFC(q) \).
Example

- \( C(q) = 125 + 5q + 5q^2 \)
- \( AC(q) = \)
- \( MC(q) = \)
- \( AFC(q) = 125/q; \quad AVC(q) = 5 + 5q \)

<table>
<thead>
<tr>
<th>q</th>
<th>AC(q)</th>
<th>MC(q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>135</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>61.67</td>
<td>35</td>
</tr>
<tr>
<td>5</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>7</td>
<td>57.86</td>
<td>75</td>
</tr>
<tr>
<td>9</td>
<td>63.89</td>
<td>95</td>
</tr>
</tbody>
</table>

- AC rises if MC exceeds it, and falls if MC is below it. Implies that MC intersects AC at the minimum of AC.
**Example**

- \( C(q) = 125 + 5q + 5q^2 \)
- \( AC(q) = \)
- \( MC(q) = \)
- \( AFC(q) = 125/q; \quad AVC(q) = 5 + 5q \)

<table>
<thead>
<tr>
<th>q</th>
<th>AC(q)</th>
<th>MC(q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>135</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>61.67</td>
<td>35</td>
</tr>
<tr>
<td>5</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>7</td>
<td>57.86</td>
<td>75</td>
</tr>
<tr>
<td>9</td>
<td>63.89</td>
<td>95</td>
</tr>
</tbody>
</table>

AC rises if MC exceeds it, and falls if MC is below it. Implies that MC intersects AC at the minimum of AC.
Example

- $C(q) = 125 + 5q + 5q^2$
- $AC(q) =$
- $MC(q) =$
- $AFC(q) = 125/q; \quad AVC(q) = 5 + 5q$

<table>
<thead>
<tr>
<th>q</th>
<th>AC(q)</th>
<th>MC(q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>135</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>61.67</td>
<td>35</td>
</tr>
<tr>
<td>5</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>7</td>
<td>57.86</td>
<td>75</td>
</tr>
<tr>
<td>9</td>
<td>63.89</td>
<td>95</td>
</tr>
</tbody>
</table>

- AC rises if MC exceeds it, and falls if MC is below it. Implies that MC intersects AC at the minimum of AC.
Example

- \( C(q) = 125 + 5q + 5q^2 \)
- \( AC(q) = \)
- \( MC(q) = \)
- \( AFC(q) = \frac{125}{q}; \quad AVC(q) = 5 + 5q \)

<table>
<thead>
<tr>
<th>q</th>
<th>AC(q)</th>
<th>MC(q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>135</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>61.67</td>
<td>35</td>
</tr>
<tr>
<td>5</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>7</td>
<td>57.86</td>
<td>75</td>
</tr>
<tr>
<td>9</td>
<td>63.89</td>
<td>95</td>
</tr>
</tbody>
</table>

AC rises if MC exceeds it, and falls if MC is below it. Implies that MC intersects AC at the minimum of AC.
Example

- \( C(q) = 125 + 5q + 5q^2 \)
- \( AC(q) = \)
- \( MC(q) = \)
- \( AFC(q) = \frac{125}{q}; \quad AVC(q) = 5 + 5q \)

<table>
<thead>
<tr>
<th>q</th>
<th>AC(q)</th>
<th>MC(q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>135</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>61.67</td>
<td>35</td>
</tr>
<tr>
<td>5</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>7</td>
<td>57.86</td>
<td>75</td>
</tr>
<tr>
<td>9</td>
<td>63.89</td>
<td>95</td>
</tr>
</tbody>
</table>

AC rises if MC exceeds it, and falls if MC is below it. Implies that MC intersects AC at the minimum of AC.
Traditional vs. internet firms

- Traditional companies vs. internet companies can have very different cost structures:
  - Compare to traditional products, some internet versions have *lower fixed costs* but *higher variable costs*
- Ex: Restaurants
  - Traditional restaurant: FC (costs of renting space; hiring workers) >> VC (cost of ingredients for each dish)
  - Food delivery service: FC (no need for physical restaurant) << (?) VC (costs for delivery)
- Ex: Internet retailers (books, electronics, clothing, etc.)
  - Lower overhead costs, but *high shipping costs*
- As a result, internet companies actively try to reduce VC
  - automation based on machine learning in warehouses;
  - using drones for delivery
Traditional companies vs. internet companies can have very different cost structures:

- Compare to traditional products, some internet versions have lower fixed costs but higher variable costs.
- Ex: Restaurants
  - Traditional restaurant: FC (costs of renting space; hiring workers) >> VC (cost of ingredients for each dish)
  - Food delivery service: FC (no need for physical restaurant) << (?) VC (costs for delivery)
- Ex: Internet retailers (books, electronics, clothing, etc.)
  - Lower overhead costs, but high shipping costs
- As a result, internet companies actively try to reduce VC
  - automation based on machine learning in warehouses;
  - using drones for delivery
Traditional vs. internet firms

- Traditional companies vs. internet companies can have very different cost structures:

- Compare to traditional products, some internet versions have lower fixed costs but higher variable costs

- Ex: Restaurants
  - Traditional restaurant: FC (costs of renting space; hiring workers) >> VC (cost of ingredients for each dish)
  - Food delivery service: FC (no need for physical restaurant) << (?) VC (costs for delivery)

- Ex: Internet retailers (books, electronics, clothing, etc.)
  - Lower overhead costs, but high shipping costs

- As a result, internet companies actively try to reduce VC
  - automation based on machine learning in warehouses;
  - using drones for delivery
Traditional vs. internet firms

- Traditional companies vs. internet companies can have very different cost structures:
- Compare to traditional products, some internet versions have lower fixed costs but higher variable costs
- Ex: Restaurants
  - Traditional restaurant: FC (costs of renting space; hiring workers) >> VC (cost of ingredients for each dish)
  - Food delivery service: FC (no need for physical restaurant) <<(?) VC (costs for delivery)
- Ex: Internet retailers (books, electronics, clothing, etc.)
  - Lower overhead costs, but high shipping costs
- As a result, internet companies actively try to reduce VC
  - automation based on machine learning in warehouses;
  - using drones for delivery
Traditional vs. internet firms

- Traditional companies vs. internet companies can have very different cost structures:
- Compare to traditional products, some internet versions have lower fixed costs but higher variable costs
- Ex: Restaurants
  - Traditional restaurant: FC (costs of renting space; hiring workers) >> VC (cost of ingredients for each dish)
  - Food delivery service: FC (no need for physical restaurant) <<(?) VC (costs for delivery)
- Ex: Internet retailers (books, electronics, clothing, etc.)
  - Lower overhead costs, but high shipping costs
- As a result, internet companies actively try to reduce VC
  - automation based on machine learning in warehouses;
  - using drones for delivery
Traditional vs. internet firms

- Traditional companies vs. internet companies can have very different cost structures:
- Compare to traditional products, some internet versions have *lower fixed costs* but *higher variable costs*
- **Ex:** Restaurants
  - Traditional restaurant: FC (costs of renting space; hiring workers) >> VC (cost of ingredients for each dish)
  - Food delivery service: FC (no need for physical restaurant) << (?) VC (costs for delivery)
- **Ex:** Internet retailers (books, electronics, clothing, etc.)
  - Lower overhead costs, but *high shipping costs*
- As a result, internet companies actively try to reduce VC
  - automation based on machine learning in warehouses;
  - using drones for delivery
Traditional vs. internet firms

- Traditional companies vs. internet companies can have very different cost structures:

- Compare to traditional products, some internet versions have *lower fixed costs* but *higher variable costs*

- **Ex:** Restaurants
  - Traditional restaurant: FC (costs of renting space; hiring workers) >> VC (cost of ingredients for each dish)
  - Food delivery service: FC (no need for physical restaurant) <<(?) VC (costs for delivery)

- **Ex:** Internet retailers (books, electronics, clothing, etc.)
  - Lower overhead costs, but *high shipping costs*

- As a result, internet companies actively try to reduce VC
  - automation based on machine learning in warehouses;
  - using drones for delivery
Traditional products vs. internet: cont’d

- Other internet ventures dominate traditional in both FC and VC
  - Ex: movies
    - Traditional movie theater: FC >> VC
    - Online movie streaming: very low FC and VC
  - Ex: music streaming, eBooks, etc.
- In these cases, however, traditional versions still surviving!
  - Traditional and internet versions are becoming differentiated products
    - Watching movie at home (with friends) vs. in theater (“date”)
  - can profitably coexist
Traditional products vs. internet: cont’d

- Other internet ventures dominate traditional in both FC and VC
- Ex: movies
  - Traditional movie theater: FC >> VC
  - Online movie streaming: very low FC and VC
- Ex: music streaming, eBooks, etc.
- In these cases, however, traditional versions still surviving!
  - Traditional and internet versions are becoming differentiated products
    - Watching movie at home (with friends) vs. in theater ("date")
  - can profitably coexist
Traditional products vs. internet: cont’d

- Other internet ventures dominate traditional in both FC and VC
- Ex: movies
  - Traditional movie theater: FC >> VC
  - Online movie streaming: very low FC and VC
- Ex: music streaming, eBooks, etc.
  - In these cases, however, traditional versions still surviving!
    - Traditional and internet versions are becoming differentiated products
      - Watching movie at home (with friends) vs. in theater (“date”)
    - can profitably coexist
Traditional products vs. internet: cont’d

- Other internet ventures dominate traditional in both FC and VC
- Ex: movies
  - Traditional movie theater: FC >> VC
  - Online movie streaming: very low FC and VC
- Ex: music streaming, eBooks, etc.
- In these cases, however, traditional versions still surviving!
  - Traditional and internet versions are becoming differentiated products
    - Watching movie at home (with friends) vs. in theater (“date”)
  - can profitably coexist
Traditional products vs. internet: cont’d

- Other internet ventures dominate traditional in both FC and VC
- Ex: movies
  - Traditional movie theater: FC >> VC
  - Online movie streaming: very low FC and VC
- Ex: music streaming, eBooks, etc.
- In these cases, however, traditional versions still surviving!
  - Traditional and internet versions are becoming differentiated products
    - Watching movie at home (with friends) vs. in theater (‘‘date’’)
    - can profitably coexist
Other internet ventures dominate traditional in both FC and VC

Ex: movies
- Traditional movie theater: FC >> VC
- Online movie streaming: very low FC and VC

Ex: music streaming, eBooks, etc.

In these cases, however, traditional versions still surviving!
- Traditional and internet versions are becoming differentiated products
  - Watching movie at home (with friends) vs. in theater (“date”)
- can profitably coexist
Economies of scale

- Magnitude of FC and VC determine the best size for a firm.
- When FC high relative to VC, there are Economies of scale:
  - Larger firms are more efficient, bc they produce at lower avg costs:
    - $AC'(q) < 0$: increasing returns to scale
    - $AC'(q) > 0$: decreasing returns to scale
    - $AC'(q) = 0$: constant returns to scale
  - Example: U-shaped AC curve
- **Minimum Efficient Scale (MES):** level of production where AC is minimized. At minimum of AC curve (more later).
- Factors affecting scale economies:
  - Fixed costs
  - Congestion
  - specialization: “division of labor limited by the extent of the market”
Economies of scale

- Magnitude of FC and VC determine the best size for a firm.
- When FC high relative to VC, there are *Economies of scale*:
- Larger firms are more *efficient*, bc they *produce at lower avg costs*:
  - $AC'(q) < 0$: increasing returns to scale
  - $AC'(q) > 0$: decreasing returns to scale
  - $AC'(q) = 0$: constant returns to scale
  - Example: U-shaped AC curve
- **Minimum Efficient Scale (MES):** level of production where AC is minimized. At minimum of AC curve (more later).
- Factors affecting scale economies:
  - Fixed costs
  - Congestion
  - specialization: “division of labor limited by the extent of the market”
Economies of scale

- Magnitude of FC and VC determine the best size for a firm.
- When FC high relative to VC, there are Economies of scale:
- Larger firms are more efficient, bc they produce at lower avg costs:
  - $AC'(q) < 0$: increasing returns to scale
  - $AC'(q) > 0$: decreasing returns to scale
  - $AC'(q) = 0$: constant returns to scale
  - Example: U-shaped AC curve

- Minimum Efficient Scale (MES): level of production where AC is minimized. At minimum of AC curve (more later).
- Factors affecting scale economies:
  - Fixed costs
  - Congestion
  - specialization: “division of labor limited by the extent of the market”
Economies of scale

- Magnitude of FC and VC determine the best size for a firm.
- When FC high relative to VC, there are *Economies of scale*:
- Larger firms are more *efficient*, bc they *produce at lower avg costs*:
  - $AC'(q) < 0$: increasing returns to scale
  - $AC'(q) > 0$: decreasing returns to scale
  - $AC'(q) = 0$: constant returns to scale
  - Example: U-shaped AC curve

- **Minimum Efficient Scale (MES):** level of production where AC is minimized. At minimum of AC curve (more later).

- Factors affecting scale economies:
  - Fixed costs
  - Congestion
  - specialization: “division of labor limited by the extent of the market”
Economies of scale

- Magnitude of FC and VC determine the best size for a firm.
- When FC high relative to VC, there are *Economies of scale*:
  - Larger firms are more *efficient*, bc they *produce at lower avg costs*:
    - \( AC'(q) < 0 \): increasing returns to scale
    - \( AC'(q) > 0 \): decreasing returns to scale
    - \( AC'(q) = 0 \): constant returns to scale
    - Example: U-shaped AC curve
- **Minimum Efficient Scale (MES)**: level of production where AC is minimized. At minimum of AC curve (more later).
- Factors affecting scale economies:
  - Fixed costs
  - Congestion
  - specialization: “division of labor limited by the extent of the market”
Multiproduct firms: Economies of Scope

- **Subadditive** cost function:

\[ C(q_1, q_2) < C_1(q_1) + C_2(q_2) \]

- Example 1: common fixed costs
  - Ricardo: rancher produces beef & leather
  - "joint production"
  - Leads to *global* EOS

\[ C_1(q_1) = 10 + 2q_1; \quad C_2(q_2) = 10 + 3q_2; \quad C(q_1, q_2) = 10 + 2q_1 + 3q_2 \]
Multiproduct firms: Economies of Scope

- **Subadditive cost function:**
  \[ C(q_1, q_2) < C_1(q_1) + C_2(q_2) \]

Example 1: common fixed costs
- Ricardo: rancher produces beef & leather
- “joint production”
- Leads to *global* EOS

\[ C_1(q_1) = 10 + 2q_1; \quad C_2(q_2) = 10 + 3q_2; \quad C(q_1, q_2) = 10 + 2q_1 + 3q_2 \]
Multiproduct firms: Economies of Scope

- **Subadditive** cost function:

\[ C(q_1, q_2) < C_1(q_1) + C_2(q_2) \]

- Example 1: common fixed costs
  - **Ricardo**: rancher produces beef & leather
  - “joint production”
  - Leads to *global* EOS

\[
C_1(q_1) = 10 + 2q_1; \quad C_2(q_2) = 10 + 3q_2; \quad C(q_1, q_2) = 10 + 2q_1 + 3q_2
\]
Multiproduct firms: Economies of Scope

- **Subadditive** cost function:

\[ C(q_1, q_2) < C_1(q_1) + C_2(q_2) \]

- Example 1: common fixed costs
  - **Ricardo**: rancher produces beef & leather
  - “joint production”
  - Leads to **global** EOS

\[ C_1(q_1) = 10 + 2q_1; \quad C_2(q_2) = 10 + 3q_2; \quad C(q_1, q_2) = 10 + 2q_1 + 3q_2 \]
More often, EOS is local & depends on levels of $q_1$, $q_2$:

\[
C_1(q_1) = 5 + 2q_1; \quad C_2(q_2) = 5 + 3q_2; \quad C(q_1, q_2) = 10 + 3q_1 + 2q_2
\]

<table>
<thead>
<tr>
<th>$(q_1, q_2)$</th>
<th>$C_1(q_1)$</th>
<th>$C_2(q_2)$</th>
<th>$C(q_1, q_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,2)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Economies of Scope, cont’d

More often, EOS is local & depends on levels of $q_1, q_2$:

\[ C_1(q_1) = 5 + 2q_1; \quad C_2(q_2) = 5 + 3q_2; \quad C(q_1, q_2) = 10 + 3q_1 + 2q_2 \]

<table>
<thead>
<tr>
<th>$(q_1, q_2)$</th>
<th>$C_1(q_1)$</th>
<th>$C_2(q_2)$</th>
<th>$C(q_1, q_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,2)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Economies of Scope: Ray Average Costs

How to measure economics of scale for multiproduct firms? Need to define appropriate notion of “average costs” for this firm.

- What is AC for a multiproduct firm?
- Not straightforward to answer, except in special cases.
- Assume production of the different products $i = 1, \ldots, N$ in fixed proportions, and let these proportions be $\lambda_1, \ldots, \lambda_N$, with $\sum_i \lambda_i = 1$.
- Let $q_1, \ldots, q_N$ denote production of the different products, and $q = q_1 + q_2 + \ldots$.
- Then define $\lambda_i = q_i / q$, the “proportion” of component $i$ in the total production. Note that $q_i = \lambda_i q$. 
How to measure economics of scale for multiproduct firms? Need to define appropriate notion of “average costs” for this firm.

- What is AC for a multiproduct firm?
- Not straightforward to answer, except in special cases.
- Assume production of the different products \( i = 1, \ldots, N \) in **fixed proportions**, and let these proportions be \( \lambda_1, \ldots, \lambda_N \), with \( \sum_i \lambda_i = 1 \).
- Let \( q_1, \ldots, q_N \) denote production of the different products, and \( q = q_1 + q_2 + \ldots \).
- Then define \( \lambda_i = q_i / q \), the “proportion” of component \( i \) in the total production. Note that \( q_i = \lambda_i q \).
Economies of Scope: Ray Average Costs

How to measure economics of scale for multiproduct firms? Need to define appropriate notion of “average costs” for this firm.

- What is AC for a multiproduct firm?
- Not straightforward to answer, except in special cases.
- Assume production of the different products $i = 1, \ldots, N$ in fixed proportions, and let these proportions be $\lambda_1, \ldots, \lambda_N$, with $\sum_i \lambda_i = 1$.
- Let $q_1, \ldots, q_N$ denote production of the different products, and $q = q_1 + q_2 + \ldots$.
- Then define $\lambda_i = q_i / q$, the “proportion” of component $i$ in the total production. Note that $q_i = \lambda_i q$. 
How to measure economics of scale for multiproduct firms? Need to define appropriate notion of “average costs” for this firm.

- What is AC for a multiproduct firm?
- Not straightforward to answer, except in special cases.
- Assume production of the different products $i = 1, \ldots, N$ in fixed proportions, and let these proportions be $\lambda_1, \ldots, \lambda_N$, with $\sum_i \lambda_i = 1$.
- Let $q_1, \ldots, q_N$ denote production of the different products, and $q = q_1 + q_2 + \ldots$.
- Then define $\lambda_i = q_i/q$, the “proportion” of component $i$ in the total production. Note that $q_i = \lambda_i q$. 
Economies of Scope: Ray Average Costs

How to measure economics of scale for multiproduct firms? Need to define appropriate notion of “average costs” for this firm.

- What is AC for a multiproduct firm?
- Not straightforward to answer, except in special cases.
- Assume production of the different products \( i = 1, \ldots, N \) in fixed proportions, and let these proportions be \( \lambda_1, \ldots, \lambda_N \), with \( \sum_i \lambda_i = 1 \).
- Let \( q_1, \ldots, q_N \) denote production of the different products, and \( q = q_1 + q_2 + \ldots \).
- Then define \( \lambda_i = q_i/q \), the “proportion” of component \( i \) in the total production. Note that \( q_i = \lambda_i q \).
Economies of Scope: Ray Average Costs

How to measure economics of scale for multiproduct firms? Need to define appropriate notion of “average costs” for this firm.

- What is AC for a multiproduct firm?
- Not straightforward to answer, except in special cases.
- Assume production of the different products \( i = 1, \ldots, N \) in **fixed proportions**, and let these proportions be \( \lambda_1, \ldots, \lambda_N \), with \( \sum_i \lambda_i = 1 \).
- Let \( q_1, \ldots, q_N \) denote production of the different products, and \( q = q_1 + q_2 + \ldots \).
- Then define \( \lambda_i = q_i/q \), the “proportion” of component \( i \) in the total production. Note that \( q_i = \lambda_i q \).
Economies of Scope: Ray Average Costs (contd)

- **Strict example: Shoe factory**
  - $q_1$ is number of right shoes
  - $q_2$ is number of left shoes
  - $\lambda_1 = 0.5$, $\lambda_2 = 0.5$

- **More general:** take $\lambda$ as quantity shares of production:
  - schools, restaurants, factory, etc.

- **Define: Ray Average Costs**

$$RAC(q) = \frac{C(\lambda_1 q, \ldots, \lambda_N q)}{q}$$

RAC considers production combinations along “rays”.

- **Example:** Shoe factory $C(q_1, q_2) = 100 + 5q_1 + 5q_2$,
  - $RAC(q) = \frac{1}{q} \times [100 + 5\lambda_1 q + 5\lambda_2 q] = \frac{100+5q}{q}$.

- **$RAC'(q)$** determines economies of scale for a multiproduct firm.

- **Weakness:** fixed proportions only *approximate*
Economies of Scope: Ray Average Costs (contd)

- **Strict example**: Shoe factory
  - \( q_1 \) is number of right shoes
  - \( q_2 \) is number of left shoes
  - \( \lambda_1 = 0.5 \), \( \lambda_2 = 0.5 \)
- **More general**: take \( \lambda \) as quantity shares of production:
  - schools, restaurants, factory, etc.
- **Define**: Ray Average Costs

\[
RAC(q) = \frac{C(\lambda_1 q, \ldots, \lambda_N q)}{q}
\]

RAC considers production combinations along “rays”.

- **Example**: Shoe factory \( C(q_1, q_2) = 100 + 5q_1 + 5q_2 \),
  - \( RAC(q) = \frac{1}{q} \times [100 + 5\lambda_1 q + 5\lambda_2 q] = \frac{100+5q}{q} \).
- \( RAC'(q) \) determines economies of scale for a multiproduct firm.
- **Weakness**: fixed proportions only approximate
Economies of Scope: Ray Average Costs (contd)

- **Strict example:** Shoe factory
  - $q_1$ is number of right shoes
  - $q_2$ is number of left shoes
  - $\lambda_1 = 0.5$, $\lambda_2 = 0.5$

- More general: take $\lambda$ as quantity shares of production:
  - schools, restaurants, factory, etc.

- **Define:** _Ray Average Costs_

  \[
  RAC(q) = \frac{C(\lambda_1 q, \ldots, \lambda_N q)}{q}
  \]

  $RAC$ considers production combinations along “rays”.

- **Example:** Shoe factory $C(q_1, q_2) = 100 + 5q_1 + 5q_2$,
  - $RAC(q) = \frac{1}{q} \times [100 + 5\lambda_1 q + 5\lambda_2 q] = \frac{100 + 5q}{q}$.

- $RAC'(q)$ determines economies of scale for a multiproduct firm.

- Weakness: fixed proportions only _approximate_
Economies of Scope: Ray Average Costs (contd)

- **Strict example:** Shoe factory
  - $q_1$ is number of right shoes
  - $q_2$ is number of left shoes
  - $\lambda_1 = 0.5$, $\lambda_2 = 0.5$

- **More general:** take $\lambda$ as quantity shares of production:
  - schools, restaurants, factory, etc.

- **Define:** Ray Average Costs

\[
RAC(q) = \frac{C(\lambda_1 q, \ldots, \lambda_N q)}{q}
\]

RAC considers production combinations along “rays”.

- **Example:** Shoe factory $C(q_1, q_2) = 100 + 5q_1 + 5q_2$,
  
  
  \[
  RAC(q) = \frac{1}{q} \times [100 + 5\lambda_1 q + 5\lambda_2 q] = \frac{100 + 5q}{q}.
  \]

- **$RAC'(q)$** determines economies of scale for a multiproduct firm.

- **Weakness:** fixed proportions only *approximate*
Economies of Scope: Ray Average Costs (contd)

- **Strict example:** Shoe factory
  - $q_1$ is number of right shoes
  - $q_2$ is number of left shoes
  - $\lambda_1 = 0.5$, $\lambda_2 = 0.5$
- **More general:** take $\lambda$ as quantity shares of production:
  - schools, restaurants, factory, etc.
- **Define:** **Ray Average Costs**

  $$RAC(q) = \frac{C(\lambda_1 q, \ldots, \lambda_N q)}{q}$$

  RAC considers production combinations along “rays”.
- **Example:** Shoe factory $C(q_1, q_2) = 100 + 5q_1 + 5q_2$,
  - $RAC(q) = \frac{1}{q} \times [100 + 5\lambda_1 q + 5\lambda_2 q] = \frac{100 + 5q}{q}$.
  - $RAC'(q)$ determines economies of scale for a multiproduct firm.
- **Weakness:** fixed proportions only *approximate*
Economies of Scope: Ray Average Costs (contd)

- **Strict example:** Shoe factory
  - $q_1$ is number of right shoes
  - $q_2$ is number of left shoes
  - $\lambda_1 = 0.5, \lambda_2 = 0.5$

- **More general:** take $\lambda$ as quantity shares of production:
  - schools, restaurants, factory, etc.

- **Define:** **Ray Average Costs**

  $$RAC(q) = \frac{C(\lambda_1 q, \ldots, \lambda_N q)}{q}$$

  RAC considers production combinations along “rays”.

- **Example:** Shoe factory $C(q_1, q_2) = 100 + 5q_1 + 5q_2$,
  - $RAC(q) = \frac{1}{q} \cdot [100 + 5\lambda_1 q + 5\lambda_2 q] = \frac{100+5q}{q}$.

- $RAC'(q)$ determines economies of scale for a multiproduct firm.

- **Weakness:** fixed proportions only *approximate*
Economies of Scope: Ray Average Costs (contd)

- **Strict example: Shoe factory**
  - $q_1$ is number of right shoes
  - $q_2$ is number of left shoes
  - $\lambda_1 = 0.5$, $\lambda_2 = 0.5$

- **More general:** take $\lambda$ as quantity shares of production:
  - schools, restaurants, factory, etc.

- **Define:** **Ray Average Costs**

\[
RAC(q) = \frac{C(\lambda_1 q, \ldots, \lambda_N q)}{q}
\]

$RAC$ considers production combinations along “rays”.

- **Example:** Shoe factory $C(q_1, q_2) = 100 + 5q_1 + 5q_2$,
  - $RAC(q) = \frac{1}{q} \cdot [100 + 5\lambda_1 q + 5\lambda_2 q] = \frac{100 + 5q}{q}$.

- $RAC'(q)$ determines economies of scale for a multiproduct firm.

- **Weakness:** fixed proportions only *approximate*
Reasons for Economies of Scope

- Classic example: common inputs (cow yields beef and leather)
- Marketing channels ("umbrella branding": Regular, Honey Nut, and Apple-Cinnamon Cheerios)
- Japanese *keiretsu*; Korean *chaebol*
  - Toyota, Sony, Sumitomo, Mitsubishi
  - Samsung, Hyundai, Kia
- Most major internet companies are involved in many products
  - Are Amazon, Google, Alibaba, Tencent, the "new keiretsu"??
    - retail, transportation, banking and finance, health insurance
- Economies of scope arise from *information* on consumers?
  - Use consumer shopping information to better price insurance?
  - Hiring armies of machine learning specialists
Reasons for Economies of Scope

- Classic example: common inputs (cow yields beef and leather)
- Marketing channels (“umbrella branding”: Regular, Honey Nut, and Apple-Cinnamon Cheerios)
- Japanese *keiretsu*; Korean *chaebol*
  - Toyota, Sony, Sumitomo, Mitsubishi
  - Samsung, Hyundai, Kia
- Most major internet companies are involved in many products
  - Are Amazon, Google, Alibaba, Tencent, the “new keiretsu”??
    - retail, transportation, banking and finance, health insurance
- Economies of scope arise from *information* on consumers?
  - Use consumer shopping information to better price insurance?
  - Hiring armies of machine learning specialists
Reasons for Economies of Scope

- Classic example: common inputs (cow yields beef and leather)
- Marketing channels (“umbrella branding”: Regular, Honey Nut, and Apple-Cinnamon Cheerios)
- Japanese *keiretsu*; Korean *chaebol*
  - Toyota, Sony, Sumitomo, Mitsubishi
  - Samsung, Hyundai, Kia
- Most major internet companies are involved in many products
  - Are Amazon, Google, Alibaba, Tencent, the “new keiretsu”??
    - retail, transportation, banking and finance, health insurance
- Economies of scope arise from *information* on consumers?
  - Use consumer shopping information to better price insurance?
  - Hiring armies of machine learning specialists
Reasons for Economies of Scope

- Classic example: common inputs (cow yields beef and leather)
- Marketing channels (“umbrella branding”: Regular, Honey Nut, and Apple-Cinnamon Cheerios)
- Japanese *keiretsu*; Korean *chaebol*
  - Toyota, Sony, Sumitomo, Mitsubishi
  - Samsung, Hyundai, Kia
- Most major internet companies are involved in many products
  - Are Amazon, Google, Alibaba, Tencent, the “new keiretsu”??
  - retail, transportation, banking and finance, health insurance
- Economies of scope arise from *information* on consumers?
  - Use consumer shopping information to better price insurance?
  - Hiring armies of machine learning specialists
Reasons for Economies of Scope

- Classic example: common inputs (cow yields beef and leather)
- Marketing channels ("umbrella branding": Regular, Honey Nut, and Apple-Cinnamon Cheerios)
- Japanese *keiretsu*; Korean *chaebol*
  - Toyota, Sony, Sumitomo, Mitsubishi
  - Samsung, Hyundai, Kia
- Most major internet companies are involved in many products
  - Are Amazon, Google, Alibaba, Tencent, the "new keiretsu"??
    - retail, transportation, banking and finance, health insurance
- Economies of scope arise from *information* on consumers?
  - Use consumer shopping information to better price insurance?
  - Hiring armies of machine learning specialists
Reasons for Economies of Scope

- Classic example: common inputs (cow yields beef and leather)
- Marketing channels (“umbrella branding”: Regular, Honey Nut, and Apple-Cinnamon Cheerios)
- Japanese *keiretsu*; Korean *chaebol*
  - Toyota, Sony, Sumitomo, Mitsubishi
  - Samsung, Hyundai, Kia
- Most major internet companies are involved in many products
  - Are Amazon, Google, Alibaba, Tencent, the “new keiretsu”??
    - retail, transportation, banking and finance, health insurance
- Economies of scope arise from information on consumers?
  - Use consumer shopping information to better price insurance?
  - Hiring armies of machine learning specialists
Reasons for Economies of Scope

- Classic example: common inputs (cow yields beef and leather)
- Marketing channels (“umbrella branding”: Regular, Honey Nut, and Apple-Cinnamon Cheerios)
- Japanese *keiretsu*; Korean *chaebol*
  - Toyota, Sony, Sumitomo, Mitsubishi
  - Samsung, Hyundai, Kia
- Most major internet companies are involved in many products
  - Are Amazon, Google, Alibaba, Tencent, the “new keiretsu”??
    - retail, transportation, banking and finance, health insurance
  - Economies of scope arise from *information* on consumers?
    - Use consumer shopping information to better price insurance?
    - Hiring armies of machine learning specialists
Short-run vs. long-run costs:

- Short run: production technology given
- Long run: can adapt production technology to market conditions
- Long-run AC curve cannot exceed short-run AC curve: it's the lower envelope
- Minimum efficient scale of a firm: smallest output which minimizes long-run AC.
Short-run vs. long-run costs:

- **Short run**: production technology given
- **Long run**: can adapt production technology to market conditions
- **Long-run AC curve cannot exceed short-run AC curve**: its the lower envelope
- **Minimum efficient scale** of a firm: smallest output which minimizes long-run AC.
Short-run vs. long-run costs:

- **Short run**: production technology given
- **Long run**: can adapt production technology to market conditions
- **Long-run AC curve cannot exceed short-run AC curve**: it's the *lower envelope*
- **Minimum efficient scale** of a firm: smallest output which minimizes *long-run AC.*
Short-run vs. long-run costs:

- Short run: production technology given
- Long run: can adapt production technology to market conditions
- Long-run AC curve cannot exceed short-run AC curve: it's the lower envelope
- Minimum efficient scale of a firm: smallest output which minimizes long-run AC.
Consider this well-known quote:

“The division of labor is limited by the extent of the market” (Adam Smith)

- Division of labor requires high fixed costs (for example, assembly line requires high setup costs).
- Firm adopts division of labor only when scale of production (market demand) is high enough.
- Graph: Price-taking firm has “choice” between two production technologies.
Consider this well-known quote:

“The division of labor is limited by the extent of the market” (Adam Smith)

- Division of labor requires high fixed costs (for example, assembly line requires high setup costs).
- Firm adopts division of labor only when scale of production (market demand) is high enough.
- Graph: Price-taking firm has “choice” between two production technologies.
Consider this well-known quote:

“The division of labor is limited by the extent of the market” (Adam Smith)

- Division of labor requires high fixed costs (for example, assembly line requires high setup costs).
- Firm adopts division of labor only when scale of production (market demand) is high enough.
- Graph: Price-taking firm has “choice” between two production technologies.
Consider this well-known quote:

“The division of labor is limited by the extent of the market” (Adam Smith)

- Division of labor requires high fixed costs (for example, assembly line requires high setup costs).
- Firm adopts division of labor only when scale of production (market demand) is high enough.
- Graph: Price-taking firm has “choice” between two production technologies.
Define: Opportunity cost

- The *opportunity cost* of a product is the value of the best forgone alternative use of the resources employed in making it.
- Normal profit of a product is its selling price minus opportunity cost. Quit when normal profit < 0.
- (vs. accounting profits: quit when revenue minus *production cost* < 0)
- Example:
  - Car factory: a worker would make $5 an hour
  - Two brothers, who make one lamp each hour, with $7 prod cost
  - What is opportunity cost of lamp?
  - Normal profit when market price of lamp is $11? $10? $9?
  - Accounting profit when market price of lamp is $11? $10? $9?
- Economics as “dismal science”
Define: Opportunity cost

- The *opportunity cost* of a product is the value of the best forgone alternative use of the resources employed in making it.
- **Normal profit** of a product is its selling price minus opportunity cost. Quit when normal profit < 0.
- (vs. **accounting profits**: quit when revenue minus *production cost* < 0)
- Example:
  - Car factory: a worker would make $5 an hour
  - Two brothers, who make one lamp each hour, with $7 prod cost
  - What is opportunity cost of lamp?
  - Normal profit when market price of lamp is $11? $10? $9?
  - Accounting profit when market price of lamp is $11? $10? $9?
- Economics as “dismal science”
Define: Opportunity cost

- The *opportunity cost* of a product is the value of the best forgone alternative use of the resources employed in making it.
- **Normal profit** of a product is its selling price minus opportunity cost. Quit when normal profit < 0.
- (vs. **accounting profits**: quit when revenue minus *production cost* < 0)

Example:
- Car factory: a worker would make $5 an hour
- Two brothers, who make one lamp each hour, with $7 prod cost
- What is opportunity cost of lamp?
- Normal profit when market price of lamp is $11? $10? $9?
- Accounting profit when market price of lamp is $11? $10? $9?

Economics as “dismal science”
Define: Opportunity cost

- The *opportunity cost* of a product is the value of the best forgone alternative use of the resources employed in making it.

- **Normal profit** of a product is its selling price minus opportunity cost. Quit when normal profit < 0.

- *(vs. accounting profits: quit when revenue minus *production cost* < 0)*

- Example:
  - Car factory: a worker would make $5 an hour
  - Two brothers, who make one lamp each hour, with $7 prod cost
    - What is opportunity cost of lamp?
  - Normal profit when market price of lamp is $11? $10? $9?
  - Accounting profit when market price of lamp is $11? $10? $9?

- Economics as “dismal science”
Define: Opportunity cost

- The *opportunity cost* of a product is the value of the best forgone alternative use of the resources employed in making it.
- **Normal profit** of a product is its selling price minus opportunity cost. Quit when normal profit < 0.
- *(vs. accounting profits: quit when revenue minus production cost < 0)*
- Example:
  - Car factory: a worker would make $5 an hour
  - Two brothers, who make one lamp each hour, with $7 prod cost
  - What is opportunity cost of lamp?
    - Normal profit when market price of lamp is $11? $10? $9?
    - Accounting profit when market price of lamp is $11? $10? $9?
  - Economics as “dismal science”
Define: Opportunity cost

- The *opportunity cost* of a product is the value of the best forgone alternative use of the resources employed in making it.

- **Normal profit** of a product is its selling price minus opportunity cost. Quit when normal profit < 0.

  - *(vs. *accounting profits*: quit when revenue minus production cost < 0)*

- **Example:**
  - Car factory: a worker would make $5 an hour
  - Two brothers, who make one lamp each hour, with $7 prod cost
  - What is opportunity cost of lamp?
  - Normal profit when market price of lamp is $11? $10? $9?
  - Accounting profit when market price of lamp is $11? $10? $9?

- Economics as “dismal science”
Define: Opportunity cost

- The *opportunity cost* of a product is the value of the best forgone alternative use of the resources employed in making it.
- **Normal profit** of a product is its selling price minus opportunity cost. Quit when normal profit $< 0$.
- (vs. **accounting profits**: quit when revenue minus *production cost* $< 0$)
- **Example:**
  - Car factory: a worker would make $5 an hour
  - Two brothers, who make one lamp each hour, with $7 prod cost
  - What is opportunity cost of lamp?
  - Normal profit when market price of lamp is $11? $10? $9?  
  - Accounting profit when market price of lamp is $11? $10? $9?  

Economics as “dismal science”
Define: Opportunity cost

- The *opportunity cost* of a product is the value of the best forgone alternative use of the resources employed in making it.
- **Normal profit** of a product is its selling price minus opportunity cost. Quit when normal profit $< 0$.
- (vs. *accounting profits*: quit when revenue minus *production cost* $< 0$)
- Example:
  - Car factory: a worker would make $5 an hour
  - Two brothers, who make one lamp each hour, with $7 prod cost
  - What is opportunity cost of lamp?
  - Normal profit when market price of lamp is $11? $10? $9?
  - Accounting profit when market price of lamp is $11? $10? $9?
- Economics as “dismal science”
Summary

- Define: cost function
- Economies of Scale
- Economies of Scope
- Long-run vs. short-run
- Accounting vs. opportunity cost