Fermionic spinon theory of square lattice spin liquids near the Néel state

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Harvard University

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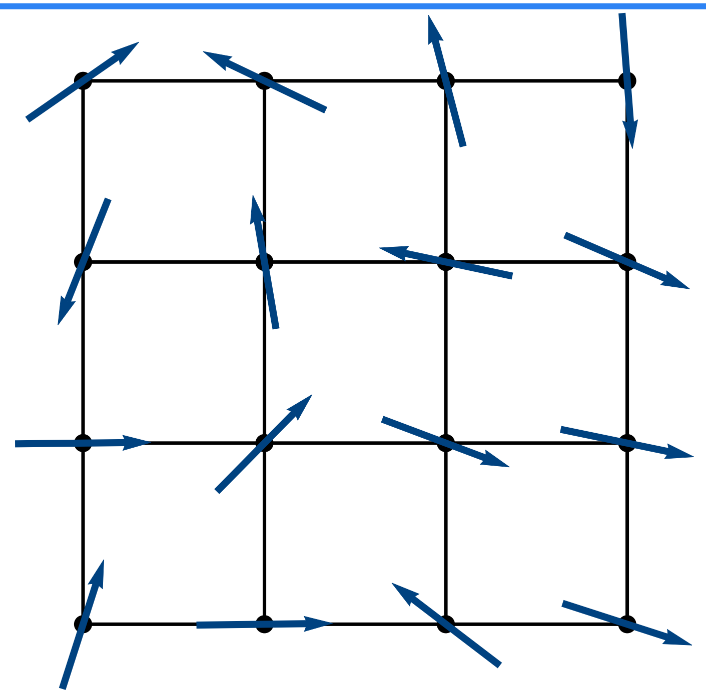
Square lattice antiferromagnet

$$H = J \sum_{\langle \boldsymbol{ij} \rangle} \boldsymbol{S_i} \cdot \boldsymbol{S_j} + \cdots$$

- Diverse phenomenology, such as:
  - symmetry broken states, *e.g.* Néel, VBS
  - topological phases, *e.g.* $\mathbb{Z}_2$ spin liquids
  - exotic (deconfined) quantum phase transitions



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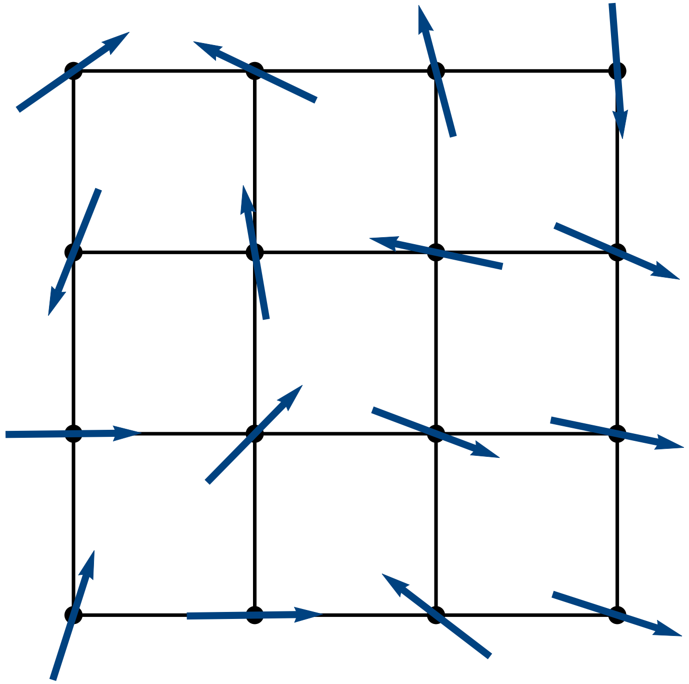

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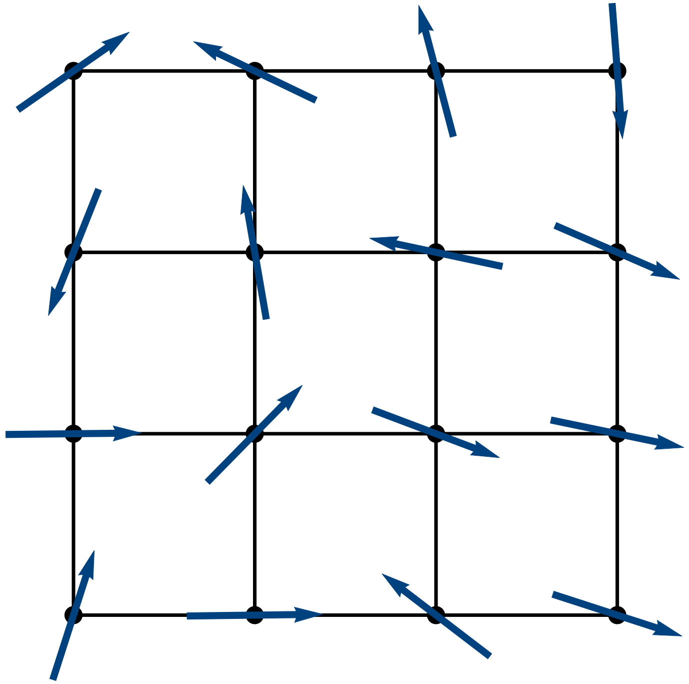

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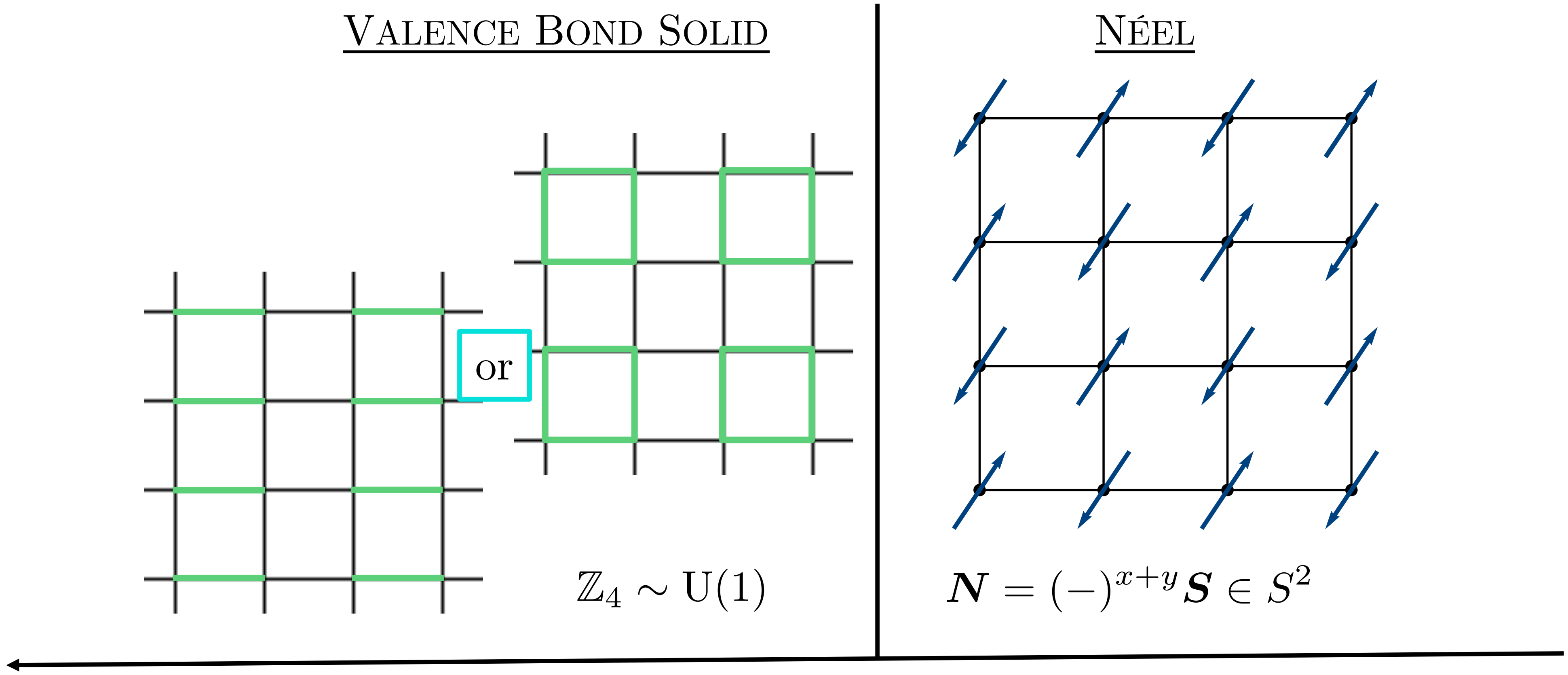
→ I will be using both of these ideas to obtain a better understanding of the Heisenberg model

Review: deconfined criticality



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Review: deconfined criticality

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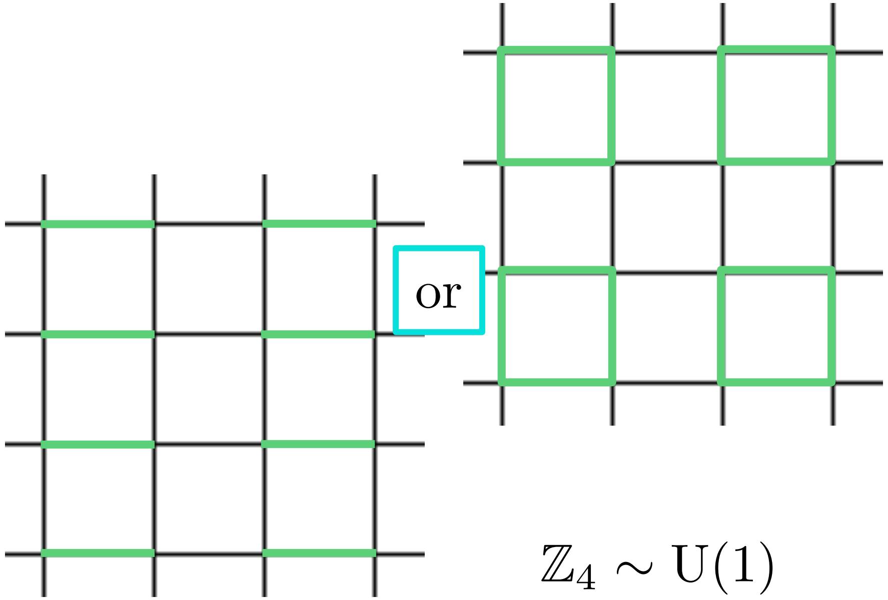
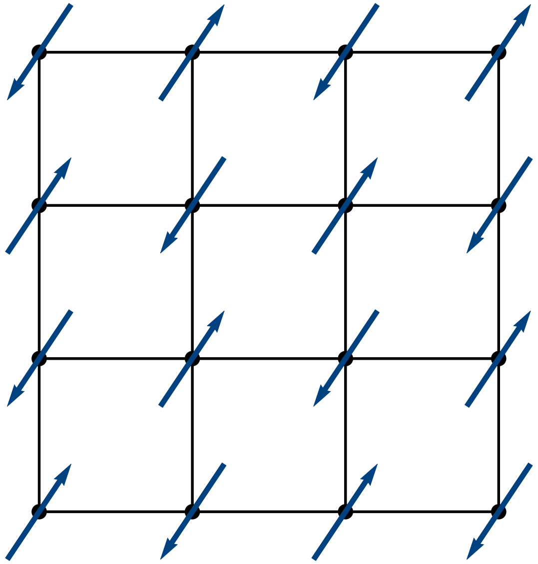


$\langle z_\alpha \rangle = 0$

$\langle \mathcal{M} \rangle \neq 0$

$\mathbb{Z}_4 \sim \mathrm{U}(1)$

NÉEL



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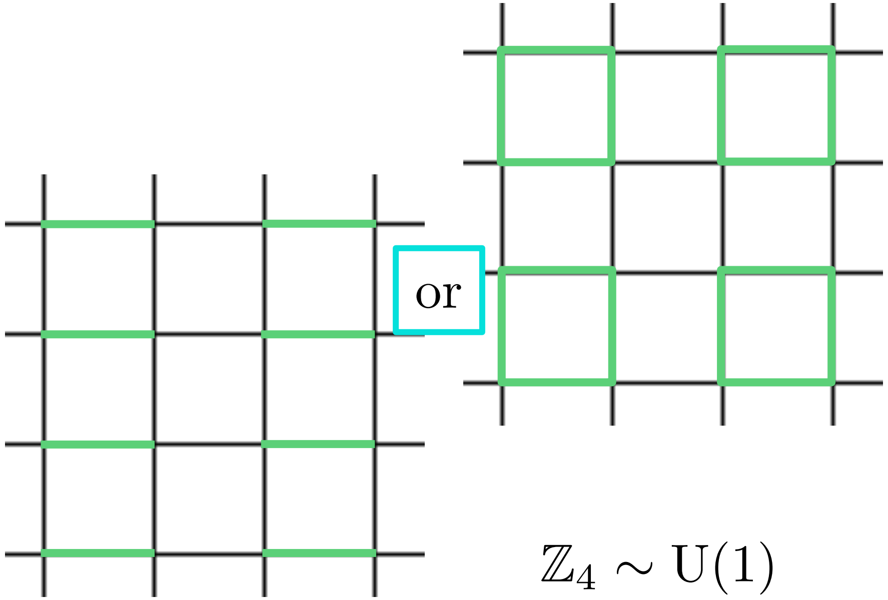

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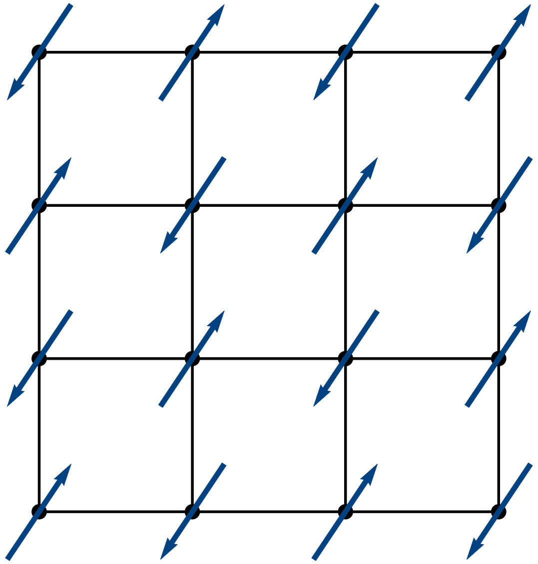

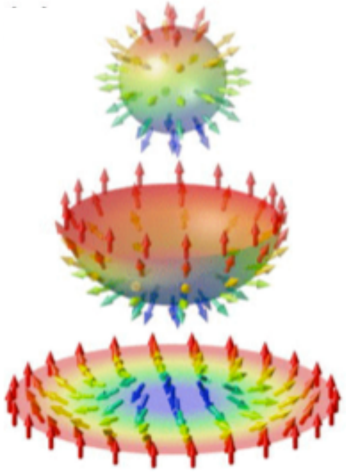

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Dualities

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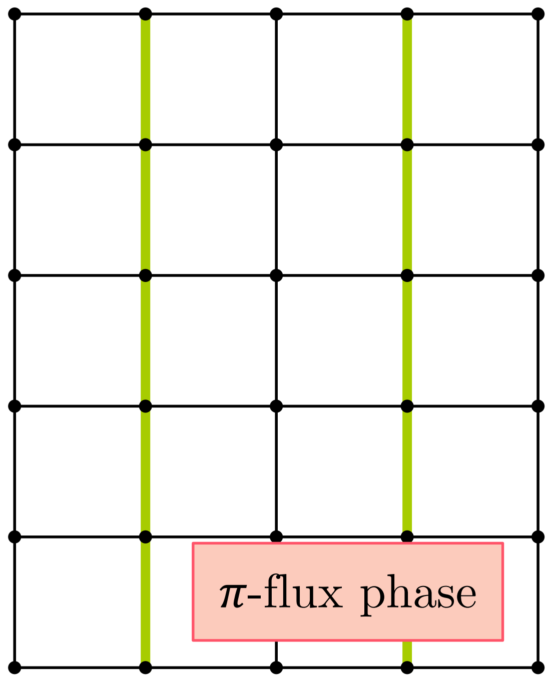

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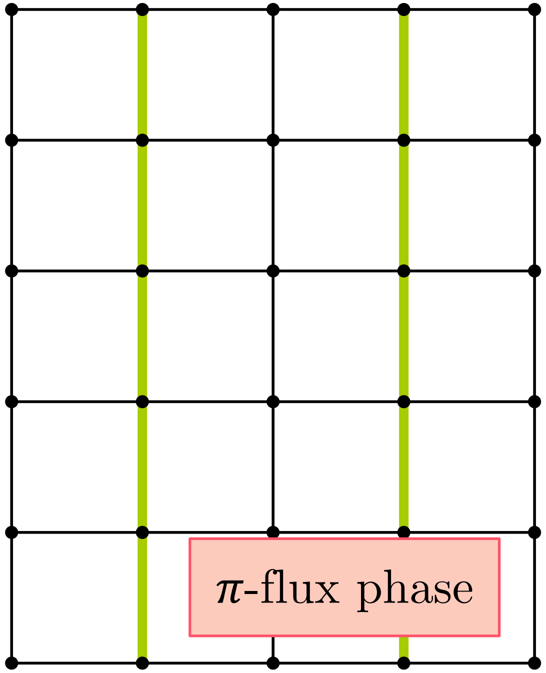

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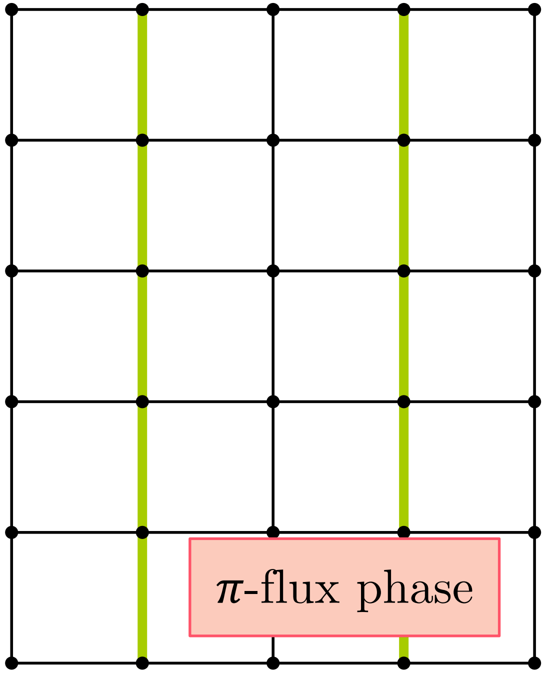

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- The symmetries are most manifest when $\mathcal{L}_{\text{QCD}}$ is expressed in terms of Majorana fermions:

$$\mathcal{L}_{\text{QCD}} = i\text{tr}\left(\bar{X}\gamma^\mu D_\mu X\right) \qquad D_\mu X = \partial_\mu X - i a_\mu^a X \sigma^a$$

$$X_v = \frac{1}{\sqrt{2}}\left(\chi_{0,v}\mathbb{1} + i\chi_{a,v}\sigma^a\right)$$

valley index, v=1,2

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Symmetries

$$\mathcal{L}_{\text{QCD}} = i\text{tr}\left(\bar{X}\gamma^{\mu}\left[\partial_{\mu}X - \frac{i}{2}a_{\mu}^{a}X\sigma^{a}\right]\right) \qquad X_{v,\alpha;\beta} = \frac{1}{\sqrt{2}}\left(\chi_{0,v}\delta_{\alpha\beta} + i\chi_{a,v}\sigma^{a}_{\alpha\beta}\right): \; 4\times 2 \text{ matrix}$$

- Global spin rotations act on the left while gauge transformations act on the right:

<u>SPIN</u>

$$\text{SU}(2)_s : X \to \exp\left(\frac{i}{2}\theta\boldsymbol{n}\cdot\boldsymbol{\sigma}\right)X$$

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- The action of the lattice symmetries can also be deduced from the parton construction:

$$e.g. \qquad T_x : \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \to \mu^x \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} X_2 \\ X_1 \end{pmatrix}$$

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- $\mathcal{L}_{\text{QCD}}$ has a manifest SO(5) symmetry, $X \to LX$, where L is a 4×4 unitary matrix

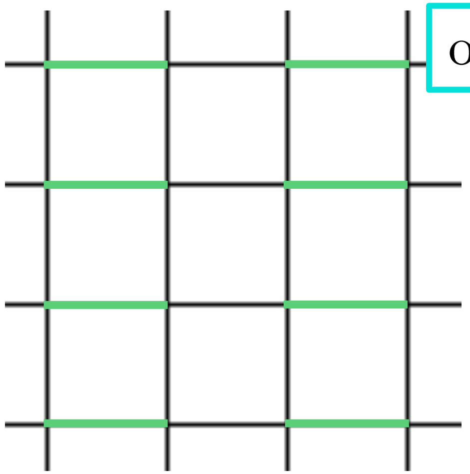
  - SO(5) order parameter: $n^a = \text{tr}\left(\bar{X}\Gamma^a X\right), \quad \Gamma^a = \{\underbrace{\mu^x, \mu^z}_{\text{VBS order parameter}}, \underbrace{\mu^y\sigma^x, \mu^y\sigma^y, \mu^y\sigma^z}_{\text{Néel order parameter}}\}$

Wang et al. (2016
Nahum et al. (2015)

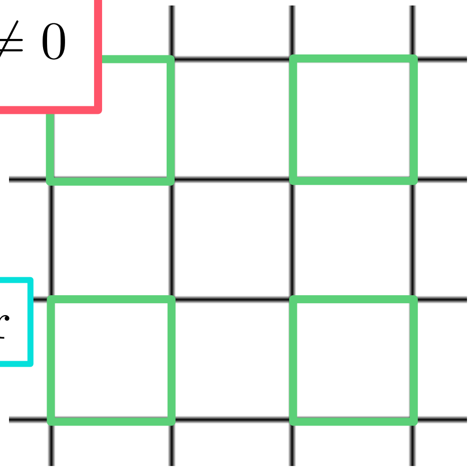
Phase diagram

Valence Bond Solid

$$\boldsymbol{V} = \Big(\mathrm{tr}\,(\bar{X}\mu^x X)\,,\mathrm{tr}\,(\bar{X}\mu^z X)\Big) \neq 0$$



or



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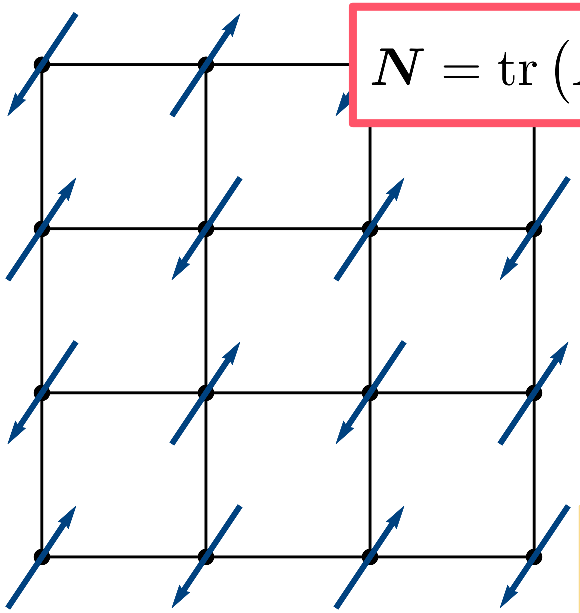

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SO(5) order parameter: $\boldsymbol{n} = (V_1, V_2, N_1, N_2, N_3)$

Proximate phase via the Higgs mechanism

$$\mathcal{L}_{\rm QCD} = i{\rm tr}\left(\bar{X}\gamma^\mu\left[\partial_\mu X - \frac{i}{2}a_\mu^a X\sigma^a\right]\right)$$

- Proximate topological phases can be accessed by Higgsing the SU(2) gauge symmetry:

$$\mathcal{L} = \mathcal{L}_{\rm QCD} + \Phi^a \underbrace{{\rm tr}\left(\sigma^a \bar{X} M X\right)}_{\text{fermion bilinear}} + \underbrace{\left|\partial_\mu \Phi^a - i\epsilon^{abc}a_\mu^b \Phi^c\right|^2}_{\Phi^a \text{ transforms under the } vector \text{ representation}} - s\left(\Phi^a\right)^2 + \cdots$$

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  - Two Higgs fields, $\mathbf{\Phi}$ and $\mathbf{\Phi}_1$, are needed to obtain a $\mathbb{Z}_2$ spin liquid:

$$\mathcal{L}_{f} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{0} + \mathcal{L}_{1}$$

$$\mathcal{L}_{0} = \lambda\mathbf{\Phi}\cdot\boldsymbol{\mathcal{O}}_{0} + \left|D_{\mu}\mathbf{\Phi}\right|^{2} - s\mathbf{\Phi}^{2} + \cdots \qquad \mathcal{L}_{1} = \lambda'\mathbf{\Phi}_{1}\cdot\boldsymbol{\mathcal{O}}_{1} + \left|D_{\mu}\mathbf{\Phi}_{1}\right|^{2} - \bar{s}_{1}\mathbf{\Phi}_{1}^{2} + \cdots$$

Projective symmetry group

- The symmetries act in a non-trivial way in the Higgs phase. For instance,

$$\mathcal{L} = i\mathrm{tr}\left(\bar{X}\gamma^{\mu}\left[\partial_{\mu}X - \frac{i}{2}a_{\mu}^{x}X\sigma^{x}\right]\right) + \underbrace{\lambda\left\langle\Phi^{x}\right\rangle \mathrm{tr}\left(\sigma^{x}\bar{X}\mu^{y}X\right)}_{\text{Gives the fermions a mass}} + \cdots$$

Wen (2002)

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  - $\mathcal{L}$ does not appear to be invariant under the lattice symmetries:

$$T_{x}:\mathrm{tr}\left(\sigma^{x}\bar{X}\mu^{y}X\right) \to \mathrm{tr}\left(\sigma^{x}\bar{X}\mu^{x}\mu^{y}\mu^{x}X\right) = -\mathrm{tr}\left(\sigma^{x}\bar{X}\mu^{y}X\right)$$

  - This can be undone through a gauge transformation:

$$V_{tx}: X \to Xi\sigma^{z}, \qquad \mathrm{tr}\left(\sigma^{x}\bar{X}\mu^{y}X\right) \to -\mathrm{tr}\left(\sigma^{x}\bar{X}\mu^{y}X\right)$$

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➡ The transformations $V_g G$, $V_g \in \mathrm{SU}(2)_g$ defines the *projective symmetry group*, which can be used to classify the spin liquid.

Wen (2002)

Gapped, symmetric $\mathbb{Z}_2$ spin liquids

$$\mathcal{L}_0 = \lambda \mathbf{\Phi} \cdot \boldsymbol{\mathcal{O}}_0 + |D_\mu \mathbf{\Phi}|^2 - s\mathbf{\Phi}^2 + \cdots \qquad \mathcal{L}_1 = \lambda' \mathbf{\Phi}_1 \cdot \boldsymbol{\mathcal{O}}_1 + |D_\mu \mathbf{\Phi}_1|^2 - \bar{s}_1 \mathbf{\Phi}_1^2 + \cdots$$

- When only bilinears containing up to a *single* derivative are considered, 5 distinct gapped, symmetric $\mathbb{Z}_2$ spin liquids are possible.

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	M
1	$i\partial_0$
2	$\mu^y \gamma^\mu i\partial_\mu$
3	$\mu^y\left(\gamma^x i\partial_x - \gamma^y i\partial_y\right)$
4	$\mu^y\left(\gamma^x i\partial_y + \gamma^y i\partial_x\right)$
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PSG 1

$$\mathcal{L}_f = \mathcal{L}_{\text{QCD}} + \lambda\, \boldsymbol{\Phi} \cdot \text{tr}\left(\boldsymbol{\sigma} \bar{X} \mu^y X\right) - \bar{s}\,|\boldsymbol{\Phi}|^2 + \lambda'\, \boldsymbol{\Phi}_1 \cdot \text{tr}\left(\boldsymbol{\sigma} \bar{X} i\partial_0 X\right) - \bar{s}_1\,|\boldsymbol{\Phi}_1|^2 + \cdots \;\; \text{with } \bar{s}, \bar{s}_1 \leq 0$$

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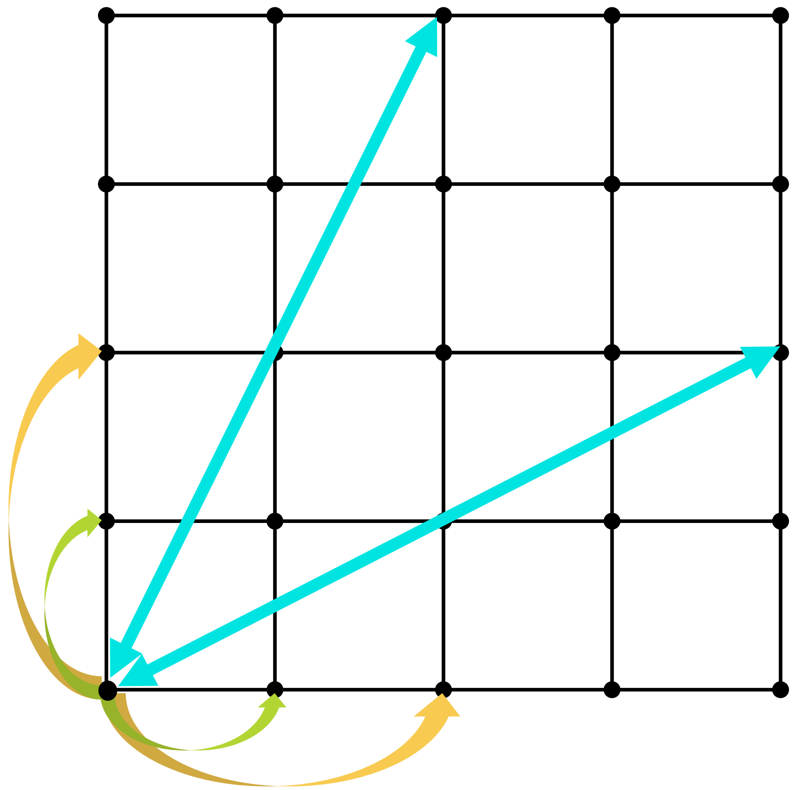

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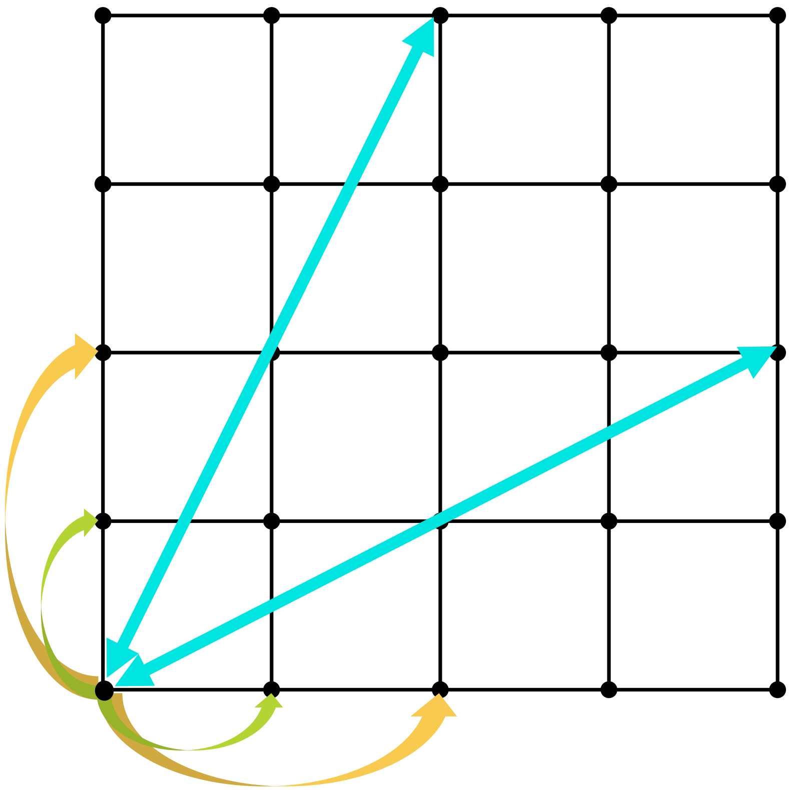

- $\mathcal{O}_1 = \text{tr}\left(\boldsymbol{\sigma}\bar{X}i\partial_0 X\right)$ has no lattice analogue

→ this phase is more energetically favourable than mean field theory seems to imply

Comparison with bosonic formulation

$$\mathcal{L}_f = \mathcal{L}_{\text{QCD}} + \lambda\, \boldsymbol{\Phi} \cdot \text{tr}\left(\boldsymbol{\sigma} \bar{X} \mu^y X\right) + \lambda' \boldsymbol{\Phi}_1 \cdot \text{tr}\left(\boldsymbol{\sigma} \bar{X} \partial_0 X\right) + \cdots$$

- #1 is the only spin liquid with a corresponding bosonic formulation

	M
1	$i\partial_0$
2	$\mu^y \gamma^\mu i\partial_\mu$
3	$\mu^y \left(\gamma^x i\partial_x - \gamma^y i\partial_y\right)$
4	$\mu^y \left(\gamma^x i\partial_y + \gamma^y i\partial_x\right)$
5	$\mu^y \left(\gamma^x i\partial_y - \gamma^y i\partial_x\right)$

Essin & Hermele (2011)
Yang & Wang (2016)
Chatterjee et al. (2017)

Comparison with bosonic formulation

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- #1 is the only spin liquid with a corresponding bosonic formulation
- Further, this spin liquid can be obtained by perturbing directly about $\mathbb{CP}^1$:

$$\mathcal{L}_P = \lambda_1 P^* \epsilon_{\alpha\beta} z_\alpha \partial_0 z_\beta + h.c. + \left|\left(\partial_\mu - i2b_\mu\right) P\right|^2 - s_1 \left|P\right|^2 + \cdots$$

$$\mathcal{L}_b = \mathcal{L}_{dcp} + \mathcal{L}_P$$

Essin & Hermele (2011)
Yang & Wang (2016)
Chatterjee et al. (2017)

Comparison with bosonic formulation

$$\mathcal{L}_f = \mathcal{L}_{\text{QCD}} + \lambda\, \mathbf{\Phi} \cdot \text{tr}\left(\boldsymbol{\sigma} \bar{X} \mu^y X\right) + \lambda' \mathbf{\Phi}_1 \cdot \text{tr}\left(\boldsymbol{\sigma} \bar{X} \partial_0 X\right) + \cdots$$

	M
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2	$\mu^y \gamma^\mu i \partial_\mu$
3	$\mu^y \left(\gamma^x i \partial_x - \gamma^y i \partial_y\right)$
4	$\mu^y \left(\gamma^x i \partial_y + \gamma^y i \partial_x\right)$
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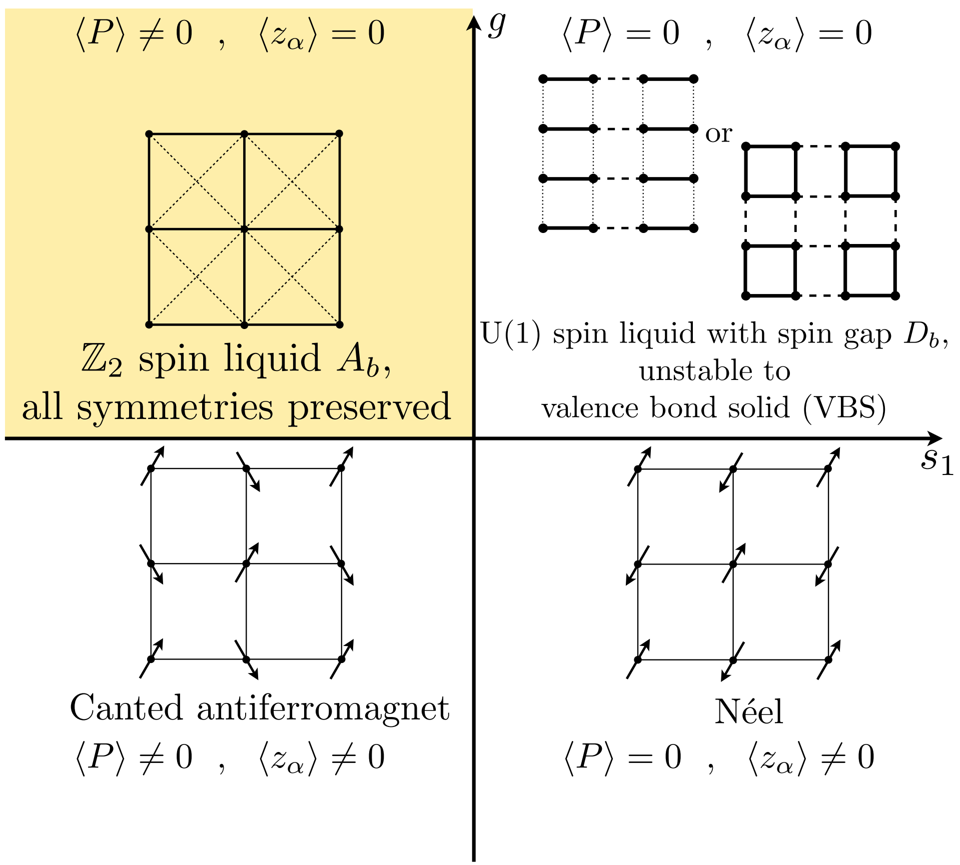
$$\mathcal{L}_b = \mathcal{L}_{dcp} + \mathcal{L}_P$$

→ Obtain the *same* $\mathbb{Z}_2$ spin liquid when $\langle z \rangle = 0, \quad \langle P \rangle \neq 0$

Essin & Hermele (2011)
Yang & Wang (2016)
Chatterjee et al. (2017)

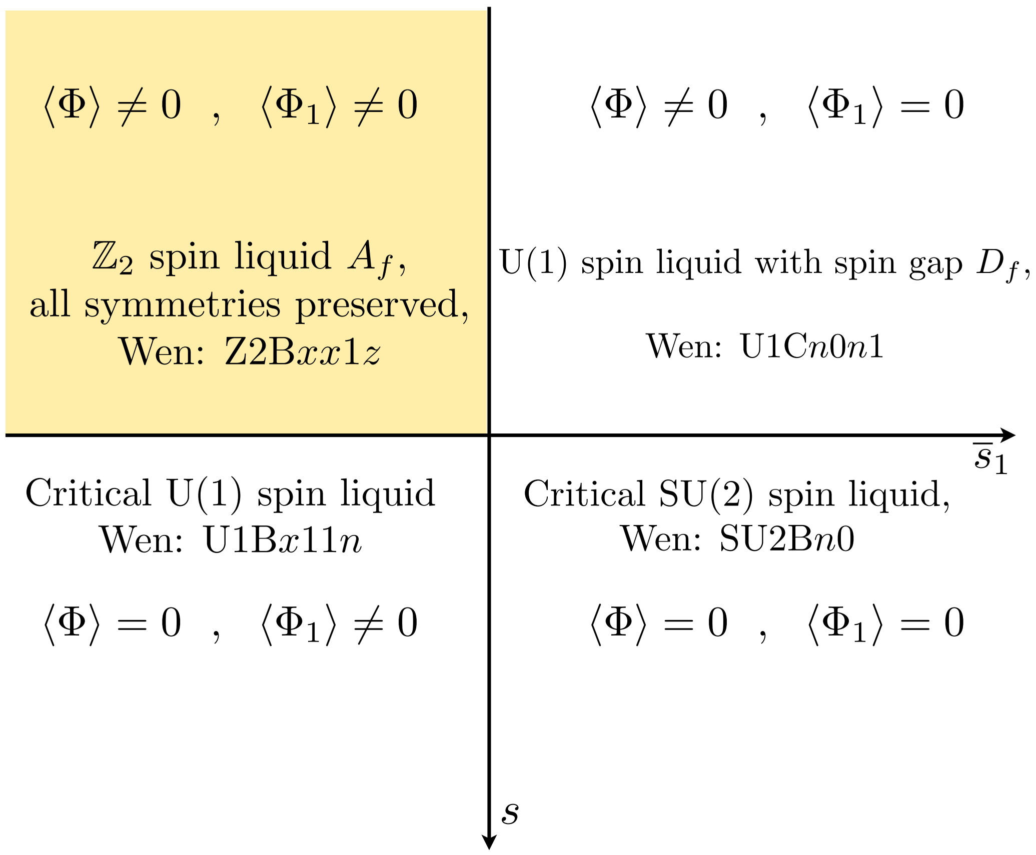
Phase diagram

Bosonic



$$\mathcal{L}_b = \mathcal{L}_{dcp} + \lambda_1 P^* \epsilon_{\alpha\beta} z_\alpha \partial_0 z_\beta + h.c. \\ - s_1 |P|^2 + \cdots$$

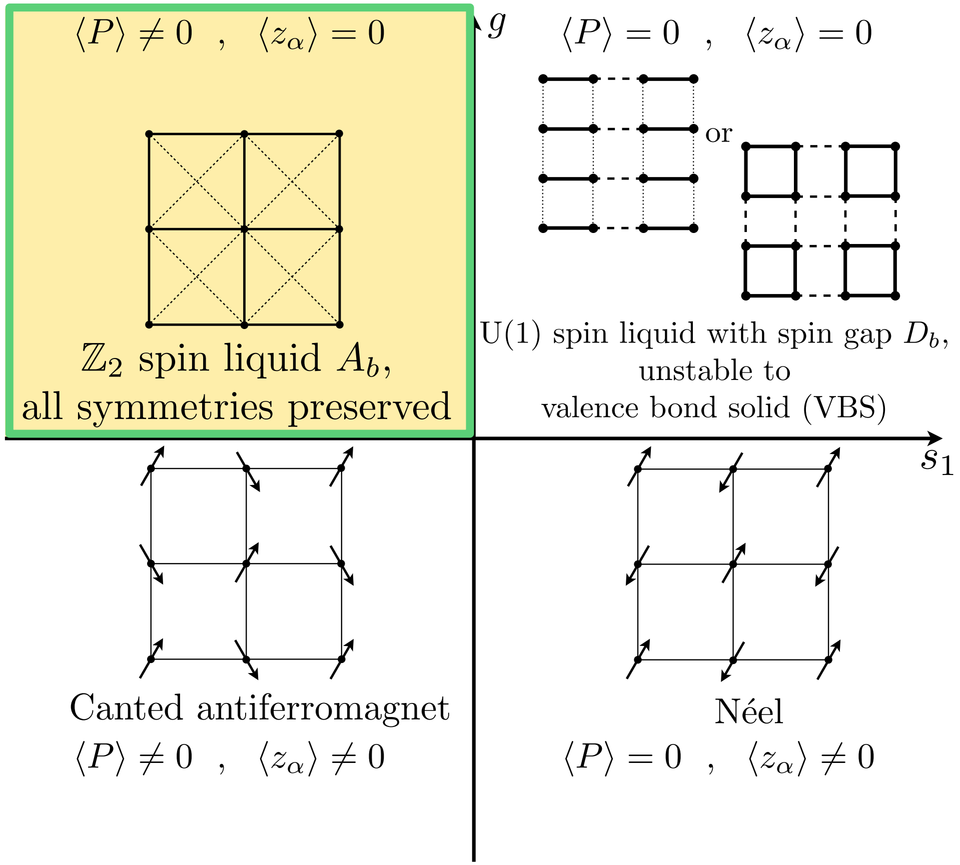
Fermionic



$$\mathcal{L}_f = \mathcal{L}_{\rm QCD} + \lambda \mathbf{\Phi} \cdot {\rm tr}\left(\boldsymbol{\sigma} \bar{X} \mu^y X\right) - s\,|\mathbf{\Phi}|^2 \\ + \lambda' \mathbf{\Phi}_1 \cdot {\rm tr}\left(\boldsymbol{\sigma} \bar{X} \partial_0 X\right) - \bar{s}_1\,|\mathbf{\Phi}_1|^2 + \cdots$$

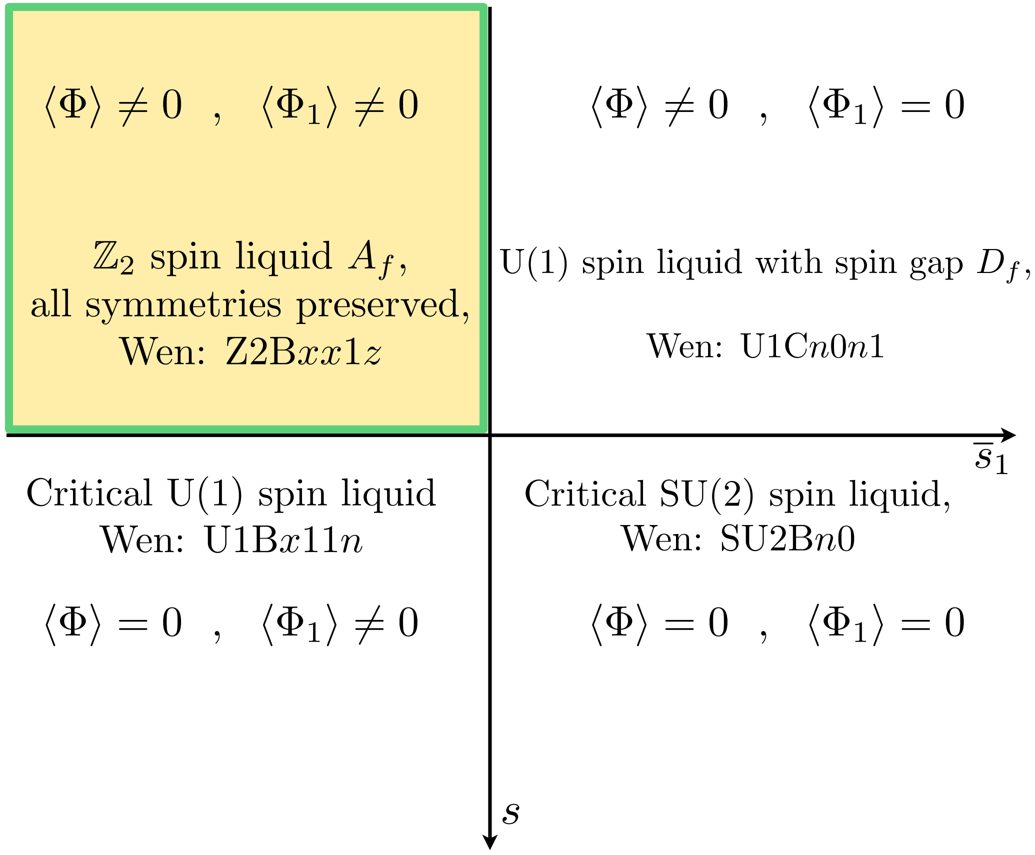
Phase diagram

BOSONIC



$$\mathcal{L}_b = \mathcal{L}_{dcp} + \lambda_1 P^* \epsilon_{\alpha\beta} z_\alpha \partial_0 z_\beta + h.c.$$
$$- s_1 |P|^2 + \cdots$$

FERMIONIC

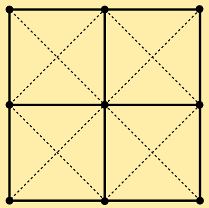


$$\mathcal{L}_f = \mathcal{L}_{\rm QCD} + \lambda \boldsymbol{\Phi} \cdot {\rm tr}\left(\boldsymbol{\sigma} \bar{X} \mu^y X\right) - s\,|\boldsymbol{\Phi}|^2$$
$$+ \lambda' \boldsymbol{\Phi}_1 \cdot {\rm tr}\left(\boldsymbol{\sigma} \bar{X} \partial_0 X\right) - \bar{s}_1 |\boldsymbol{\Phi}_1|^2 + \cdots$$

Phase diagram

Bosonic

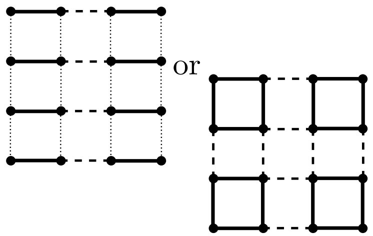
$\langle P \rangle \neq 0 \;,\; \langle z_\alpha \rangle = 0$



$\mathbb{Z}_2$ spin liquid A_b,
all symmetries preserved

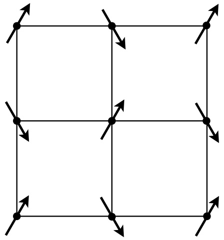
g

$\langle P \rangle = 0 \;,\; \langle z_\alpha \rangle = 0$

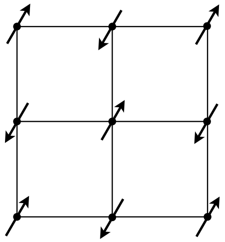


U(1) spin liquid with spin gap D_b,
unstable to
valence bond solid (VBS)

s_1



Canted antiferromagnet
$\langle P \rangle \neq 0 \;,\; \langle z_\alpha \rangle \neq 0$



Néel
$\langle P \rangle = 0 \;,\; \langle z_\alpha \rangle \neq 0$

$$\mathcal{L}_b = \mathcal{L}_{dcp} + \lambda_1 P^* \epsilon_{\alpha\beta} z_\alpha \partial_0 z_\beta + h.c. - s_1 |P|^2 + \cdots$$

Fermionic

$\langle \Phi \rangle \neq 0 \;,\; \langle \Phi_1 \rangle \neq 0$

$\mathbb{Z}_2$ spin liquid A_f,
all symmetries preserved,
Wen: Z2Bxx1z

$\langle \Phi \rangle \neq 0 \;,\; \langle \Phi_1 \rangle = 0$

U(1) spin liquid with spin gap D_f

Wen: U1Cn0n1

$\bar{s}_1$

Critical U(1) spin liquid
Wen: U1Bx11n

$\langle \Phi \rangle = 0 \;,\; \langle \Phi_1 \rangle \neq 0$

Critical SU(2) spin liquid,
Wen: SU2Bn0

$\langle \Phi \rangle = 0 \;,\; \langle \Phi_1 \rangle = 0$

s

$$\mathcal{L}_f = \mathcal{L}_{\rm QCD} + \lambda \boldsymbol{\Phi} \cdot {\rm tr}\left(\boldsymbol{\sigma} \bar{X} \mu^y X\right) - s\, |\boldsymbol{\Phi}|^2 + \lambda' \boldsymbol{\Phi}_1 \cdot {\rm tr}\left(\boldsymbol{\sigma} \bar{X} \partial_0 X\right) - \bar{s}_1 |\boldsymbol{\Phi}_1|^2 + \cdots$$

Flux response

$$\mathcal{L}_{\mathrm{U}(1)} = i\mathrm{tr}\left(\bar{X}\gamma^\mu\left[\partial_\mu X - \frac{i}{2}a_\mu X\sigma^x\right]\right) + +m\mathrm{tr}\left(\sigma^x\bar{X}\mu^y X\right) + \cdots$$

- Response to insertion of flux: $\langle\mathcal{O}(r)\rangle = \int d^3r\,\left\langle\mathcal{O}(r)\mathrm{tr}\left(\sigma^x\bar{X}(r')\gamma^\mu X(r')\right)\right\rangle a_\mu^{cl}(r')$

$\longrightarrow$ $\mathcal{O} = \mathrm{tr}\left(\bar{X}\gamma^\mu\mu^y X\right)$ $\quad\left\langle\mathrm{tr}\left(\bar{X}\gamma^\mu\mu^y X\right)\right\rangle \sim \frac{1}{\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda^{cl}$

Fu, Sachdev, Xu (2011)

Flux response

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$$\rightarrow \quad \mathcal{O} = \mathrm{tr}\left(\bar{X}\gamma^\mu\mu^y X\right) \qquad \boxed{\langle\mathrm{tr}\left(\bar{X}\gamma^\mu\mu^y X\right)\rangle \sim \frac{1}{\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a^{cl}_\lambda}$$

- This is exactly the current associated with the $\mathrm{U}(1)_v$ VBS

$$n^a = \mathrm{tr}\left(\bar{X}\Gamma^a X\right), \quad \Gamma^a = \{\underbrace{\mu^x, \mu^z}_{\text{VBS order parameters}}, \mu^y\sigma^x, \mu^y\sigma^y, \mu^y\sigma^z\} \quad \Rightarrow \quad \underbrace{J^\mu_{vbs} = \frac{1}{2}\mathrm{tr}\left(\bar{X}\gamma^\mu\mu^y X\right)}_{\text{VBS conserved current}}$$

Fu, Sachdev, Xu (2011)

Flux response

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- Response to insertion of flux: $\langle\mathcal{O}(r)\rangle = \int d^3r \left\langle\mathcal{O}(r)\mathrm{tr}\left(\sigma^x\bar{X}(r')\gamma^\mu X(r')\right)\right\rangle a^{cl}_\mu(r')$

$$\longrightarrow \mathcal{O} = \mathrm{tr}\left(\bar{X}\gamma^\mu\mu^y X\right) \qquad \boxed{\left\langle\mathrm{tr}\left(\bar{X}\gamma^\mu\mu^y X\right)\right\rangle \sim \frac{1}{\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a^{cl}_\lambda}$$

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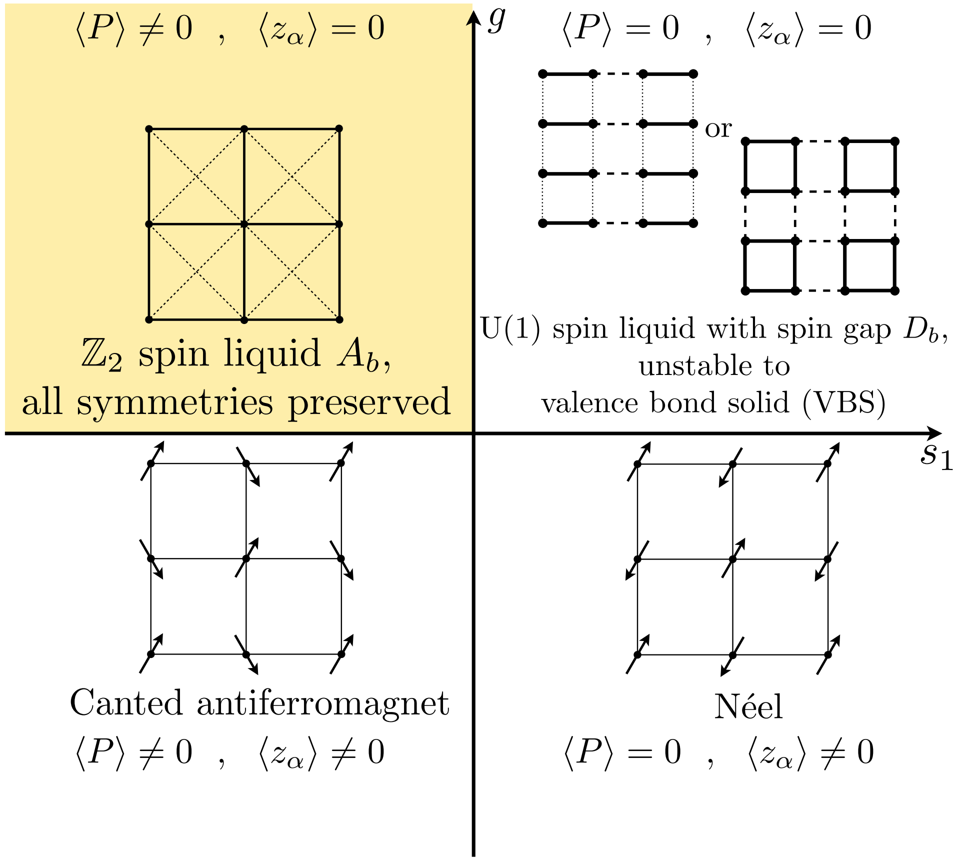
- Monopole proliferation results in *large fluctuations* of the charge Q associated to J^μ_{vbs} which will *suppress* fluctuations of operators *conjugate* to Q, *i.e.* the order parameters

$$\Rightarrow \boxed{\left\langle\mathrm{tr}\left(\bar{X}\mu^z X\right)\right\rangle \neq 0 \quad \text{or} \quad \left\langle\mathrm{tr}\left(\bar{X}\mu^x X\right)\right\rangle \neq 0.}$$

Fu, Sachdev, Xu (2011)

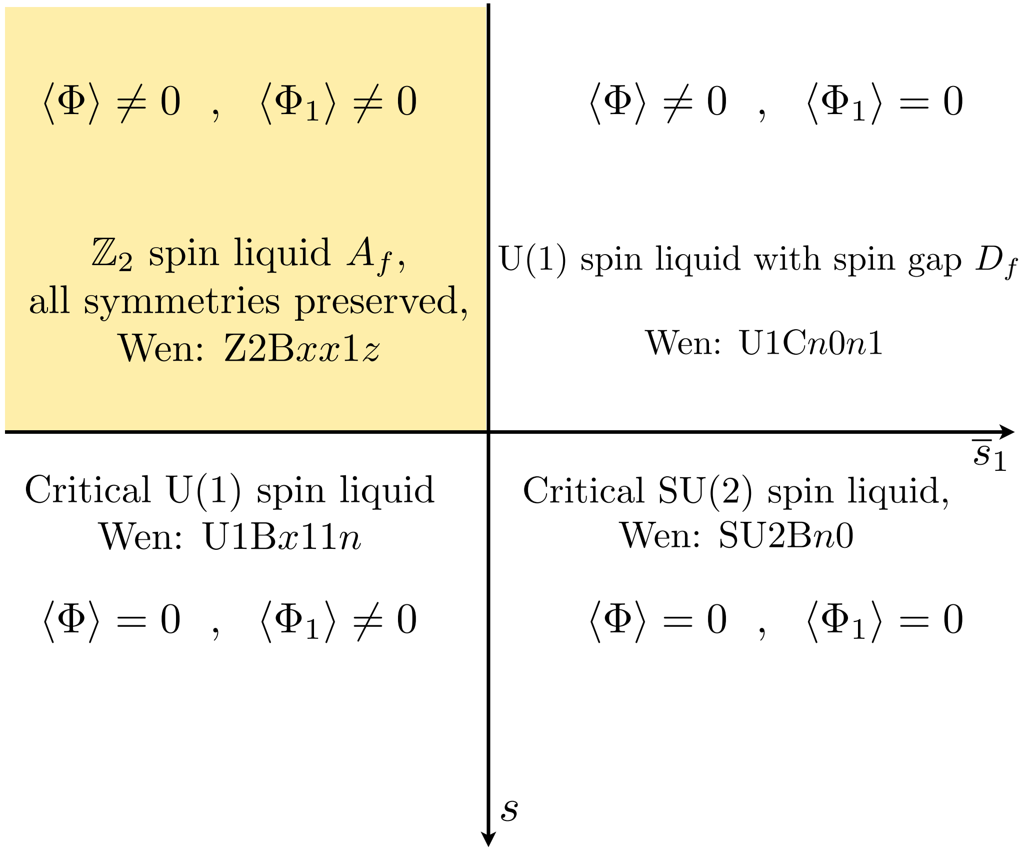
Phase diagram

Bosonic



$$\mathcal{L}_b = \mathcal{L}_{dcp} + \lambda_1 P^* \epsilon_{\alpha\beta} z_\alpha \partial_0 z_\beta + h.c. - s_1 |P|^2 + \cdots$$

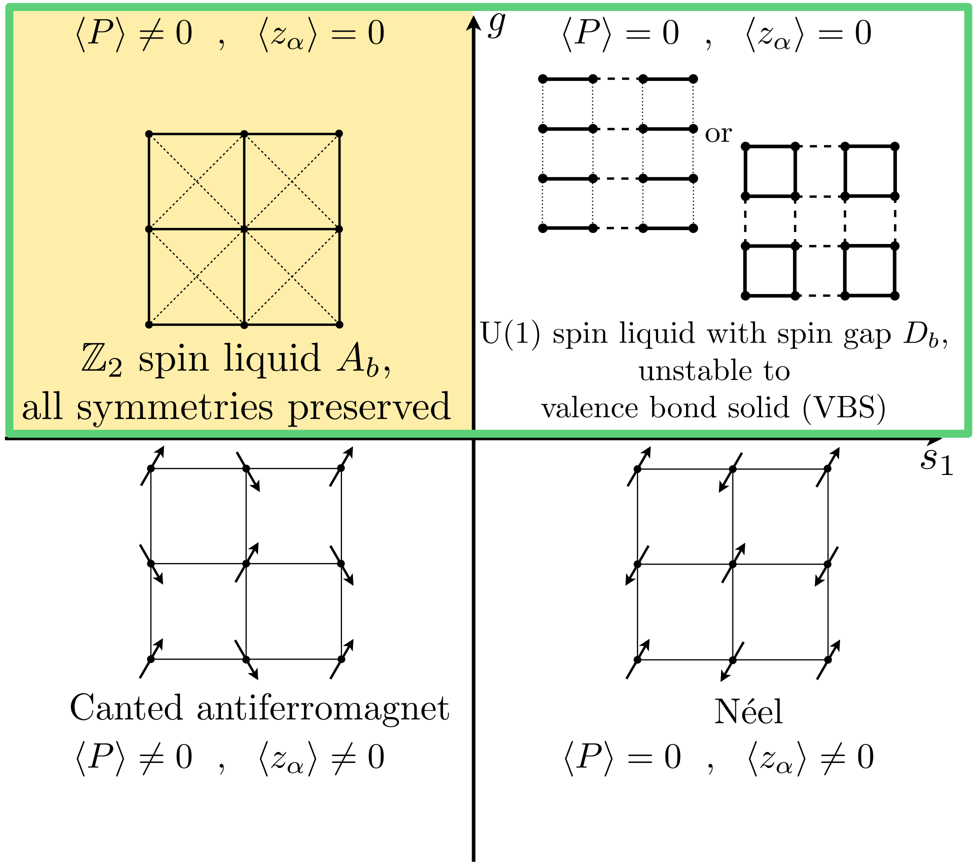
Fermionic



$$\mathcal{L}_f = \mathcal{L}_{\text{QCD}} + \lambda \mathbf{\Phi} \cdot \text{tr}\left(\boldsymbol{\sigma} \bar{X} \mu^y X\right) - s |\mathbf{\Phi}|^2 + \lambda' \mathbf{\Phi}_1 \cdot \text{tr}\left(\boldsymbol{\sigma} \bar{X} \partial_0 X\right) - \bar{s}_1 |\mathbf{\Phi}_1|^2 + \cdots$$

Phase diagram

Bosonic



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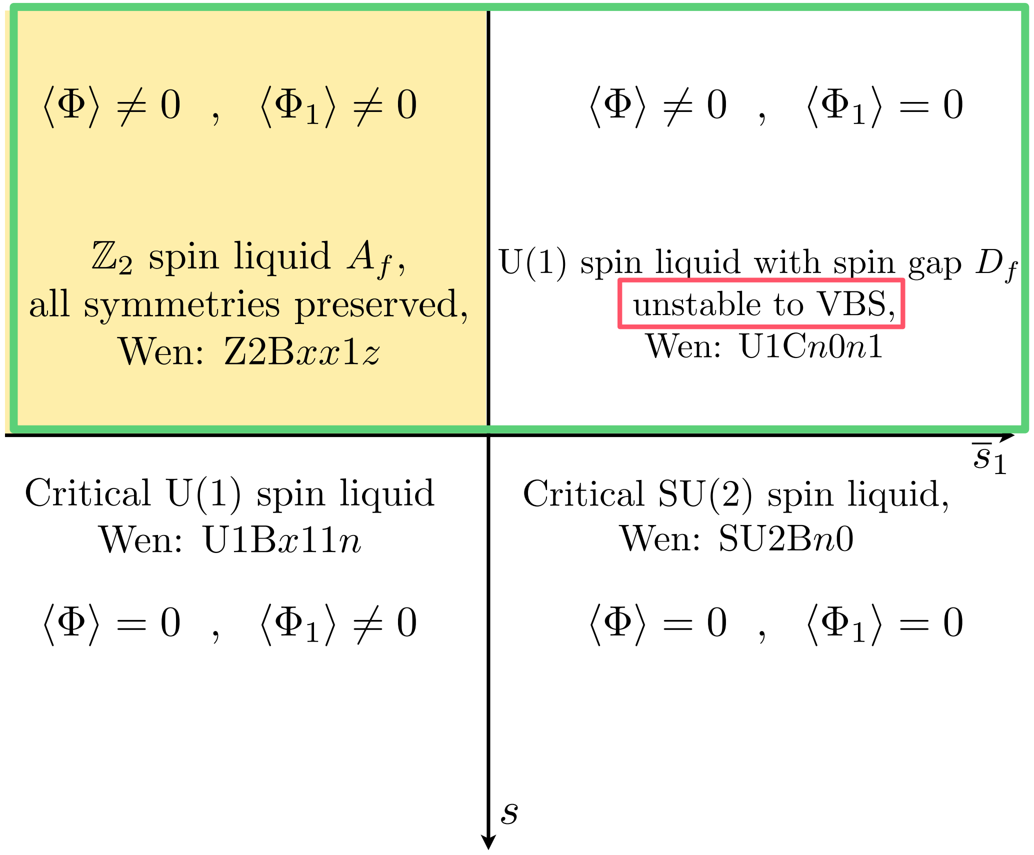
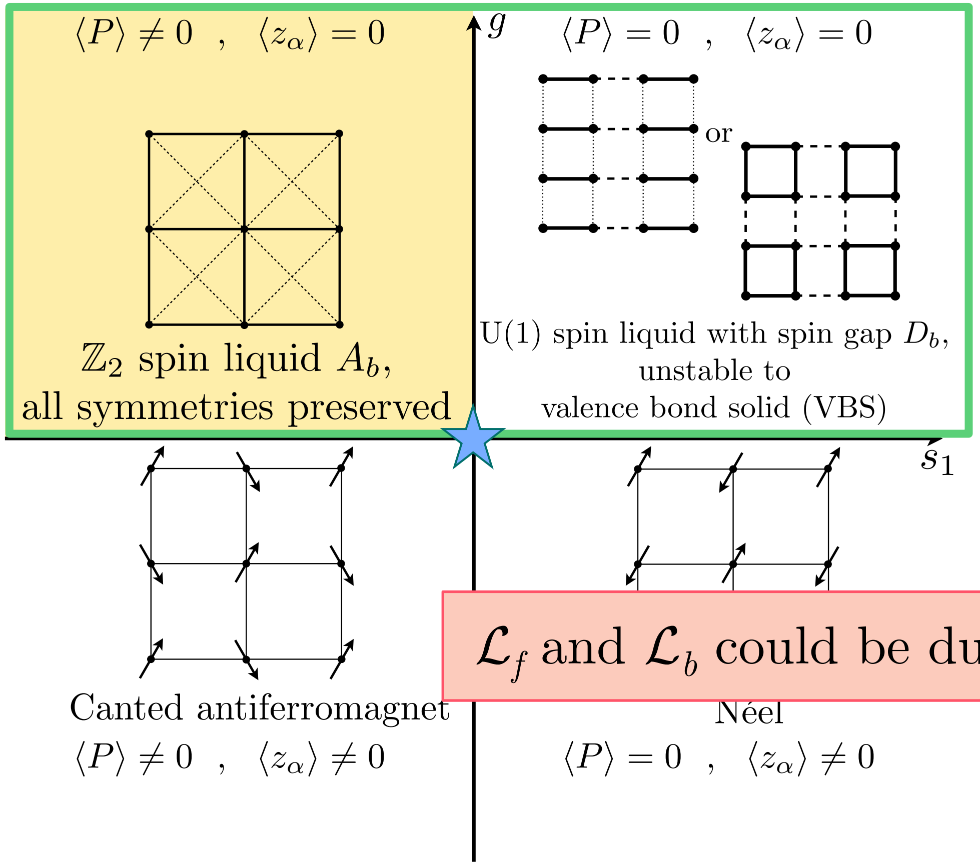
Fermionic



$$\mathcal{L}_f = \mathcal{L}_{\mathrm{QCD}} + \lambda \boldsymbol{\Phi} \cdot \mathrm{tr}\left(\boldsymbol{\sigma} \bar{X} \mu^y X\right) - s |\boldsymbol{\Phi}|^2 + \lambda' \boldsymbol{\Phi}_1 \cdot \mathrm{tr}\left(\boldsymbol{\sigma} \bar{X} \partial_0 X\right) - \bar{s}_1 |\boldsymbol{\Phi}_1|^2 + \cdots$$

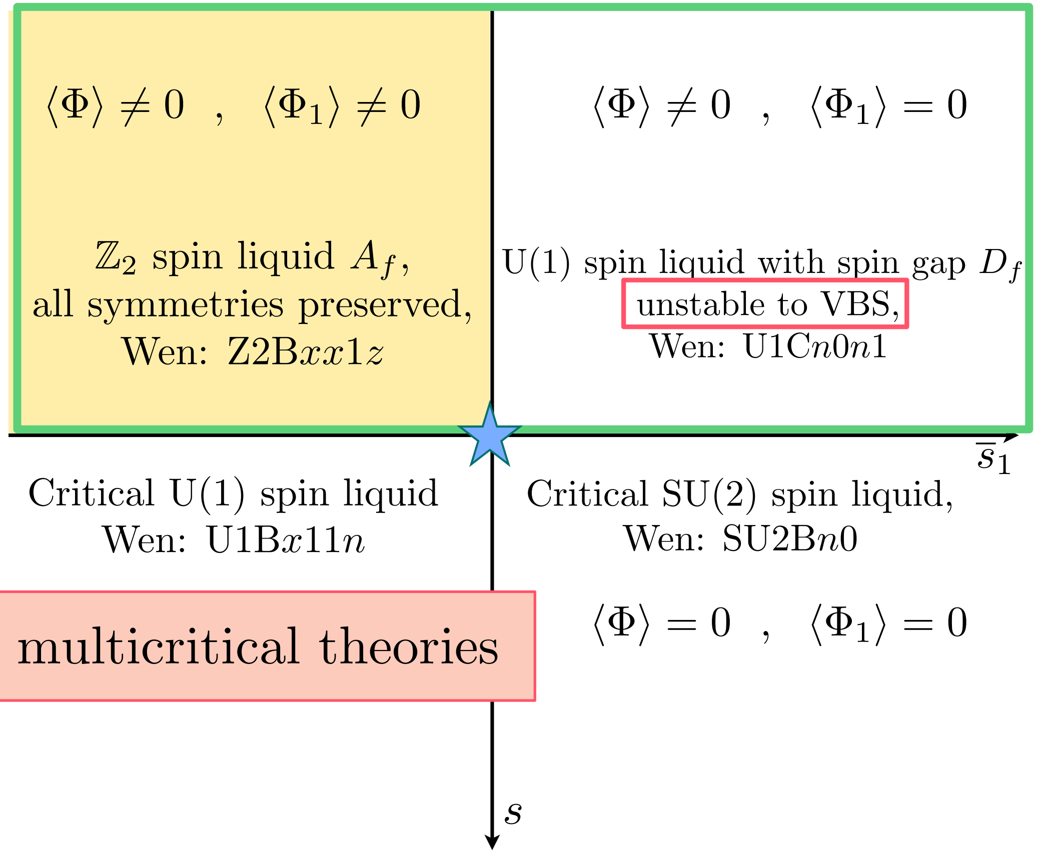
Phase diagram

Bosonic



$$\mathcal{L}_b = \mathcal{L}_{dcp} + \lambda_1 P^* \epsilon_{\alpha\beta} z_\alpha \partial_0 z_\beta + h.c.$$
$$- s_1 |P|^2 + \cdots$$

Fermionic



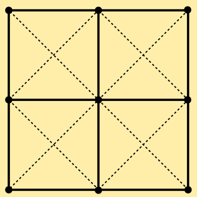
$$\mathcal{L}_f = \mathcal{L}_{\rm QCD} + \lambda \mathbf{\Phi} \cdot {\rm tr}\left(\boldsymbol{\sigma} \bar{X} \mu^y X\right) - s |\mathbf{\Phi}|^2$$
$$+ \lambda' \mathbf{\Phi}_1 \cdot {\rm tr}\left(\boldsymbol{\sigma} \bar{X} \partial_0 X\right) - \bar{s}_1 |\mathbf{\Phi}_1|^2 + \cdots$$

$\mathcal{L}_f$ and $\mathcal{L}_b$ could be dual multicritical theories

Symmetry broken $\mathbb{Z}_2$ phases

Bosonic

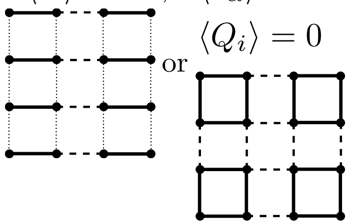
$\langle P \rangle \neq 0 \quad , \quad \langle z_\alpha \rangle = 0$
$\langle Q_i \rangle = 0$



$\mathbb{Z}_2$ spin liquid A_b,
all symmetries preserved

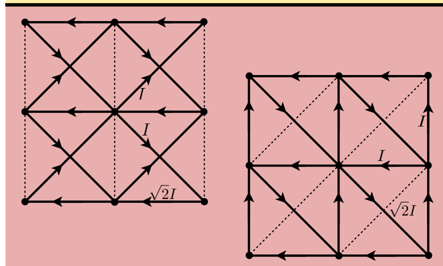
s_2

$\langle P \rangle = 0 \quad , \quad \langle z_\alpha \rangle = 0$
$\langle Q_i \rangle = 0$

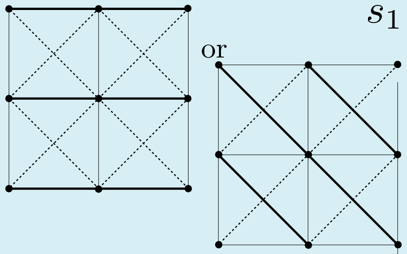


U(1) spin liquid with spin gap D_b,
unstable to
valence bond solid (VBS)

s_1



$\mathbb{Z}_2$ spin liquid C_b,
current loop order
$\langle P \rangle \neq 0 \quad , \quad \langle z_\alpha \rangle = 0$
$\langle Q_i \rangle \neq 0$



$\mathbb{Z}_2$ spin liquid B_b,
Ising-nematic order
$\langle P \rangle = 0 \quad , \quad \langle z_\alpha \rangle = 0$
$\langle Q_i \rangle \neq 0$

$$\mathcal{L}_b' = \mathcal{L}_{dcp} + \lambda_1 P^* \epsilon_{\alpha\beta} z_\alpha \partial_0 z_\beta + h.c. - s_1 |P|^2 + \lambda_2 Q_i^* \epsilon_{\alpha\beta} z_\alpha \partial_i z_\beta + h.c. - s_2 |Q_i|^2 + \cdots$$

Fermionic

$\bar{s}_2$

$\langle \Phi \rangle \neq 0 \quad , \quad \langle \Phi_1 \rangle \neq 0$
$\langle \Phi_{2i} \rangle = 0$

$\mathbb{Z}_2$ spin liquid A_f,
all symmetries preserved,
Wen: Z2Bxx1z

$\langle \Phi \rangle \neq 0 \quad , \quad \langle \Phi_1 \rangle = 0$
$\langle \Phi_{2i} \rangle = 0$

U(1) spin liquid with spin gap D_f,
unstable to VBS,
Wen: U1Cn0n1

$\bar{s}_1$

$\mathbb{Z}_2$ spin liquid C_f,
current loop order

$\langle \Phi \rangle \neq 0 \quad , \quad \langle \Phi_1 \rangle \neq 0$
$\langle \Phi_{2i} \rangle \neq 0$

$\mathbb{Z}_2$ spin liquid B_f,
Ising-nematic order

$\langle \Phi \rangle \neq 0 \quad , \quad \langle \Phi_1 \rangle = 0$
$\langle \Phi_{2i} \rangle \neq 0$

$$\mathcal{L}_f' = \mathcal{L}_{\text{QCD}} + \lambda \boldsymbol{\Phi} \cdot \text{tr}\left(\boldsymbol{\sigma} \bar{X} \mu^y X\right) - s |\boldsymbol{\Phi}|^2 + \lambda' \boldsymbol{\Phi}_1 \cdot \text{tr}\left(\boldsymbol{\sigma} \bar{X} \partial_0 X\right) - \bar{s}_1 |\boldsymbol{\Phi}_1|^2 + \lambda'' \boldsymbol{\Phi}_{2i} \cdot \text{tr}\left(\boldsymbol{\sigma} \bar{X} \partial_i X\right) - \bar{s}_2 |\boldsymbol{\Phi}_{2i}|^2 + \cdots$$

Chatterjee et al. (2017)

Summary

- Determined spin liquid states proximate to N_f=2 QCD (π-flux phase)
- Established a nontrivial correspondence between $\mathbb{Z}_2$ phases surrounding both QCD and $\mathbb{CP}^1$
- Determined a fermionic counterpart to the U(1) spin liquid with gapped matter which is unstable to a VBS
- Proposed additional dualities for multicritical points

Thank you!

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arXiv:1708.04626