Multi-flavor QED3 at Plateau Transitions of Fractional Chern Insulators

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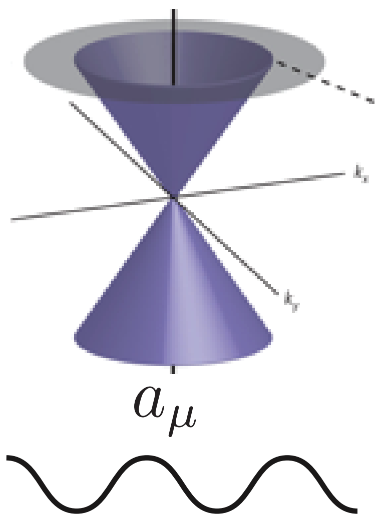
Aspen winter conference

GORDON AND BETTY MOORE FOUNDATION

arXiv:1802.09538

with Jong Yeon Lee, Chong Wang, Mike Zaletel, Ashvin Vishwanath

QED3-Chern-Simons theory



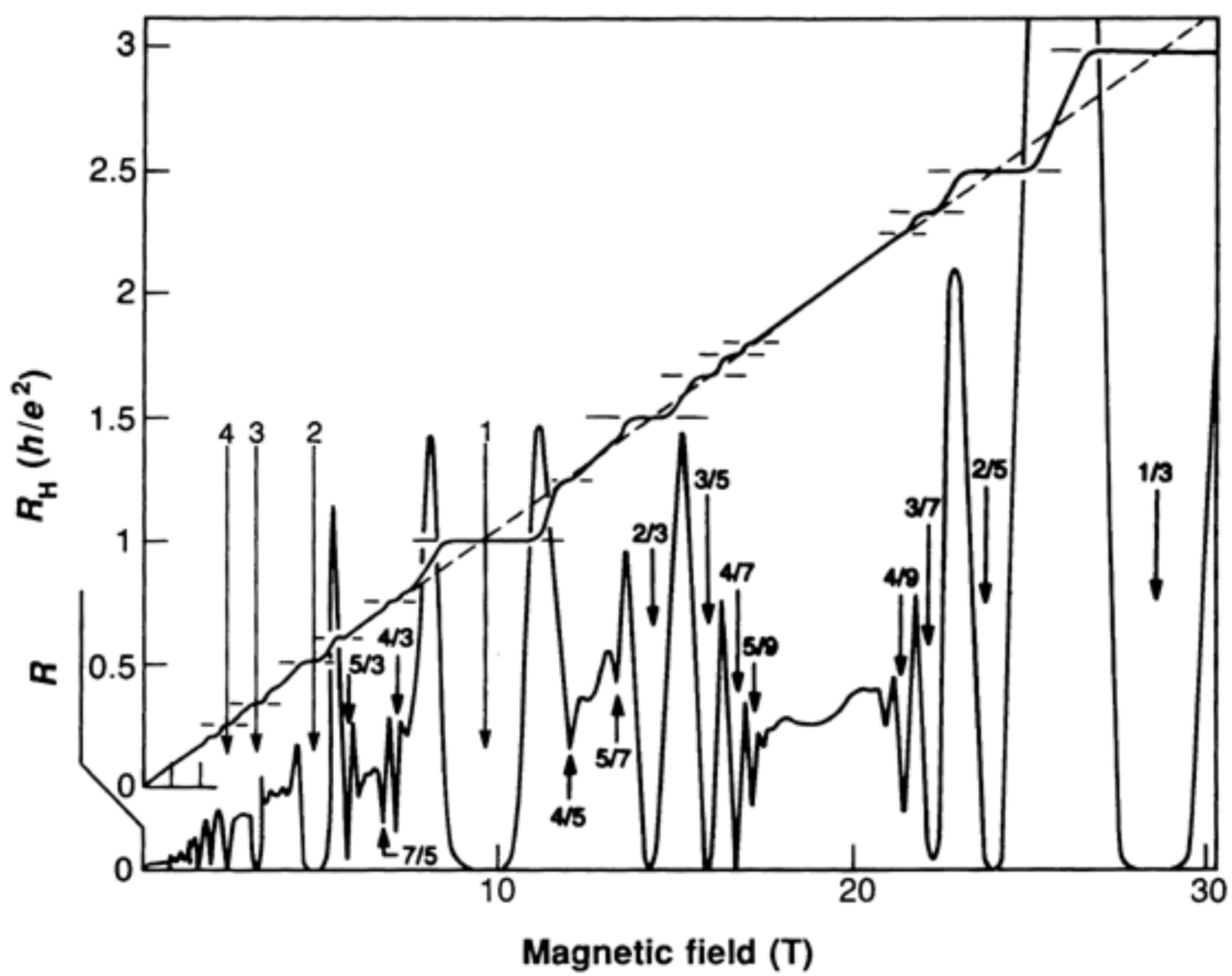
$$\mathcal{L} = \sum_{I=1}^{N_f} \bar{\psi}_I (i\displaystyle{\not}\partial + \displaystyle{\not}a - m)\psi_I + \frac{K}{4\pi} ada + \cdots$$

Simulate the entire family in condensed matter experiments!

Lee, Wang, Zaletel, Vishwanath, YCH, arXiv: 1802.09538

- A family of interacting 2+1D conformal field theories.
- Interesting duality properties.
- Open problems after several decades efforts.

Fractional quantum Hall state

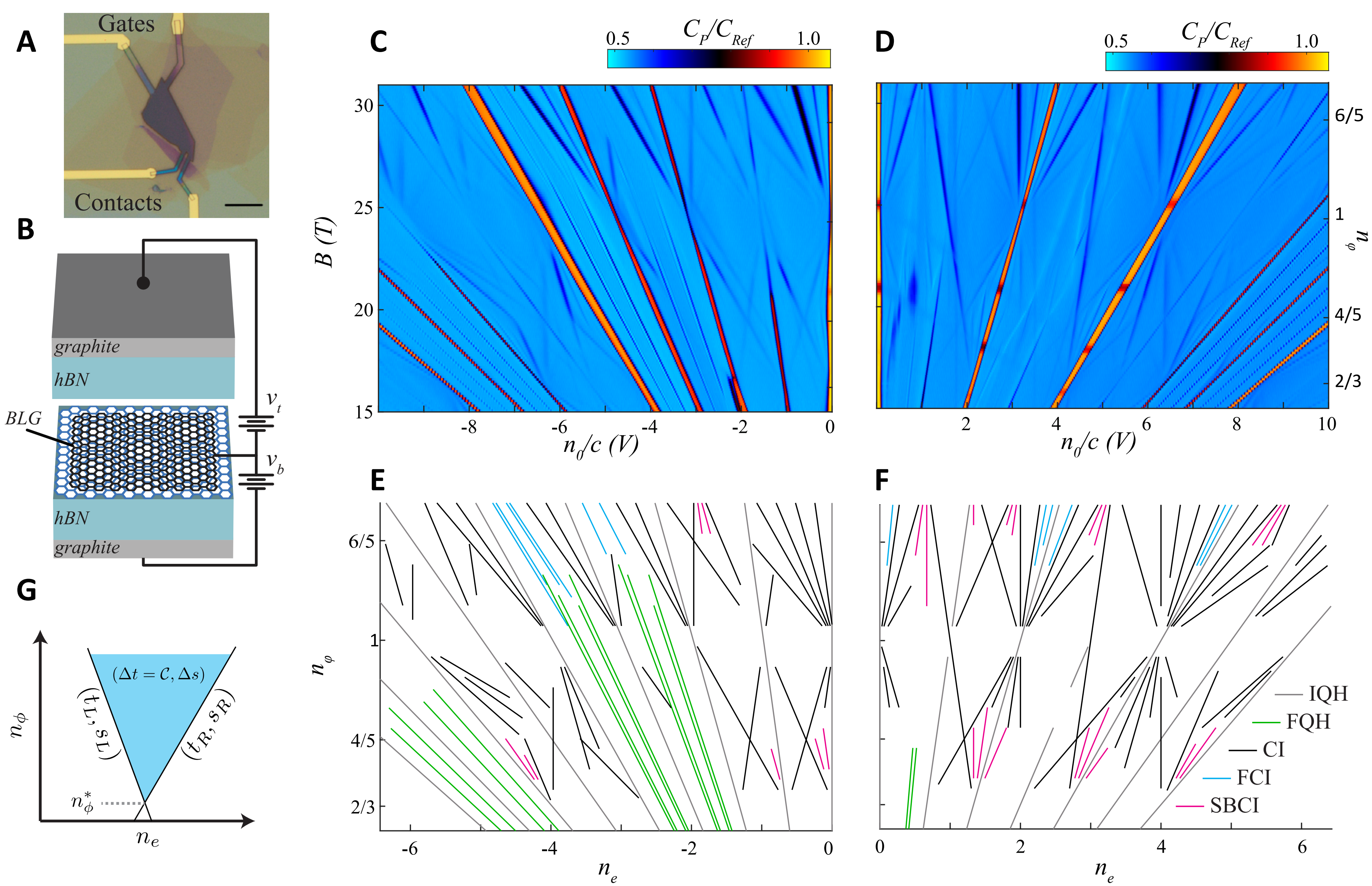


TQFT: Chern-Simons theory

Fractional Chern insulator (FCI)

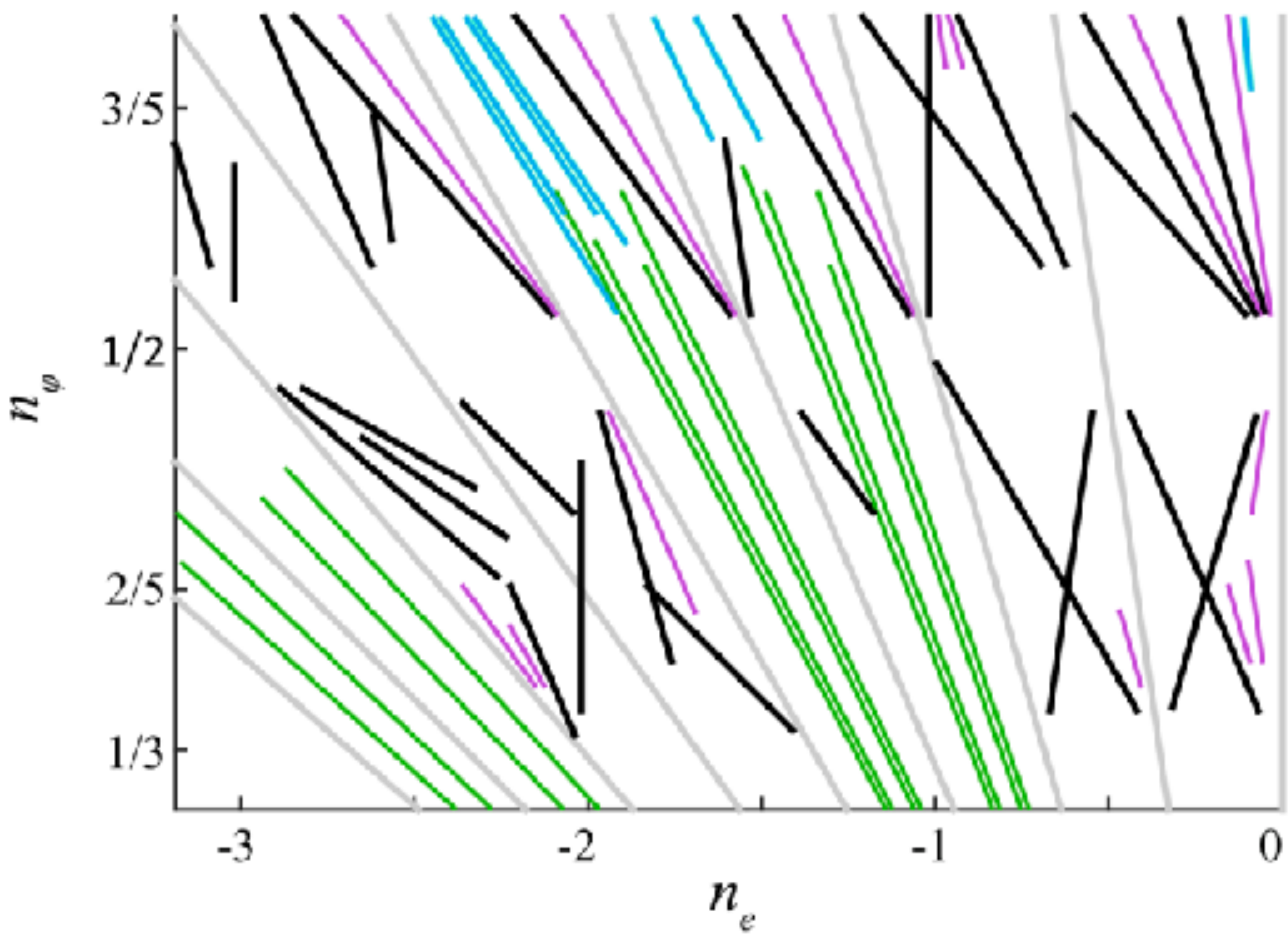
Fractional quantum Hall state on the lattice

A. Young group, arXiv: 1706.06116



Phase transition between FCI/FQH

$$n = C\phi + s$$



- Disorder is not necessary
- Lattice symmetry

New family of quantum critical points

$$\mathcal{L} = \sum_{I=1}^{N_f} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/ - m)\psi_I + \frac{K}{4\pi} ada + \cdots$$

Lee, Wang, Zaletel, Vishwanath, YCH, arXiv: 1802.09538

Plan

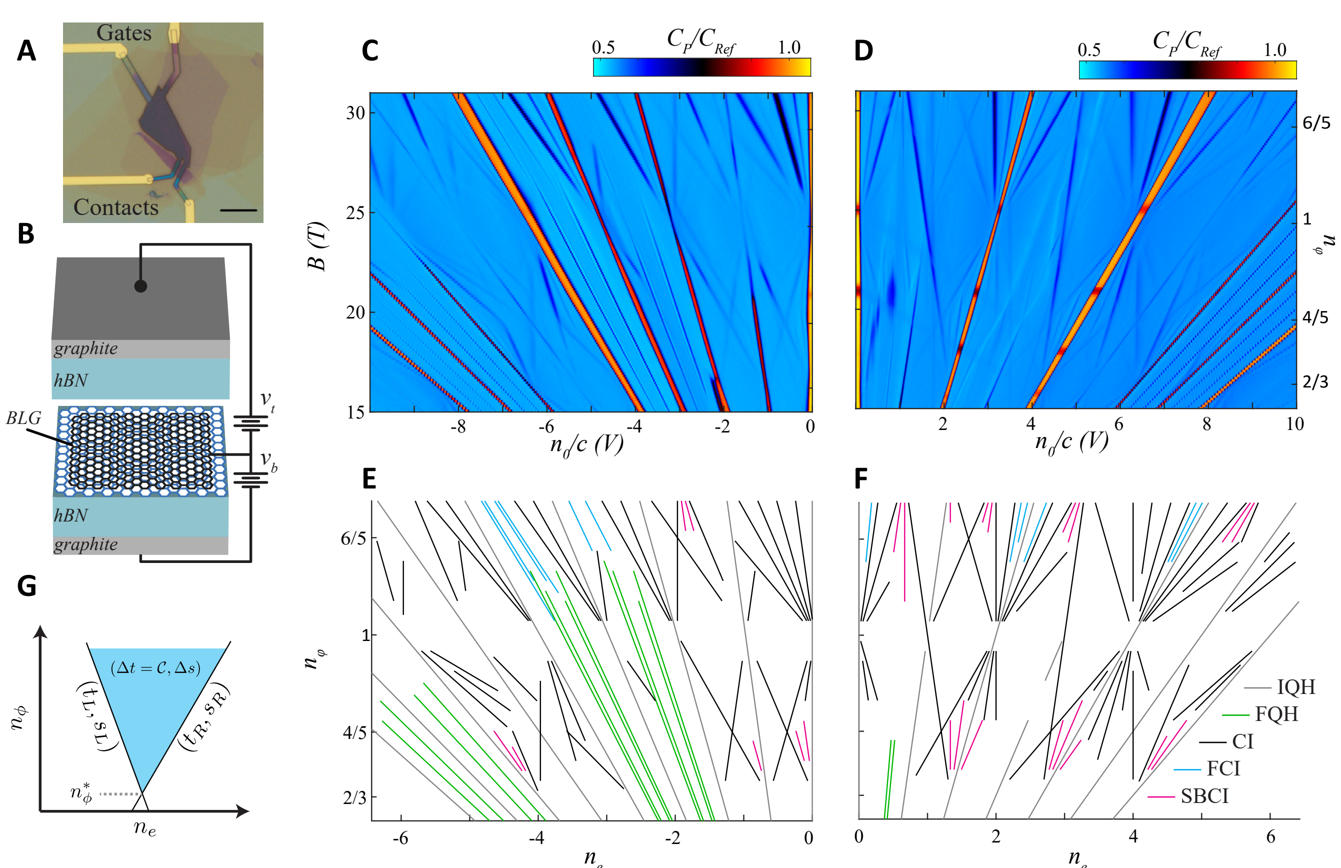
- 1/3-2/3 transitions: Nf=3 QED3
- Physical properties and experimental detection
- The whole family of QED3-Chern-Simons
- Application of level-rank duality: a fun example

Artificial superlattice on graphene

Potential strength is experimentally tunable

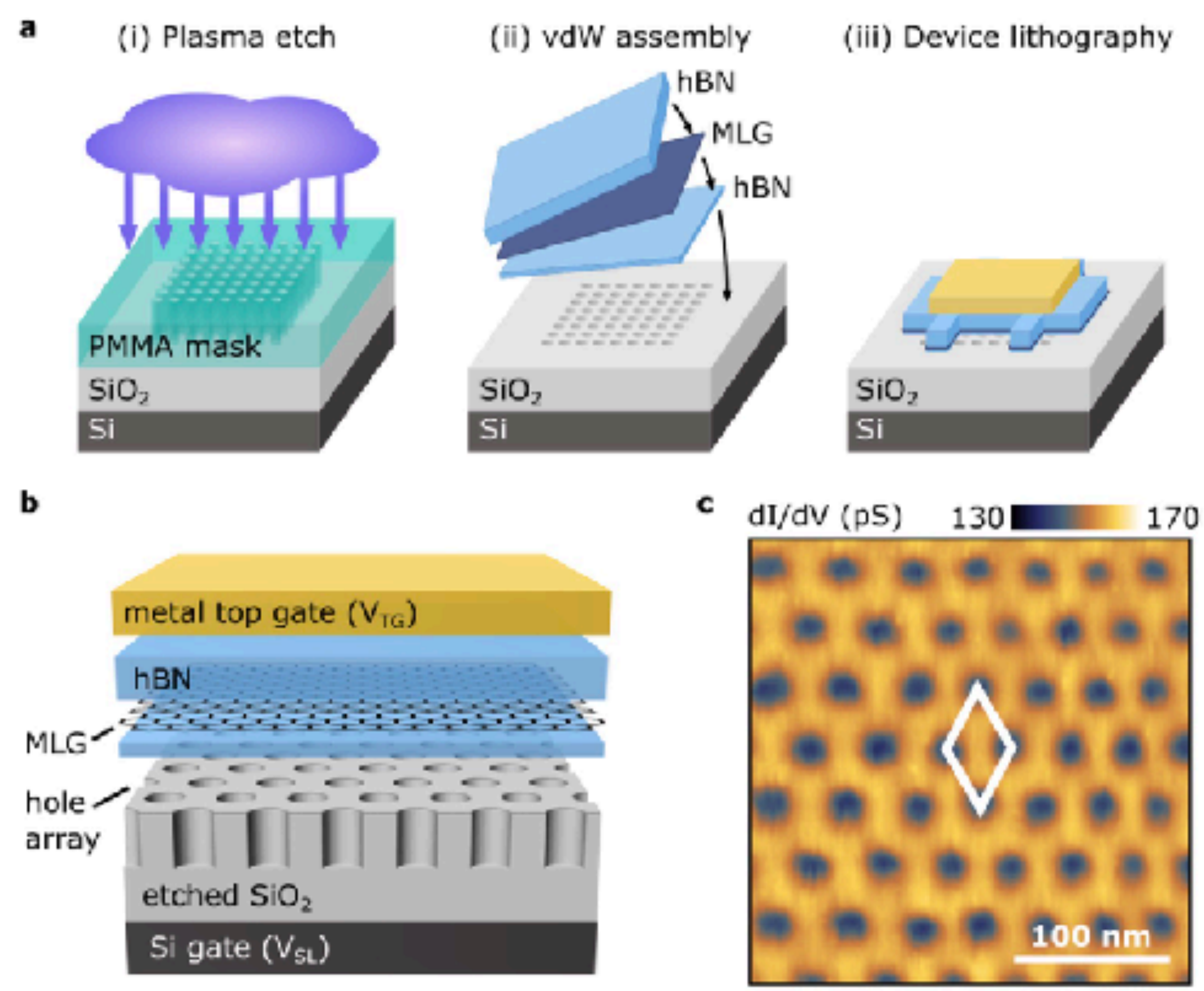
A. Young group, arXiv: 1706.06116

Moire potential



C. Dean group, arXiv:1710.01365

Electrostatic gate patterning



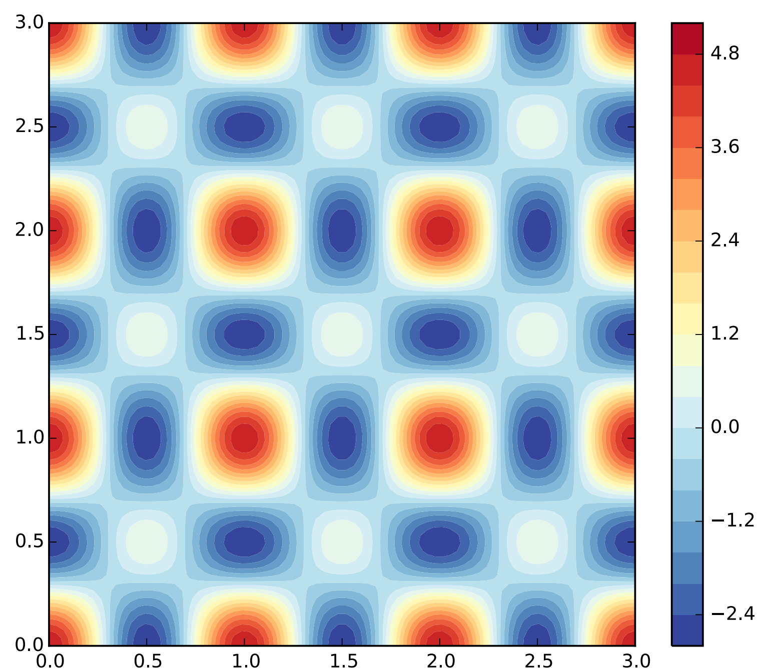
Microscopic model

Landau level with superlattice potential

$$\mu(\vec{r}) = U_0 \int d\vec{r} \sum_m (V_m e^{i\vec{r}\cdot\vec{G}_m} n_{\vec{r}} + h.c.),$$

$$\vec{G}_1 = \frac{2\pi}{a}(1,0), \quad \vec{G}_2 = \frac{2\pi}{a}(0,1)$$

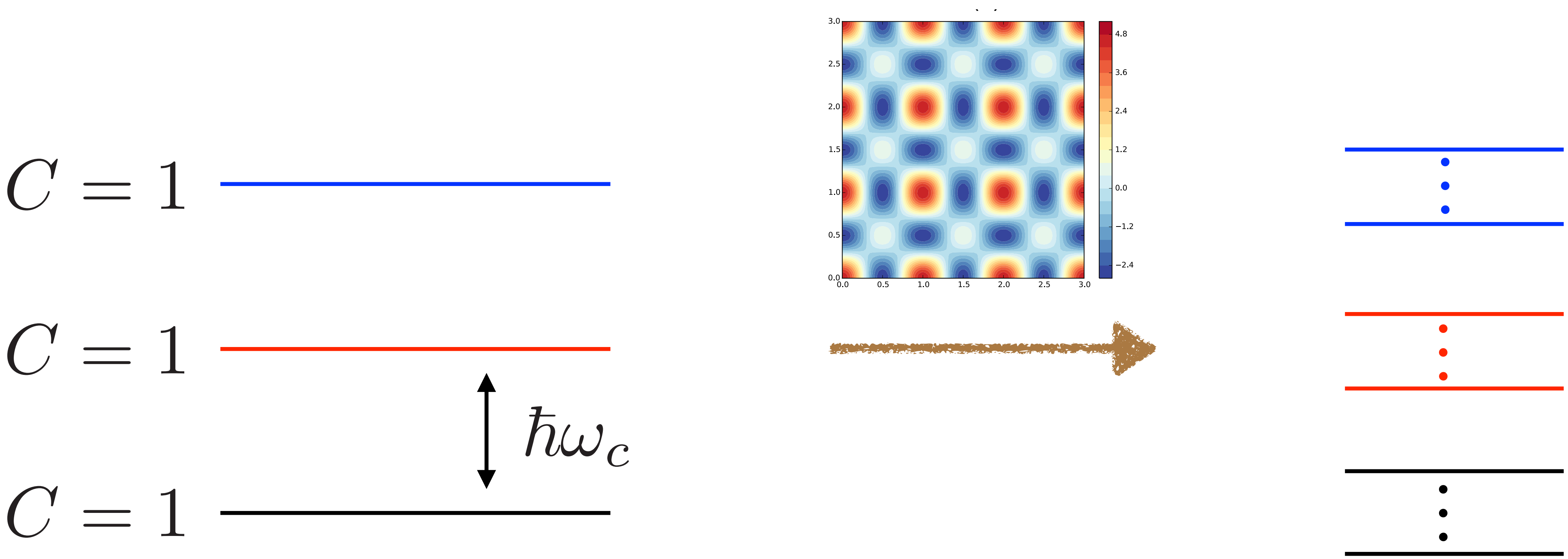
$$\vec{G}_3 = \frac{2\pi}{a}(1,1), \quad \vec{G}_2 = \frac{2\pi}{a}(1,-1), \cdots$$



Weak potential limit: potential strength smaller/comparable to cyclotron gap

Physical effects: Landau level gets broadened and split

Landau levels under lattice potential

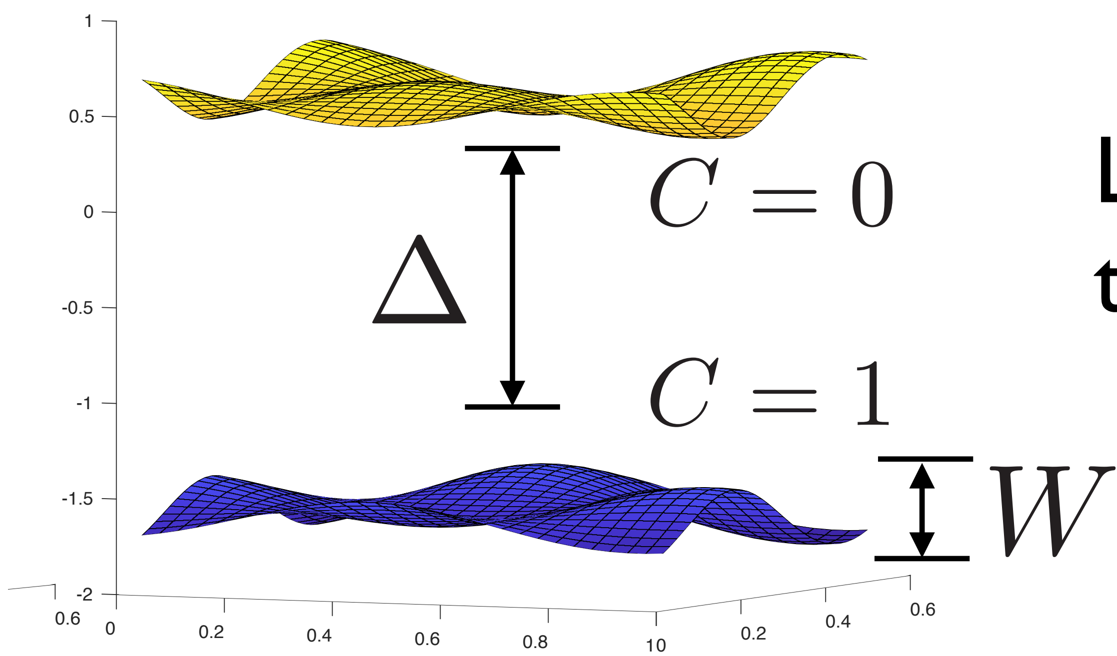


$\phi = 2\pi \frac{p}{q}$ Each Landau level splits into p sub-bands

Streda, MacDonald, Pfannkuche, Gerhardts, Usov,…

1/3 FQH to 2/3 FCI transition

Flux: $\phi = Ba^2 = 4\pi$

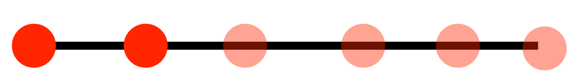


LLL splits to two sub-bands

Energy scale

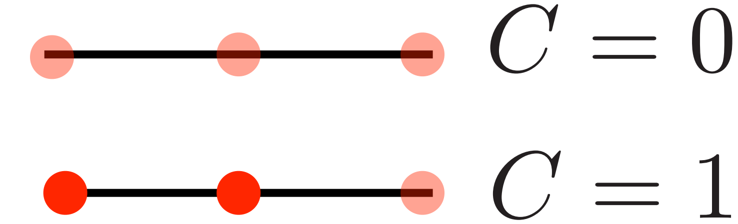
Cyclotron gap: ω_0
Coulomb energy: E_c
band width: W
band gap: Δ

$C = 1$



Flux: $\phi = Ba^2 = 4\pi$
Density: $n = 2/3$

$\Delta, W \propto U_0$



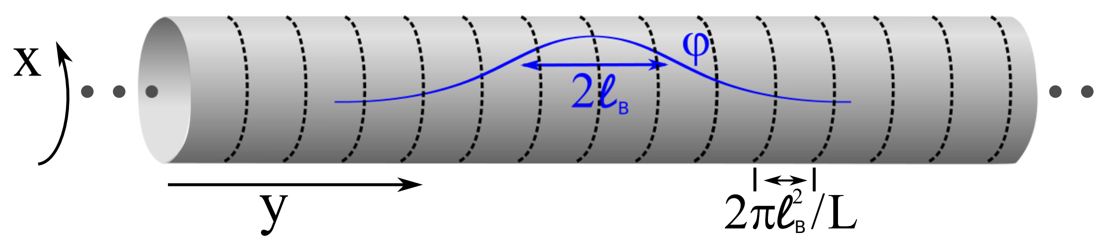
1/3 FQH state:
$\omega_0 \gg E_c \gg \Delta, W$

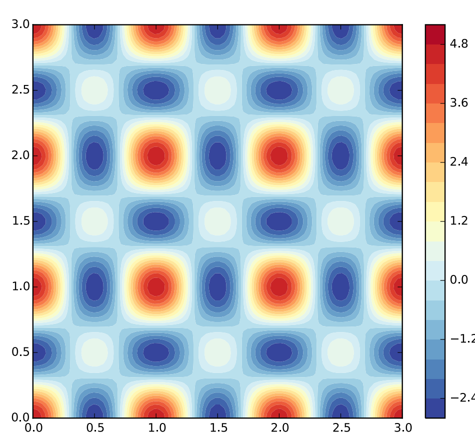
2/3 FCI state:
$\omega_0 \gg \Delta \gg E_c \gg W$

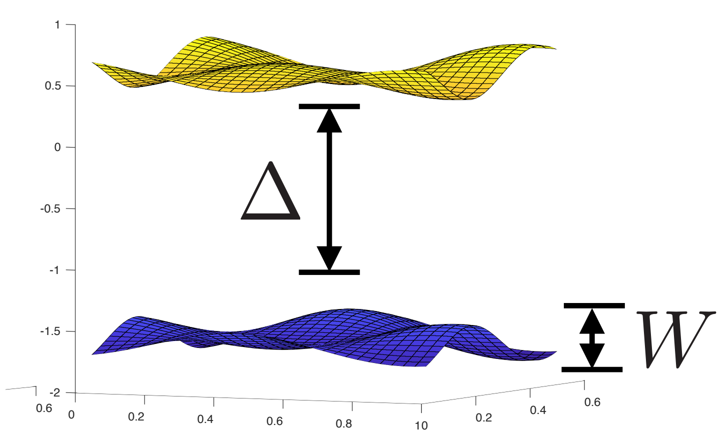
Numerical results: DMRG

DMRG: White

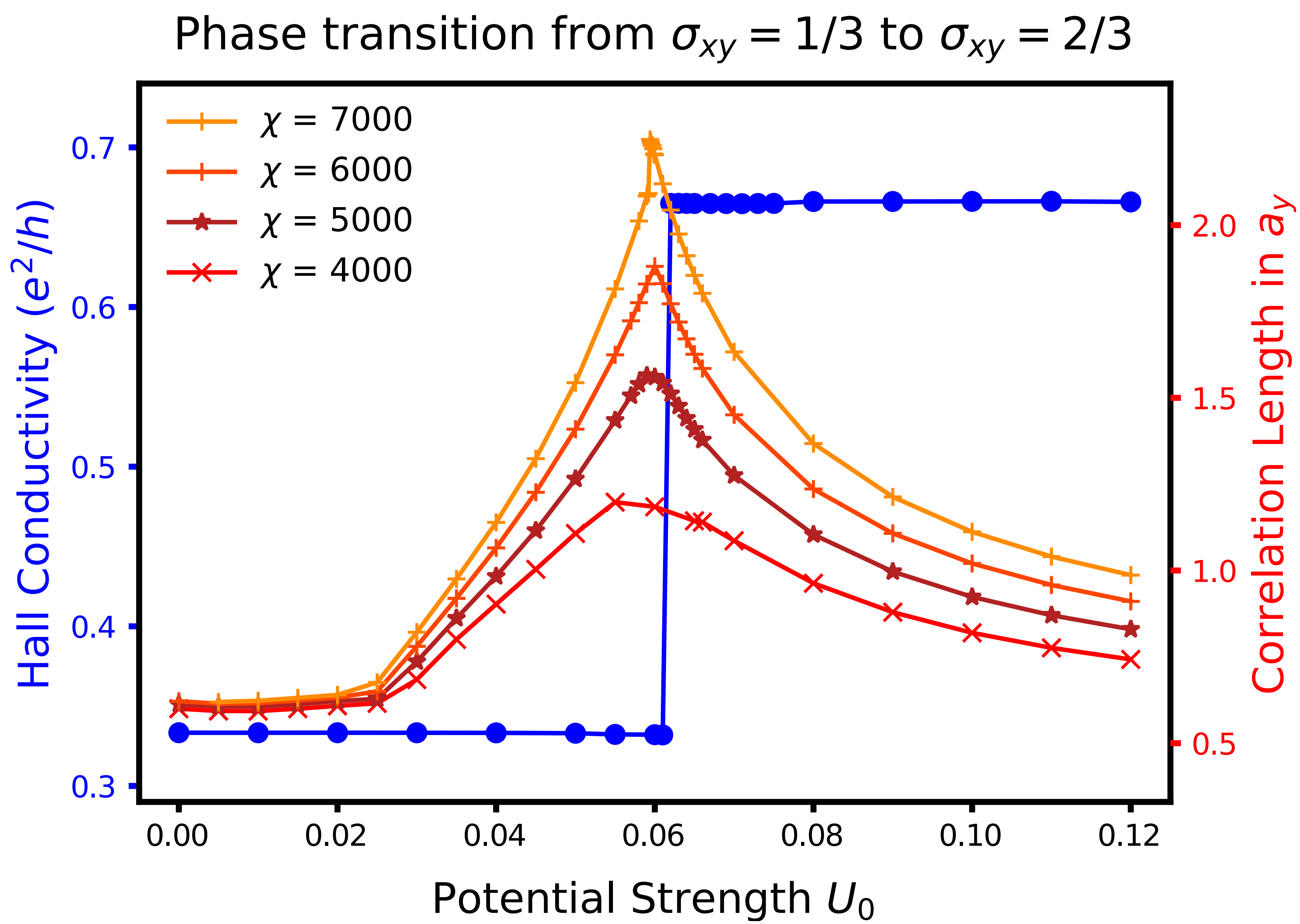
FQH DMRG:
Zaletel, Mong, Pollmann







$$E_c = 1$$

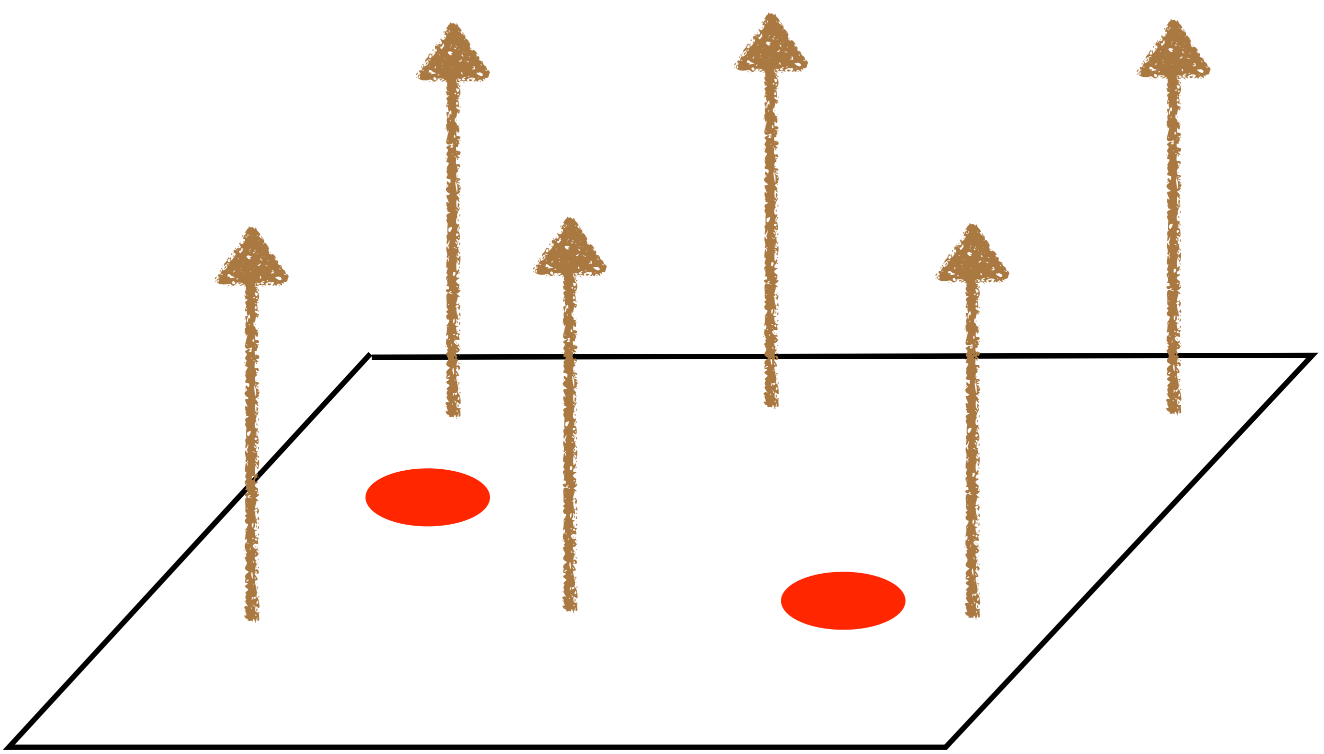


Lee, Wang, Zaletel, Vishwanath, YCH, arXiv: 1802.09538

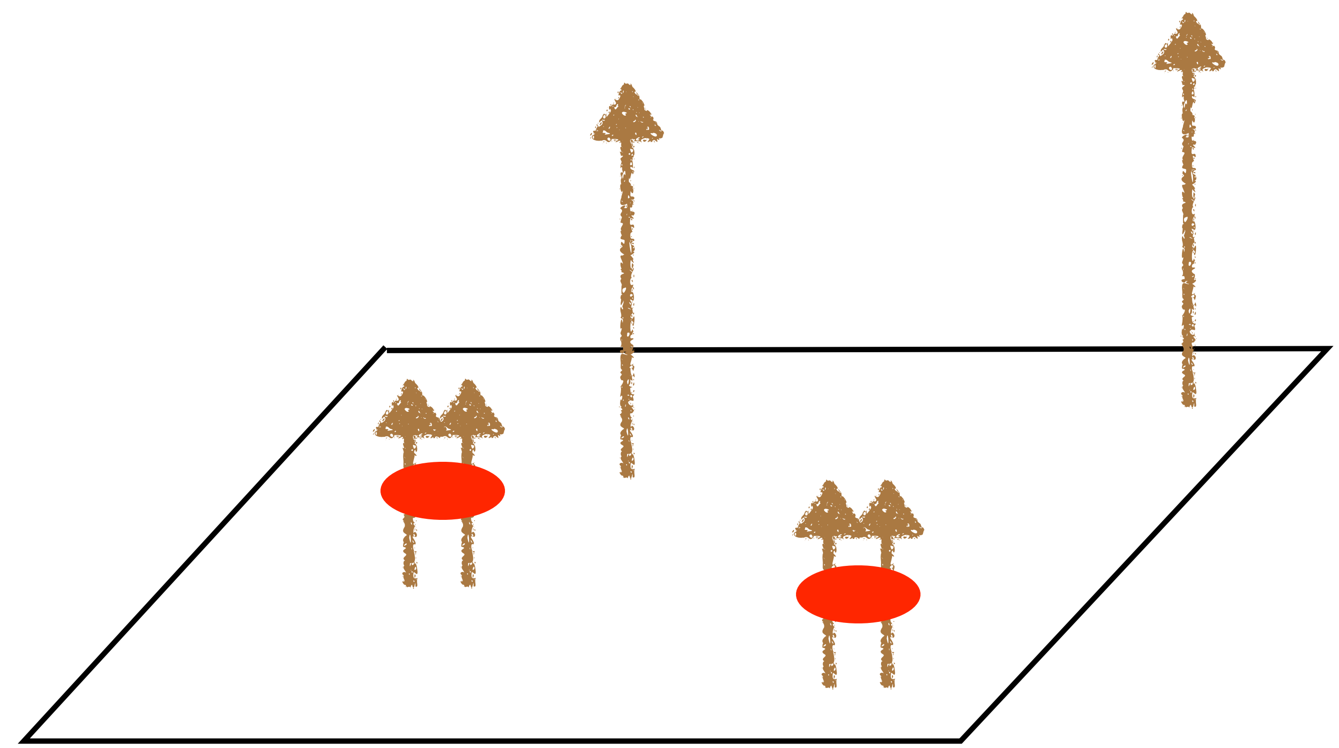
Composite fermion for the FQH/FCI

Eg: 1/3 Fractional quantum Hall state

$n/B = 1/3$

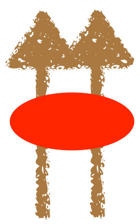


$n/B_{\text{eff}} = 1$



Jain; Lopez, Fradkin; Halperin, Lee, Read;…

Composite fermion for the transition



Composite fermion

$\nu_{\rm CF} = C$ Integer QH

Original electron

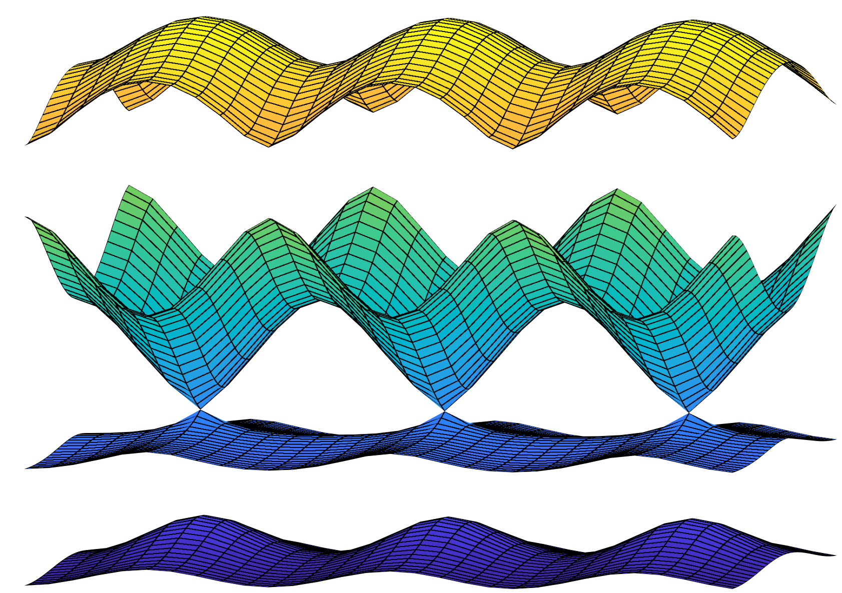
$\sigma^{xy} = \dfrac{C}{2C+1}$ FQH/FCI

Electron:

$\sigma^{xy} = 1/3 \longleftrightarrow \sigma^{xy} = 2/3$

Composite fermion:

$\nu_{\rm CF} = 1 \longleftrightarrow \nu_{\rm CF} = -2$



Critical theory:

$$\mathcal{L} = \sum_{I=1}^{3} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I - \frac{1}{8\pi} ada + \frac{1}{8\pi}(a-A)d(a-A)$$

$$= \sum_{I=1}^{3} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I - \frac{1}{4\pi} Ada + \frac{1}{8\pi} AdA.$$

Nf=3 QED3

Critical theory

$$\mathcal{L} = \sum_{I=1}^{3} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I - \frac{1}{4\pi} Ada + \frac{1}{8\pi} AdA$$

Tuning parameter: $m \sum_{I=1}^{3} \bar{\psi}_I \psi_I \Longrightarrow \mathrm{sgn}(m) \frac{3}{8\pi} ada$

1. m>0, $\mathcal{L}_{\mathrm{res}} = \frac{1}{3}\frac{1}{4\pi} AdA$

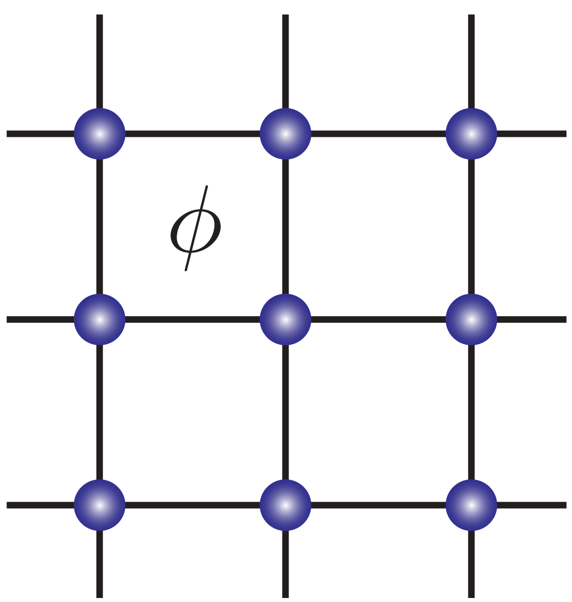
2. m<0, $\mathcal{L}_{\mathrm{res}} = \frac{2}{3}\frac{1}{4\pi} AdA$

We need symmetry to forbid other relevant term.

eg. Barkeshli, McGreevy 2014

Magnetic translation symmetry

$$T_x T_y = e^{i\phi} T_y T_x$$



$$\mathcal{L} = \sum_{I=1}^{3} \bar{\psi}_I (i\not{\partial} + \not{a}) \psi_I - \frac{1}{4\pi} A da + \frac{1}{8\pi} A dA$$

Composite fermions see a flux $2\pi/3$

$$T_x : \psi_I \to e^{i2\pi I/3} \psi_I \qquad T_y : \psi_I \to \psi_{I+1}$$

Only symmetry allowed mass term: $\sum_{I=1}^{3} \bar{\psi}_I \psi_I$

Emergent SU(3) symmetry

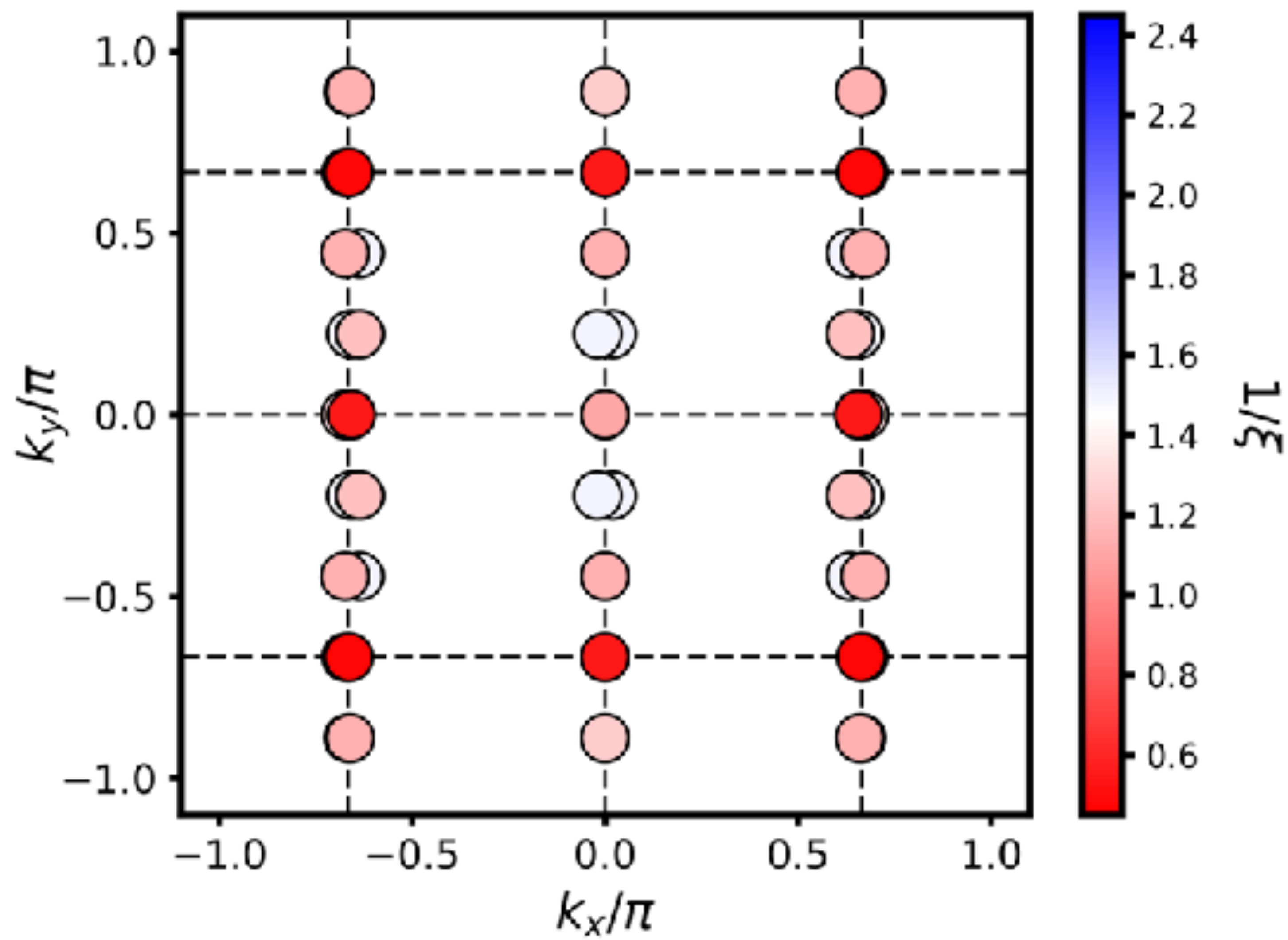
8 mass terms in the adjoint rep of SU(3): $\bar{\psi}_I M_{IJ} \psi_J$

Charge-density-wave order at 8 distinct momenta

$$\langle n_0(\vec{k}_\alpha) n_r(\vec{k}_\alpha) \rangle$$

$$\vec{k}_\alpha = (\frac{2l_x\pi}{3}, \frac{2l_y\pi}{3})$$
$$(l_x, l_y) \neq (0,0)$$

DMRG evidence:
correlation length spectra



Lee, Wang, Zaletel, Vishwanath, YCH, arXiv: 1802.09538

Monopoles

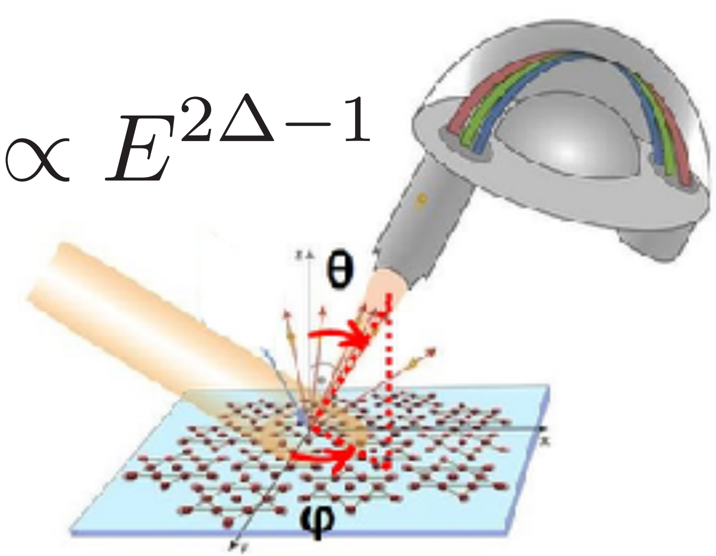
Critical theory: $\mathcal{L} = \sum_{I=1}^{3} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I - \frac{1}{4\pi} A da + \frac{1}{8\pi} A dA$

4π monopole is a local electron

20 monopoles
- spin-1/2, SU(3) adjoint $\vec{k}_\alpha = (\frac{2n_x\pi}{3}, \frac{2n_y\pi}{3})$
- spin-3/2, SU(3) singlet $\vec{k} = (0,0)$

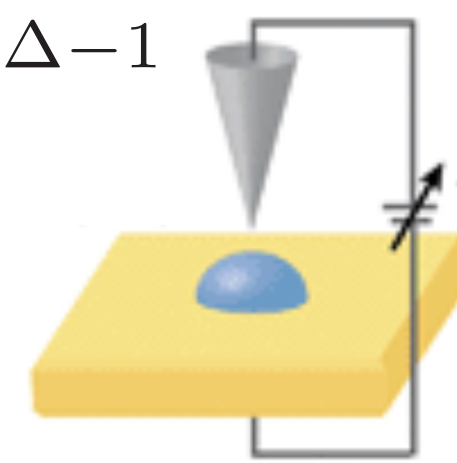
ARPES

$A(E,k) \propto E^{2\Delta-1}$

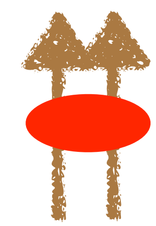


STM

$dI/dV \propto V^{2\Delta-1}$



The entire family of transition

Between two composite fermion states:  bound k flux quanta

$$\sigma^{xy} = \frac{C_1}{kC_1 + 1} \quad \longleftrightarrow \quad \sigma^{xy} = \frac{C_2}{kC_2 + 1}$$

Bosonic system has odd k, fermionic system has even k

$$\mathcal{L} = \sum_{I=1}^{|C_2 - C_1|} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/ - m)\psi_I + \frac{C_2 + C_1}{8\pi} ada + \frac{1}{4k\pi}(a - A)d(a - A).$$

Protected by (magnetic) translation symmetry

Monopoles

Critical theory: QED3-Chern-Simons

$$\mathcal{L} = \sum_{I=1}^{N_f} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/ - m)\psi_I + \frac{K}{4\pi} ada - \frac{1}{2k\pi} adA + \frac{1}{4k\pi} AdA$$

Dirac flavor: $N_f = |C_2 - C_1|$

Chern-Simons level: $K = (C_2 + C_1)/2 + 1/k$

Minimally allowed monopole

Strength-k monopole creates $2k\pi$ flux
- Boson if k is odd
- Fermion if k is even

Even flavor QED3 theory

Between bosonic particle-hole partner:

$$\sigma^{xy} = \frac{N-1}{N} \longleftrightarrow \sigma^{xy} = \frac{N+1}{N}$$

Critical theory:

$$\mathcal{L} = \sum_{I=1}^{2N} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I - \frac{1}{2\pi} Ada + \frac{1}{4\pi} AdA$$

i) 0-2 transition: Nf=2, SPT transition Grover, Vishwanath; Lu, Lee
   Self-dual, possible emergent O(4) symmetry
   Senthil, Fisher; Xu, You; Tong, Karch; Wang, Nahum, Metlitski, Xu, Senthil

ii) 1/2-3/2 transition: Nf=4

iii) …

Odd flavor QED3 theory

Between fermionic particle-hole partner

$$\sigma^{xy} = \frac{N}{2N+1} \longleftrightarrow \sigma^{xy} = \frac{N+1}{2N+1}$$

Critical theory:

$$\mathcal{L} = \sum_{I=1}^{2N+1} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/)\psi_I - \frac{1}{4\pi} Ada + \frac{1}{8\pi} AdA$$

i) 0-1 transition: Nf=1, vortex dual of free Dirac

Son; Wang & Senthil; Metlitski & Vishwanath…

ii) 1/3-2/3 transition: Nf=3

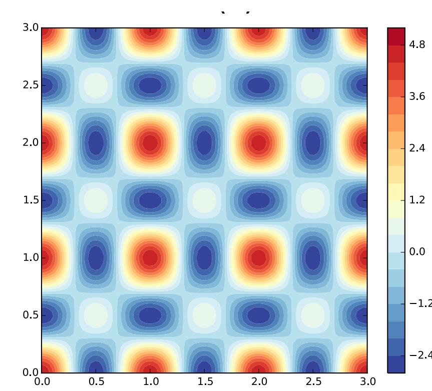
iii) ….

More about microscopic realization

$C = 1$

Partially filled Landau level

$\nu = \frac{p}{2p+1}$ FQH



Potential strength

(Partially) filled sub-bands

Chern insulator

Transition to	N_f	K	ϕ	n
$\sigma_{xy} = \frac{p+1}{2p+1}$ FCI	$2p+1$	0	$2p$	$\frac{2p^2}{2p+1}$
$\sigma_{xy} = 1$ ICI	$p+1$	$p/2$	$\frac{2p+1}{p+1}$	$\frac{p}{p+1}$
$\sigma_{xy} = 0$ ICI	p	$(p+1)/2$	$\frac{2p+1}{p}$	1

Application of level-rank duality

1/2 bosonic Laughlin state to a trivial insulating phase:

$$\mathcal{L} = \bar{\psi}(i\not{\partial} + \not{a})\psi + \frac{3}{8\pi} ada - \frac{1}{2\pi} Ada + \frac{1}{4\pi} AdA - m\bar{\psi}\psi$$

Explicit U(1) global symmetry

Level-rank duality

Hsin, Seiberg

$$\mathcal{L} = \bar{\Psi}(i\not{\partial} + \not{\alpha} + \not{A}/2)\Psi + \frac{1}{8\pi}(\mathrm{CS}[\alpha]) + \frac{1}{16\pi} AdA - m\bar{\Psi}\Psi$$

Explicit SO(3) global symmetry

Benini, Hsin, Seiberg

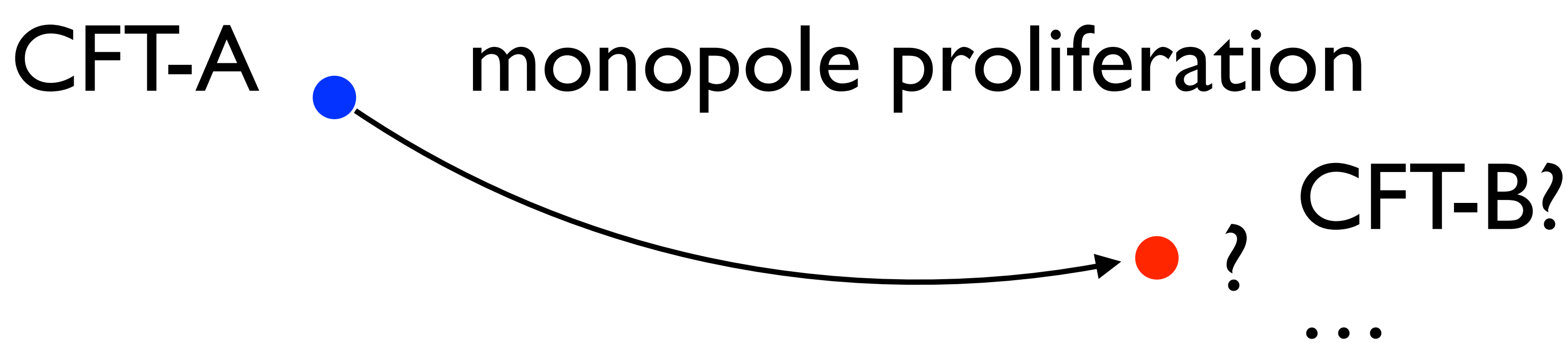
$(\nabla \times a, \mathrm{Re}(\mathcal{M}_a \psi^\dagger), \mathrm{Im}(\mathcal{M}_a \psi^\dagger))$ is a SO(3) vector

Prediction: $\langle n_0 n_r \rangle \sim 1/r^4, \quad \langle b_0^\dagger b_r \rangle \sim 1/r^4$

Outlook: monopole dynamics

Critical theory: $N_f/2 + K = \mathbf{Z} + 1/k$

$$\mathcal{L} = \sum_{I=1}^{N_f} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/ - m)\psi_I + \frac{K}{4\pi} ada - \frac{1}{2k\pi} adA + \frac{1}{4k\pi} AdA$$



e.g. Nf=4 QED3 may flow to SO(5) deconfined phase transition.

Wang, Nahum, Metlitski, Xu, Senthil

Accessible by proximity to superconductor/superfluid.

Outlook: duality

Critical theory: $N_f/2 + K = \mathbf{Z} + 1/k$

$$\mathcal{L} = \sum_{I=1}^{N_f} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/ - m)\psi_I + \frac{K}{4\pi} ada - \frac{1}{2k\pi} adA + \frac{1}{4k\pi} AdA$$

Bosonic theory: Nf/2+K is integer.

1. Nf=1, K=1/2 is dual to O(2) Wilson-Fisher.
2. Nf=2, K=0 is self-dual and has emergent O(4) symmetry.
3. Nf=1, K=3/2 has emergent SO(3) symmetry.

Fermionic theory: Nf/2+K is half integer.

1. Nf=1, K=0 is dual to a free Dirac.
2. More to be found?

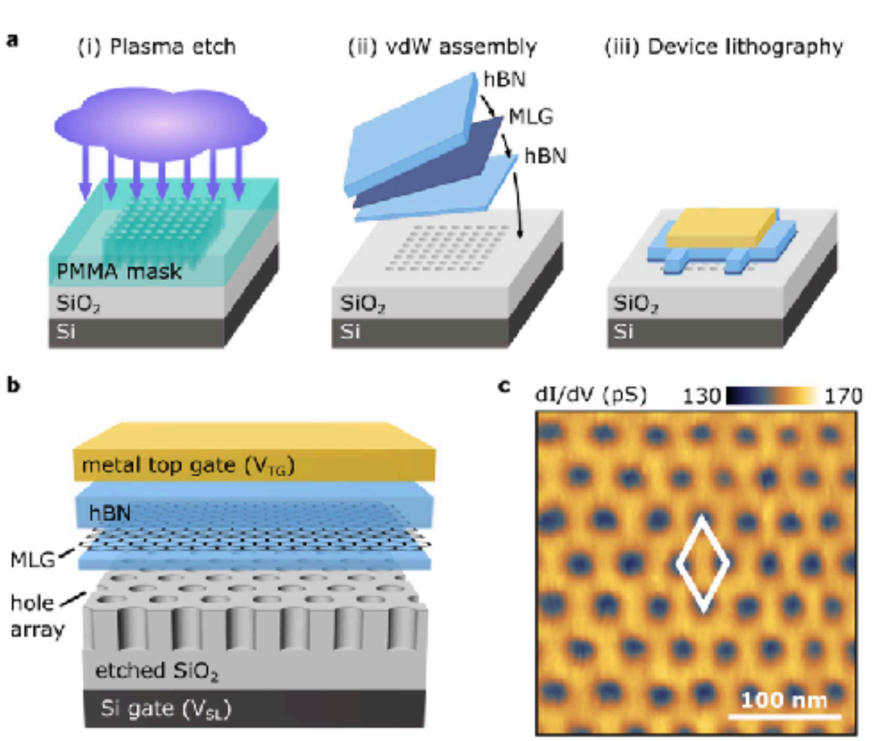
Outlook: lattice simulation

Critical theory: $N_f/2 + K = \mathbf{Z} + 1/k$

$$\mathcal{L} = \sum_{I=1}^{N_f} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/ - m)\psi_I + \frac{K}{4\pi} ada - \frac{1}{2k\pi} adA + \frac{1}{4k\pi} AdA$$

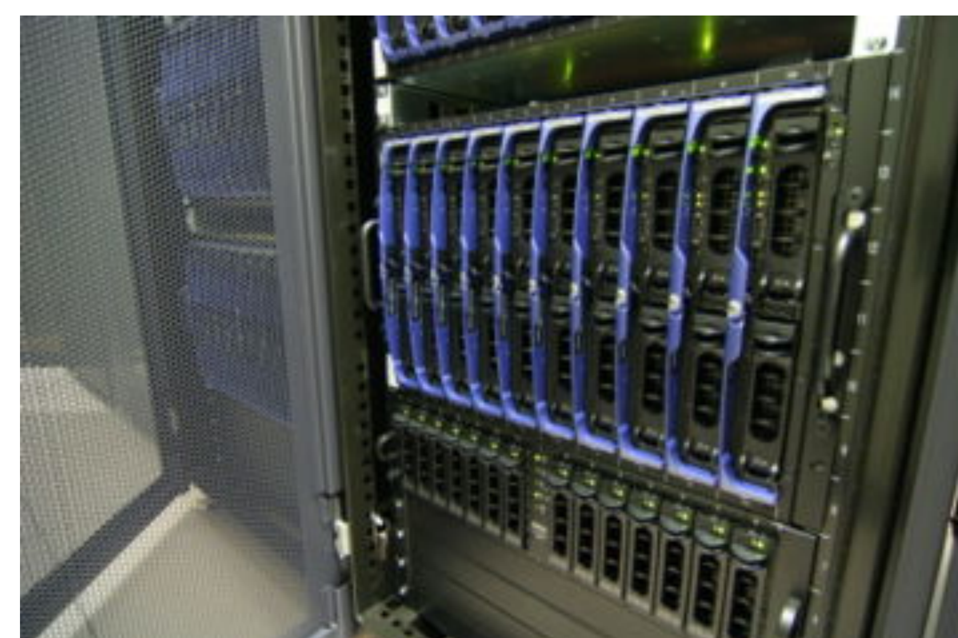
Nf odd, K=0 is parity invariant

Experiments



Numerical simulation

Karthik, Narayanan



Summary

- Fractional Chern insulator transitions can be directly realized in experiments by tuning the potential strength.
- The transition is described by the QED3-Chern-Simons theory, and remarkably the entire family can be realized.
- We provide the numerical evidence for a continuous transition and emergent SU(N) symmetry.
- The physical properties (e.g. adjoint mass, magnetic monopoles) are straightforward to detect experimentally.

Lee, Wang, Zaletel, Vishwanath, YCH, arXiv: 1802.09538

Thank you!