Dynamical Signatures of Deconfined Quantum Critical Point

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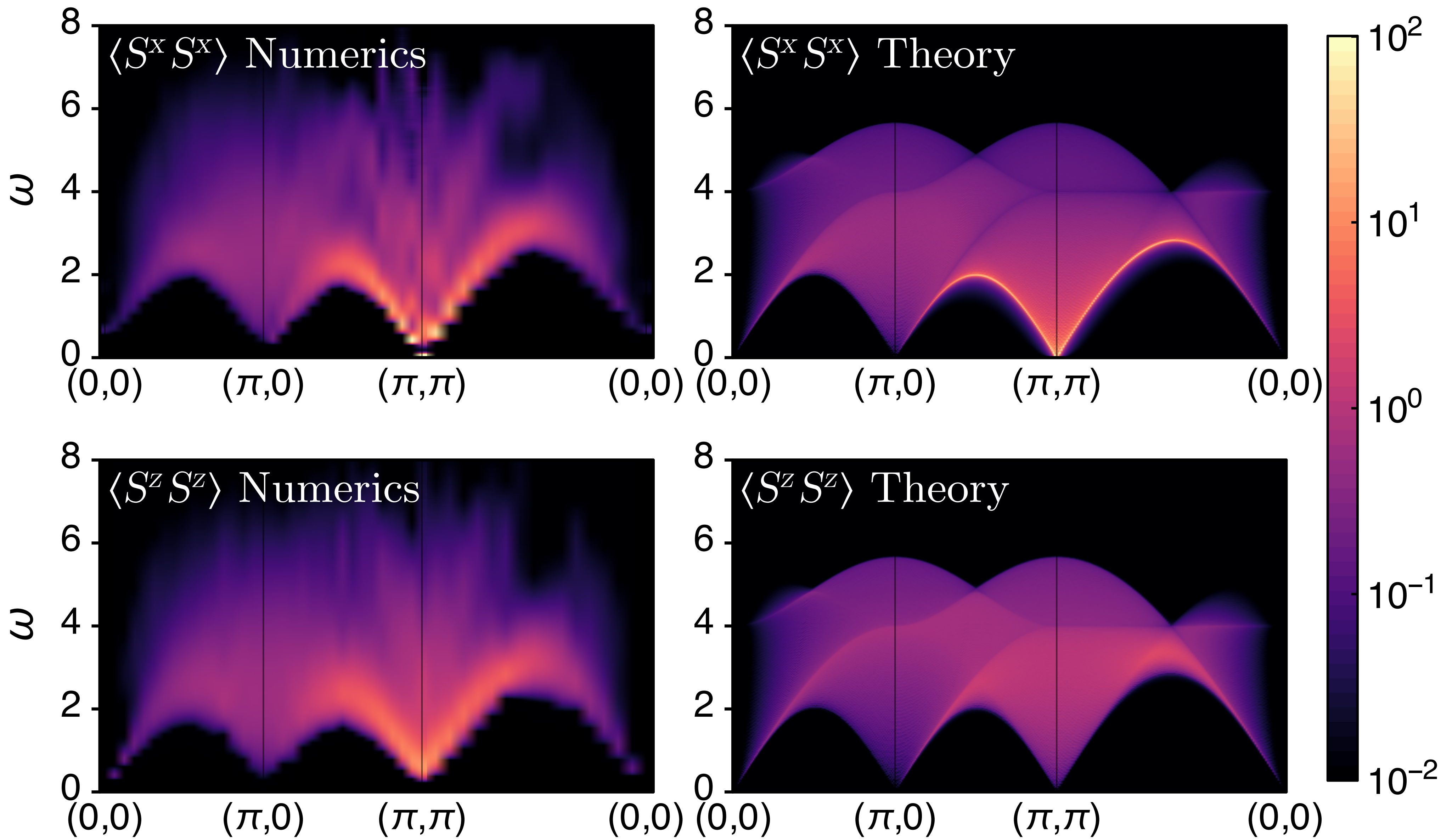
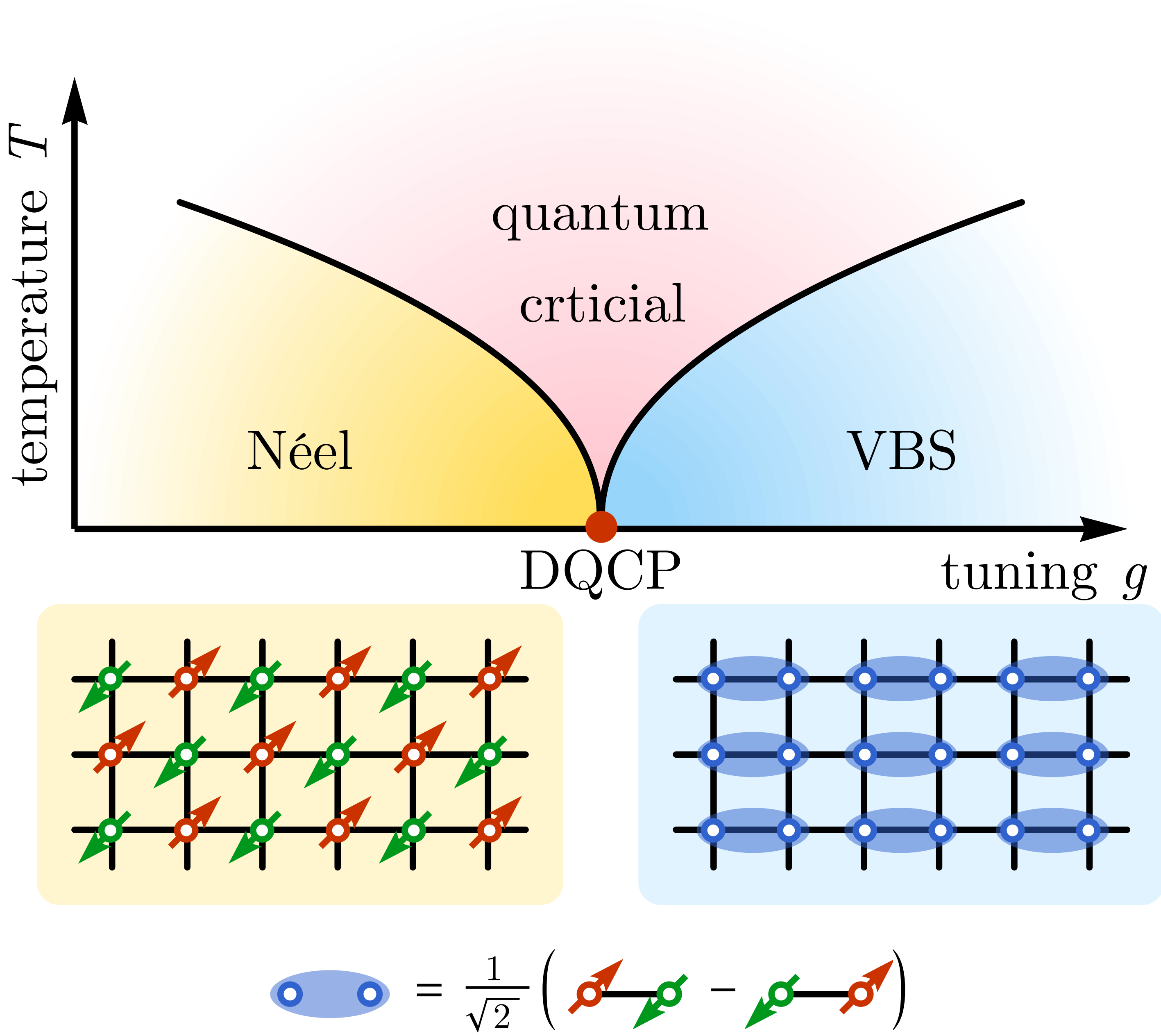


Field Theory Dualities and Strongly Correlated Matter

Aspen, March 2018

Deconfined Quantum Critical Point (DQCP)

- Exotic quantum critical point between two conventional phases: Néel and valence bond solid (VBS) in (2+1)D



Senthil, Vishwanath, Balents, Sachdev, Fisher (2004)

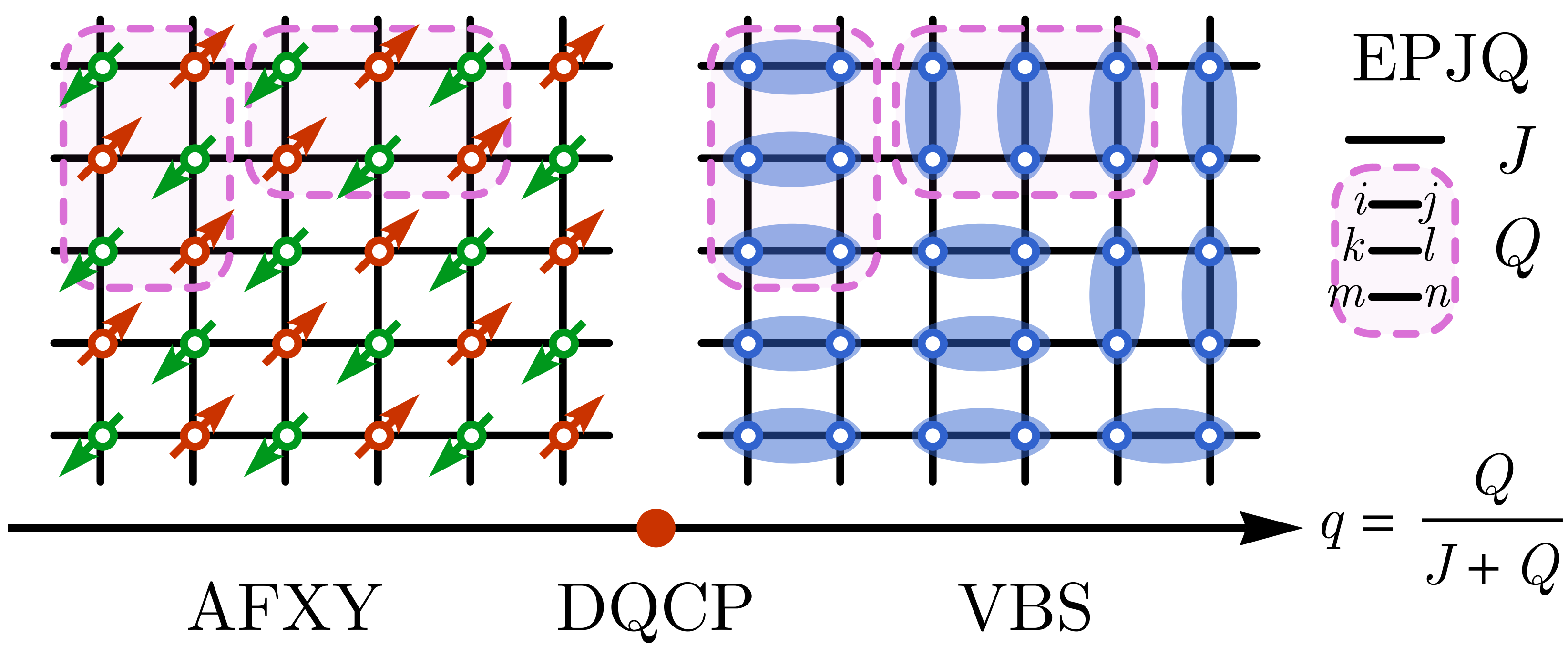
- Beyond Landau-Ginzburg-Wilson (LGW) paradigm.
- Emergent, fractionalized degrees of freedom (instead of fluctuating order parameters)
- A quantum spin liquid at and only at the critical point.
- **Question**: What does the spin excitation spectrum look like?

Lattice Models to Simulate

- Much of the work relies on large-scale Quantum Monte Carlo (QMC) simulation + stochastic analytic continuation (SAC)
- **Easy-plane J-Q (EPJQ) Model**

$$H_{\text{EPJQ}} = -J \sum_{\langle ij \rangle} \left(P_{ij} + \Delta\, S_i^z\, S_j^z \right) - Q \sum_{\langle ijklmn \rangle} P_{ij}\, P_{kl}\, P_{mn}.$$

  - Square lattice, spin-1/2 $\boldsymbol{S}_i$ per site, anisotropy $\Delta = 0.5$
  - Singlet projection operator on bond $P_{ij} = \frac{1}{4} - \boldsymbol{S}_i \cdot \boldsymbol{S}_j$



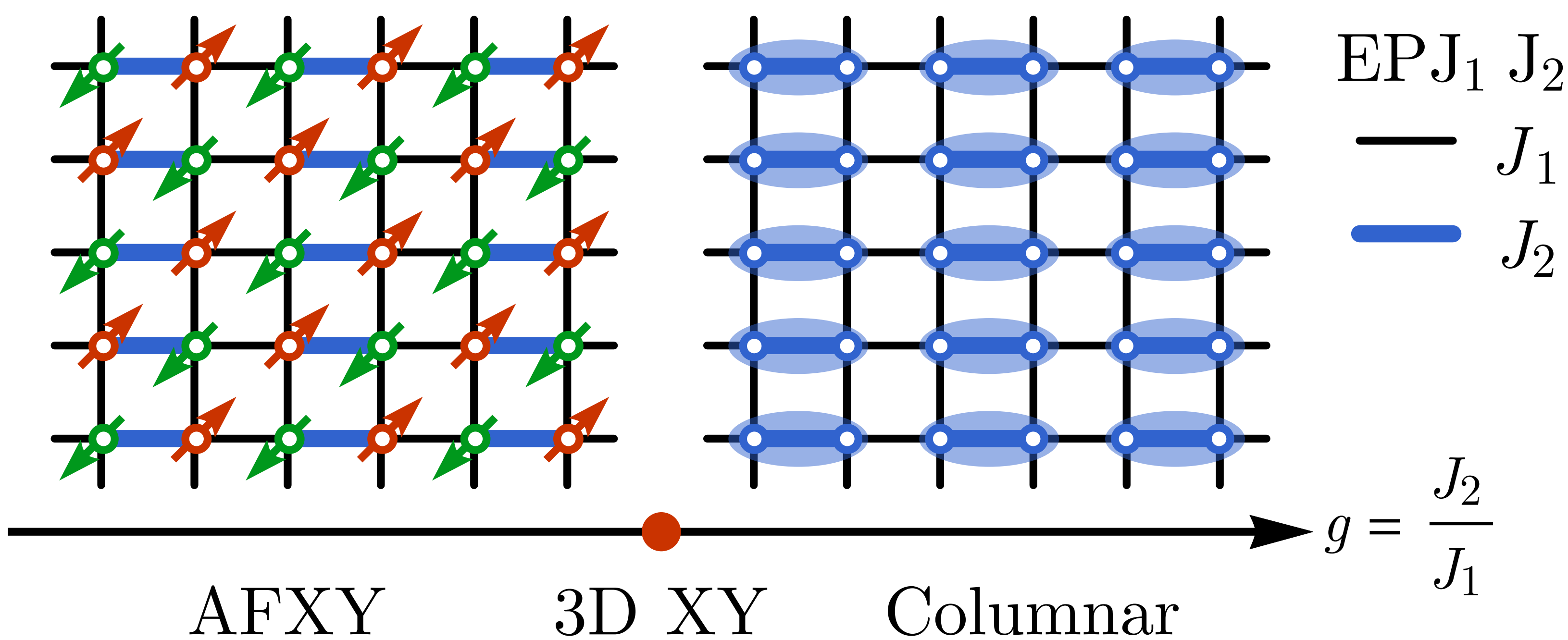
Qin, He, You, Lu, Sen, Sandvik, Xu, Meng (2017)

Lattice Models to Simulate

- Much of the work relies on large-scale Quantum Monte Carlo (QMC) simulation + stochastic analytic continuation (SAC)
- **Easy-plane J_1-J_2 (EPJ_1J_2) Model** (as "control group")

$$H_{\mathrm{EP\ J_1\ J_2}} = J_1 \sum_{\langle ij\rangle'} D_{ij} + J_2 \sum_{\langle ij\rangle''} D_{ij}.$$

  - Square lattice, spin-1/2 $\boldsymbol{S}_i$ per site + easy-plane anisotropy
  - XXZ coupling on bond $D_{ij} = S_i^x\ S_j^x + S_i^y\ S_j^y + \frac{1}{2}\ S_i^z\ S_j^z$



Dynamic Spin Structure Factor

- Measure spin correlation function in the imaginary time

$$S^{ab} = \left\langle S^a_{-\boldsymbol{q}}(\tau)\, S^b_{\boldsymbol{q}}(0) \right\rangle \;\; (a,\; b = x,\; y,\; z) \qquad S^a_{\boldsymbol{q}} = L^{-1} \sum_i e^{-\boldsymbol{i}\,\boldsymbol{q}\cdot\boldsymbol{r}_i}\, S^a_i$$

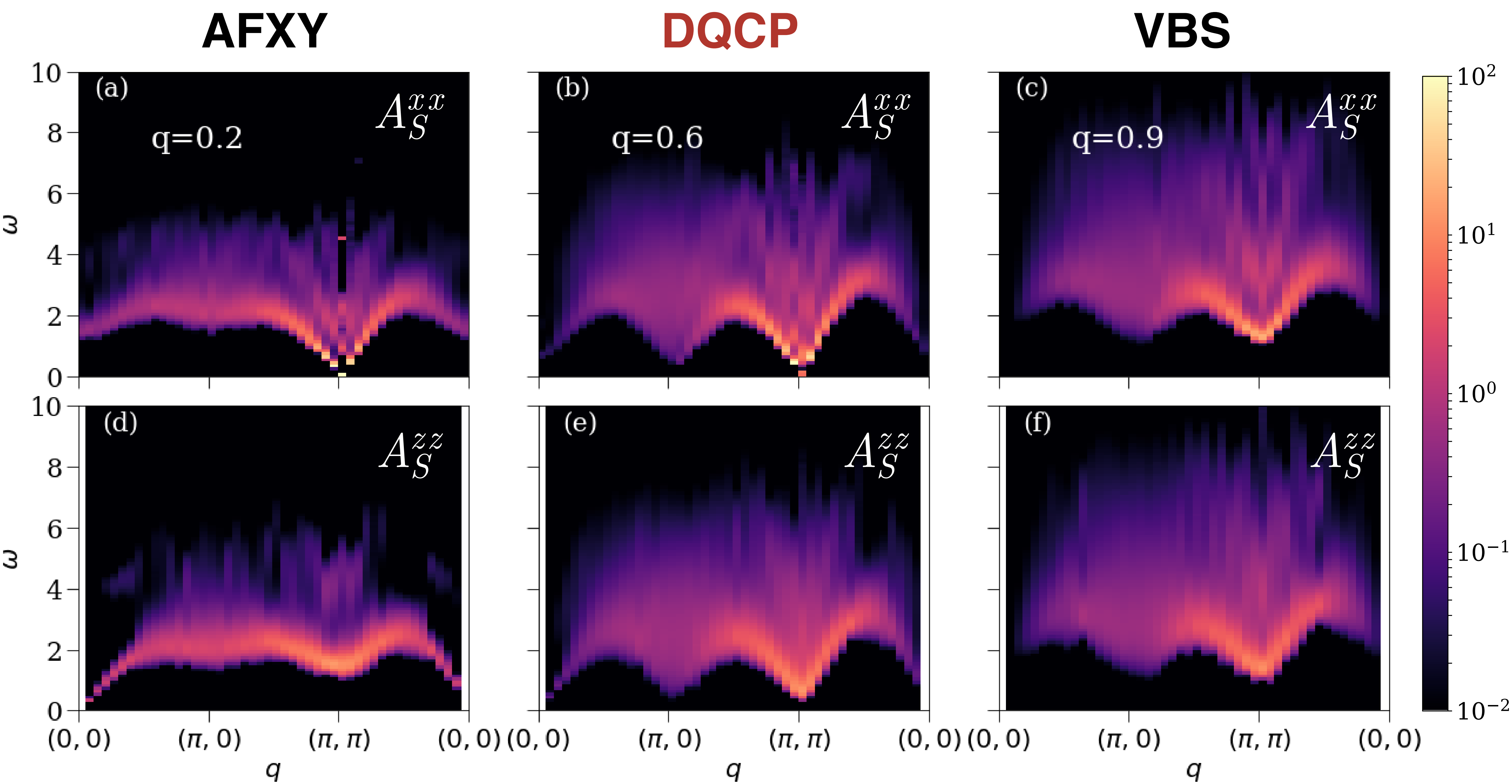
- Stochastically propose candidate dynamic spin structure factors $A^{ab}_S(\boldsymbol{q},\, \omega)$ to fit the imaginary time data

$$S^{ab}(\boldsymbol{q}, \tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \frac{e^{-\tau\omega} + e^{-(\beta-\tau)\omega}}{1 - e^{-\beta\omega}} A^{ab}_S(\boldsymbol{q}, \omega)$$

- Dynamic spin structure factor $A^{ab}_S(\boldsymbol{q},\, \omega)$
  - Detectable by multiple experimental techniques: INS, RIXS, NMR …
  - Spectral features can be observed at finite temperature (inside the quantum critical fan)

Numerical Result (QMC+SAC)

- Spin excitation spectrum for EPJQ model



Ma, Sun, You, Xu, Vishwanath, Sandvik, Meng, arXiv:1803.01180

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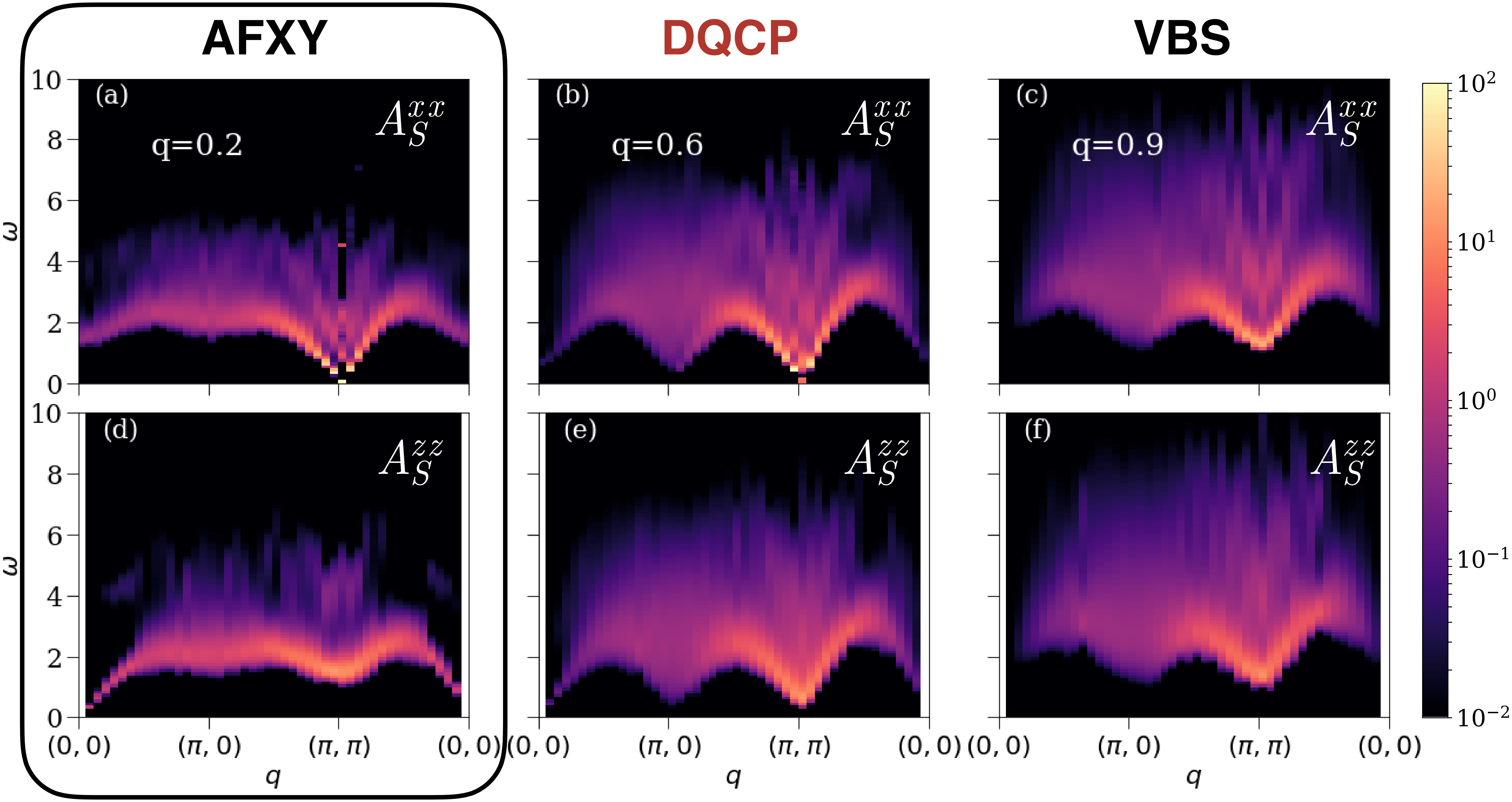
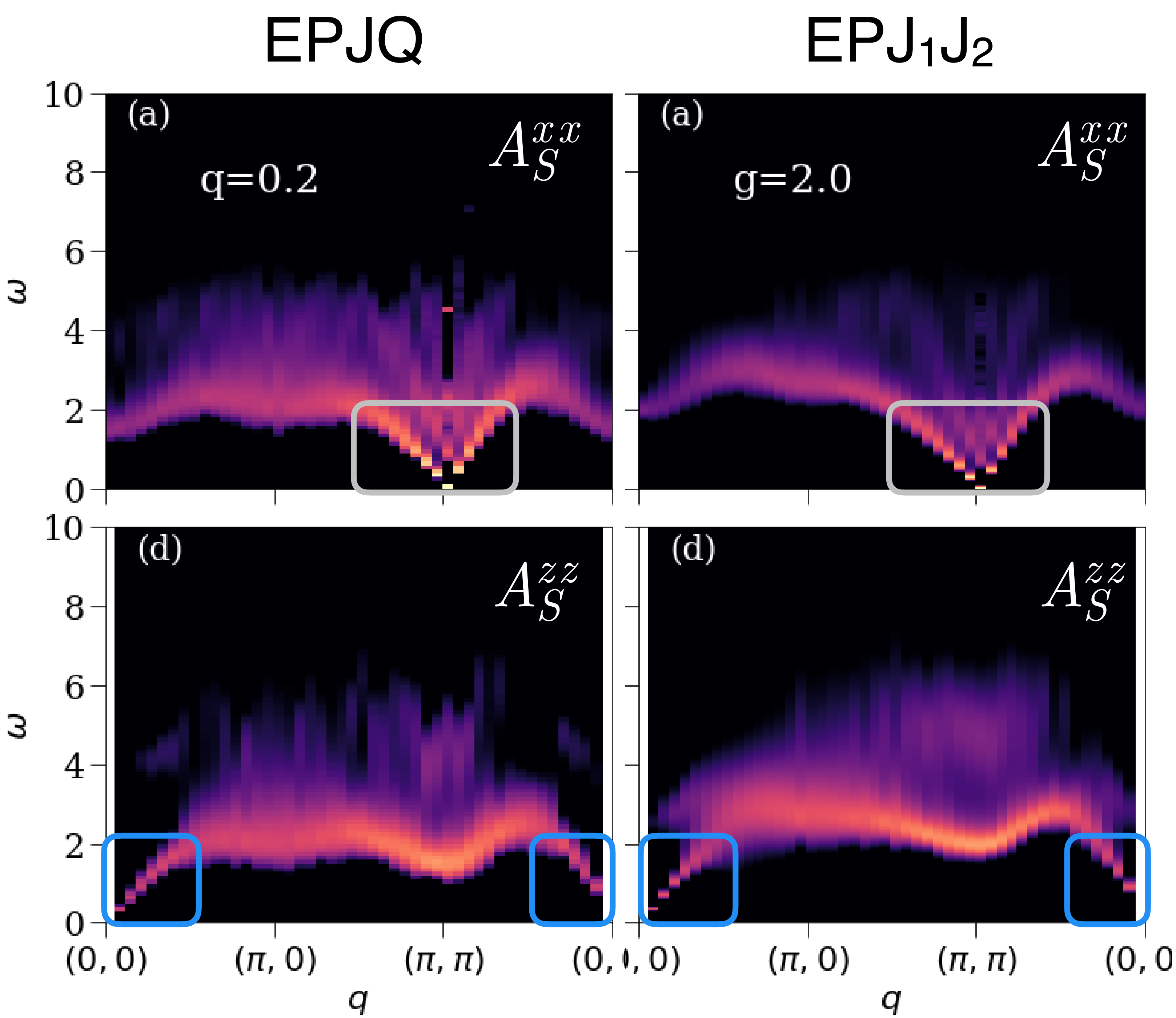


Ma, Sun, You, Xu, Vishwanath, Sandvik, Meng, arXiv:1803.01180

Antiferromagnetic XY Phase

- Spectrum in the XY ordered phase



- Order parameter

$$n_x + i\, n_y \sim e^{i\,\theta}$$

$$(\boldsymbol{n} \sim (-)^i\, \boldsymbol{S}_i)$$

- XY model

stiffness

$$\mathcal{L} = \frac{\rho_s}{2}(\partial_\mu \theta)^2$$

- **Goldstone mode** at (π, π) in A_S^{xx}
- **Spin density mode**

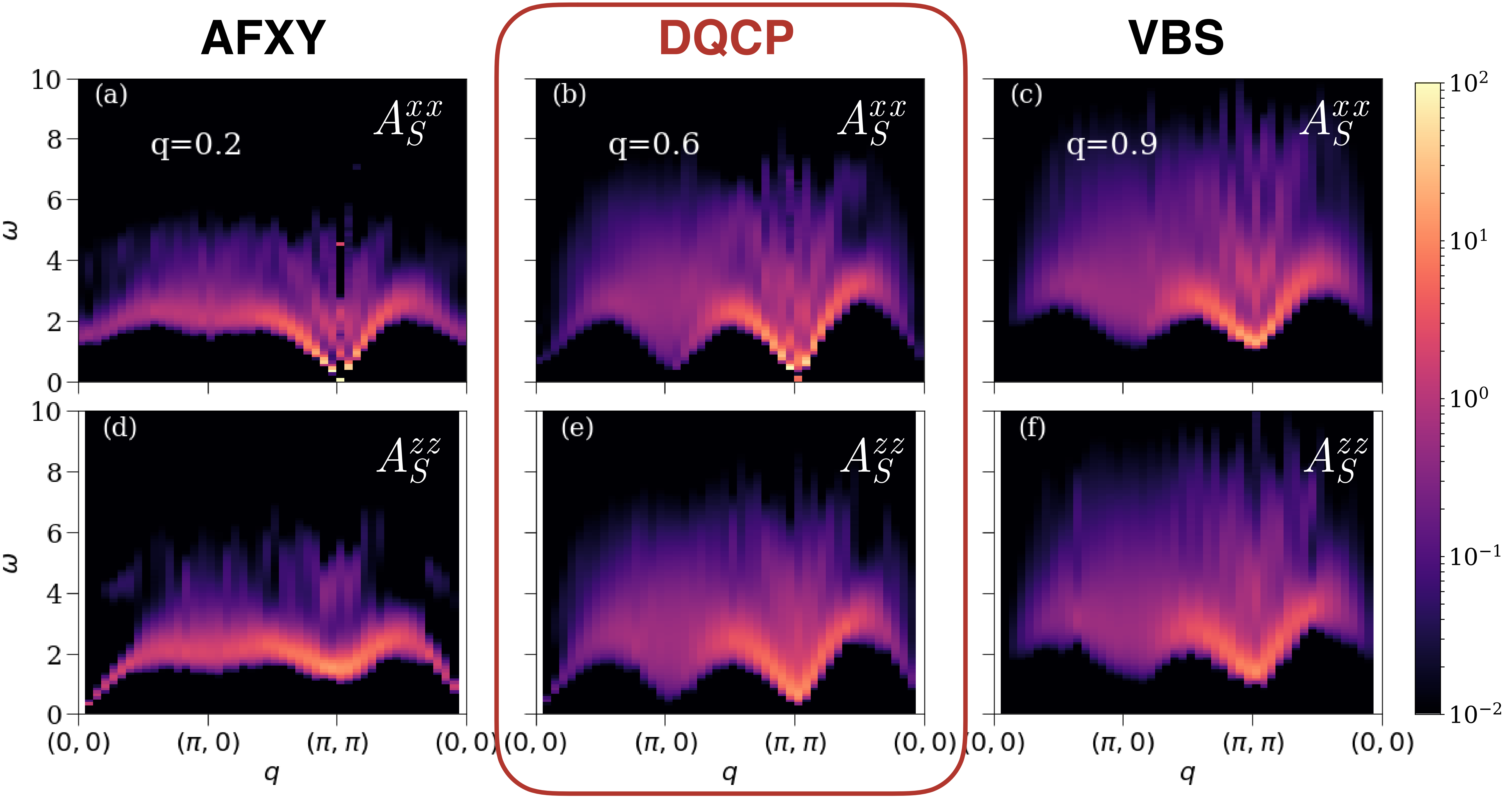
$$\partial_t \theta = n_x\, \partial_t n_y - n_y\, \partial_t n_x$$

at $(0, 0)$ in A_S^{zz}

$$\langle \partial_t \theta\, \partial_t \theta \rangle \sim \frac{\omega^2}{\boldsymbol{q}^2 - \omega^2}$$

Numerical Result (QMC+SAC)

- Spin excitation spectrum for EPJQ model

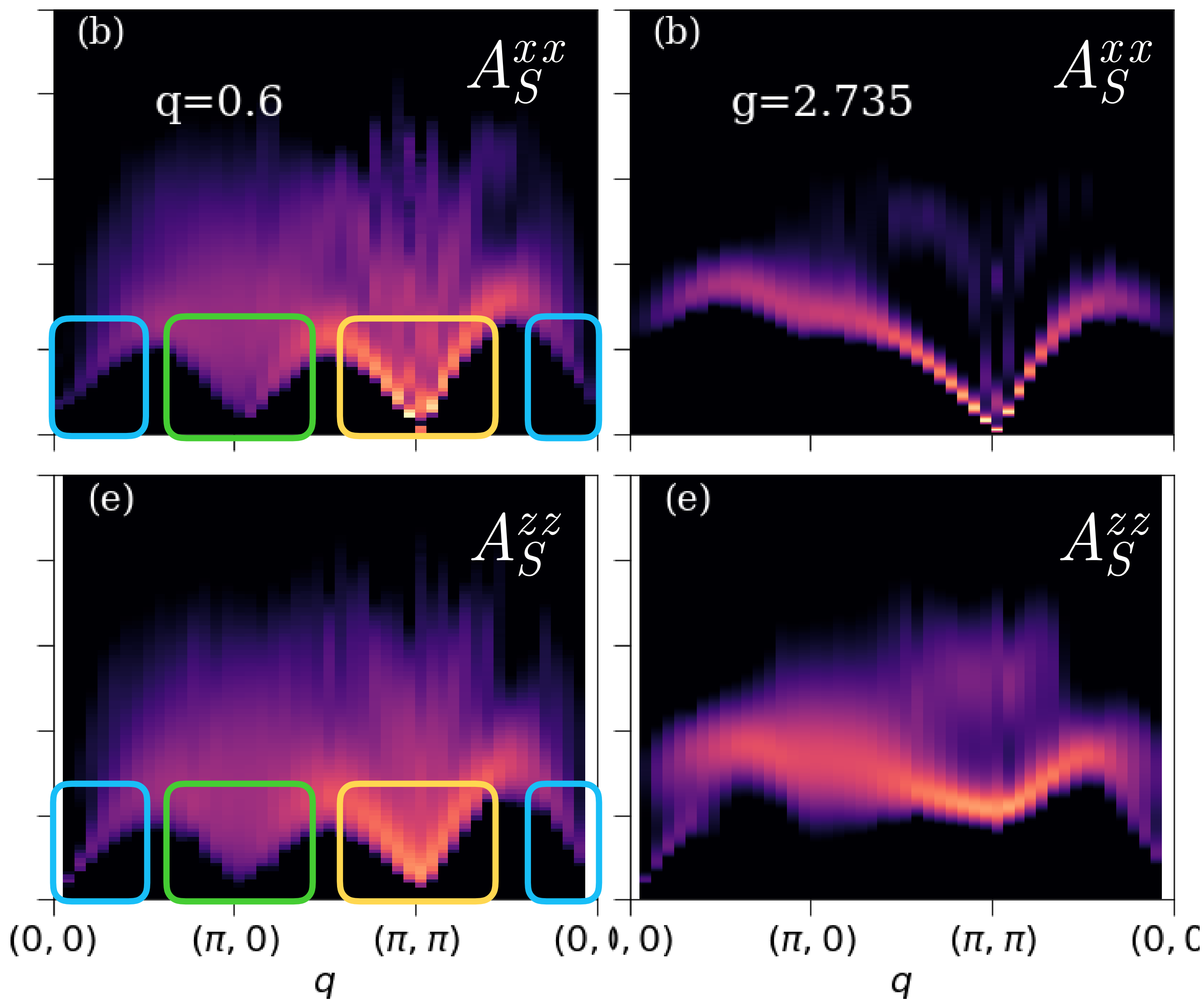


Ma, Sun, You, Xu, Vishwanath, Sandvik, Meng, arXiv:1803.01180

Deconfined Quantum Critical Point (DQCP)

- Spectrum: DQCP v.s. 3D XY

EPJQ (**DQCP**) EPJ$_1$J$_2$ (3DXY)



Néel: n_x, n_y, n_z; VBS: v_x, v_y

- **Spinon continuum** at (π, π) in A_S^{xx}, A_S^{zz}

$$\boldsymbol{n} \sim z^\dagger \boldsymbol{\sigma} z$$

- **Spin density mode** at $(0, 0)$ in A_S^{xx}, A_S^{zz}

$$\boldsymbol{n} \times \partial_t \boldsymbol{n}$$

- **XY-VBS current fluctuation** at $(\pi, 0)$, $(0, \pi)$ in A_S^{xx}, A_S^{zz}

$$\boldsymbol{n} \nabla \cdot \boldsymbol{v} - \boldsymbol{v} \cdot \nabla \boldsymbol{n}$$

Deconfined Quantum Critical Point (DQCP)

- **Spinon continuum**

$$\langle n_x n_x \rangle \sim (\boldsymbol{q}^2 - \omega^2)^{-1+\eta_{xy}/2}$$

$$\langle n_z n_z \rangle \sim (\boldsymbol{q}^2 - \omega^2)^{-1+\eta_z/2}$$

DQCP

$$\eta_{xy} \approx 0.13, 0.33$$

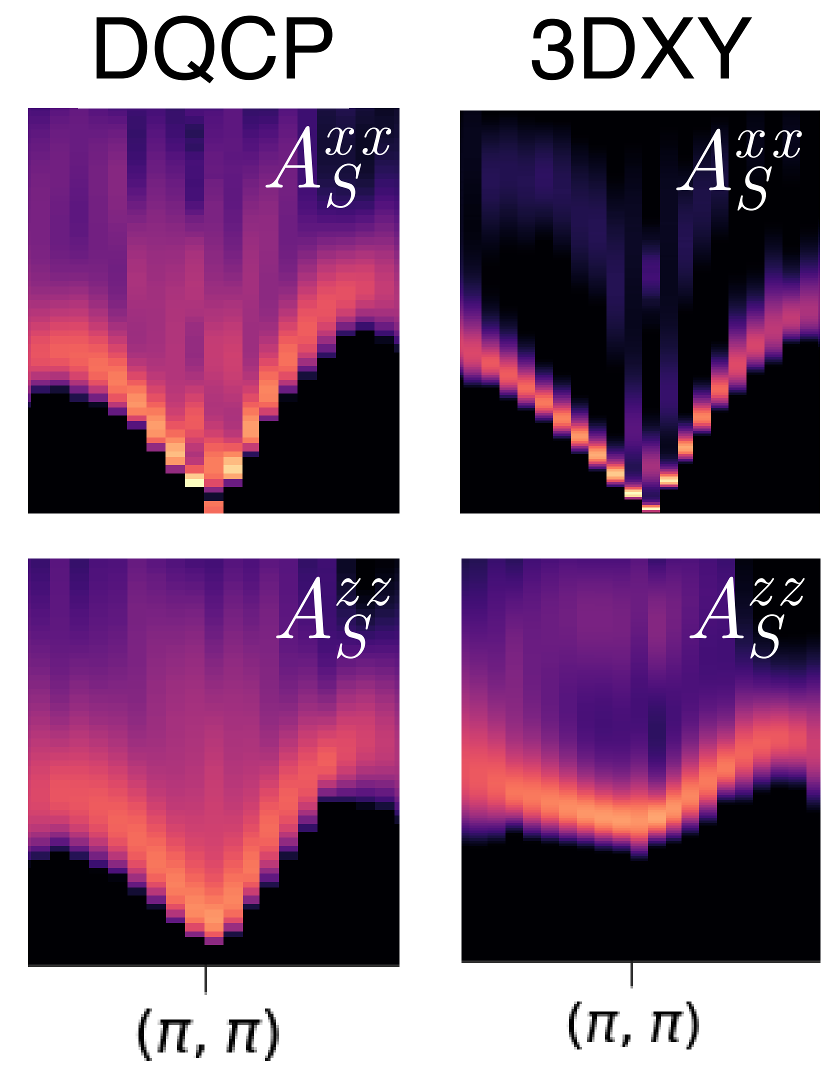
$$\eta_z \approx 0.91$$

Qin, He, You, Lu, Sen, Sandvik, Xu, Meng (2017)
Zhang, He, Eggert, Moessner, Pollmann (2017)

3D XY

$$\eta_{xy} \approx 0.04$$

Guida, Zinn-Justin (1998)



- DQCP: $n_z \sim z^\dagger \sigma^z z \sim \mathrm{d}b$

$$\mathcal{L}[z,a] = |(\partial - \mathrm{i}a - \mathrm{i}A_s \tfrac{\sigma^z}{2})z|^2 + \tfrac{\mathrm{i}}{2\pi} A_v \wedge \mathrm{d}a + \cdots$$

$$\mathcal{L}[w,b] = |(\partial - \mathrm{i}b - \mathrm{i}A_v \tfrac{\sigma^z}{2})w|^2 + \tfrac{\mathrm{i}}{2\pi} A_s \wedge \mathrm{d}b + \cdots$$

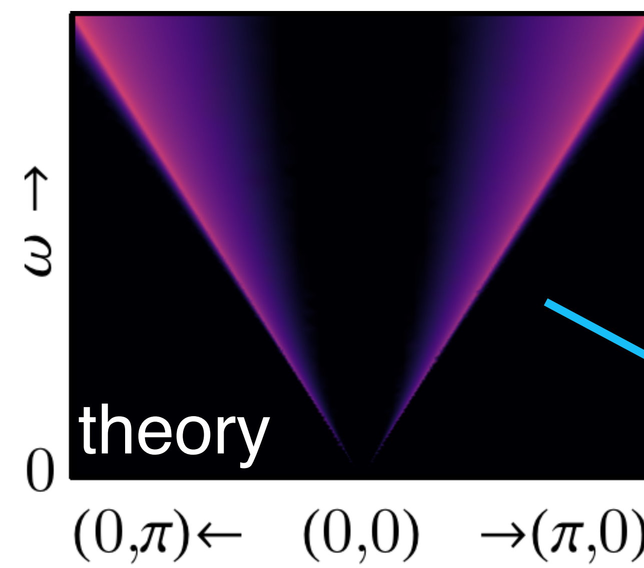
- 3D XY: n_z not associated with the critical order parameter

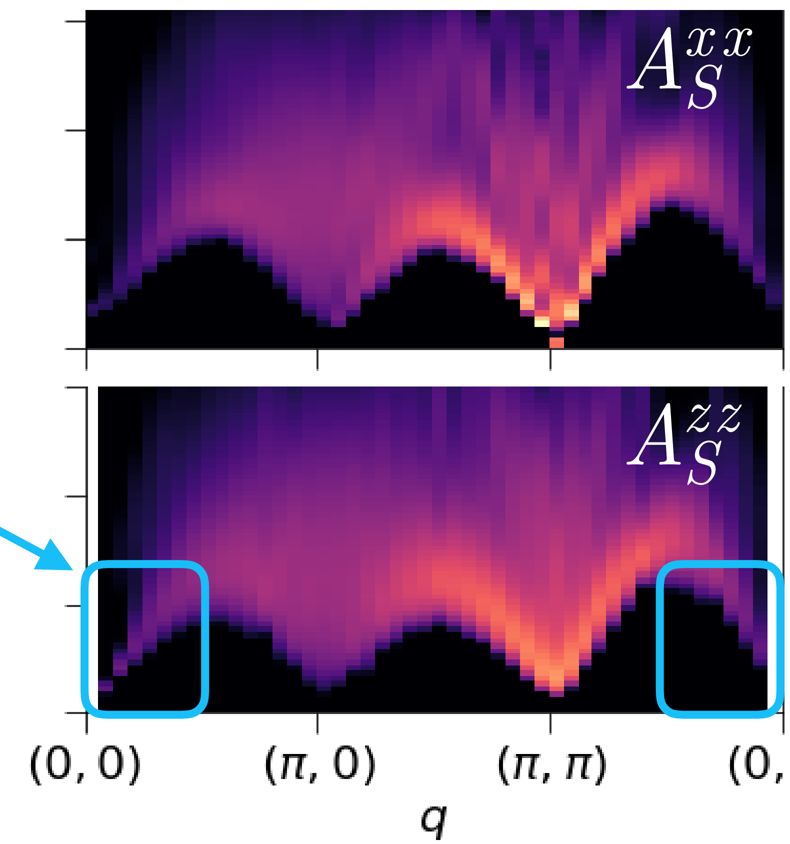
Deconfined Quantum Critical Point (DQCP)

- **Spin density fluctuation** $\boldsymbol{n} \times \partial_t \boldsymbol{n}$
  - Always appear at momentum $(0,0) = (\pi,\pi) + (\pi,\pi)$
- In A_S^{zz} channel, $j_{12}^0 = n_x \partial_t n_y - n_y \partial_t n_x$
  conserved charge of $\mathrm{U}(1)_\mathrm{s}$

$$\langle j_{12}^0 j_{12}^0 \rangle \sim \frac{\boldsymbol{q}^2}{(\boldsymbol{q}^2 - \omega^2)^{1/2}}$$

scaling dim = 2 fixed





- In A_S^{xx} channel,
  $j_{23}^0 = n_y \partial_t n_z - n_z \partial_t n_y$
  not conserved, but still critical (given n_y, n_z criticality)

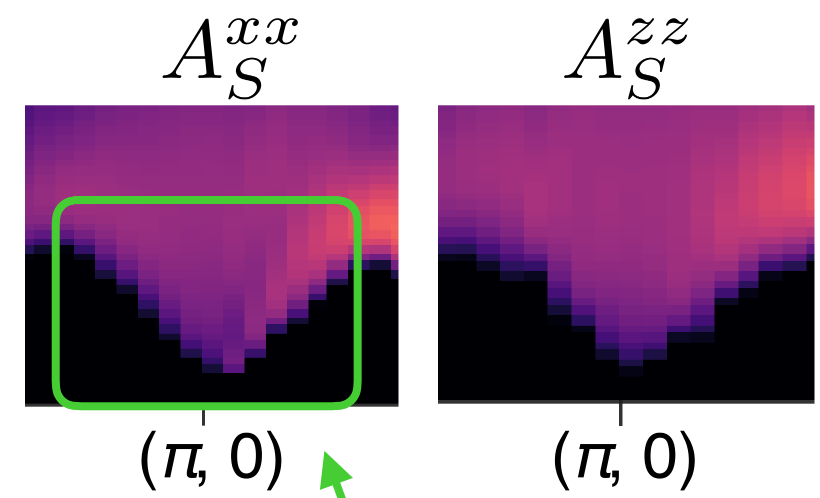
$$\langle j_{23}^0 j_{23}^0 \rangle \sim \frac{\boldsymbol{q}^2}{(\boldsymbol{q}^2 - \omega^2)^{(1-\eta_j)/2}}$$

← same form factor

← acquires anomalous dim η_j

Deconfined Quantum Critical Point (DQCP)

- **XY-VBS current fluctuation** $\boldsymbol{n}\nabla\cdot\boldsymbol{v}-\boldsymbol{v}\cdot\nabla\boldsymbol{n}$
  - Appears at momentum $(\pi,0),(0,\pi)$
  - Gapless at and only at the DQCP
  - Unique for DQCP (absent in 3D XY)

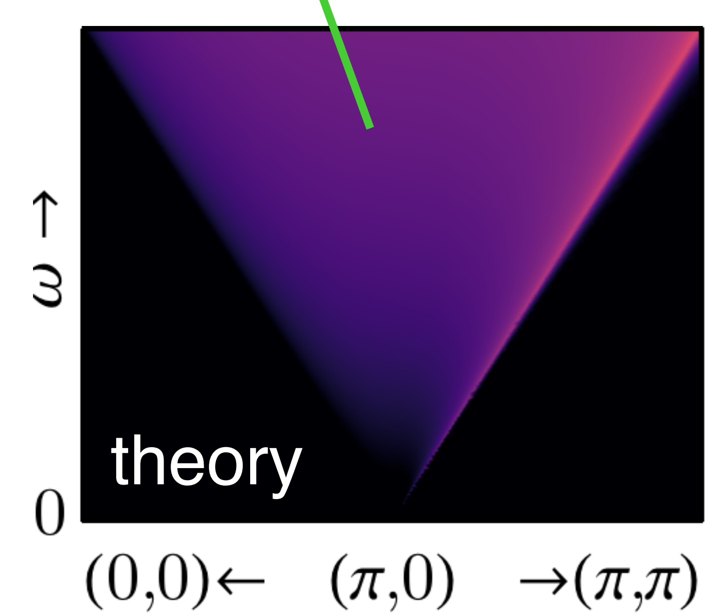


- Take the $(\pi,0)$ continuum for example
  - A_S^{xx} channel: $j^2_{15}=n_x\partial_y v_y-v_y\partial_y n_x$
    emergent O(4) symmetry → emergent conserved current

$$\langle j^2_{15}j^2_{15}\rangle\sim\frac{\omega^2-q_x^2}{(\boldsymbol{q}^2-\omega^2)^{1/2}}$$

  - A_S^{zz} channel: $j^2_{35}=n_z\partial_y v_y-v_y\partial_y n_z$

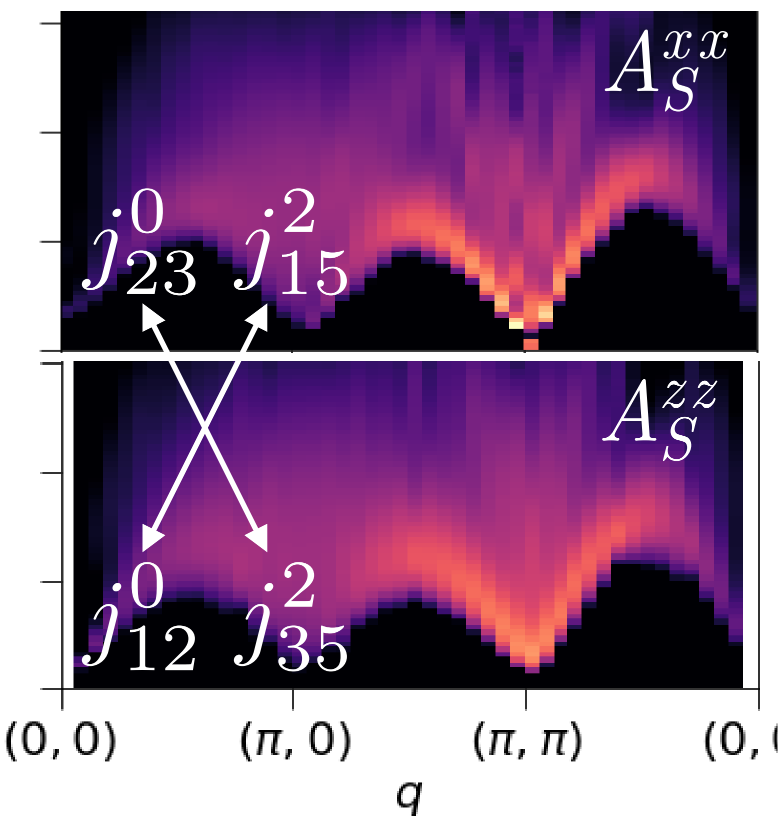
$$\langle j^2_{35}j^2_{35}\rangle\sim\frac{\omega^2-q_x^2}{(\boldsymbol{q}^2-\omega^2)^{(1-\eta_j)/2}}$$



Deconfined Quantum Critical Point (DQCP)

- Most generally, consider the O(5) order parameter
$$\boldsymbol{N} = (N_1, N_2, N_3, N_4, N_5) = (n_x, n_y, n_z, v_x, v_y)$$
  - SO(5) currents $j^{\mu}_{ab} = N_a \partial_\mu N_b - N_b \partial_\mu N_a$
  - With anisotropy, $N_3 = n_z$ singles out, emergent symmetry is only O(4), that rotates (N_1, N_2, N_4, N_5)

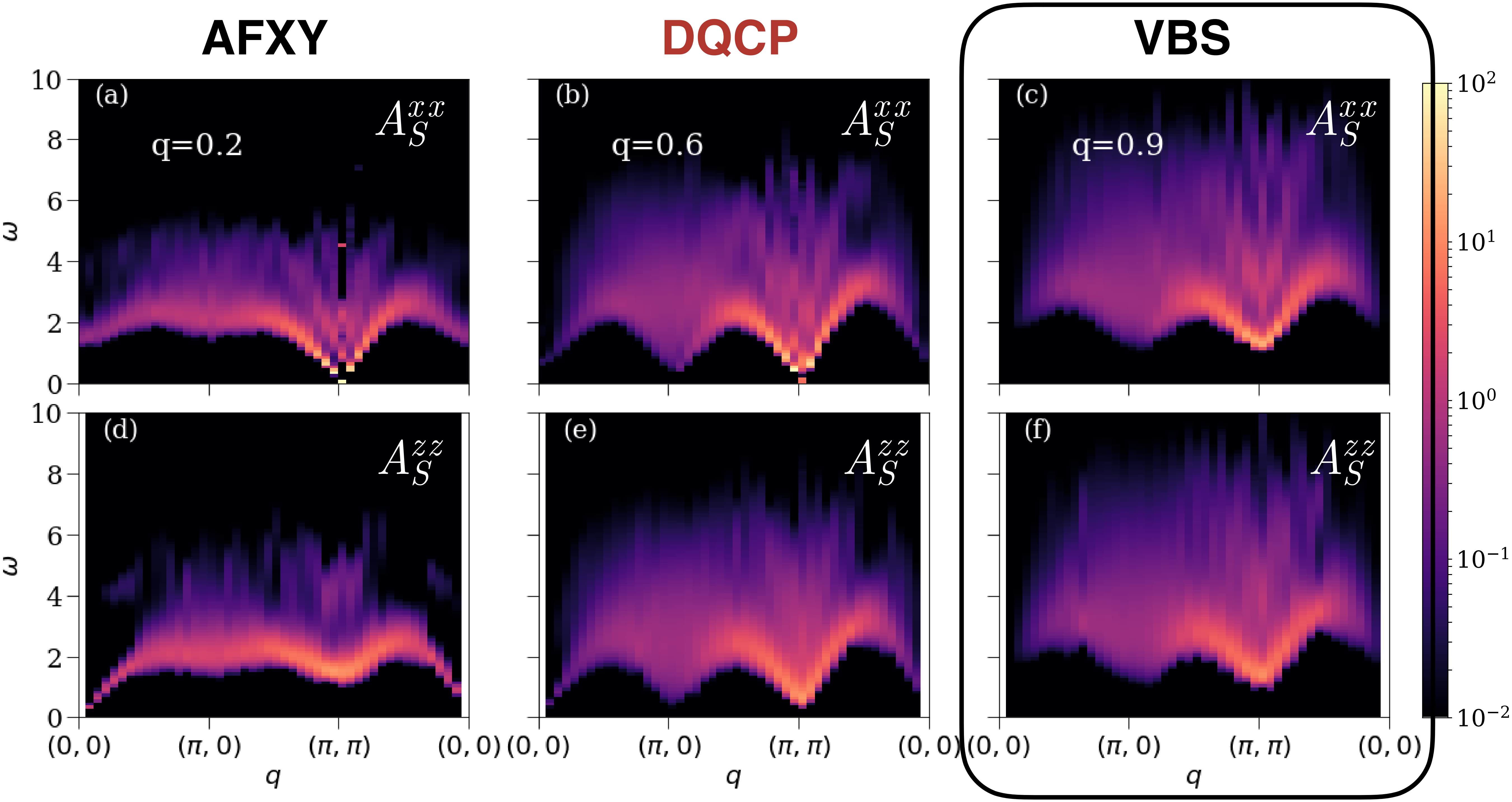
EPJQ (**DQCP**)



- (Emergent) conserved current
$$\langle j^0_{12} j^0_{12} \rangle \sim \frac{\boldsymbol{q}^2}{(\boldsymbol{q}^2 - \omega^2)^{1/2}},$$
$$\langle j^2_{15} j^2_{15} \rangle \sim \frac{\omega^2 - q_x^2}{(\boldsymbol{q}^2 - \omega^2)^{1/2}},$$
- Non-conserved current
$$\langle j^0_{23} j^0_{23} \rangle \sim \frac{\boldsymbol{q}^2}{(\boldsymbol{q}^2 - \omega^2)^{(1-\eta_j)/2}},$$
$$\langle j^2_{35} j^2_{35} \rangle \sim \frac{\omega^2 - q_x^2}{(\boldsymbol{q}^2 - \omega^2)^{(1-\eta_j)/2}}.$$

Numerical Result (QMC+SAC)

- Spin excitation spectrum for EPJQ model

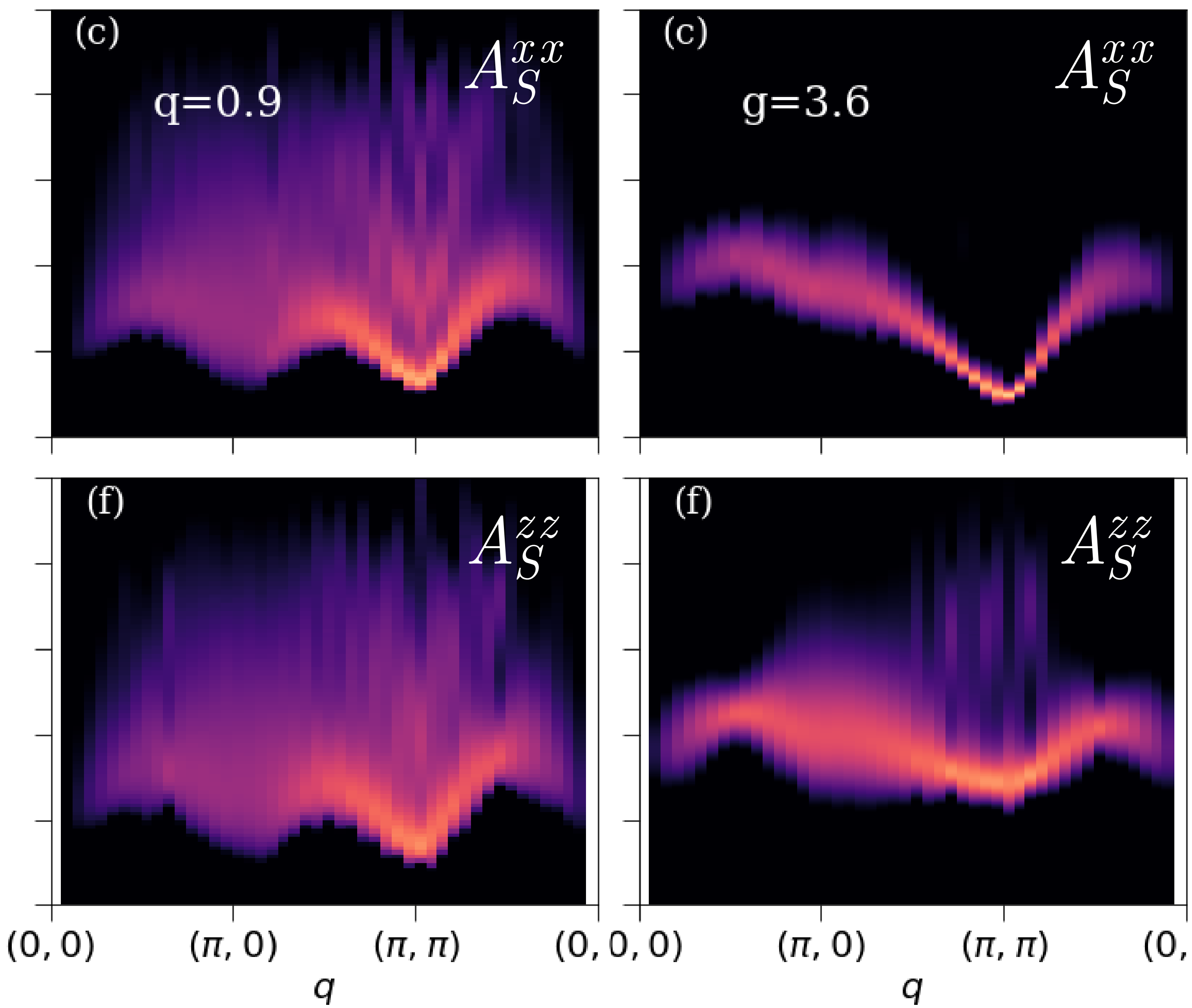


Ma, Sun, You, Xu, Vishwanath, Sandvik, Meng, arXiv:1803.01180

Valence Bond Solid and Columnar Dimmer

- Spectrum in dimmer phases (spontaneous or explicit SB)

EPJQ (VBS) EPJ$_1$J$_2$ (Dimmer)



- EPJQ: spontaneous symmetry breaking
- EPJ$_1$J$_2$: dimmer pattern pinned by J$_2$
- All spin excitations are gapped.
- Broader spectrum in VBS phase, may be related to the large confinement length scale.

Spinon Mean-Field Theory

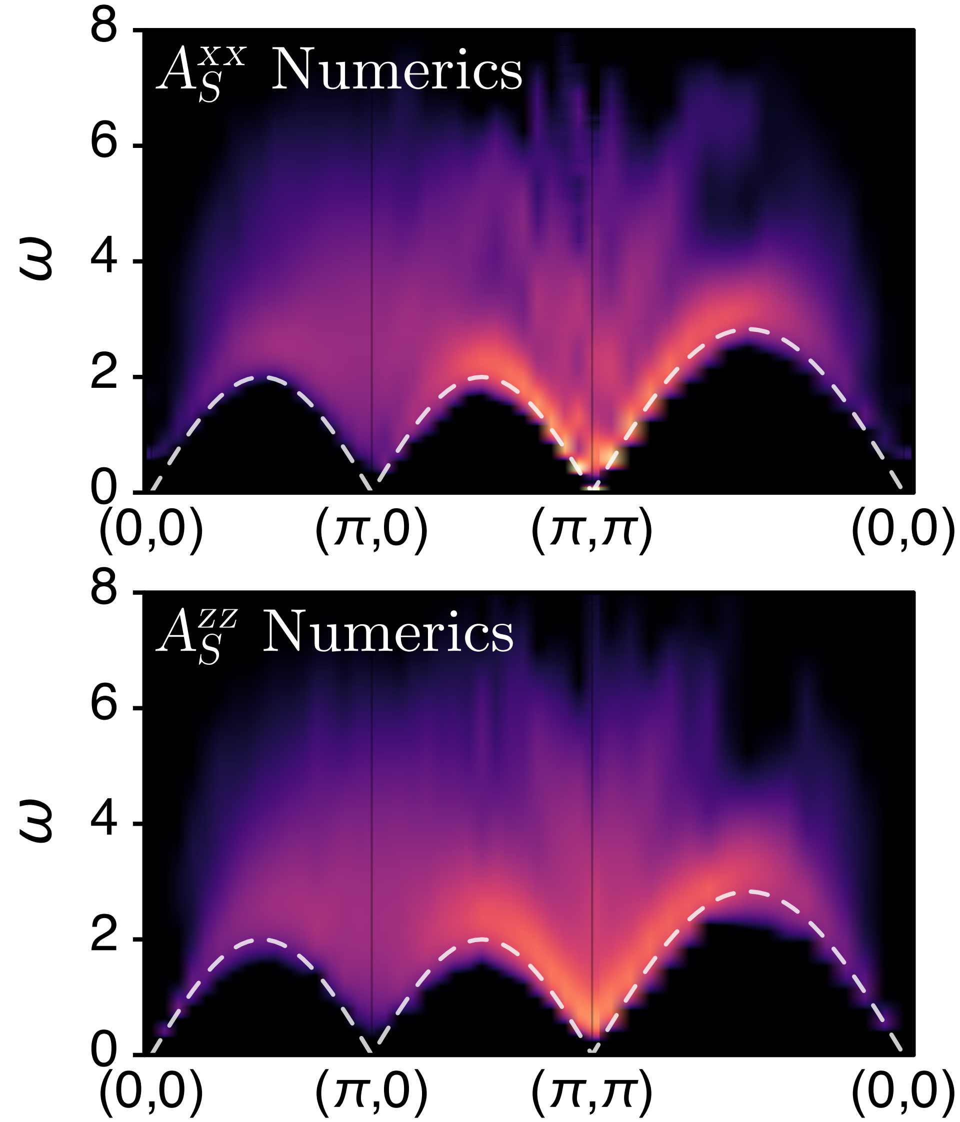
- The lower edge of the continuum can be nicely fitted by

$$\epsilon_{\boldsymbol{k}} = t(\sin^2 k_x + \sin^2 k_y)^{1/2}$$

The parton dispersion of square lattice π-flux state.

Hermele, Senthil, Fisher (2005), Ran, Wen (2006)

at DQCP

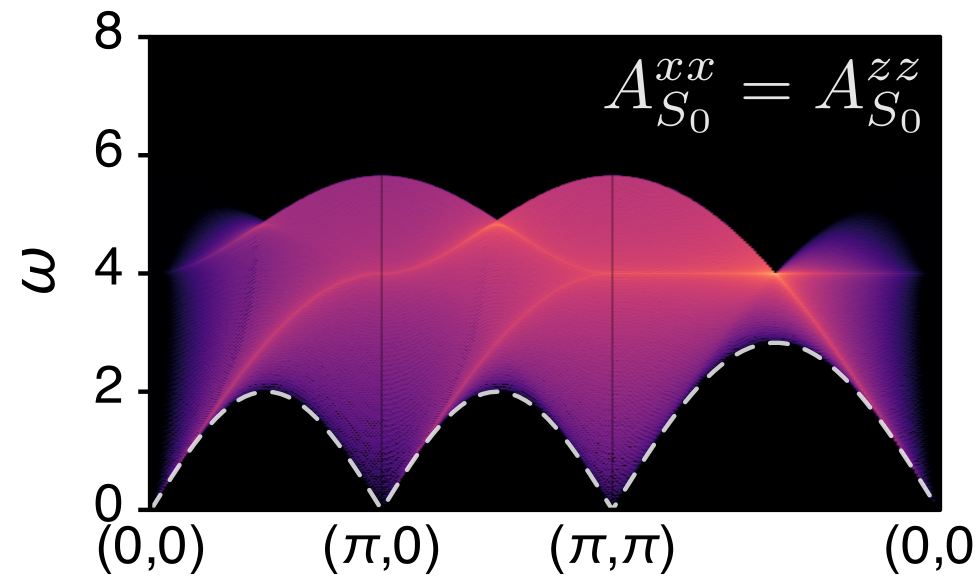


- $N_f = 2$ QCD$_3$ (or $N_f = 4$ QED$_3$) theory + easy-plane anisotropy
  - Fermionic spinon $\boldsymbol{S}_i = \frac{1}{2} f_i^\dagger \boldsymbol{\sigma} f_i$
  - Mean-field Hamiltonian

$$H_{\mathrm{MF}} = \sum_i \mathrm{i}(f_{i+\hat{x}}^\dagger f_i + (-)^x f_{i+\hat{y}}^\dagger f_i) + \mathrm{H.c.}$$

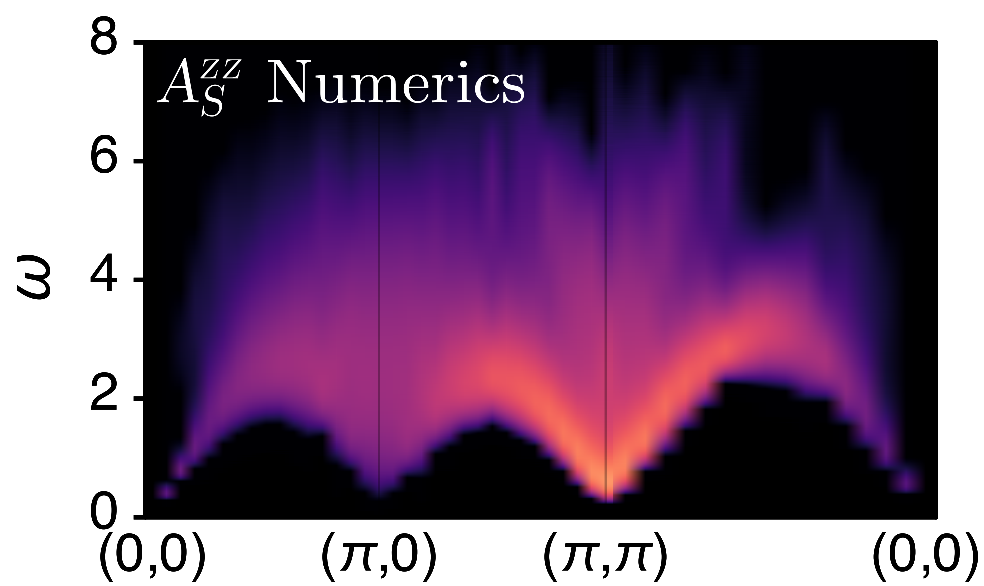
Spinon Mean-Field Theory

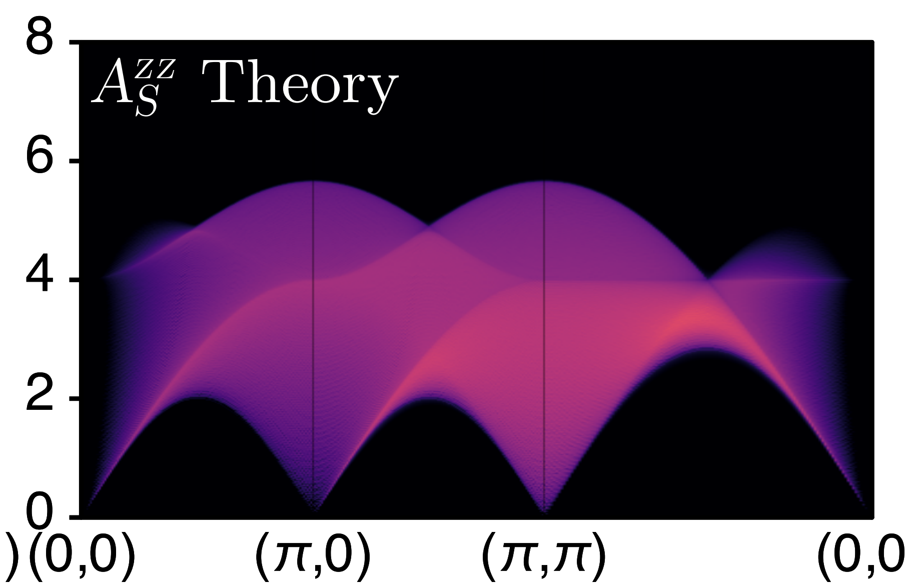
- Ignore gauge fluctuation, calculate dynamic spin structure factor on the mean-field (free fermion) level
  - Captures the overall shape
  - Can be improved by simple RPA correction



$$S^{aa}(\boldsymbol{q},\omega) = \frac{S_0^{aa}(\boldsymbol{q},\omega)}{1 + J_a S_0^{aa}(\boldsymbol{q},\omega)}$$

Anisotropy enters as $J_x = J_y > J_z$





$N_f = 4$ QED Theory

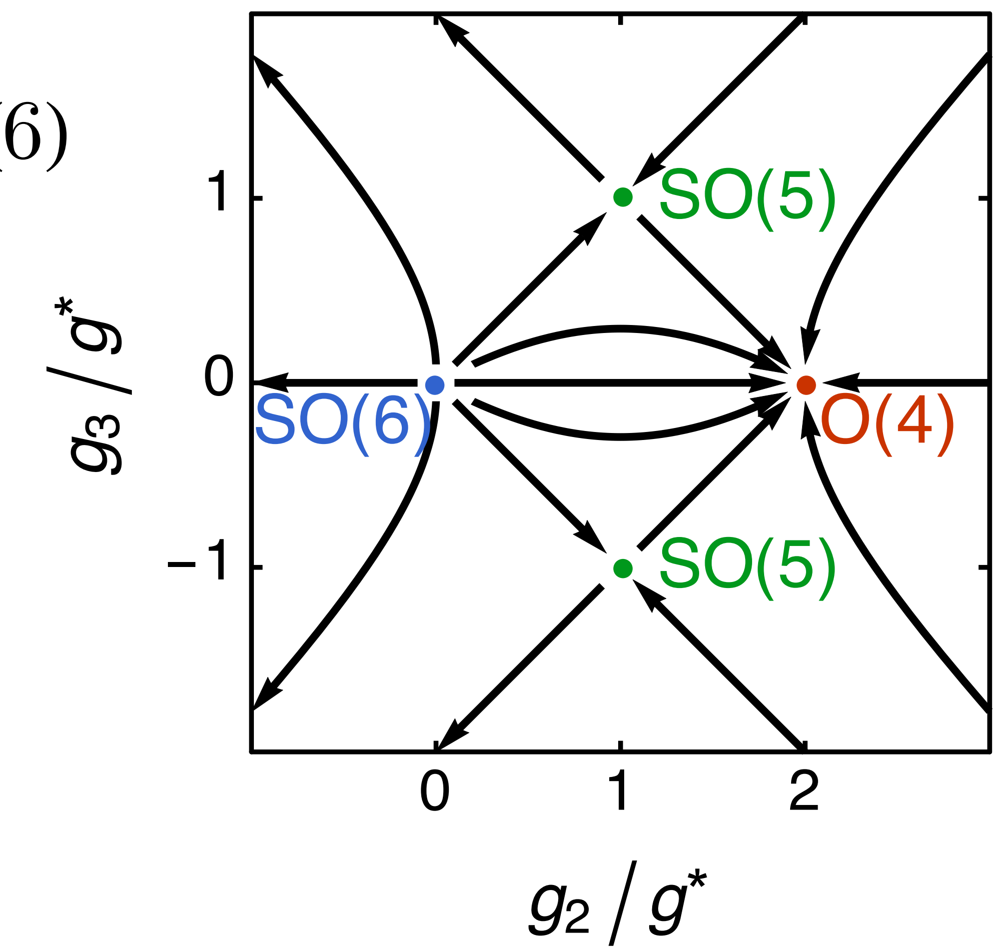
- This perspective is consistent with RG analysis of the $N_f = 4$ QED theory (large-N_f expansion, one-loop)

$$\bar{\psi}_i \gamma^\mu D_\mu \psi_i + g \bar{\psi}_i \bar{\psi}_j \psi_k \psi_l + \cdots \qquad (i,j,k,l = 1,2,3,4)$$

- Flavor symmetry $\mathrm{SU}(4)/\mathbb{Z}_2 = \mathrm{SO}(6)$ broken down by interaction

$$g_2\big(\overline{\psi}_1\,\overline{\psi}_2\,\psi_2\,\psi_1 + \overline{\psi}_3\,\overline{\psi}_4\,\psi_4\,\psi_3\big)$$
$$g_3\big(\overline{\psi}_1\,\overline{\psi}_2\,\psi_4\,\psi_3 + \overline{\psi}_3\,\overline{\psi}_4\,\psi_2\,\psi_1\big)$$

- RG fixed point found at finite interaction with emergent O(4) symmetry.



You, He, Vishwanath, Xu
PRB **97**, 125112 (2018)

The Mini-Web of Dualities

- Many dual descriptions for easy-plane DQCP
  - NCCP[1]

    $\sum_{i=1}^{2} |D_a\, z_i|^2 + |z_1|^4 + |z_2|^4 + \ldots$

    $\sum_{i=1}^{2} |D_b\, w_i|^2 + |w_1|^4 + |w_2|^4 + \ldots$

    self-dual

  - $N_f = 2$ QED

    $\sum_{i=1}^{2} \overline{\psi}_i\, \gamma \cdot D_a\, \psi_i + \ldots$

    $\sum_{i=1}^{2} \overline{\chi}_i\, \gamma \cdot D_b\, \chi_i + \ldots$

    self-dual

  NCCP[1] ⟷ dual ⟷ $N_f = 2$ QED

  Easy-plane DQCP

  - O(4) NLSM + topological Θ = π
  - $N_f = 2$ QCD (or $N_f = 4$ QED)

    $\mathrm{Tr}\, \overline{\Psi} \gamma \cdot D_a\, \Psi + g\, \overline{\Psi}\,\overline{\Psi}\Psi\Psi + \ldots$

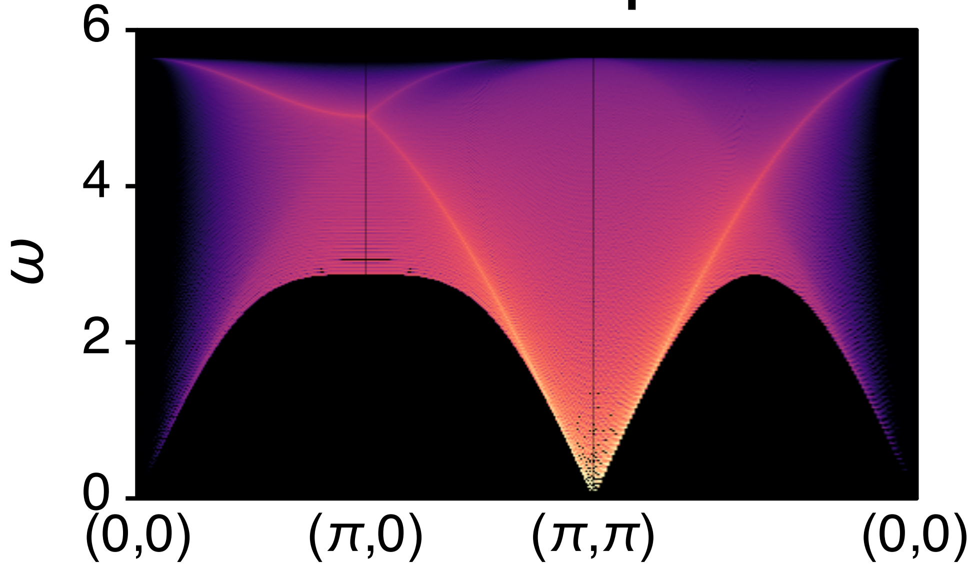
Wang, Nahum, Metlitski, Xu, Senthil (2017)

- All theories are believed to be equivalent, but some are more convenient to handle by mean-field treatment.

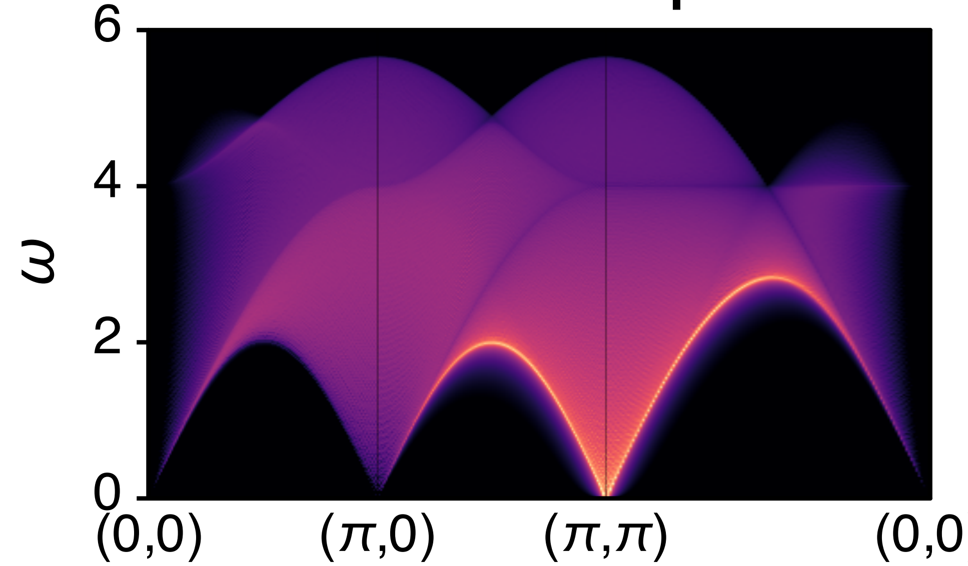
Spinon Mean-Field Theory

- For example, if we take the NCCP¹ theory
  - Swinger boson $\boldsymbol{S}_i = \frac{1}{2} b_i^\dagger \boldsymbol{\sigma} b_i$ Read, Sachdev (1990)
  - Mean-field $H_{\mathrm{MF}} = -\sum_{\langle ij \rangle} (\epsilon^{\alpha\beta} b_{i\alpha} b_{j\beta} + \mathrm{h.c.}) - \mu \sum_i b_i^\dagger b_i$
  - Will not capture the XY-VBS current continuum

Bosonic spinon



Fermionic spinon

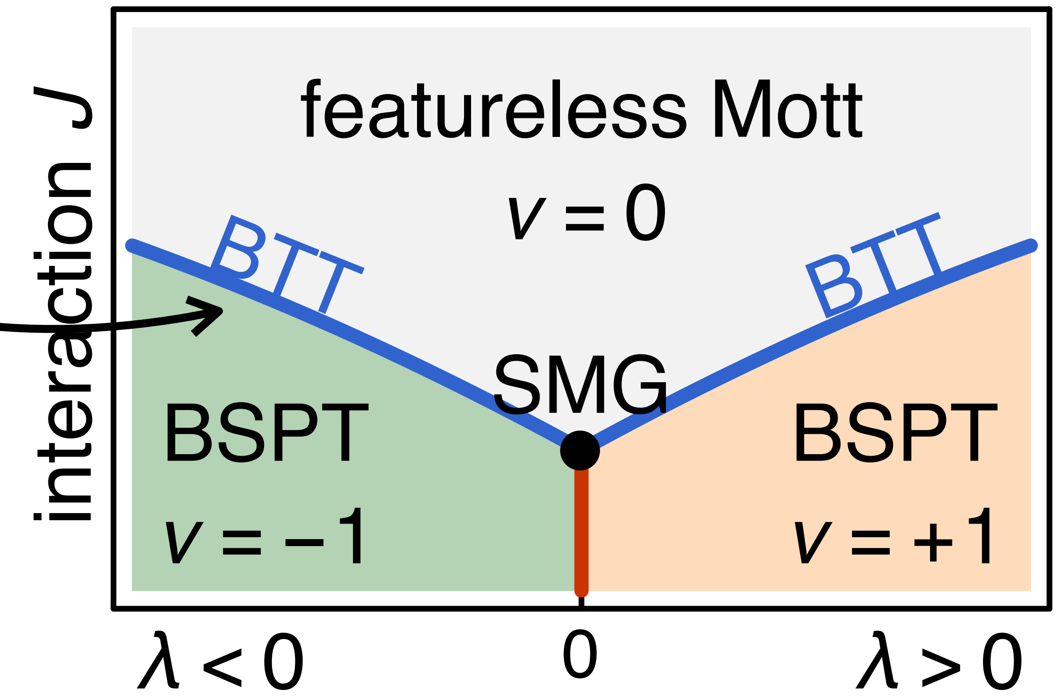


- No matter in CP¹ or dual-CP¹ theory, either XY or VBS corresponds to gauge fluctuation, which is beyond the mean-field description.

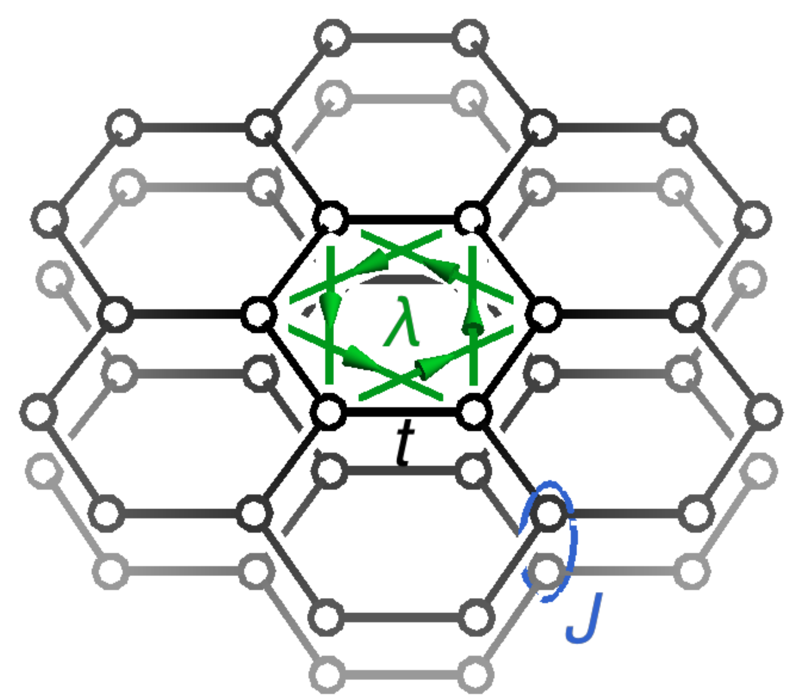
Bosonic Topological Transition

- The easy-plane DQCP is dual to a bosonic SPT transition in a lattice fermion model.

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger \, c_j + \sum_{\langle\langle ij \rangle\rangle} i \, \lambda_{ij} \, c_i^\dagger \, \sigma^z \, c_j + J \sum_i \boldsymbol{S}_{i1} \cdot \boldsymbol{S}_{i2} + \dots$$



- Exact SO(4) symmetry between in-plane SDW and interlayer SC
- SO(4) SPT phases separated by phase transitions
- Transition lines joint at a tricritical point: symmetric mass generation (SMG)



Slagle, You, Xu (2014)

Symmetric Mass Generation

- Field Theory: $N_f = 4$ SU(2) QCD$_3$-Higgs

You, He, Vishwanath, Xu, PRB **97**, 125112 (2018)

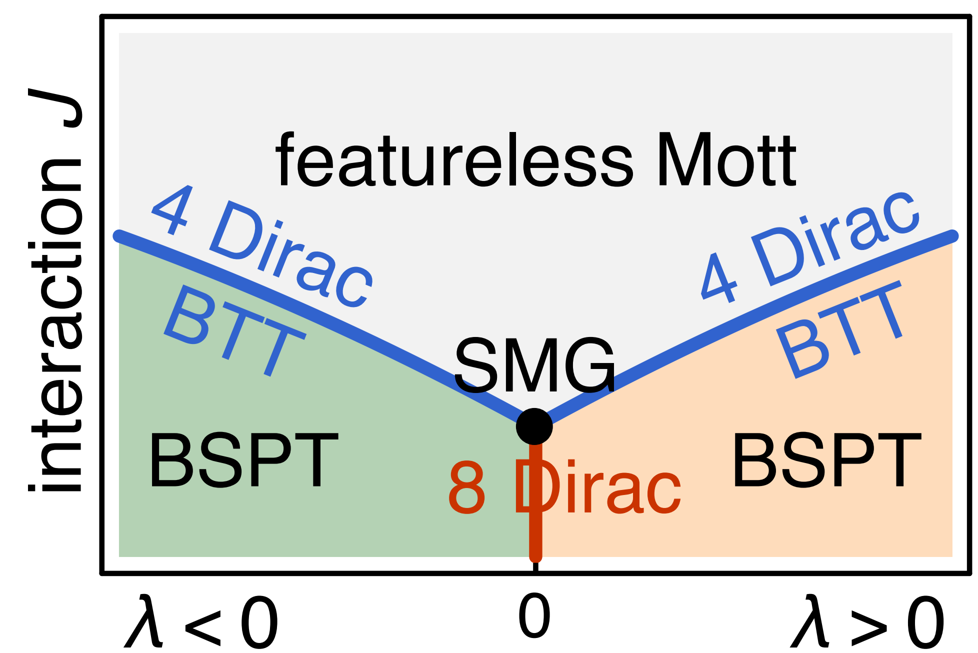
$$\bar{\psi}_i \gamma^\mu (\partial_\mu - \mathrm{i}\boldsymbol{a}_\mu \cdot \boldsymbol{\tau})\psi_i + |(\partial_\mu - \mathrm{i}\boldsymbol{a}_\mu \cdot \boldsymbol{\tau})\phi|^2 + r|\phi|^2 + u|\phi|^4$$

  - $r < 0$: boson condense → Higgs out gauge field, Dirac SM
  - $r > 0$: boson gapped → remaining QCD is confining (conjecture)

Higgs ($r < 0$) | deconfine ($r = 0$) | confine ($r > 0$) → r

semimetal | **SMG** | trivial Mott

- The theory of SMG naturally leads to the $N_f = 4$ QED theory for BTT
- Fermionic and bosonic excitation spectra can be estimated by the mean-field approach



Summary

- QMC + SAC allows us to explore spectral features of DQCP in both bosonic and (sign-problem-free) fermionic systems.
- Duality provides us different mean-field understandings of the excitation spectrum. Putting different perspectives together gives us a more comprehensive picture.
- The spectral features are relatively easy to probe by INS, RIXS or NMR, and are robust in a range of temperature, which may guide the search for DQCP in real materials.