Problem 1: Phases of the Sine-Gordon model in $d$-dimensions

Consider the Sine-Gordon theory

$$S = \int d\mathbf{r} \left[ \frac{\kappa}{2} (\nabla \chi)^2 - \lambda \cos(\alpha \chi) \right].$$

Argue that the cosine potential is always relevant in dimension $d \geq 3$.

Hint: Recall correlations of $e^{i\chi}$ in the $\lambda = 0$ theory and deduce its scaling dimension.

Remark: The corresponding $\pm 1$ system of charges from the previous problem set is therefore always in plasma phase in $d \geq 3$. This result implies, for example, that so-called Compact Electrodynamics is always confining in $(2+1)d$ – see Polyakov’s book and one of the presentation topics.

Problem 2: Running “coupling function.” (Problem 3.5.2 from Wen’s book.)

Consider a model

$$S = \int d^2 r \left[ \frac{\kappa}{2} (\nabla \theta)^2 + V(\theta) \right],$$

where $V(\theta)$ is a small $2\pi$-periodic function: $V(\theta + 2\pi) = V(\theta)$. Find the RG equations for the flow of the “coupling function” $V$. You may ignore the flow of $\kappa$ because we assumed that $V$ is small. Discuss the form of $V$ after a long flow if we have started with a very small $V$.

Hint: Expand in harmonics $V(\theta) = \sum_m V_m e^{im\theta}$ and discuss flows of $V_m$. 