A zoo of bosonization dualities

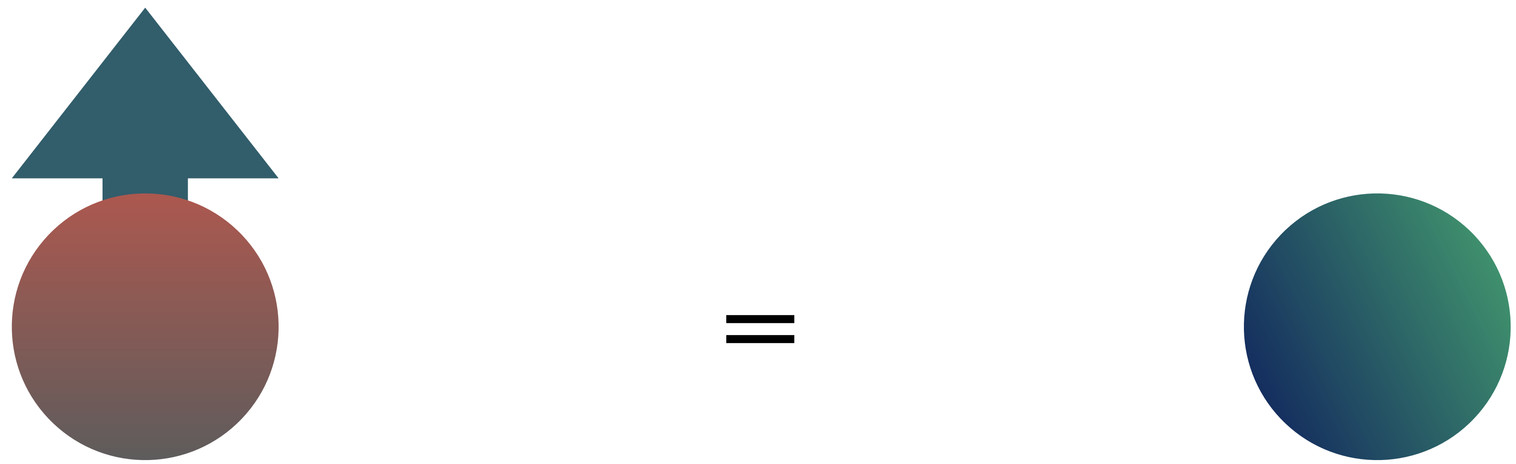
Field Theory Dualities and Strongly Correlated Matter (Aspen, March 2018)

Kristan Jensen (SFSU)

Based on:

JHEP 11 (2017), 018 (with A. Karch) (arXiv:1709.01083)
JHEP 01 (2018), 031 (arXiv:1712.04933)

Flux attachment



boson + flux

fermion

what about criticality?

lorentz invariance?

Relativistic flux attachment at criticality

Significant evidence for **3d bosonization:**

Theory A: $$\int d^3x\; \bar{\psi} i(\partial\!\!\!/ + iA\!\!\!/)\psi$$ (free fermion)

Theory B: $$\int d^3x \Big(\frac{i}{4\pi}\epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho + |(\partial + ia)\Phi|^2 + V(|\Phi|) + \frac{i}{2\pi}\epsilon^{\mu\nu\rho} a_\mu \partial_\nu A_\rho \Big)$$

($U(1)_1$ + “WF” scalar)

[Aharony `15]

fermion = monopole

An abelian duality web

Treat A = B as a **seed duality**

$$\text{free } \psi \quad \leftrightarrow \quad U(1)_1 \text{ with critical } \Phi$$

[Karch, Tong]
[Seiberg, Senthil, Wang, Witten]
(see also [Murugan, Nastase])

An abelian duality web

Treat A = B as a **seed duality**

$$\text{free } \psi \quad \leftrightarrow \quad U(1)_1 \text{ with critical } \Phi$$

Particle/vortex

[Peskin '78]
[Dasgupta, Halperin '81]

[Karch, Tong]
[Seiberg, Senthil, Wang, Witten]
(see also [Murugan, Nastase])

An abelian duality web

Treat A = B as a **seed duality**

$$\text{free } \psi \quad \leftrightarrow \quad U(1)_1 \text{ with critical } \Phi$$

Particle/vortex

[Peskin '78]
[Dasgupta, Halperin '81]

$$\text{free } \psi \quad \leftrightarrow \quad U(1)_{-\frac{1}{2}} \text{ with } \Psi$$

[Son], [Metlitski, Vishwanath] '15

[Karch, Tong]
[Seiberg, Senthil, Wang, Witten]
(see also [Murugan, Nastase])

An abelian duality web

Treat A = B as a **seed duality**

$$\text{free } \psi \quad \leftrightarrow \quad U(1)_1 \text{ with critical } \Phi$$

(3d bosonization)^2 = particle/vortex duality

Non-abelian bosonization in three dimensions

Abelian seed is $N=k=N_f=1$ limit of a sequence:

$$SU(N)_{-k+\frac{N_f}{2}} \text{ with } N_f\,\psi \quad \leftrightarrow \quad U(k)_N \text{ with } N_f\,\Phi$$

$(N_f \leq k)$

baryons = monopoles

[Aharony, Benini, Giombi, Gur-Ari, Hsin, Jain, Maldacena, Minwalla, Seiberg, Sharma, Trivedi, Wadia, Yacoby, Yin, Yokoyama, Zhiboedov, …]

Some evidence:

- Large N correlation functions, high T
- N_f=1: Vasiliev dual; inherited from Seiberg duality
- Massive phases are TFTs, match
- global symmetries, `t Hooft anomalies match
- embed into AdS/CFT [KJ, Karch]

Q: Non-abelian duality web?

Outline

1. Abelian bosonization
2. Quivers
3. A master duality
4. A fun example
5. The future

Outline

1. Abelian bosonization
2. **Quivers**
3. A master duality
4. A fun example
5. The future

New dualities from old

$$SU(N)_{-k+\frac{N_f}{2}} \text{ with } N_f\,\psi \quad \leftrightarrow \quad U(k)_N \text{ with } N_f\,\Phi$$

Global symmetry: $SU(N_f) \times U(1) \to U(N_f)/\mathbb{Z}_N$

[Benini, Hsin, Seiberg]

Gauge global symmetries on both sides

[KJ, Karch]

Quiver duality

set $N \to N_1\,,\ k \to k_1\,,\ N_f \to k_2$ gauge U(k_2) global symmetry:

Theory A

	$U(k_1)_{N_1}$	$U(k_2)_{N_2}$
X	$\square$	$\square$

Theory B

	$SU(N_1)_{-k_1+\frac{k_2}{2}}$	$U(k_2)_{N_2+\frac{N_1}{2}}$
ψ	$\square$	$\square$

Quiver triality

set $N \to N_1\,,\ k \to k_1\,,\ N_f \to k_2$ gauge U(k_2) global symmetry:

Theory A

	$U(k_1)_{N_1}$	$U(k_2)_{N_2}$
X	$\square$	$\square$

Theory B

	$SU(N_1)_{-k_1+\frac{k_2}{2}}$	$U(k_2)_{N_2+\frac{N_1}{2}}$
ψ	$\square$	$\square$

Theory C

	$SU(N_1)_{-k_1+k_2}$	$SU(N_1+N_2)_{-k_2}$
Y	$\square$	$\square$

A web of quiver dualities

Nice result: one can dualize quivers node-by-node

However, unlike 3d SUSY dualities, there do not appear to be quiver dualities that do not come from gauging global symmetries

Outline

1. Abelian bosonization
2. Quivers
3. **A master duality**
4. A fun example
5. The future

A master bosonization duality

Dualities with both fundamental bosons, fermions?

A master bosonization duality

Dualities with both fundamental bosons, fermions?

Proposal:

$$SU(N)_{-k+\frac{N_f}{2}} \text{ with } N_f\,\psi\,,\, N_s\,\phi \;\leftrightarrow\; U(k)_{N-\frac{N_s}{2}} \text{ with } N_s\,\Psi\,,\, N_f\,\Phi$$

Reduce to Aharony's when N_s=0

Flavor bounds:

$(N_f \leq k\,,\; N_s \leq N\,,\; (N_f, N_s) \neq (k, N))$

[KJ]
[Benini]

(see also [Jain, Minwalla, Yokoyama])

A master bosonization duality

Dualities with both fundamental bosons, fermions?

Proposal:

$$SU(N)_{-k+\frac{N_f}{2}} \text{ with } N_f\,\psi\,,\ N_s\,\phi \ \leftrightarrow\ U(k)_{N-\frac{N_s}{2}} \text{ with } N_s\,\Psi\,,\ N_f\,\Phi$$

$$V(|\phi|^2) - \lambda(\bar{\psi}\cdot\phi)(\phi^\dagger\cdot\psi) \qquad \tilde{V}(|\Phi|^2) + \tilde{\lambda}(\bar{\Psi}\cdot\Phi)(\Phi^\dagger\cdot\Psi)$$

A master bosonization duality

Dualities with both fundamental bosons, fermions?

$$V(|\phi|^2) - \lambda(\bar{\psi}\cdot\phi)(\phi^\dagger\cdot\psi) \qquad \tilde{V}(|\Phi|^2) + \tilde{\lambda}(\bar{\Psi}\cdot\Phi)(\Phi^\dagger\cdot\Psi)$$

Marginal at large N

Sign is important

A master bosonization duality

Dualities with both fundamental bosons, fermions?

$$V(|\phi|^2) - \lambda(\bar{\psi}\cdot\phi)(\phi^\dagger\cdot\psi) \qquad \tilde{V}(|\Phi|^2) + \tilde{\lambda}(\bar{\Psi}\cdot\Phi)(\Phi^\dagger\cdot\Psi)$$

In Higgs phase,

	$SU(N) \to SU(N-N_s)$
ψ	$N_f\,\square \to N_f\,\square + N_f N_s\,\mathbf{1}$

A master bosonization duality

Dualities with both fundamental bosons, fermions?

$$V(|\phi|^2) - \lambda(\bar{\psi} \cdot \phi)(\phi^\dagger \cdot \psi) \qquad \tilde{V}(|\Phi|^2) + \tilde{\lambda}(\bar{\Psi} \cdot \Phi)(\Phi^\dagger \cdot \Psi)$$

In Higgs phase,

	$SU(N) \to SU(N - N_s)$
ψ	$N_f \, \Box \to N_f \, \Box + N_f N_s \, \mathbf{1}$

get mass ~ $-\lambda|v|^2$

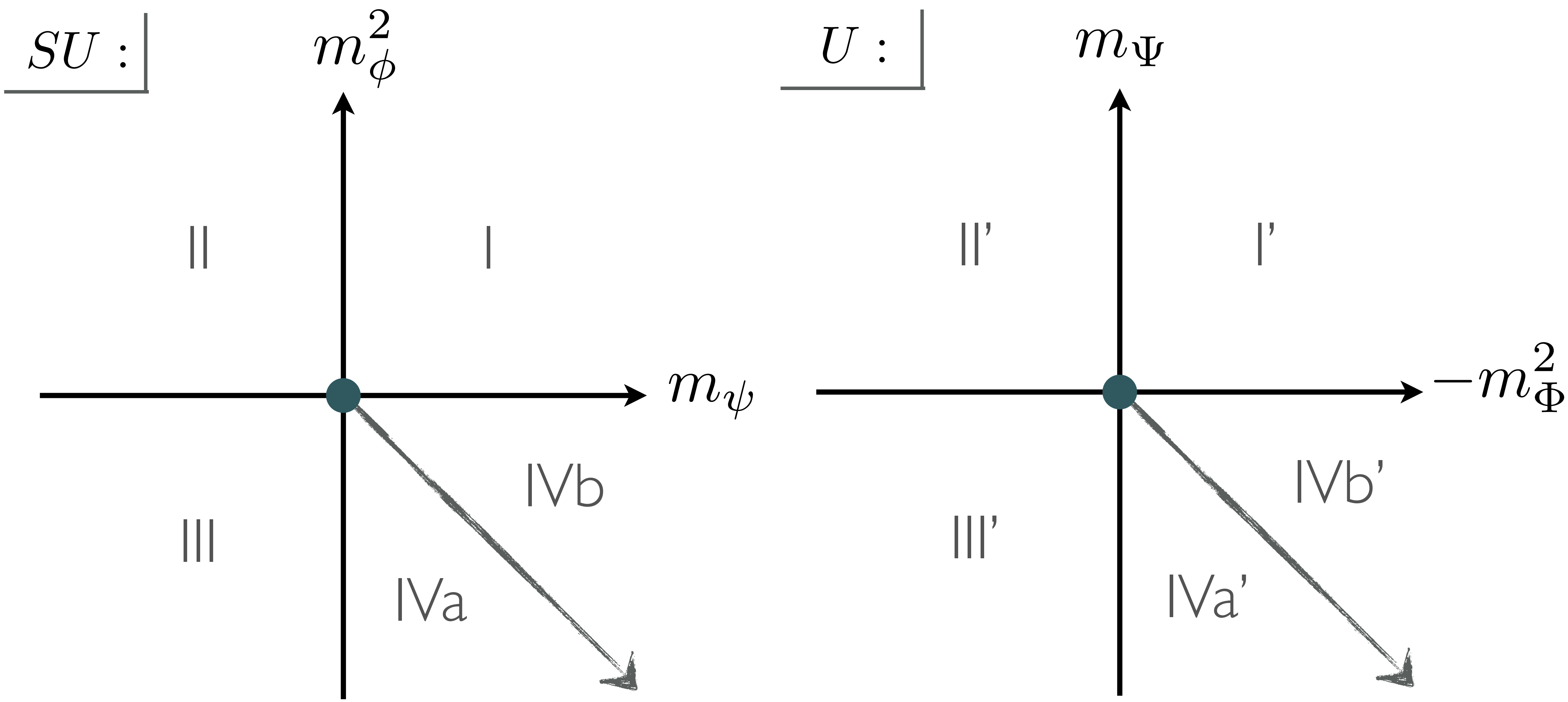
Phase diagram

At large N, two relevant deformations: $\bar{\psi}\psi\,,\ |\phi|^2$

Lead to five semiclassically visible phases:

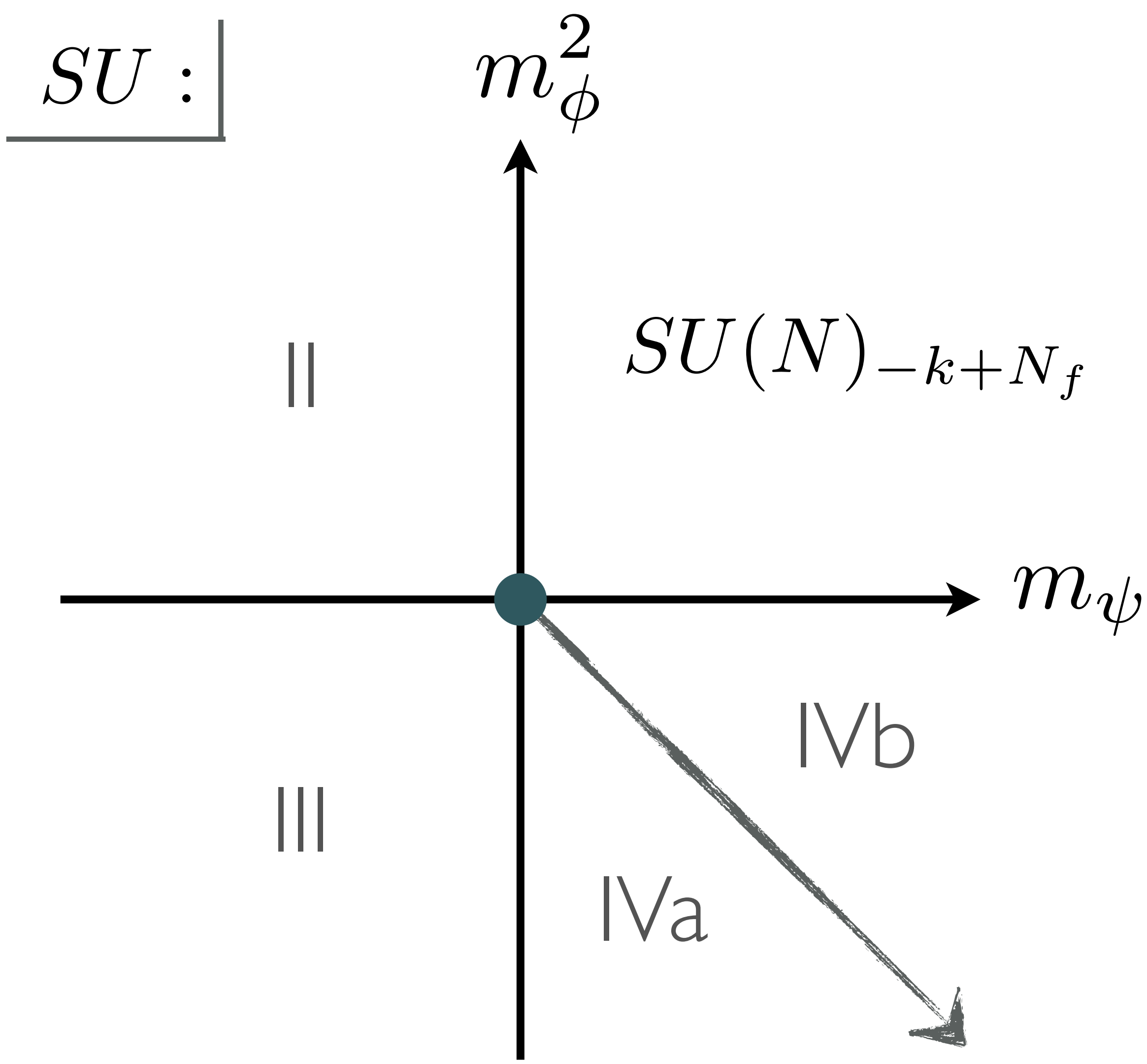
Phase diagram

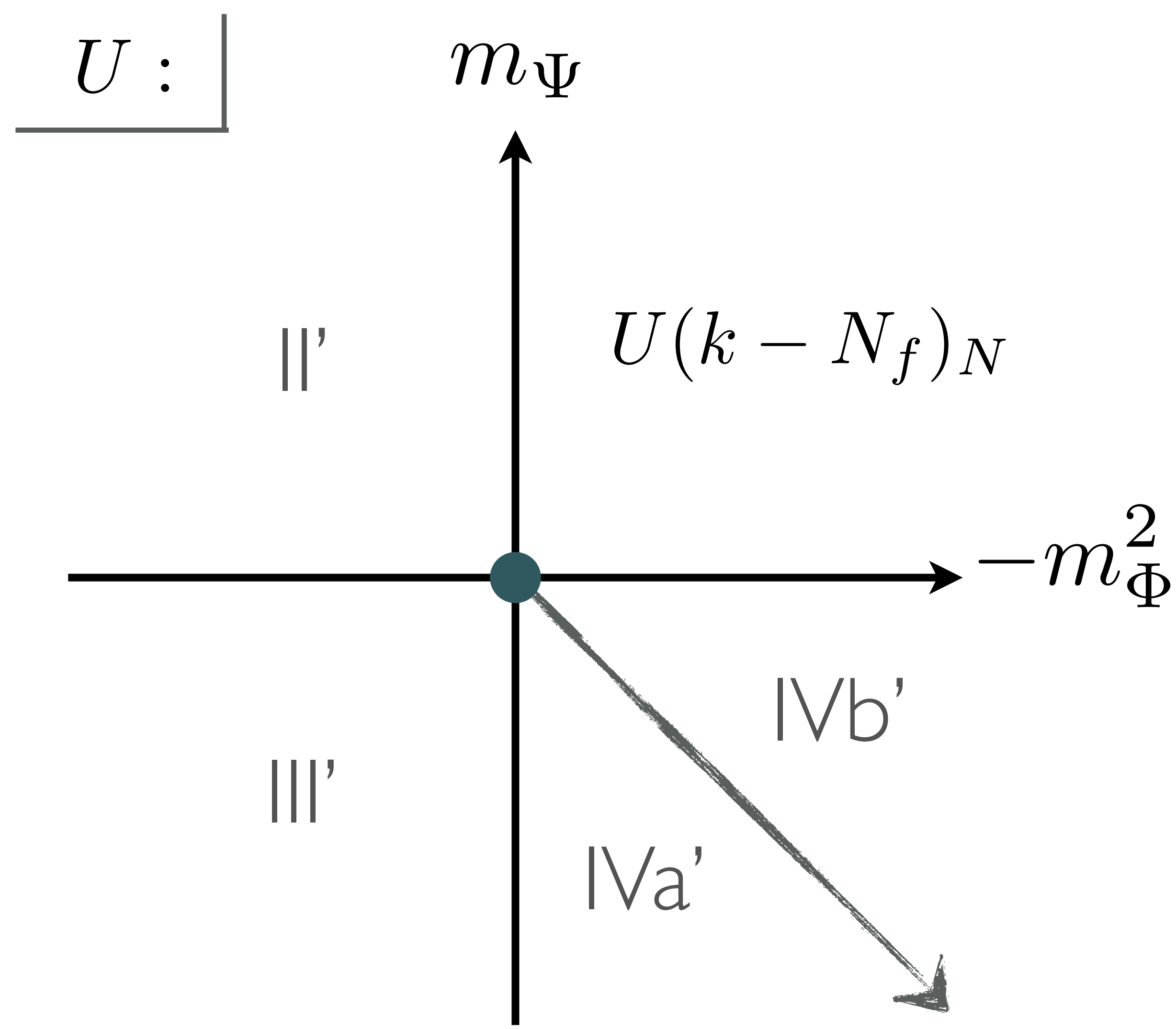
Lead to five semiclassically visible phases:



Phase diagram

Lead to five semiclassically visible phases:





Duality map

$$m_\psi \sim -m_\Phi^2\,, \quad m_{\phi^2} \sim m_\Psi$$

Tests

1. Massive phases, critical lines match
2. Global symmetries, flavor Hall response
3. Baryons = monopoles
4. Dual boundary conditions [Aitken, Karch, Robinson]

Outline

1. Abelian bosonization
2. Quivers
3. A master duality
4. **A fun example**
5. The future

Emergent symmetries

Common in IR dualities for there to be emergent symmetries on one side that are manifest in another

Emergent symmetries

Common in IR dualities for there to be emergent symmetries on one side that are manifest in another

Example: $N = 1\,,\; k \geq 3\,,\; N_f = 2\,,\; N_s = 1$

Emergent symmetries

Example: $N = 1\,,\ k \geq 3\,,\ N_f = 2\,,\ N_s = 1$

Theory A: $2 \text{ free Dirac } \psi + \text{WF}\,\phi$

Theory B: $U(k)_{\frac{1}{2}} \text{ with } 2\Phi\,,\ \Psi$

Emergent symmetries

Example: $N = 1\,,\ k \geq 3\,,\ N_f = 2\,,\ N_s = 1$

Theory A: $2 \text{ free Dirac } \psi + \text{WF}\,\phi$

T-invariant
SO(4) x U(1)

Theory B: $U(k)_{\frac{1}{2}} \text{ with } 2\Phi\,,\ \Psi$

SU(2) x U(1)^2

Emergent symmetries

Example: $N = 1\,,\; k \geq 3\,,\; N_f = 2\,,\; N_s = 1$

Theory A: $2 \text{ free Dirac } \psi + \text{WF}\, \phi$

T-invariant
SO(4) x U(1)

Theory B: $U(k)_{\frac{1}{2}} \text{ with } 2\Phi\,,\; \Psi$

SU(2) x U(1)^2

Quantum T-, SO(4)-invariance?

Emergent symmetries

Remarkably, this might work!

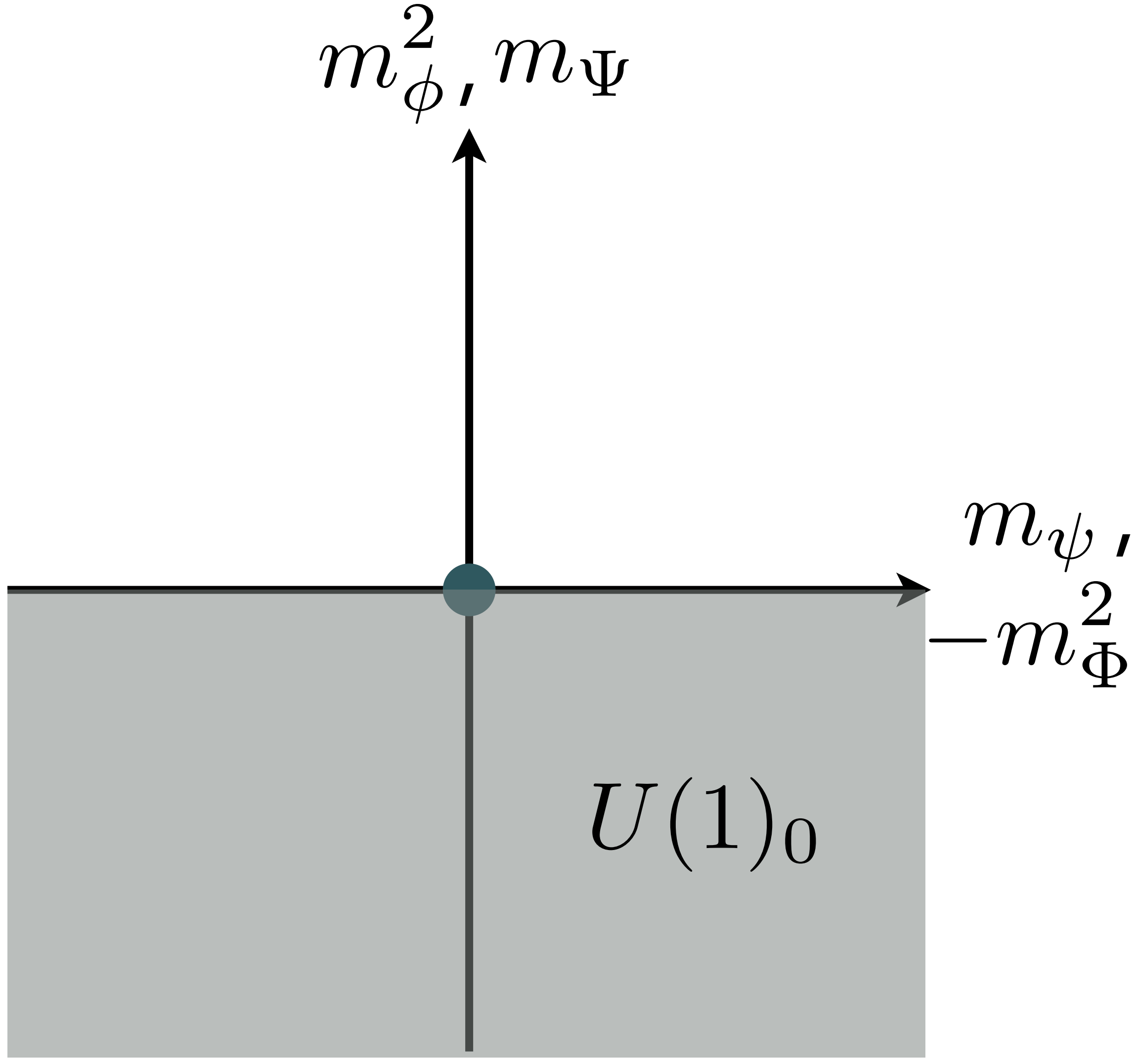
Emergent symmetries

Remarkably, this might work!

Emergent T-invariance implies four phases, which match:

A: $2\ \text{free Dirac}\ \psi + \text{WF}\ \phi$

B: $U(k)_{\frac{1}{2}}\ \text{with}\ 2\Phi\,,\ \Psi$



Emergent symmetries

Remarkably, this might work!

		spin	SU(2)
Dual fermions?	$(\text{flux})\Phi$	1/2	$\mathbf{2}$
Extra conserved currents?	$(\text{flux})\bar{\Psi}\Phi\Phi$	1	$\mathbf{1} \oplus \mathbf{3}$

Emergent symmetries

Remarkably, this might work!

		spin	SU(2)
Dual fermions?	$(\text{flux})\Phi$	1/2	$\mathbf{2}$
Extra conserved currents?	$(\text{flux})\bar{\Psi}\Phi\Phi$	1	$\mathbf{1} \oplus \mathbf{3}$

right quantum numbers
to enhance global symmetry

$$SU(2) \times U(1)^2 \rightarrow SO(4) \times U(1)$$

Outline

1. Abelian bosonization
2. Quivers
3. A master duality
4. A fun example
5. **The future**

N=1 dualities

Very recently conjectured dualities for N=1 CSM

[Bashmakov, Gomis, Komargodski, Sharon]
[Benini, Benvenuti]

N=1 dualities

Very recently conjectured dualities for N=1 CSM

[BGKS]:

$$U(N)_{k+N/2+1/2,k+1/2} + \Phi \longleftrightarrow SU(k+1)_{-N-k/2} + \Psi$$

+ radiatively induced

$$W = +|\Phi|^4$$

[B2]:

$$U(k)_{N+\frac{k}{2}-\frac{1}{2},\,N-\frac{1}{2}} \text{ with 1 flavor } Q \quad \mathcal{W} = -\tfrac{1}{4}\big(\textstyle\sum_{i=1}^{k} Q_i Q_i^\dagger\big)^2 \quad \longleftrightarrow \quad SU(N)_{-k-\frac{N}{2}+\frac{1}{2}} \text{ with 1 flavor } P \text{ and a gauge-singlet } H \quad \mathcal{W} = H\,\textstyle\sum_{i=1}^{N} P_i P_i^\dagger - \tfrac{1}{3}H^3\,.$$

[Bashmakov, Gomis, Komargodski, Sharon]
[Benini, Benvenuti]

Tests at large N, high temperature

Large N limit is soluble: CSM is vector-like

High-temperature free energy computable

[Aharony, Giombi, Gur-Ari, Maldacena, Yacoby]

[Jain, Minwalla, Sharma, Takimi, Wadia, Yokoyama]

currently: adapting to theories with multiple ψ, ϕ

Goal: test N=0,1,2 dualities, compute to O(1/N)

Thank you!