Entanglement signatures of QED_3 in the kagome spin liquid

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Chronologically:

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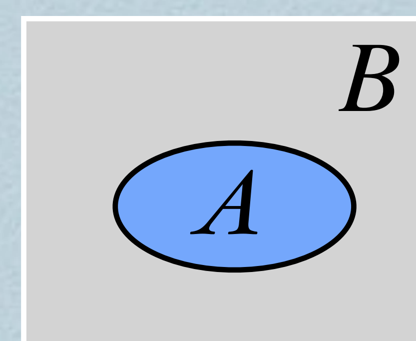
<u>**W. Zhu**</u>, Los Alamos

Plan

- Entanglement entropy in 2+1D CFTs
- Quantum critical transition from Dirac semimetal to charge density wave: **Gross-Neveu-Yukawa**
- Kagome Heisenberg model: N_f**=4 QED3**

Entanglement entropy



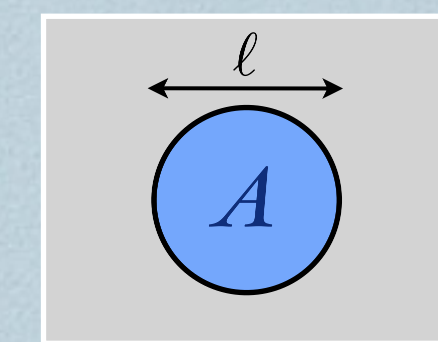
$$\rho_A = \mathrm{tr}_B |\psi\rangle\langle\psi|$$

$$\boxed{S = -\mathrm{tr}(\rho_A \ln \rho_A)}$$

- S = measure of quantum entanglement between A & B

- Versatile way to characterize quantum many-body systems; useful to detect fractionalization
  [Levine, Wen; Kitaev, Preskill; Li, Haldane; Swingle, Senthil; etc]

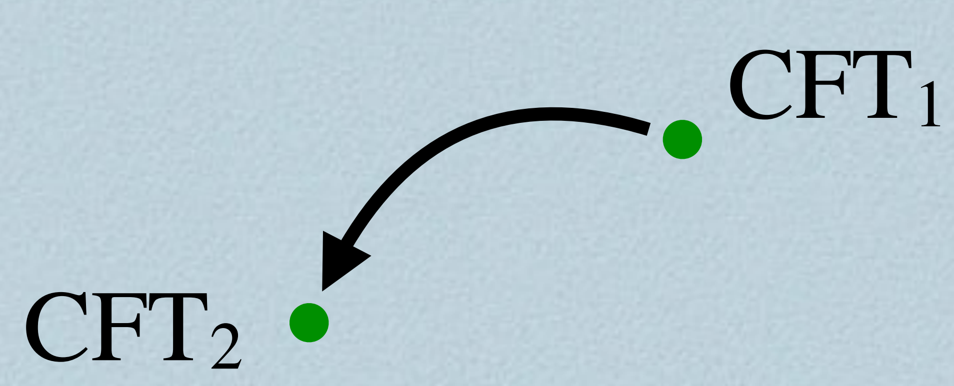
A = disk



- At relativistic RG fixed point (could be TQFT):

$$S_A = \alpha \frac{\ell}{a} - F + O(\delta/\ell)$$

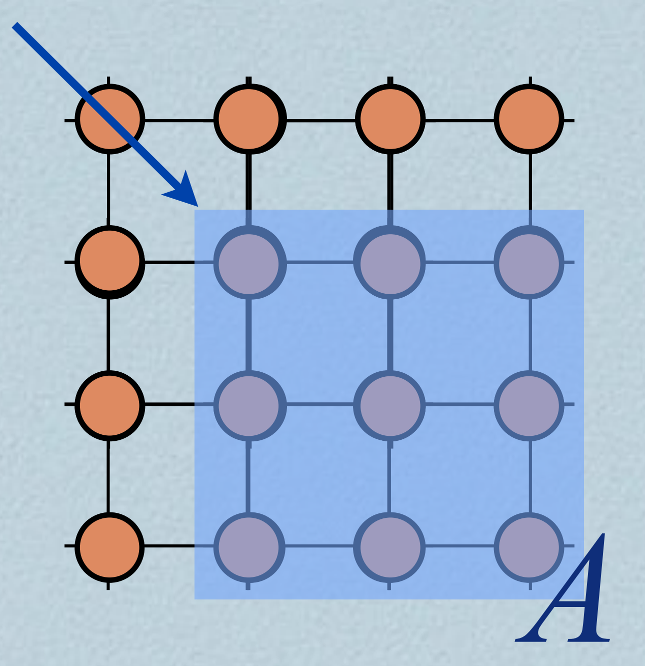
- F is RG monotone for 2+1D CFTs, aka *F-theorem* (like c in 1+1D)
  [Myers *et al* ; Casini, Huerta]



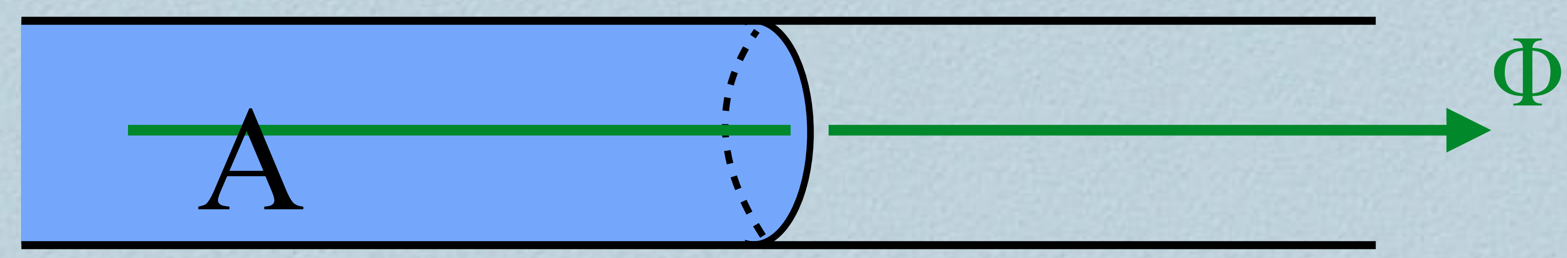
$$\boxed{F_1 \geq F_2}$$

World is not a smooth place

- Often work on a lattice
- Pixelated disk has corners that overwhelm F



Alternative: space = cylinder



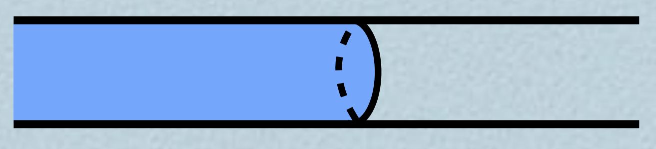
$$S_A = \alpha \frac{L_y}{a} - \gamma + \cdots$$

- Avoid corners
- If have symmetry G, insert flux Φ in cylinder
- Entire function worth of info: $S_A(\Phi)$

Free Dirac CFT

[Chen, WK, Faulkner, Fradkin; Arias, Blanco, Casini]

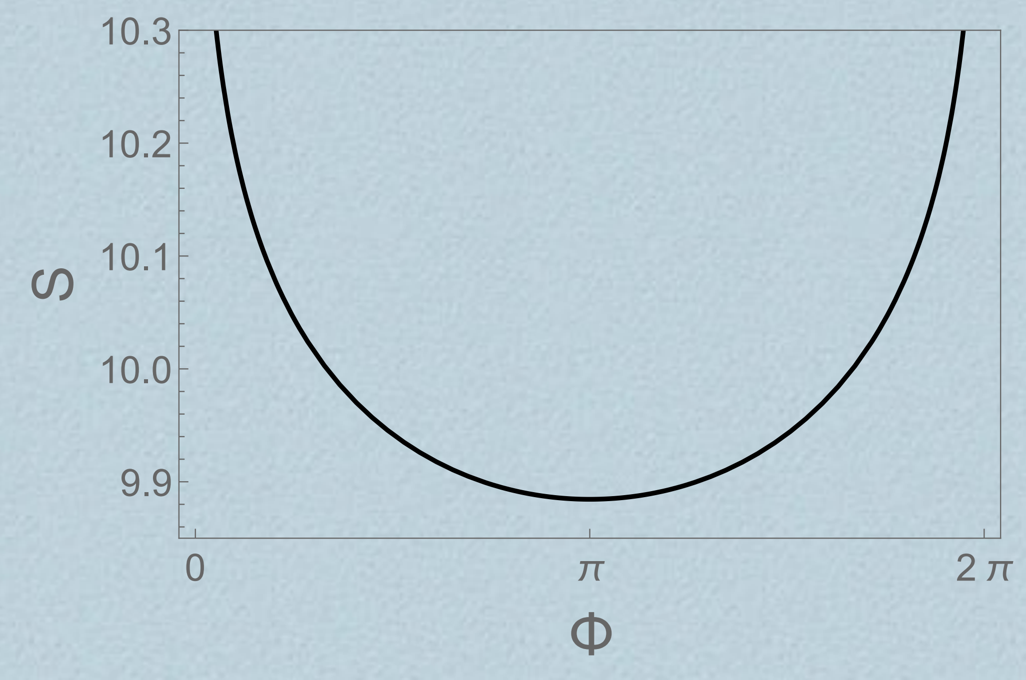
- 2-component massless Dirac fermion in the continuum



$$\boxed{S = \alpha \frac{L_y}{a} - B \ln \left| 2 \sin \left(\frac{\Phi}{2} \right) \right|}$$

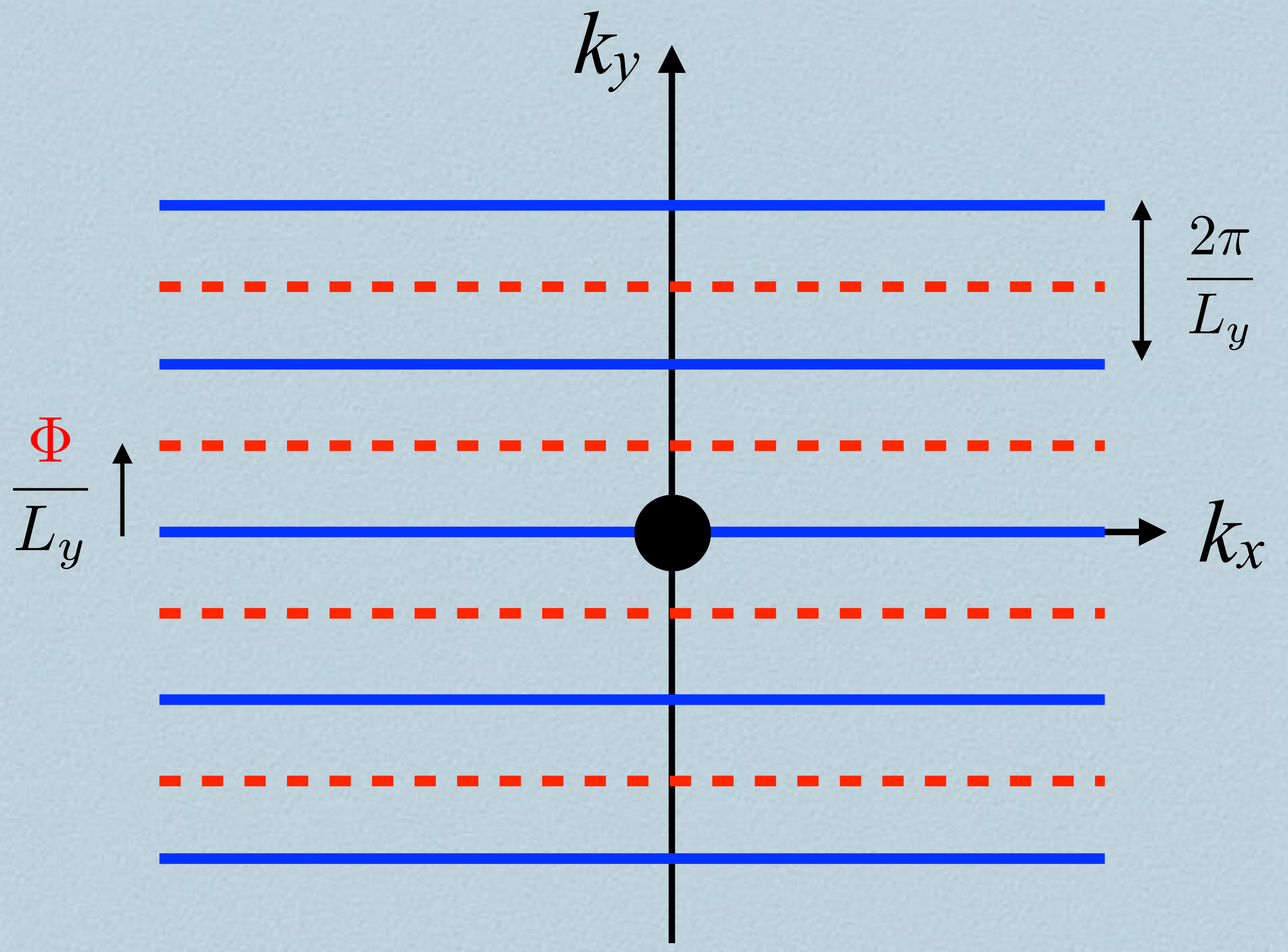
$B = 1/6 = 0.17$

- Sum EE of 1+1D fermions with mass given by k_y
[Holzhey, Larsen, Wilczek; Cardy, Calabrese]



- Transverse momenta quantized:

$$k_y = \frac{2\pi n + \Phi}{L_y}, \quad n = 0, \pm 1, \pm 2, \ldots$$



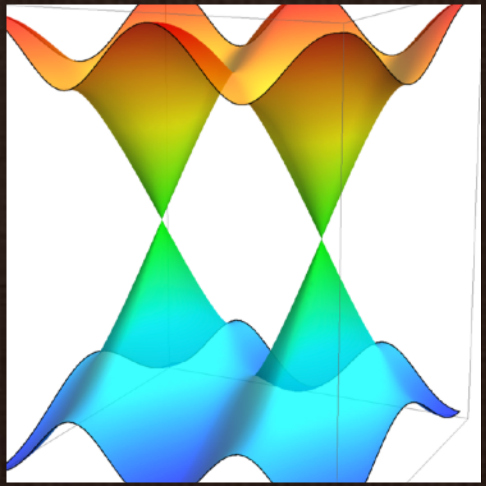
From continuum to the lattice

[Zhu, Chen, He, WK]

$$S = \alpha \frac{L_y}{a} - B \sum_{n=1}^{N} \ln \left| 2 \sin \left[\frac{1}{2} (\Phi - \Phi_n^c) \right] \right|$$

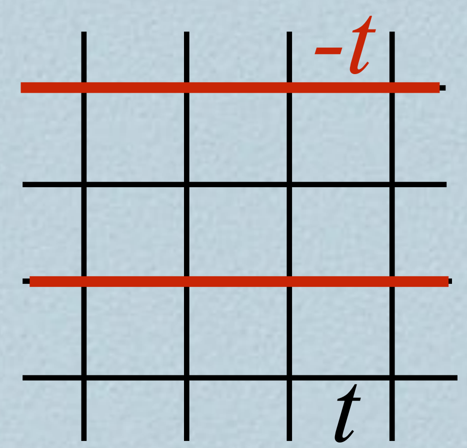
- N Dirac cones in the Brillouin Zone
- Dirac points can be a different positions in BZ
  => $\Phi^c{}_n$: flux at which nth Dirac fermion goes gapless
- Interacting Dirac fermions: treat B as **fitting parameter**

Dirac semi-metal to CDW critical point

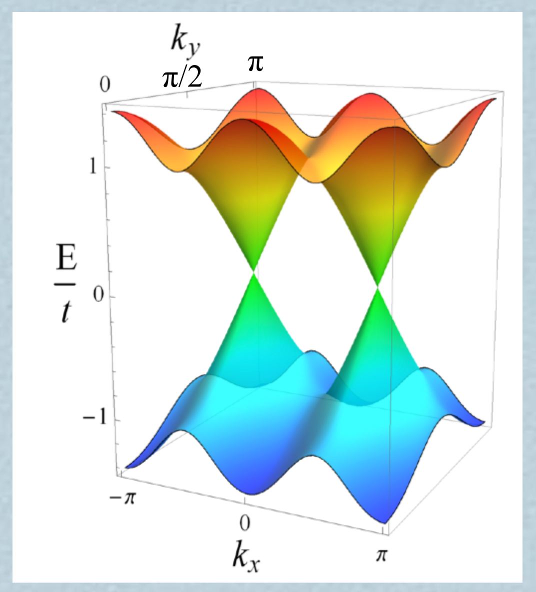


Model with Dirac points

- π-flux square lattice model;
  0.5 el/site (no spin)



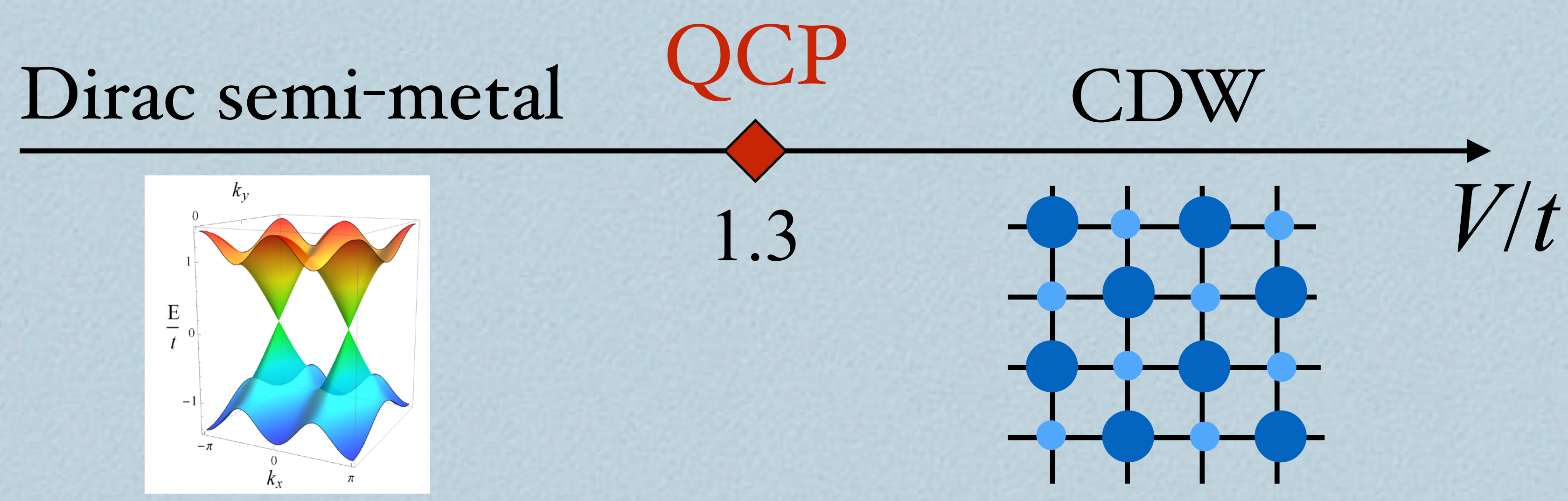
$$H_0 = t\sum_{\langle ij\rangle}\left((-1)^{s_{ij}}c_i^\dagger c_j + \mathrm{h.c.}\right)$$



- Dirac points: **k** = (±π/2, π/2)

- Repulsion

$$H = H_0 + V\sum_{\langle ij\rangle} n_i n_j$$



- CDW is gapped; breaks Z_2 symmetry
  => real order parameter $\varphi(x)$

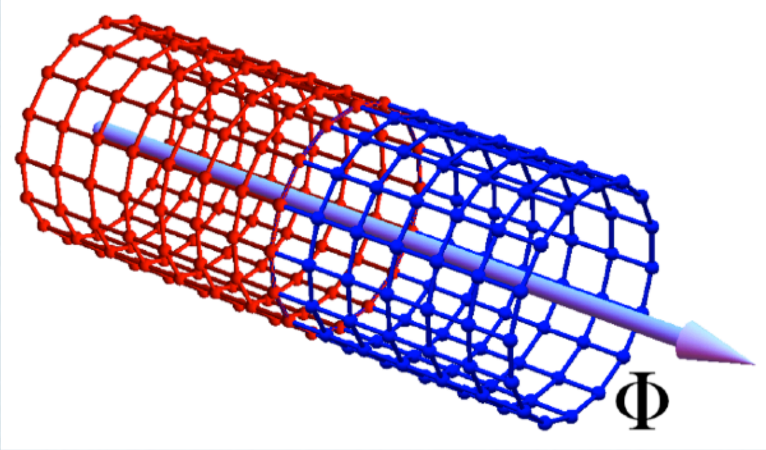
- QCP = fluctuating Ising order parameter coupled to 2 Dirac cones (Gross-Neveu-Yukawa)

$$\boxed{\begin{aligned}\mathcal{L} = \bar{\psi}_a \partial\!\!\!/ \psi_a + \left(\partial_\mu \phi \partial^\mu \phi + r\phi^2 + \lambda \phi^4\right) \\ + g\phi(\bar{\psi}_1\psi_1 - \bar{\psi}_2\psi_2)\end{aligned}}$$

[Wang *et al*; Li *et al*; Zhu, Chen, He, WK]

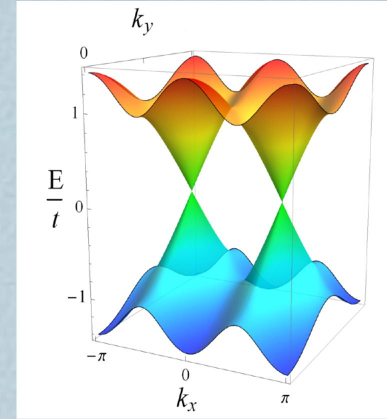
Study entanglement with DMRG

- Previous studies used Monte Carlo to study critical exponents [Wang *et al*; Li *et al*]
- Density Matrix Renormalization Group (DMRG) on infinitely long cylinders to analyze entanglement
- Extra knob: magnetic flux Φ inside cylinder (Aharonov-Bohm)

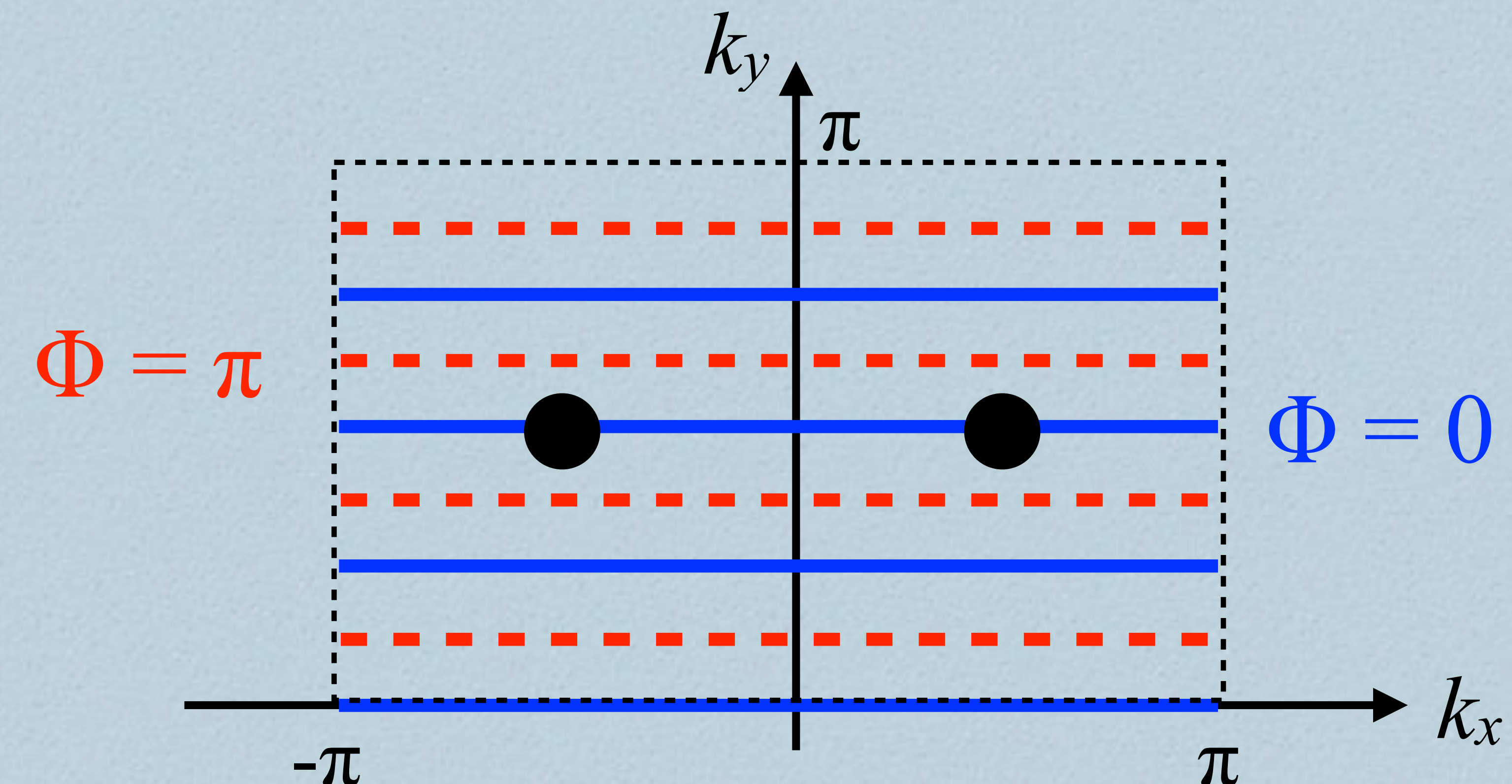


$L_y = 8$ (4 unit cells)

Allowed momenta



- $L_y = 8$ sites (4 unit cells)



$$\Phi_1^c = \Phi_2^c = 0$$

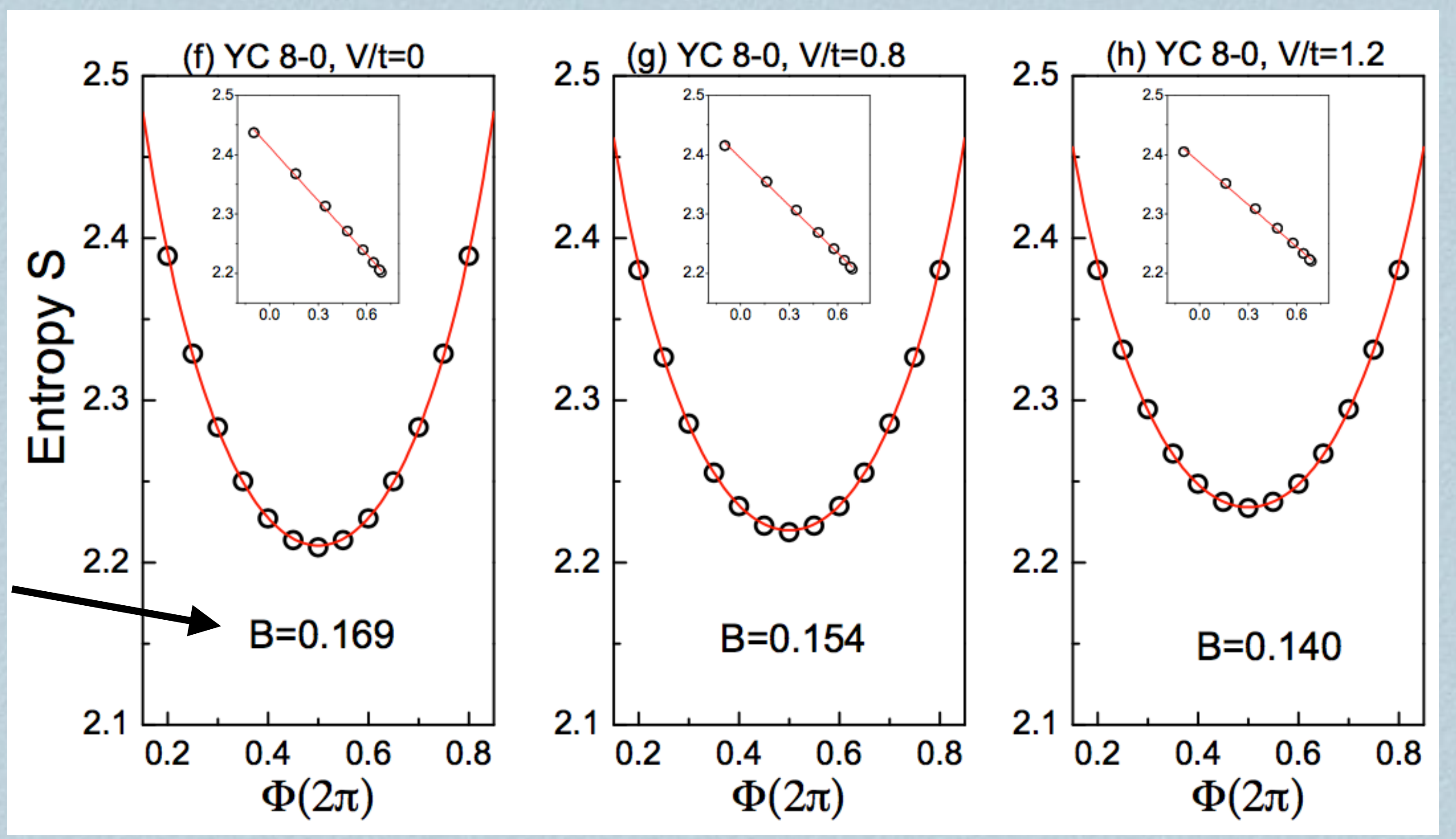
[Zhu, Chen, He, WK]

DMRG on cylinder

[Zhu, Chen, He, WK]

- $V/t < (V/t)_c = 1.3$

free fermion
$B = 1/6 = 0.167$

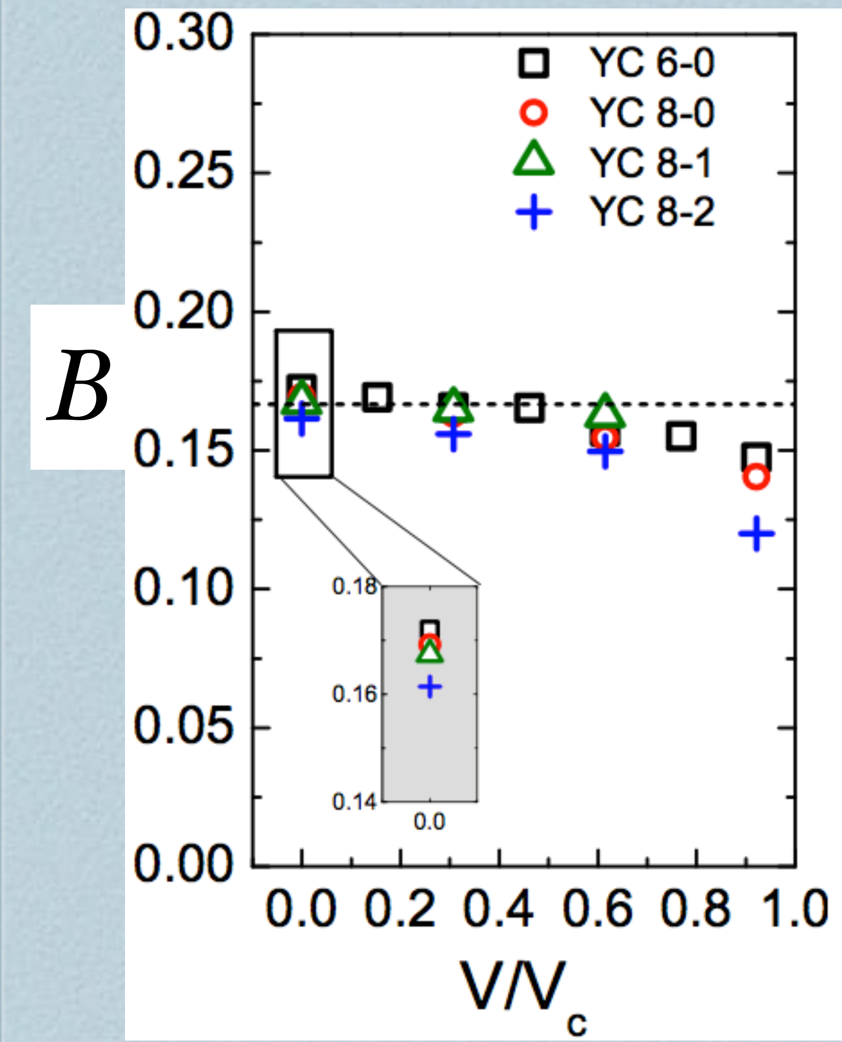


$$S = \alpha \frac{L_y}{a} - 2B \ln \left| \sin\left(\frac{\Phi}{2}\right) \right|$$

***B* decreases approaching QCP**

Entanglement via field theory at N≫1

[Whitsitt, WK, Sachdev]



[Zhu, Chen, He, WK]

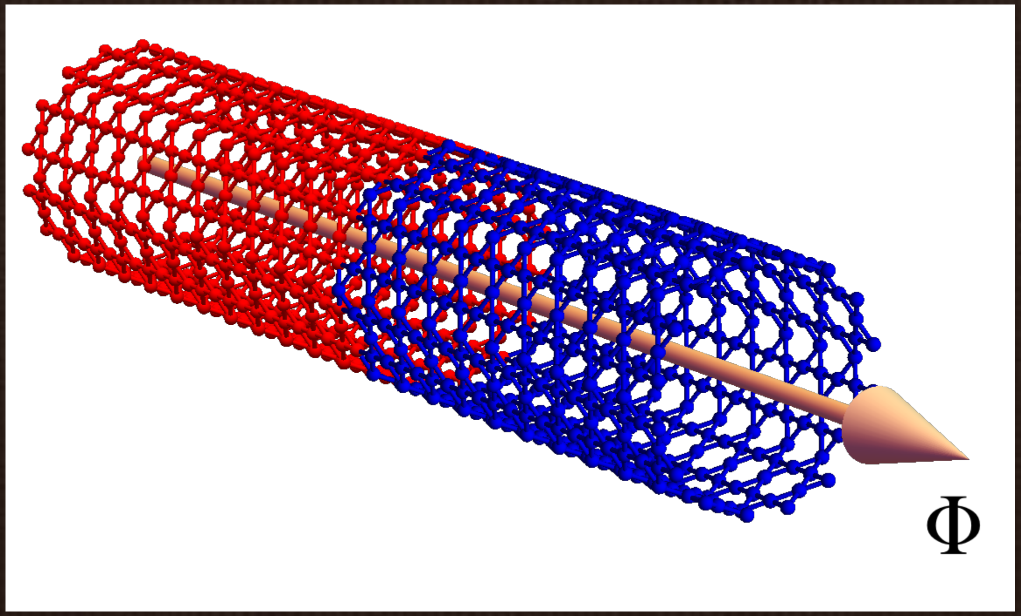
- Disk:
  $F = N\, F_{\mathrm{Dirac}} + 0.0152 + O(1/N)$
  [Klebanov, Pufu, Safdi]

- Naively expect
  $\gamma = N\, \gamma_{\mathrm{Dirac}}(\Phi) + \mathrm{g}(\Phi) + O(1/N)$

- Instead, drastic **decrease**
  $\gamma = \cancel{N\, \gamma_{\mathrm{Dirac}}(\Phi)} + \mathrm{g}(\Phi) + O(1/N)$

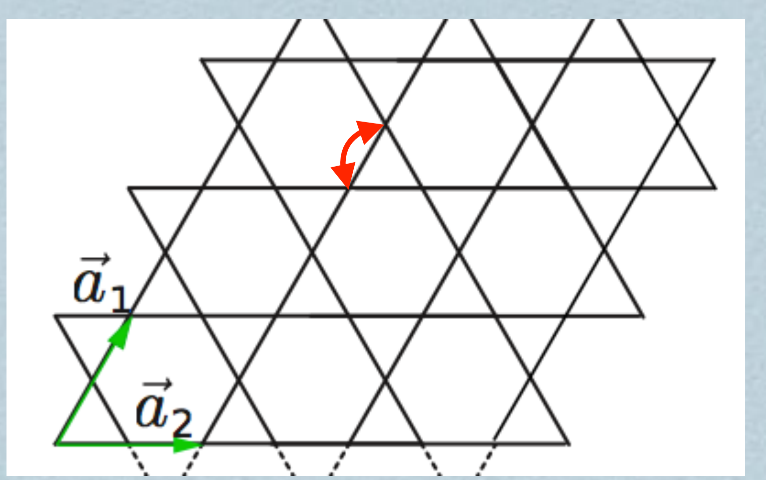
- Expect decrease to persist to finite N
  => smaller B

Kagome spin liquid



Spin 1/2 Heisenberg model on Kagome

$$H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



- Strong geometric frustration
  => promising candidate for quantum spin liquid
- Some materials have layered kagome structure
  (e.g. herberthsmithite)
- More than 2 decades of numerics (ED, VMC, DMRG, PEPS, etc): **no order**

[Reviews: Balents; Normand]

What kind of spin liquid?

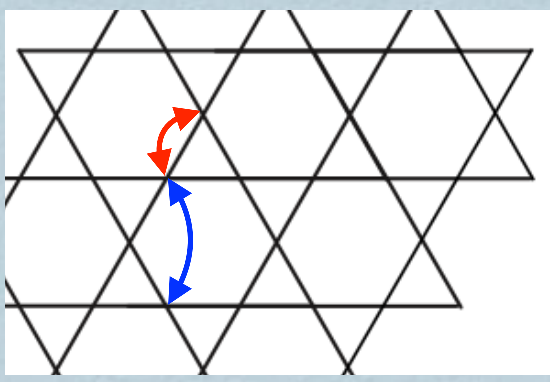
- Main competitors:
  Z2 — gapped [Sachdev 92; Yan, Huse, White 11]
  Dirac *aka* **QED3** — gapless [Hastings 00; Ran, Hermele, Lee, Wen 07]

- VMC [Iqbal *et al*; Tay, Motrunich] & latest DMRG [He, Zaletel *et al*] suggest Dirac

- Should see signatures of Dirac spinons in entanglement entropy

- DMRG on infinitely long cylinders
  [He, Zaletel *et al*; Zhu, Chen, He, WK]

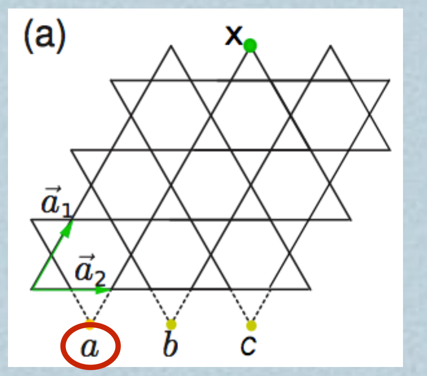
$$H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$

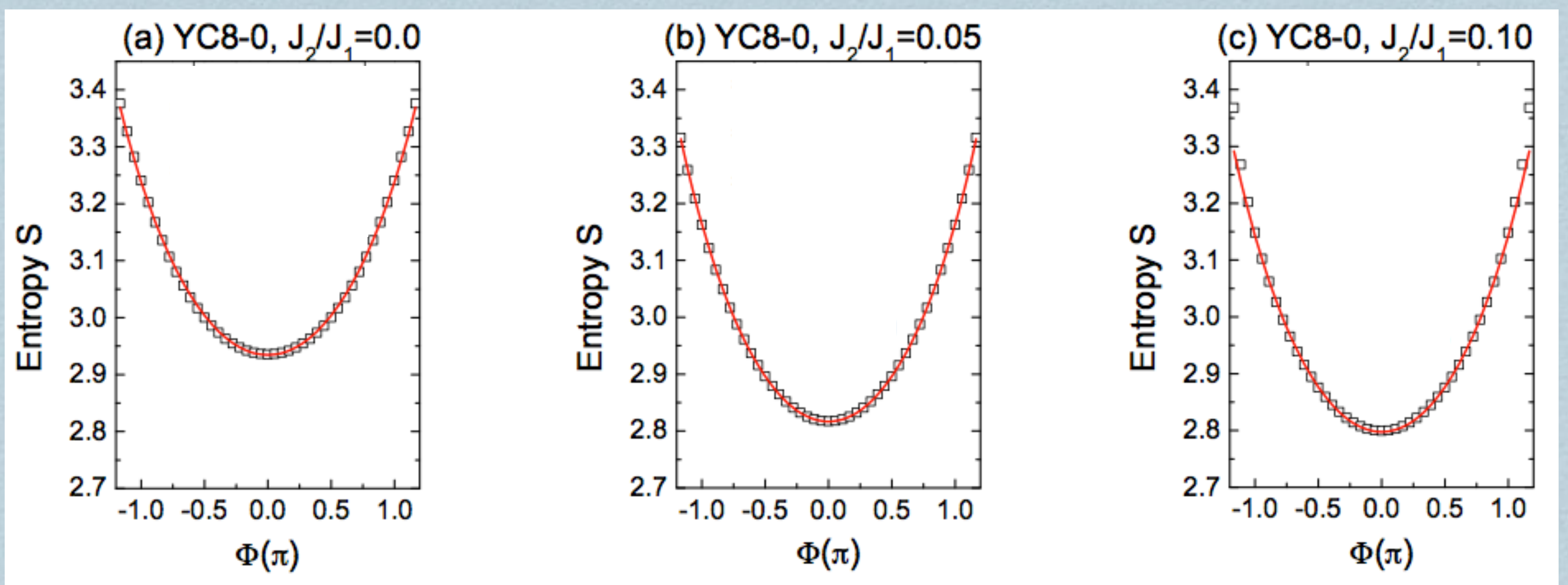


DMRG on cylinder

[Zhu, Chen, He, WK]

- YC8-0 cylinder





strong flux dependence: not expected for gapped QSL

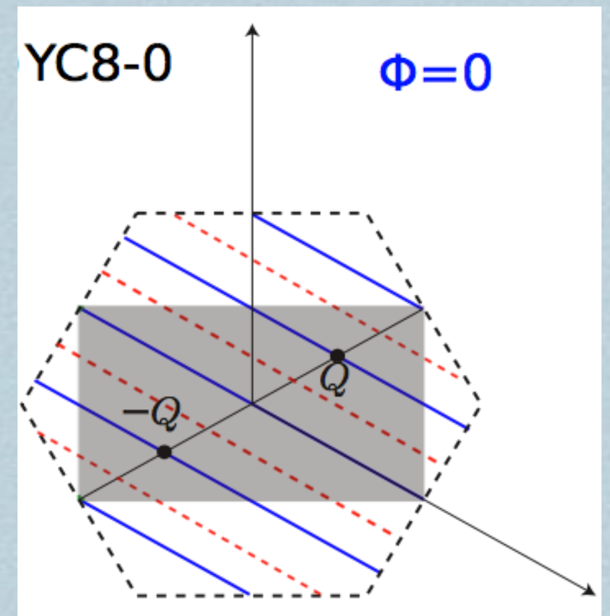
Entanglement of Dirac **spinons**

- Dirac spinons are neutral fermions with spin 1/2
  => feel half the flux compared to usual S=1 quasiparticles

$$\boxed{S = \alpha \frac{L_y}{a} - B \sum_{n=1}^{N} \ln \left| 2 \sin \left[\frac{1}{2} (s\Phi - \Phi_n^c) \right] \right|} \qquad s = 1/2$$

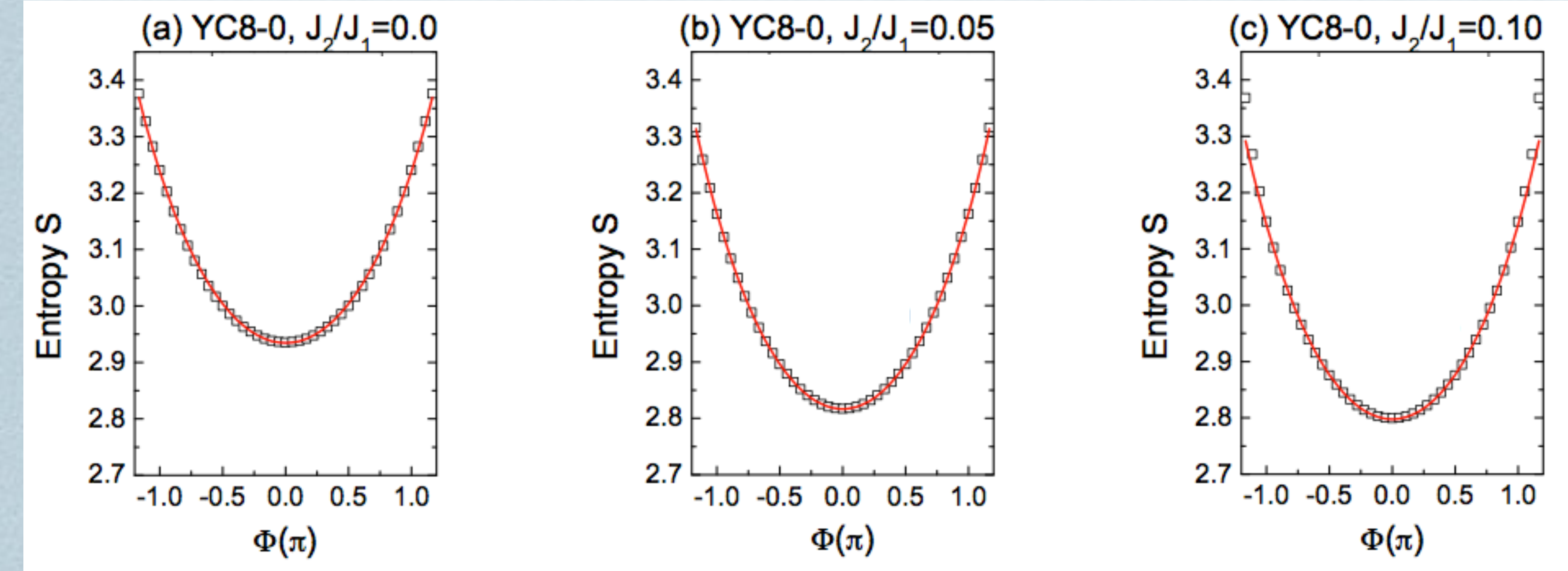
- Dirac QSL has *N*=4 Dirac cones [Hastings; Hermele *et al*]

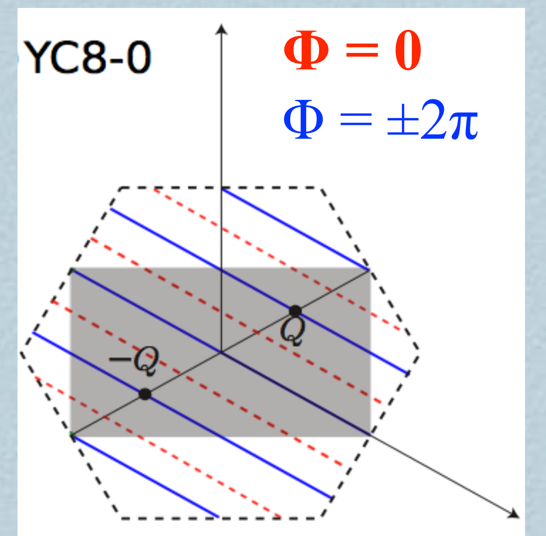
Q = (π/2, π/2)



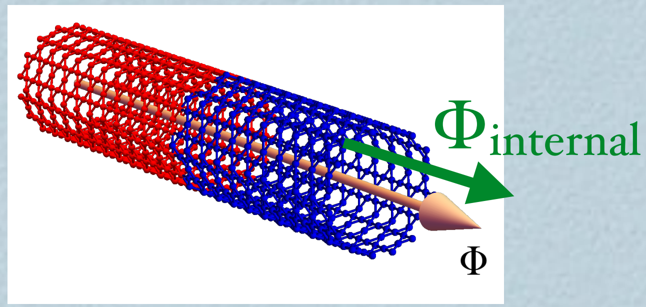
S should become max as Φ⟶0

S min at Φ=0 !

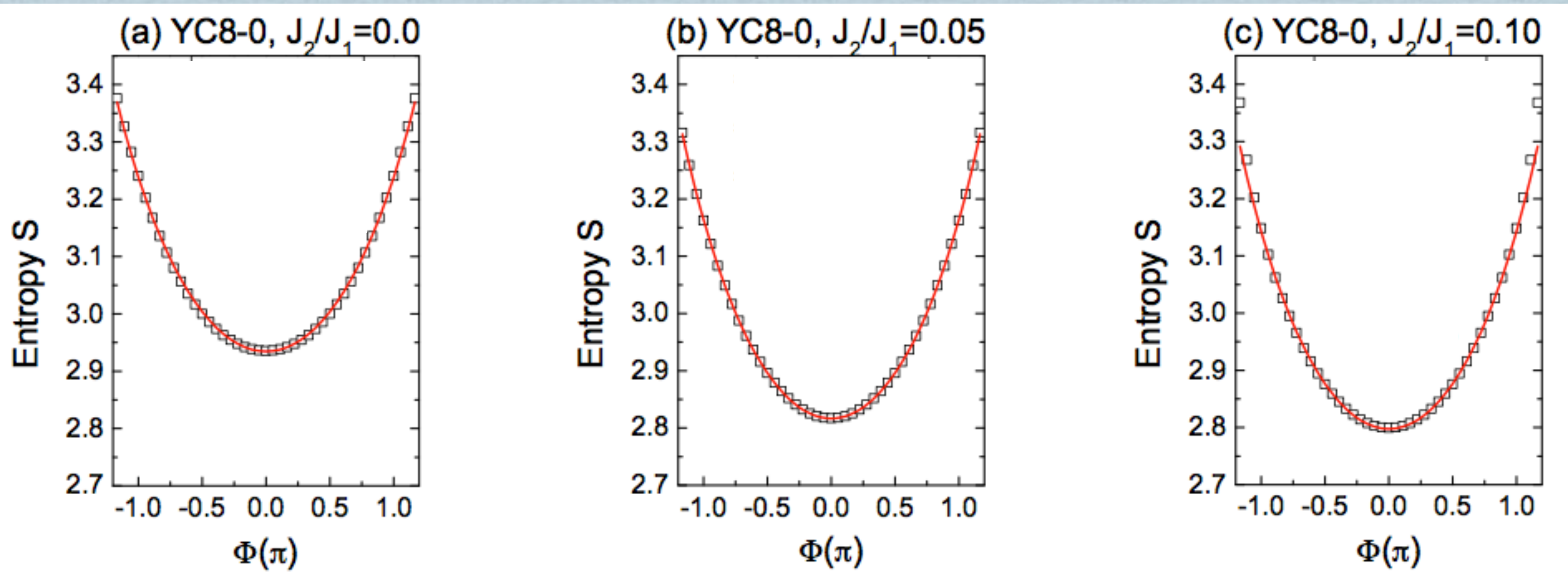


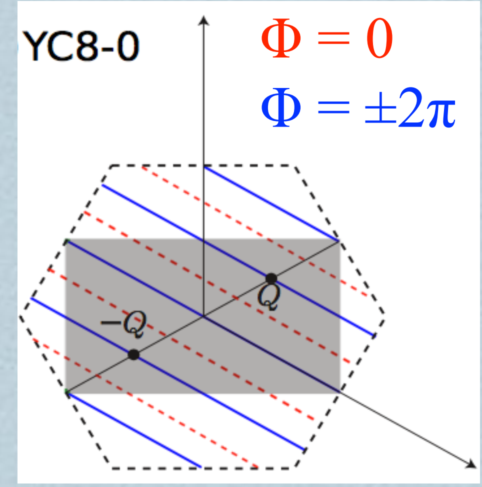


- spinons coupled to emergent gauge field that can create additional flux
- system would be exactly gapless at Φ=0, **instead** generates internal π flux



[He *et al*; Zhu, Chen, He, WK]

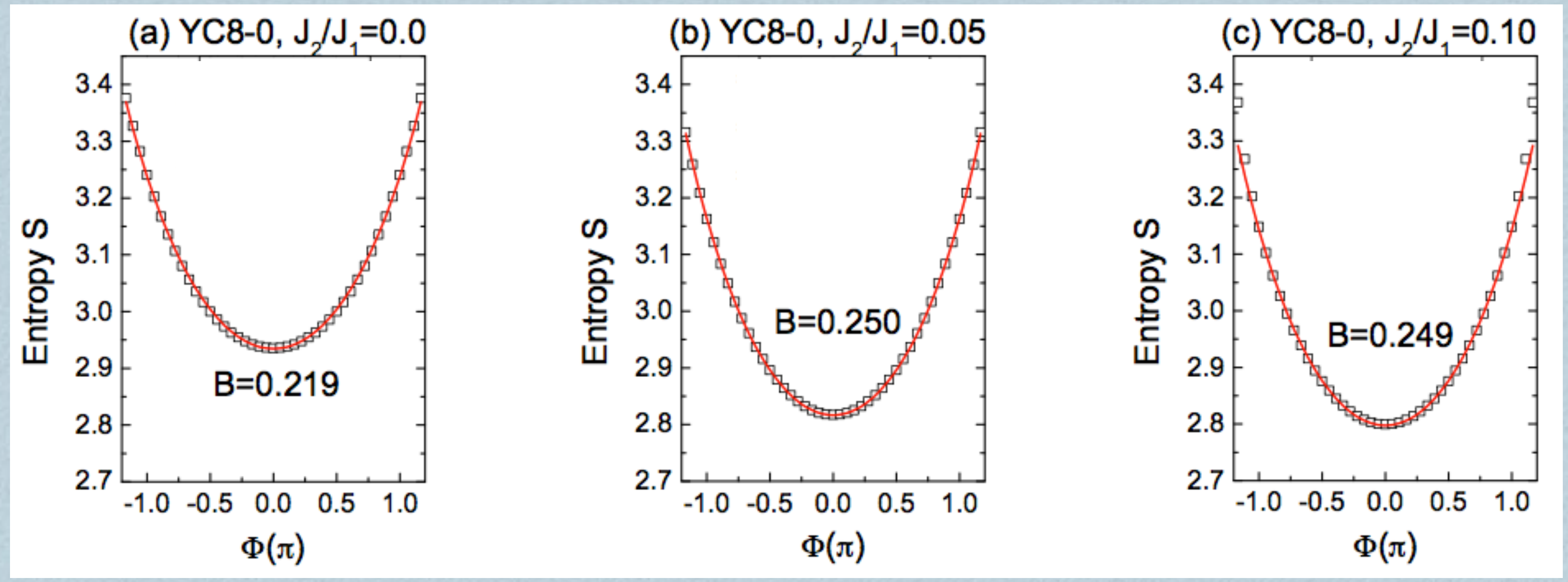




$$S = \alpha \frac{L_y}{a} - B \sum_{n=1}^{4} \ln \left| 2 \sin \left[\frac{1}{2} \left(\frac{1}{2} \Phi - \Phi_n^c \right) \right] \right|$$

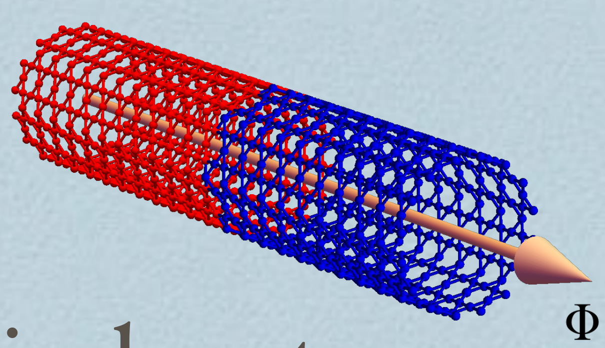
- $\Phi^c{}_n$ = *half* external flux at which *n*th Dirac fermion goes gapless

Dirac flavor		YC8-0
$\uparrow, Q$	Φ_1^c	π
$\uparrow, -Q$	Φ_2^c	π
$\downarrow, Q$	Φ_3^c	π
$\downarrow, -Q$	Φ_4^c	π



- As expected, $B \neq$ free value (1/6=0.17) since Dirac spinons interact with gauge field
- Strongly interacting system described by Quantum Electrodynamics (**QED**$_3$) with N_f=4 fermions in 2+1D
- $N_f \gg 1$: B=1/6 [Zhu, Chen, He, WK]
  Next step, compute $1/N_f$ correction to $S(\Phi)$

Recap



- Used entanglement entropy on cylinders to study:

- Quantum critical transition from Dirac semimetal to CDW (GNY) => quantum fluctuations strongly renormalize $S(\Phi)$

- Kagome Heisenberg model signatures of fractionalized Dirac fermions *(QED3)*

Outlook

- $1/N$ corrections using field theory (both systems)
- Kagome: test other properties to see if consistent with Dirac
- Study $S(\Phi)$ in other systems:
  $O(N)$ Wilson-Fisher, cWZ with emergent N=2 SUSY, etc

Thanks !

- X. Chen, WK, T. Faulkner, E. Fradkin, "***Two-cylinder entanglement entropy under a twist***", Journal of Statistical Mechanics: Theory and Experiment (2017)

- S. Whitsitt, WK, S. Sachdev, "***Entanglement entropy of the large N Wilson-Fisher conformal field theory***" (2017)

- W. Zhu, X. Chen, Y.-C. He, WK, "***Entanglement signatures of emergent Dirac fermions: kagome spin liquid & quantum criticality***", arXiv:1801.06177 (2018)