

Orbital magnetic field effects in spin liquid with spinon Fermi sea: Possible application to $\kappa\text{-(ET)}_2\text{Cu}_2(\text{CN})_3$

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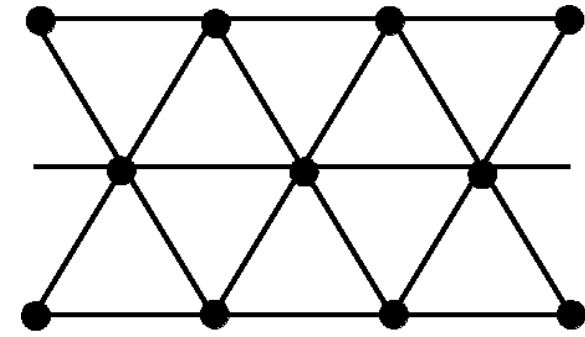
with many thanks to T.Senthil

Outline

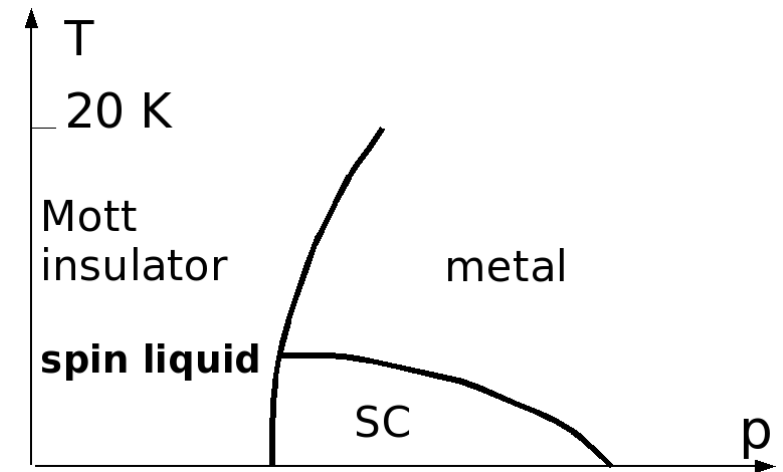
1. $\kappa\text{-(ET)}_2\text{Cu}_2(\text{CN})_3$ - material facts
2. Spinon Fermi sea proposal
 - Microscopic energetics motivation (mean field)
 - Low-energy description
3. Coupling to the external magnetic field
 - Induced internal gauge field
 - Direct Fermi surface probes?
 - Fragility of the spin liquid state
4. Conclusions

Material facts

- Modeled as triangular Hubbard at half-filling McKenzie 98, Imada 02
- Just on the Insulator side
- No magnetic order down to 20mK $\sim 10^{-4}$ J
- Many gapless spin excitations – as many as in a metal with Fermi surface
- Large spin entropy – more than in a metal!



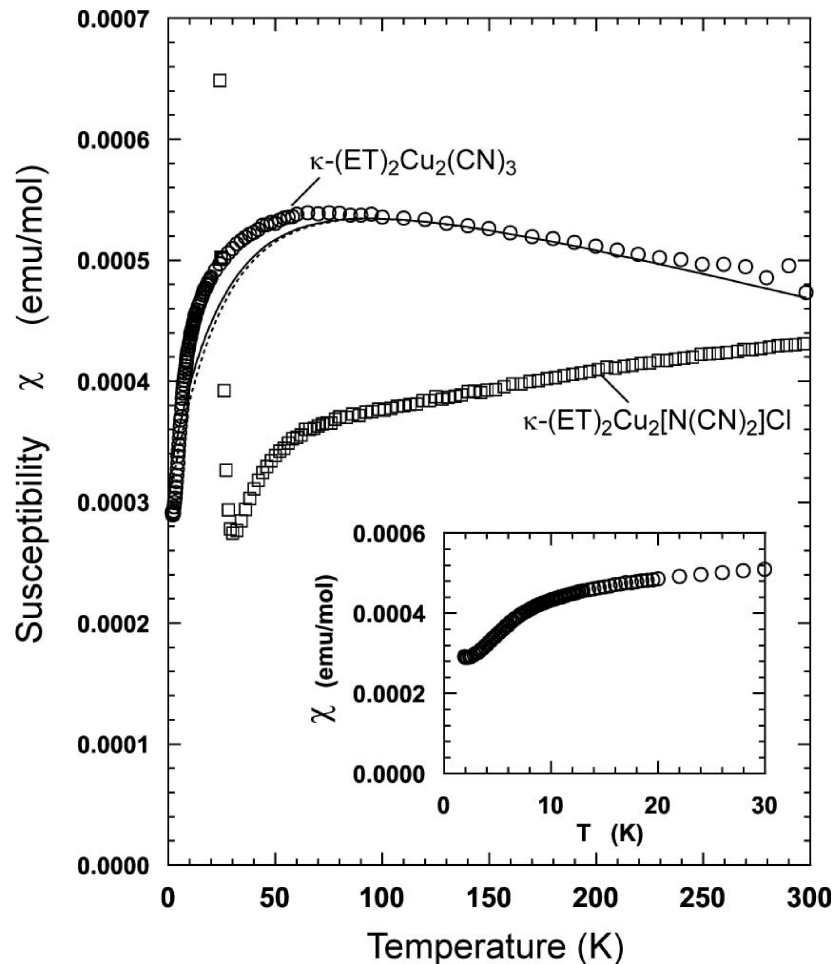
$t = 55\text{meV}$; $U/t = 8$
 $\rightarrow J \sim 250\text{K}$



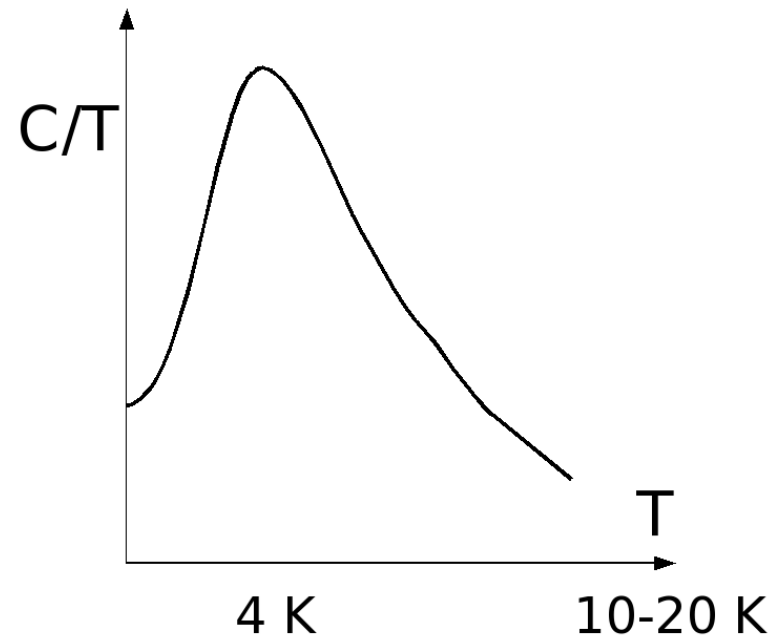
Kurosaki et.al. 05; Shimizu et.al. 03

Material facts

Spin susceptibility



Specific heat



Kanoda, APS March Meeting 2006

Gutzwiller-projected Fermi Sea

$$P_G (|\text{Fermi Sea}\rangle = \prod_{k < k_F} c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger |vac\rangle)$$

$$= a|(\uparrow) (\uparrow) (\downarrow) (\uparrow) (\downarrow) (\uparrow)\rangle + b|(\uparrow\downarrow) (0) (\downarrow) (\uparrow) (\uparrow) (\downarrow)\rangle \\ + c|(\uparrow) (\downarrow) (\downarrow) (\uparrow\downarrow) (\uparrow) (0)\rangle + d|(\downarrow) (\uparrow) (\uparrow) (\downarrow) (\uparrow) (\downarrow)\rangle + \dots$$

real-space configurations

-- insulator wave function (Brinkman-Rice picture of Mott transition)

$$\Psi_{\text{spin}}(\{R \uparrow\}, \{R' \downarrow\}) = \det[R \uparrow] \det[R' \downarrow] (-1)^{p(\{R \uparrow\}, \{R' \downarrow\})}$$

Spinon Fermi sea spin liquid (aka “uniform RVB”)

- Fermi surface of spinons
 - “metallic” spin susceptibility $\chi(T \rightarrow 0) = \text{const}$;
 - metallic $1/(T_1 T)$
- Emergent U(1) gauge field
 - large spin entropy (overdamped “photon”)

$$C_{\text{uRVB}} \sim k_B t_{\text{spinon}}^{1/3} (k_B T)^{2/3}$$
$$> C_{\text{metal}} \sim k_B (k_B T)$$

Microscopic thinking

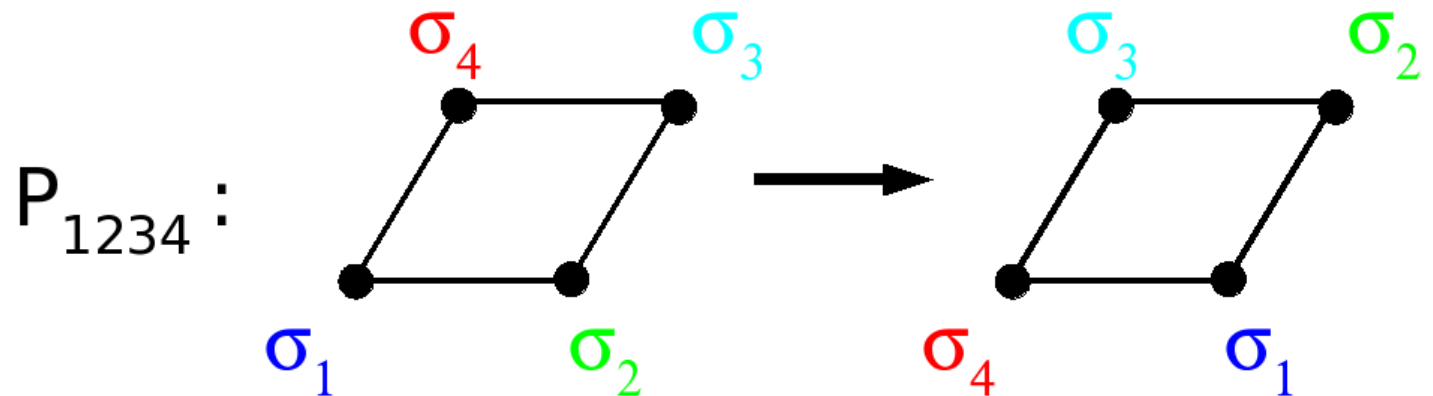
Hubbard model

$$\hat{H}_{\text{Hubbard}} = -t \sum_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Insulator --> effective spin model

$$\hat{H}_{\text{eff}} = \frac{2t^2}{U} \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + \frac{20t^4}{U^3} \sum_{\text{rhombi}} (P_{ijkl} + \text{h.c.}) + \dots$$

Ring exchange:

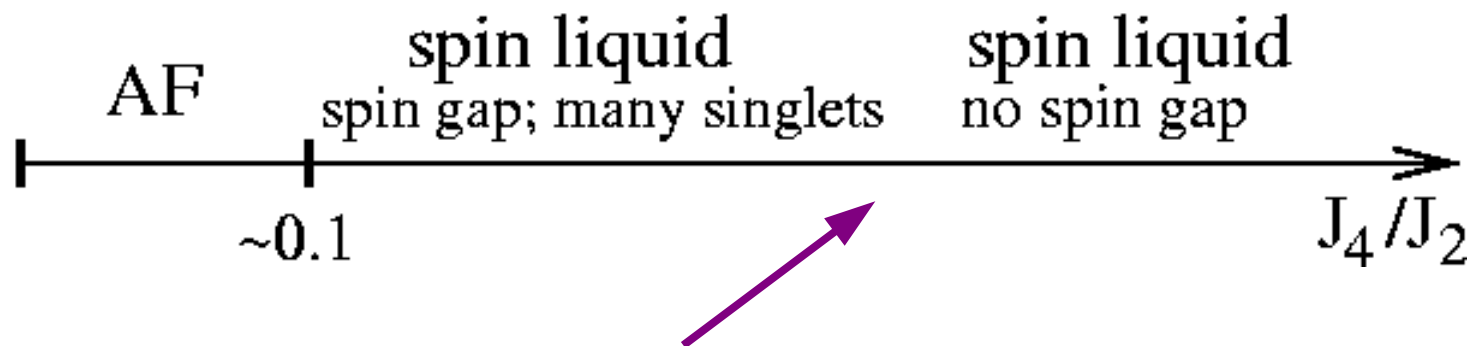


J_2 - J_4 ring exchange model

$$\hat{H}_{\text{ring}} = J_2 \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + J_4 \sum_{\text{rhombi}} (P_{ijkl} + \text{h.c.})$$

$\kappa\text{-(ET)}_2\text{Cu}_2(\text{CN})_3$: $J_4/J_2 \sim 0.3$

Exact diagonalization study: (LiMing et.al. 2000, Misguich et.al. 1999)



Which spin liquid is realized?

(also numerical study of the Hubbard model, Imada et.al. 2002)

Slave-fermion trial spin liquids

Fermionic representation of spin-1/2

$$\mathbf{S}_i = f_i^\dagger \frac{\boldsymbol{\sigma}}{2} f_i; \quad f_{i\alpha}^\dagger f_{i\alpha} = 1;$$

General “Hartree-Fock” in the singlet channel

$$\mathcal{H}_{\text{trial}} = - \sum_{ij} t_{ij} f_{i\alpha}^\dagger f_{j\alpha} - \sum_{ij} \Delta_{ij} (f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger - f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger)$$

$$\longrightarrow |\Psi_0\rangle \longrightarrow |\Psi_{\text{spin}}\rangle = \mathbf{P}_G(|\Psi_0\rangle)$$

free fermions

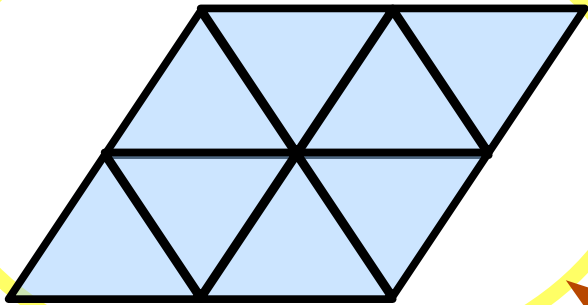
spins

Gutzwiller
projection

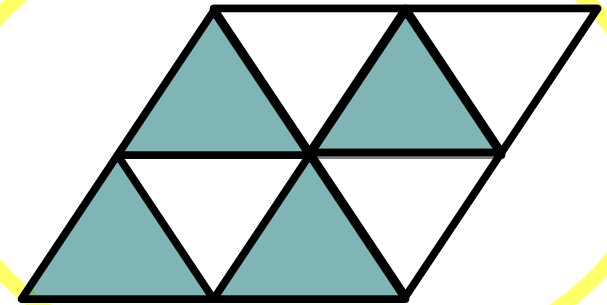
- easy to work with numerically – VMC (Ceperley 77, Gros 89)

Examples of “fermionic” spin liquids

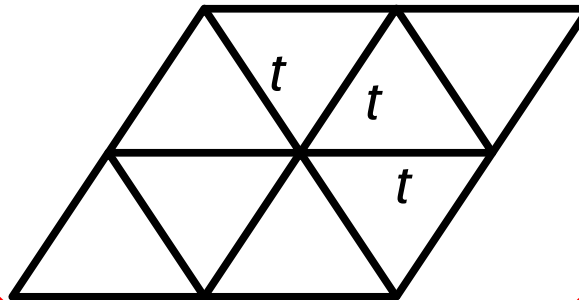
uniform flux



staggered flux

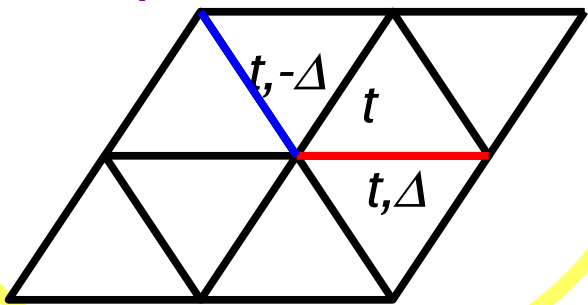


uRVB
real hopping

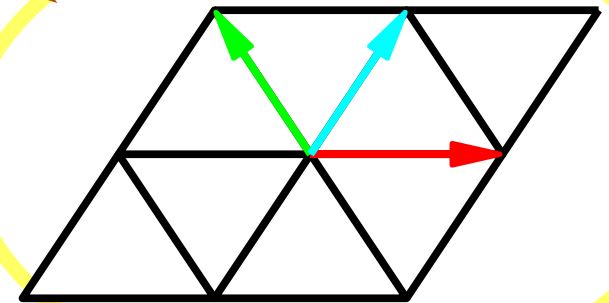


Kalmeyer
-Laughlin

$d_{x^2-y^2}$ Z2 spin liquid



d+id chiral SL

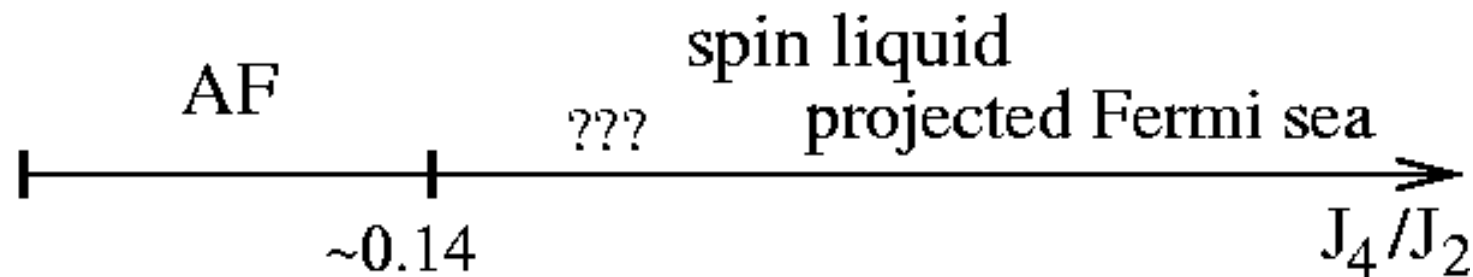


can be all classified!

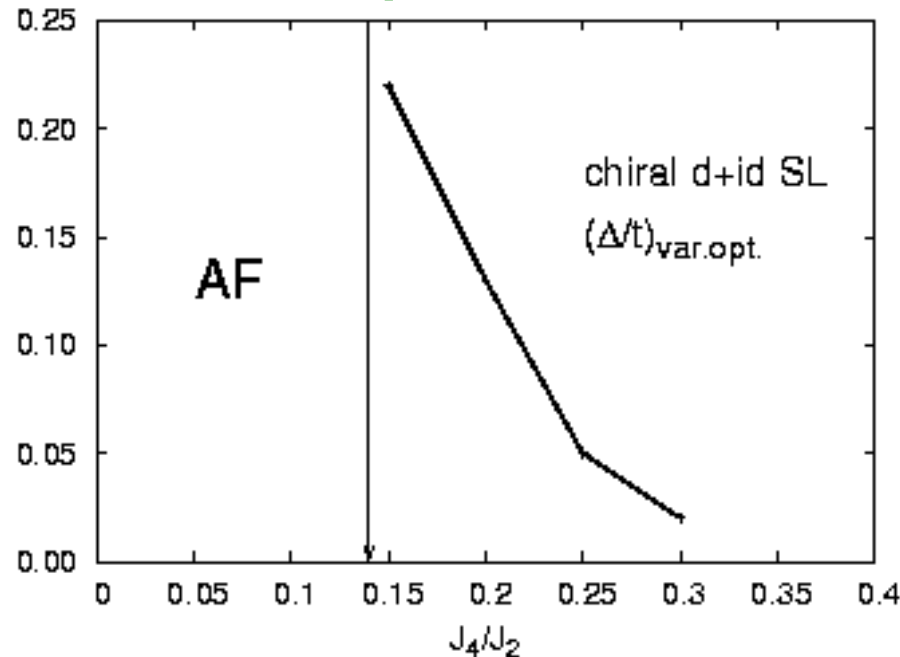
Wen 2001; Zhou and Wen 2002

Variational results: J_2 - J_4 model

“Phase diagram:”

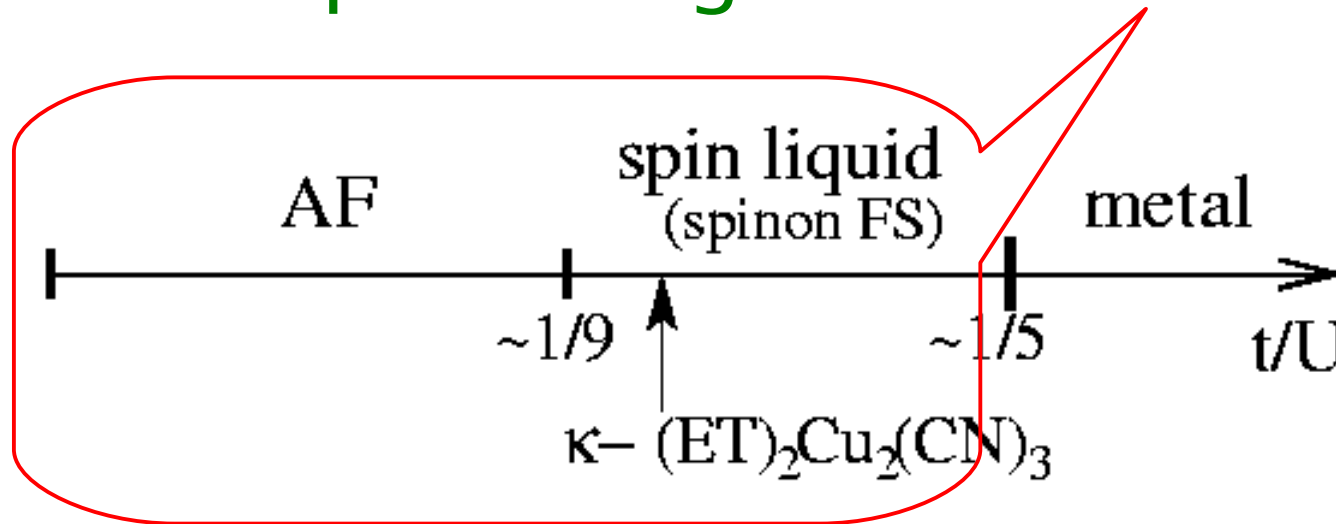


Optimization example:



Hubbard $\rightarrow H_{\text{eff}}$ including all t^4/U^3 terms

“Variational phase diagram” - insulator side



Charge fluctuations are included:

$$\hat{H}_{\text{eff}} = e^{-iS} \hat{H}_{\text{Hubbard}} e^{iS}$$

Unitary transformation that systematically separates sectors controlled by U

$\rightarrow |\Psi_{\text{electrons}}\rangle = e^{-iS} |\Psi_{\text{spins}}\rangle$

“rotate back” to recover electron wave function

Mean field guide

Ring exchanges in the fermionic representation

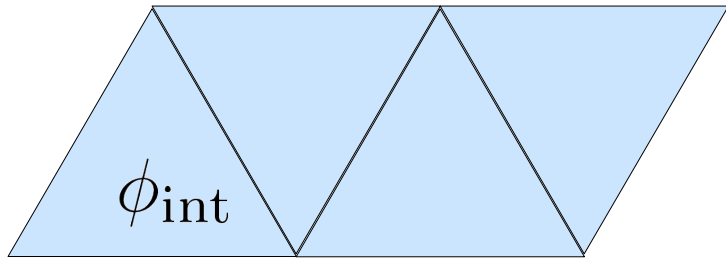
$$H = J_2 \sum_{\text{links}} \overbrace{(f_{1\alpha}^\dagger f_{1\beta})(f_{2\beta}^\dagger f_{2\alpha})} \\ + J_4 \sum_{\text{rhombi}} \overbrace{[(f_{1\alpha}^\dagger f_{1\beta})(f_{2\beta}^\dagger f_{2\gamma})(f_{3\gamma}^\dagger f_{3\delta})(f_{4\delta}^\dagger f_{4\alpha}) + h.c.]}$$

“Hopping” (U(1)) trial mean field

$$\mathcal{H}_{\text{trial}} = - \sum_{ij} t_{ij} f_{i\alpha}^\dagger f_{j\alpha} \quad \longrightarrow \quad \chi_{ij} \equiv \langle f_j^\dagger f_i \rangle$$

$$E_{\text{mf}} = -4J_2 \sum_{\text{links}} |\chi_{12}|^2 - 16J_4 \sum_{\text{rhombi}} [\chi_{12}\chi_{23}\chi_{34}\chi_{41} + c.c.]$$

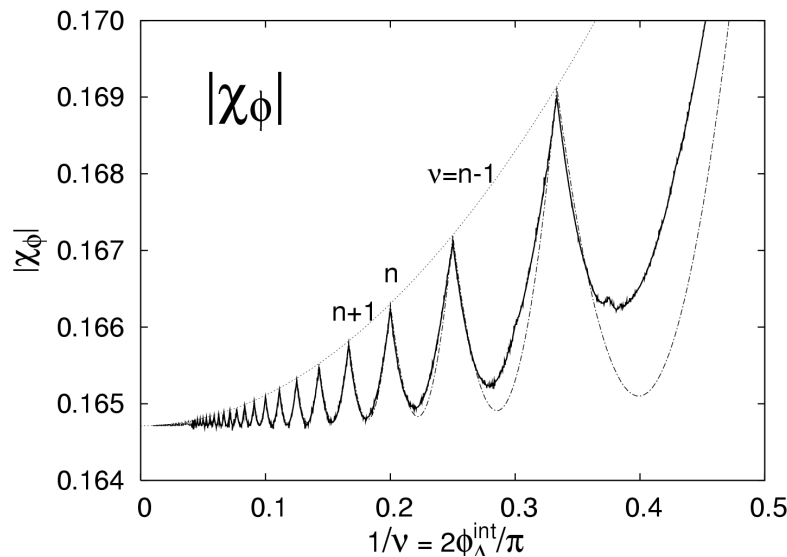
Mean field example: flux states



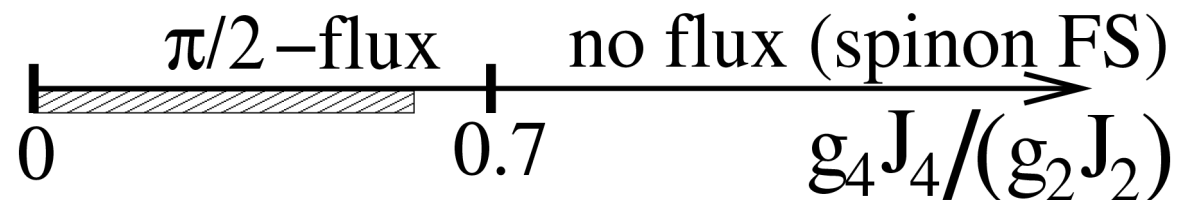
$$\epsilon_{\text{mf}} = -12J_2|\chi_\phi|^2 - 96J_4|\chi_\phi|^4 \cos(2\phi)$$

likes large flux

dislikes fluxes

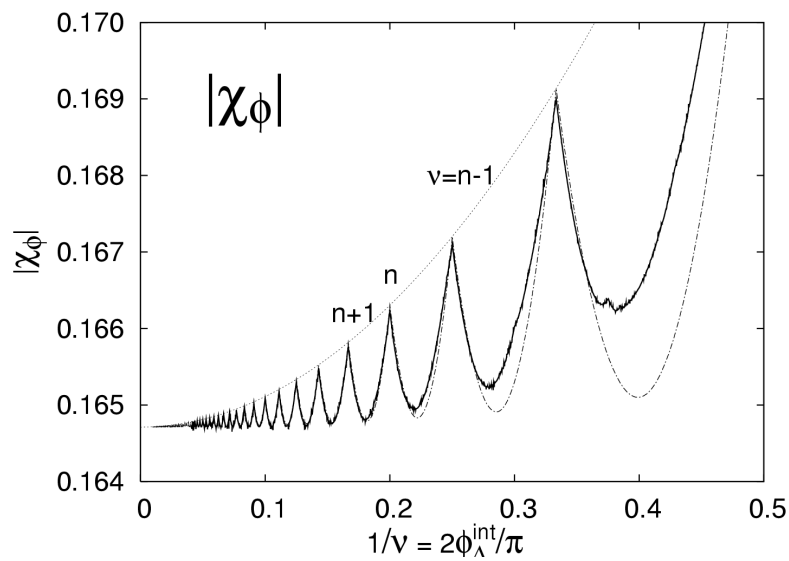


“Phase diagram”

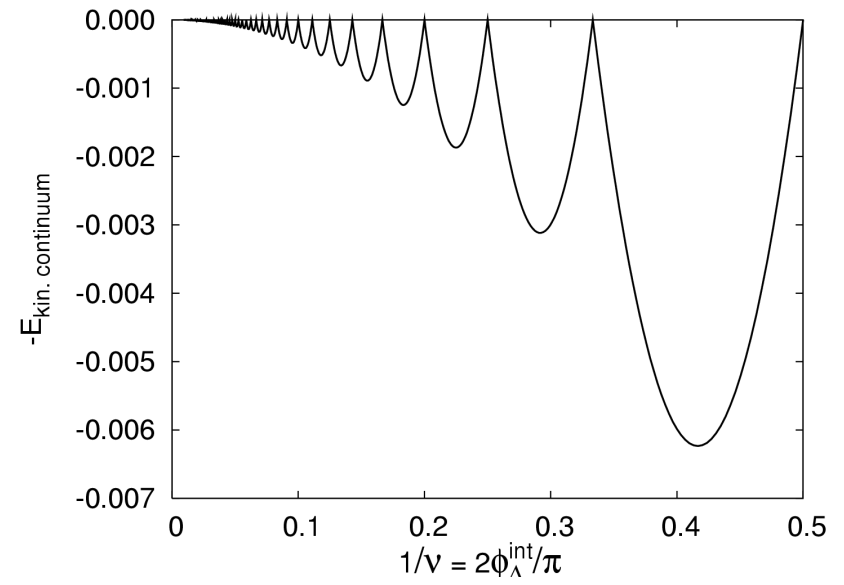


Flux states analysis

Triangular lattice



Continuum Landau problem



Approximate

$$|\chi_\phi| = |\chi_0|(1 + c\phi_\Delta^2) - A \nu^{-2}(\nu - k)(k + 1 - \nu)$$



$$\epsilon_{\text{mf}} = \kappa(\phi_\Delta^{\text{int}})^2 + t_{\text{spinon}}\epsilon^{\text{osc}}[\phi^{\text{int}}]$$

Estimates for κ -(ET)₂Cu₂(CN)₃

$$\epsilon_{\text{mf}} = \kappa(\phi_{\Delta}^{\text{int}})^2 + t_{\text{spinon}}\epsilon^{\text{osc}}[\phi^{\text{int}}]$$

Spinon hopping $t_{\text{spinon}} = 2J_2|\chi_0| + 32J_4|\chi_0|^3 \sim 100\text{K}$

- compare with $t_{\text{electron}} \sim 500\text{K}$

Gauge field stiffness $\kappa = 192J_4|\chi_0|^4 - 24cJ_2|\chi_0|^2 \sim \text{meV}$

- compare with $\kappa_{\text{EM}} = \frac{V\hbar^2c^2}{8\pi A_{\Delta}^2e^2} \sim 10^4 \text{ eV}$

Gauge structure

$$\mathcal{H}_{\text{trial}} = - \sum_{ij} t_{ij} f_{i\alpha}^\dagger f_{j\alpha} = - \sum_{ij} \boxed{|t_{ij}| e^{ia_{ij}}} f_{i\alpha}^\dagger f_{j\alpha}$$

variational parameter

Slow spatial variation of the phases a_{ij} produces only small trial energy change $\sim (\text{curl } a)^2$



need to include a_{ij} as dynamical variables

Spinon-gauge theory (uRVB)

$$\mathcal{H}_{\text{lattice}} \sim - \sum |t_{r,r'}| \boxed{e^{ia_{rr'}}} f_{r\sigma}^\dagger f_{r'\sigma} + \frac{\kappa}{2} (\nabla \times \mathbf{a})^2$$

variational low-energy field

$$\mathcal{H}_{\text{continuum}} \sim \int_{\mathbf{r}} f_\sigma^\dagger \left(- \frac{(\nabla - i\mathbf{a})^2}{2m_{\text{spinon}}} - \mu \right) f_\sigma + \frac{\kappa}{2} (\nabla \times \mathbf{a})^2$$

- 2+1 D “quantum electrodynamics in metal”, studied as uRVB in high- T_c literature (Reizer 89, Lee 89, Nagaosa et.al. 92, Polchinski 94, Altshuler et.al. 94, Nayak et.al. 94, Kim et.al. 94)
- Closely related theory arises in composite-fermion description of $\nu=1/2$ Quantum Hall (Halperin, Lee, Read 93)

Detecting spinon Fermi surface

- Indirect:
 - Thermal / magnetic properties
 - Transport by spinons (heat, spin)
- Direct Fermi surface probes:
 - Magneto-oscillations?
 - Geometric resonances?

Coupling to magnetic field

Zeeman spin coupling (naively, spinons carry no charge)

$$H_{\text{Zeeman}} = - \sum_{\mathbf{r}} \mu S_{\mathbf{r}}^z \cdot B$$

... benign effects – Pauli spin paramagnetism
in the spinon liquid

“Orbital” coupling

$$H_{\text{eff}}[B] = H_{\text{eff}}[0] + H_{\text{Zeeman}} + \frac{eBA_{\Delta}}{\hbar c} J_3 \sum_{\Delta} i(P_{123} - \text{h.c.})$$

$$J_2 = \frac{2t^2}{U} \quad , \quad J_4 = \frac{20t^4}{U^3} \quad , \quad J_3 = \frac{6t^3}{U^2}$$

Sen et.al. 95

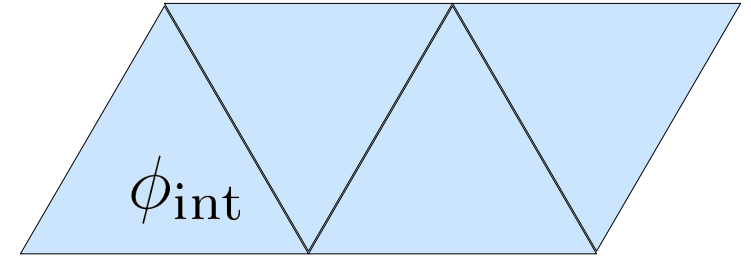
$$\kappa - (\text{ET})_2\text{Cu}_2(\text{CN})_3 : J_4/J_2 \sim 0.3, \quad J_3/J_2 \sim 0.7$$

$$i(P_{123} - \text{h.c.}) = -4\mathbf{S}_1 \cdot \mathbf{S}_2 \times \mathbf{S}_3$$

Magnetic field couples to the spin chirality!

Mean field in the presence of B

Trial flux states



$$i[P_{123} - \text{h.c.}] = i[(f_{1\alpha}^\dagger f_{1\beta})(f_{2\beta}^\dagger f_{2\gamma})(f_{3\gamma}^\dagger f_{1\alpha}) - \text{h.c.}]$$
$$\sim i(\chi_{12}\chi_{23}\chi_{31} - \text{c.c.}) = -|\chi|^3 \sin(\phi_{\text{int}})$$

Spin chirality $\sim$ internal gauge flux
- couples linearly to B!

Mean field in the presence of B

$$\epsilon_{\text{mf}} = \kappa[(\phi_{\Delta}^{\text{int}})^2 - 2\gamma\phi_{\Delta}^{\text{int}}\Phi_{\Delta}^{\text{ext}}] + t_{\text{spinon}}\epsilon^{\text{osc}}[\phi^{\text{int}}]$$

$$\gamma = \frac{J_3}{12J_4|\chi_0|} \sim 1 - 2 \text{ for the } \kappa\text{-ET}$$

Minimize the energy $\rightarrow \phi^{\text{int}} \simeq \gamma\Phi^{\text{ext}}$

Spinons see static internal field
comparable to the external B!

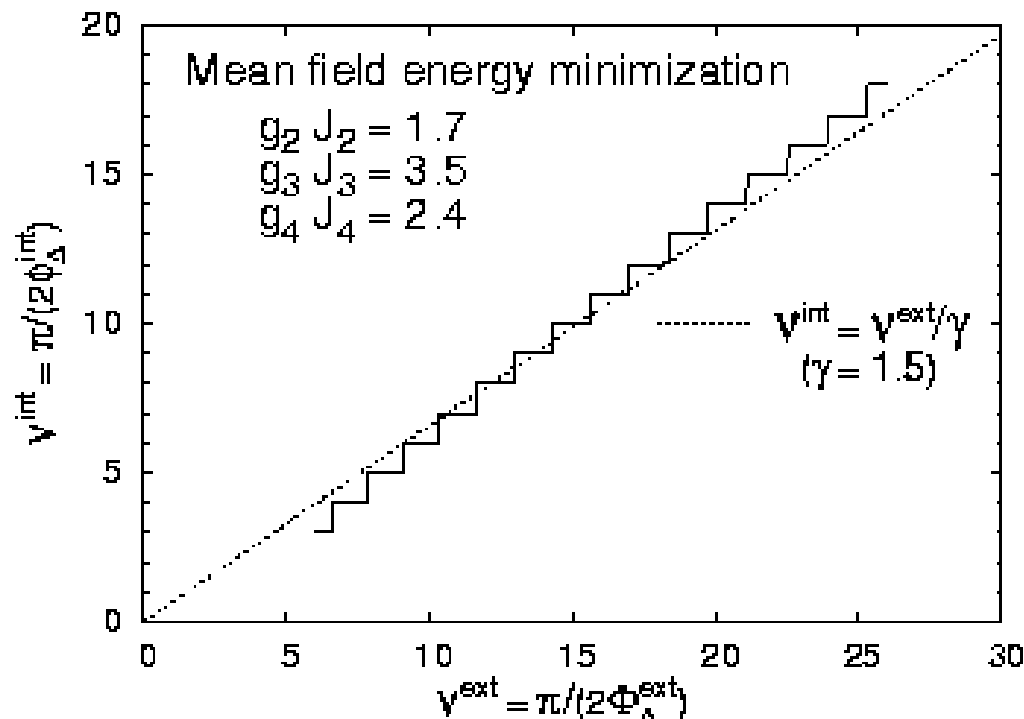
... perhaps can use to probe Fermi surface

... but fragile because of the very soft internal gauge field

Effect of Landau level discreteness

Integer Landau level fillings of spinons are special

-> soft internal gauge field readily adjusts to achieve this!



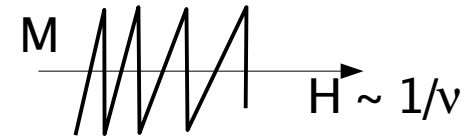
At $T=0$ sequence of
1st-order transitions
stepping through **chiral**
spin liquids

$\kappa\text{-(ET)}_2\text{Cu}_2(\text{CN})_3$:

$\nu \sim 500$ **for** $B=8$ Tesla

Analogy with magnetic interaction effects in metals + electromagnetic field

Landau problem with static field



$$\mathcal{M}^{\text{osc}}(H) = R_T R_S \frac{ne\hbar}{\pi m^* c} \sin\left(\frac{2\pi F}{H}\right); \quad F/H = \nu$$

Electrons + dynamic EM field + applied field

$$\Omega_{\text{mf}} = \Omega_{\text{ferm}}(B) + \frac{\kappa}{2}(B - H^{\text{ext}})^2$$

Minimize energy

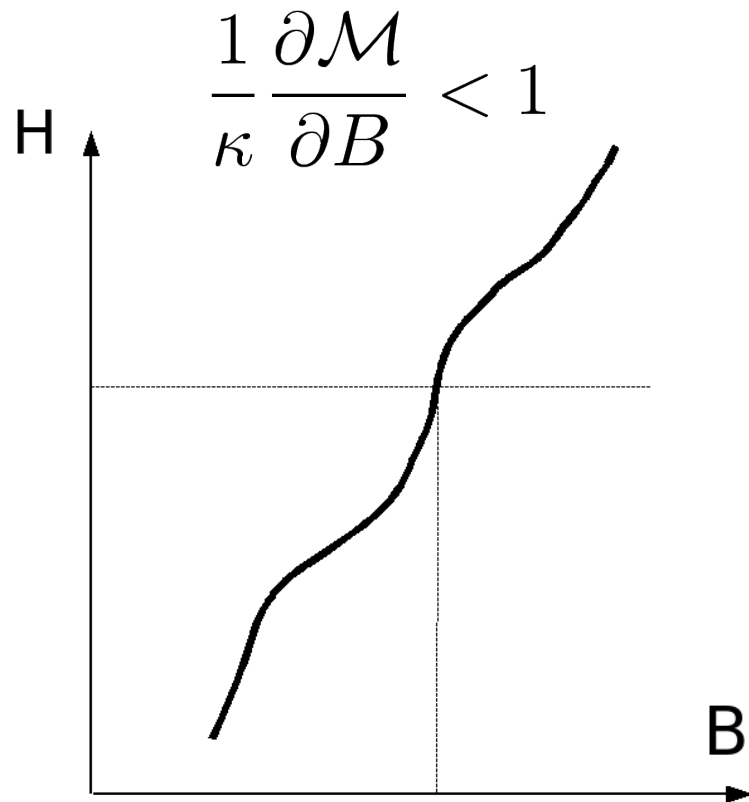
$$H^{\text{ext}} = B + \frac{1}{\kappa} \frac{\partial \Omega_{\text{ferm}}}{\partial B} \equiv B - \frac{1}{\kappa} \mathcal{M}(B)$$

electrons see average field

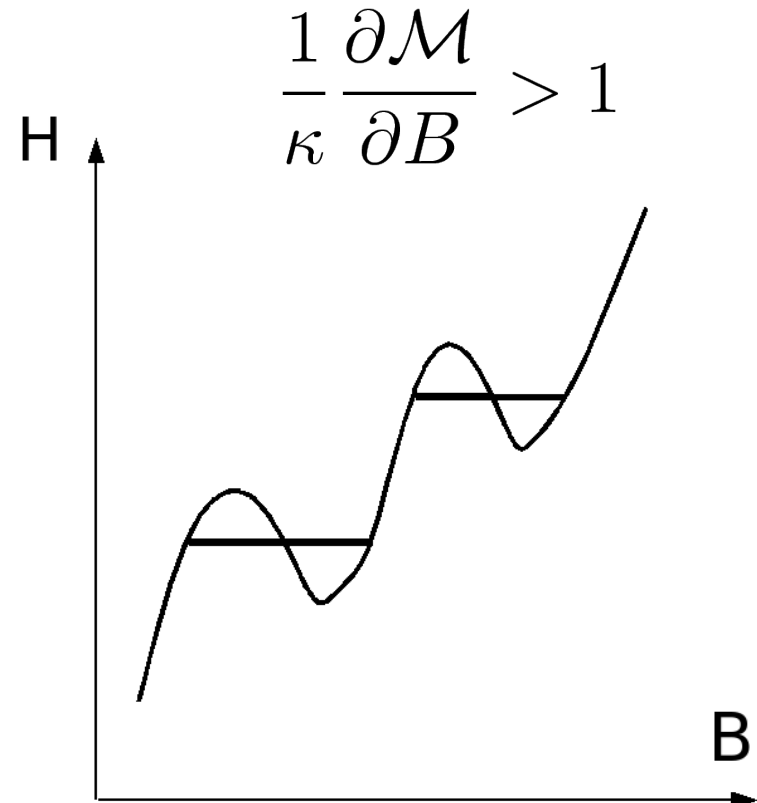


Magnetic interaction effects (contd)

$$H^{\text{ext}} = B - \frac{1}{\kappa} \mathcal{M}(B)$$



unique stable solution



multiple solutions

-> “diamagnetic instability”, Condon domains (e.g. observed in silver)

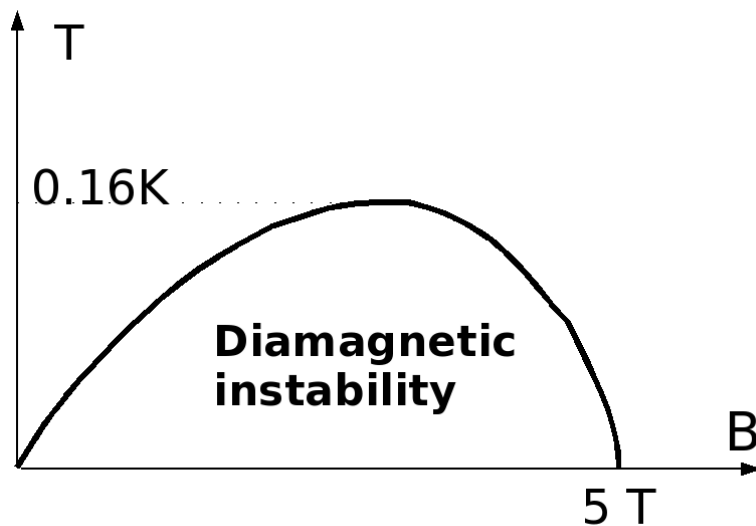
$\kappa\text{-(ET)}_2\text{Cu}_2(\text{CN})_3$ electron liquid + EM vs spinon liquid + internal gauge field

$$R_T \sim \frac{\hbar^2 k_F^2}{2m^*} \gtrsim \frac{\kappa}{2} H^2, \quad R_T = \frac{2\pi^2 T / \hbar \omega_c}{\sinh(2\pi^2 T / \hbar \omega_c)}$$

$$\kappa_{\text{EM}} / \kappa_{\text{int}} \sim 10^7$$

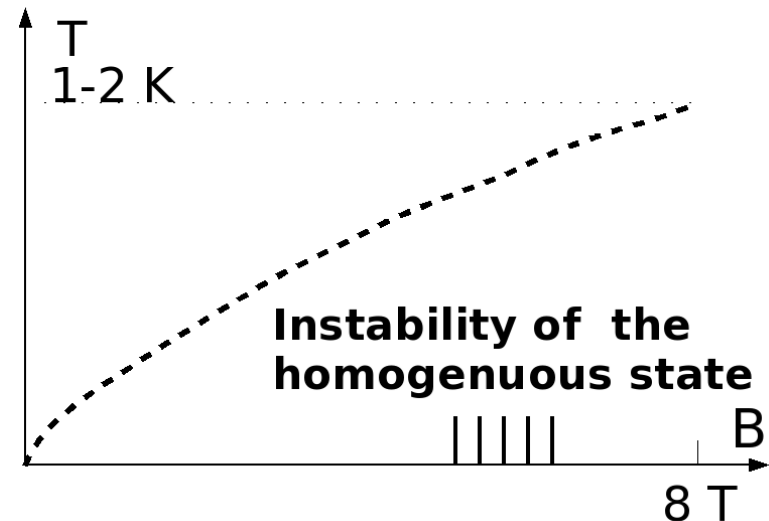
theoretical:

$\kappa\text{-(ET)}_2\text{Cu}_2(\text{CN})_3$ metal (0.8GPa)



Itskovsky et.al. (1997)

spinon liquid



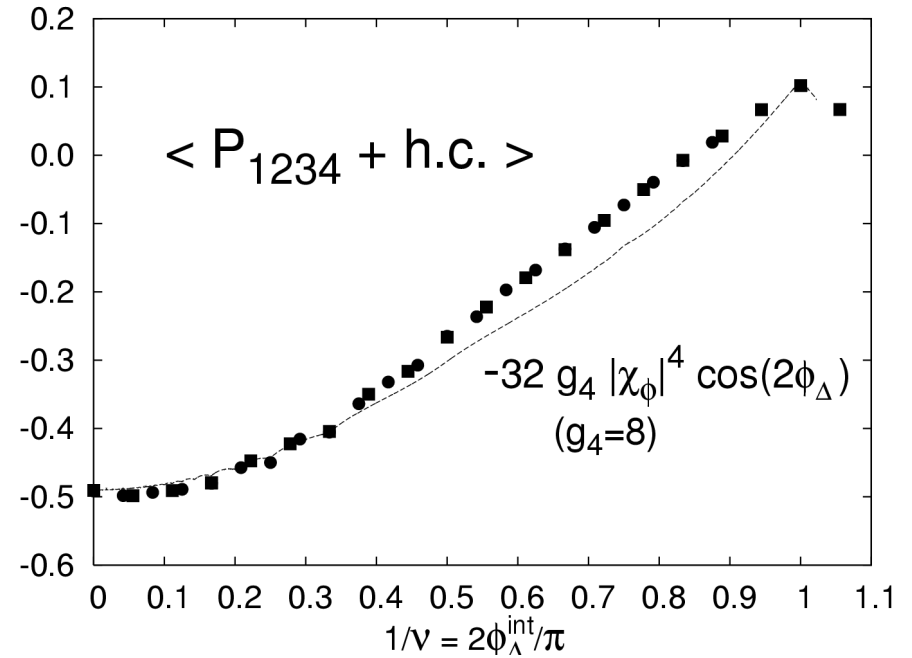
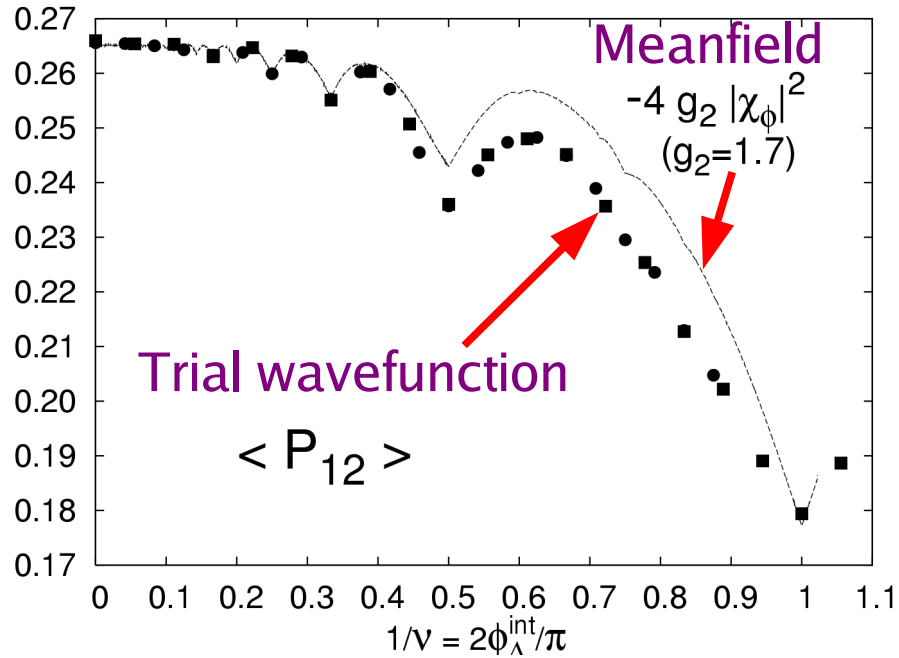
Summary on response to magnetic field

- Spinons see large internal field
- ... but the homogeneous flux state is unstable
- Instability becomes stronger for larger B and smaller T ; preempts magneto-oscillations?
Perhaps can still look for geometric resonances.
- Important issues:
 - Field orientation with respect to planes
 - Large-scale inhomogeneities / crystal mosaic

Conclusions

- $\kappa\text{-(ET)}_2\text{Cu}_2(\text{CN})_3$ - unique material that realizes spin liquid state near Mott transition
- Spinon Fermi sea state – a framework to discuss the phenomenon / make proposals
 - Thermal, transport
 - Response to magnetic field
 - A good start and not the last word - “In praise of unstable fixed points”
- More experiments / more theory input
 - Impurities in the critical spin liquid?
 - Spinon pairing at low T?

“Renormalized mean field”

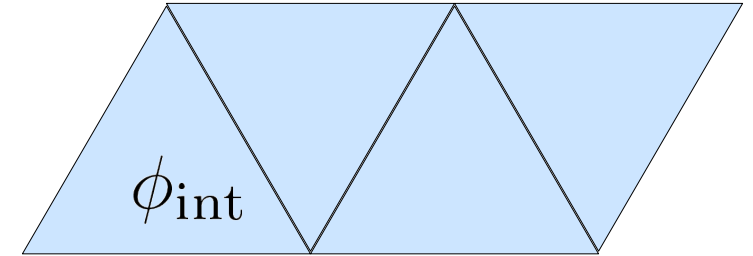


Mean field estimate of trial energy

$$\epsilon_{\text{mf}} = - \mathbf{g_2} 12J_2 |\chi_{\phi}|^2 - \mathbf{g_4} 96J_4 |\chi_{\phi}|^4 \cos(2\phi)$$

Mean field in the presence of B

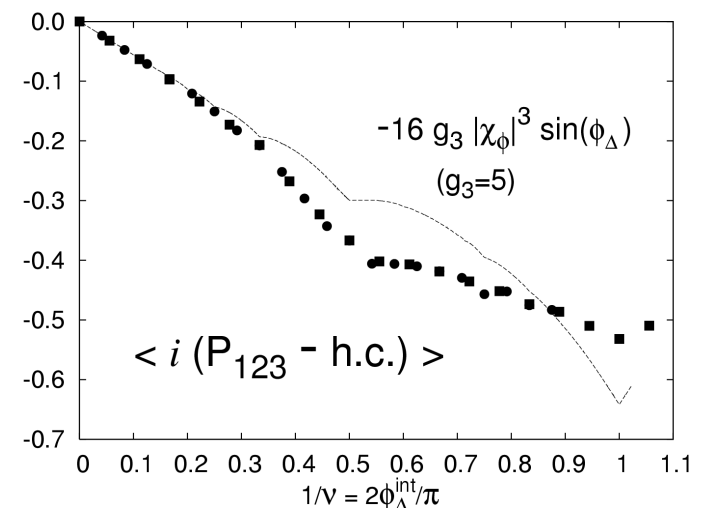
Trial flux states



$$i[P_{123} - \text{h.c.}] = i[(f_{1\alpha}^\dagger f_{1\beta})(f_{2\beta}^\dagger f_{2\gamma})(f_{3\gamma}^\dagger f_{1\alpha}) - \text{h.c.}]$$

$$\sim i(\chi_{12}\chi_{23}\chi_{31} - \text{c.c.}) = -|\chi|^3 \sin(\phi_{\text{int}})$$

Spin chirality $\sim$ internal gauge flux
- couples to B!



Finite temperature

In the mean field, t_{spinon} vanishes above $T \sim 50\text{-}100\text{ K}$

Crude phenomenology:

- $T > 50\text{-}100\text{ K}$ – Curie-Weiss paramagnet
- $T < 50\text{ K}$ – correlations grow; spinon liquid?
- $T < \text{few K}$ - ???

