Gauge fields coupled to massless fermions in three dimensions

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Euclidean field theory in three dimensions

Two component massless Dirac operator in three dimensions in a gauge field background

$$C = \sum_{k=1}^{3} \sigma_k \left[\partial_k + iA_k(x)\right] \qquad A_k^\dagger(x) = A_k(x)$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \qquad \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Parity transformation

$$\begin{aligned} x_k &\rightarrow -x_k \\ A_k(x) &\rightarrow -A_k(-x) \\ C &\rightarrow C^\dagger = -C \\ m &\rightarrow -m \end{aligned}$$

Parity anomaly

Independent of regularization that preserves gauge invariance: $\det(C+m) = \det(C^\dagger - m)e^{2i\mathcal{A}}$

On perturbative configurations: $2\mathcal{A} \sim \frac{1}{4\pi}\int d^3x \epsilon^{ijk} \mathrm{Tr}\left(A_i\left[\partial_j A_k + \frac{2}{3}A_j A_k\right]\right)$

A fine point: Periodic 3-torus L^3 $\quad F_{12} = \frac{2\pi}{L^2} \quad A_3 = \frac{2\pi h}{L} \quad h \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$$e^{2i\mathcal{A}} \neq e^{i\pi h}$$

$$e^{2i\mathcal{A}} = e^{2i\pi h}$$

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Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar\eta,\eta;m) = \det C_0(m) e^{\bar\eta G_0(m)\eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m\in[-1,1]$$

$$G_o(m) = \frac{1}{1-m}\left[C_o^{-1}(m) - 1\right] = \frac{A}{1+mA}$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V$$

$$A = \frac{1-V}{1+V} \qquad A = -A^\dagger$$

Parity

$$C_o^\dagger(m) = \frac{C_o(-m)}{V}$$

Central character
A regularized unitary operator, V, that depends on the background gauge fields

Parity

$$G_o^\dagger(m) = -G_o(-m)$$

It is unitary: $T_k T_k^\dagger = 1$

Parallel transporter on the lattice: $T_k : [T_k\phi](n) = U_k(n)\phi\left(n+\hat{k}\right)$

Under a gauge transformation:

$$T_k \to G T_k G^\dagger$$

Under parity

$$T_k \to T_k^\dagger$$

Under parity

$$V \to V^\dagger$$

$$V = X\frac{1}{\sqrt{X^\dagger X}}$$

$$X = B + D$$

$$D = \frac{1}{2}\sum_{k=1}^{3}\sigma_k\left[T_k - T_k^\dagger\right] \qquad B = \frac{1}{2}\sum_{k=1}^{3}\left[2 - T_k - T_k^\dagger\right] - m_w \qquad 0 < m_w < 2$$

$$D^\dagger = -D \qquad B^\dagger = B$$

Phase of the fermion determinant

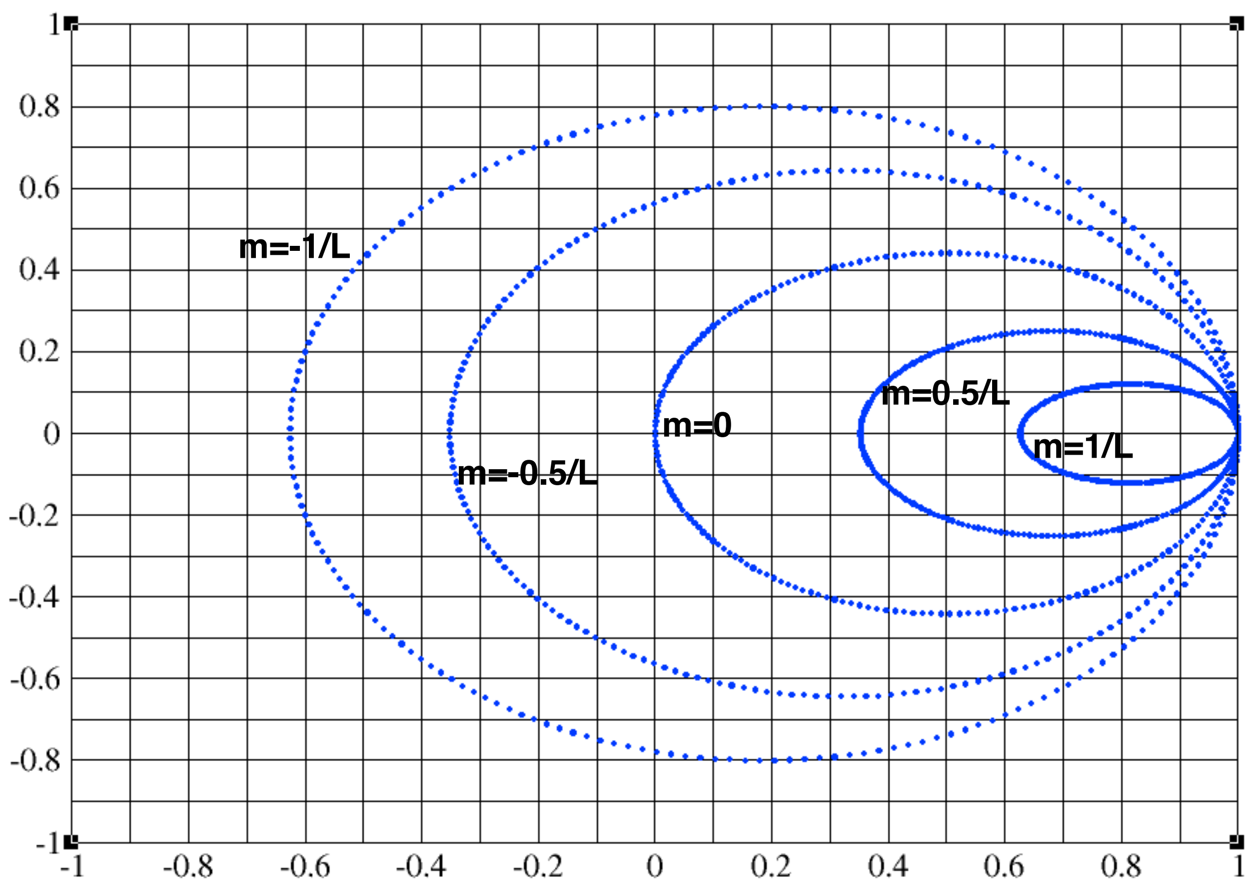
$$\det C_o(m) = \det\left[\frac{1+V}{2} + m\frac{1-V}{2}\right] \qquad \det C_o(-1) = \det V$$

Perturbative gauge fields: $$\det V = \exp\left[i\frac{1}{4\pi}\int d^3x\ \epsilon_{\mu\nu\rho}A_\mu\partial_\nu A_\rho\right]$$

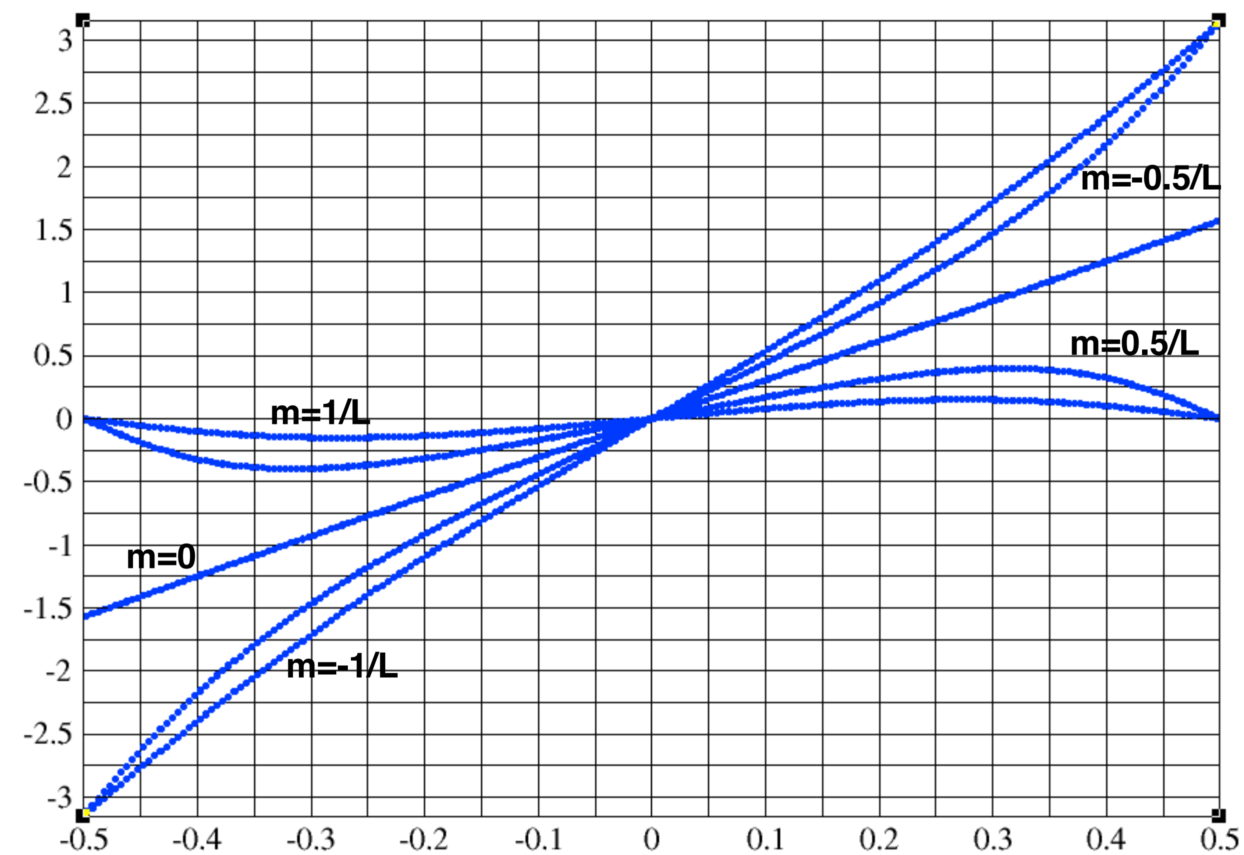
A standard non-perturbative gauge field: $$F_{12} = \frac{2\pi}{L^2} \quad A_3 = \frac{2\pi h}{L} \quad h \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$\det V = \exp[i2\pi h]$ is gauge invariant

Orbit of the determinant as h is varied over a full period



Phase of the determinant as a function of h



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Abelian theories

Compact gauge field seen by fermion of charge q: $U_k^{(q)}(\mathbf{n}) = e^{iq\theta_k(\mathbf{n})}$

Fermionic determinant of a charge q fermion with mass M in a gauge field background: $\det\left[\frac{1+M}{2} + \frac{1-M}{2}V_{q\theta}\right]$ $M \in [-1,1]$

Set q=1 as the unit of charge. All fermions will have integer charges.

Vector-like theory: A pair of fermions with charge 1 and equal and opposite masses

$$\det\left[\frac{1+M}{2} + \frac{1-M}{2}V_\theta\right] \det\left[\frac{1-M}{2} + \frac{1+M}{2}V_\theta\right] \det V_\theta^\dagger = \left|\det\left[\frac{1+M}{2} + \frac{1-M}{2}V_\theta\right]\right|^2$$

Fermion of mass M | Fermion of mass -M | Infinite mass fermion | Parity invariant theory

Can be simply extended to non-abelian theories

Chiral-like theory: One fermion of charge q with mass M

$$Z(M,\theta) = \det\left[\frac{1+M}{2} + \frac{1-M}{2}V_{q\theta}\right]\left[\det V_{p\theta}^\dagger\right]^r$$

$$q^2 = 2rp^2 \quad \Rightarrow \quad \det\left[V_{q\theta}\left(V_{p\theta}^\dagger\right)^{2r}\right] = 1 \quad \Rightarrow \quad Z(-M,\theta_p) = Z(M,\theta)$$

Chiral-like anomaly cancellation

Massless theory is parity invariant
q has to be even

An example of interest: q=2, p=1, r=2

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Continuum physics from lattice

- We worked on a symmetric periodic lattice with L sites in each direction.
- We used a non-compact single plaquette action for the U(1) gauge fields to suppress monopoles.

$$S_g(\theta) = \frac{L}{\ell} \sum_{\mathbf{n}} \sum_{j<k=1}^{3} \left(\Delta_j \theta_k(\mathbf{n}) - \Delta_k \theta_j(\mathbf{n})\right)^2 ,$$

- We used the standard Wilson plaquette action for SU(N_c) gauge fields.

$$S_g = -\frac{2L}{\ell} \sum_{n} \sum_{\mu>\nu=1}^{3} \mathrm{Tr}\; P_{\mu\nu}(n), \quad P_{\mu\nu} = U_\mu(n) U_\nu(n+\hat{\mu}) U_\mu^\dagger(n+\hat{\nu}) U_\nu^\dagger(n)$$

- ℓ is the physical size of the box measured in units of the physical coupling.
- We worked at a fixed ℓ and first took the continuum limit by extracting the results at several values of L.
- We then varied ℓ to study the behavior as a function of the dimensionless volume.

Strategy for showing if the theory breaks the scale or not

ℓ is the size of the periodic box and sets the scale

We are working with massless fermions even away from the continuum limit

Ask if a scale appears in the theory that remains finite as we take ℓ to infinity

A useful set of physical quantities are the eigenvalues of the massless Dirac operator

$$iL\frac{1+V}{1-V}\psi_j = \pm \ell \lambda_j \psi_j$$

$$0 < \lambda_1 < \lambda_2 < \cdots$$

The spectrum is not symmetric about zero in a fixed gauge field background

The spectrum will be symmetric and discrete when averaged over the gauge field measure at a fixed ℓ

Asymptotic behavior of the low lying spectrum as a function of ℓ

Free field behavior: $\lambda_j \sim \frac{1}{\ell}$

Easiest to see numerically since there is no change from small ℓ to large ℓ

Scale invariant interacting theory: $\lambda_j \sim \frac{1}{\ell^{1+\gamma_m}}$, $0 < \gamma_m < 2$

Hardest to see since the exponent will grow from γ_m=0 at small ℓ and never reach γ_m=2

Breaking of scale invariance: $\lambda_j \sim \frac{1}{\ell^3}$

Not so difficult to see since we expect universal behavior of distribution of eigenvalues to set in at finite but large ℓ

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Strategy to compute the bilinear condensate if scale invariance is broken

Universality

$$\lambda_i \ell^3 \Sigma = z_i$$

There is a universal joint probability distribution of $\{z_i, i=1,2,\ldots\}$ in the infinite volume limit given by Random Matrix Theory

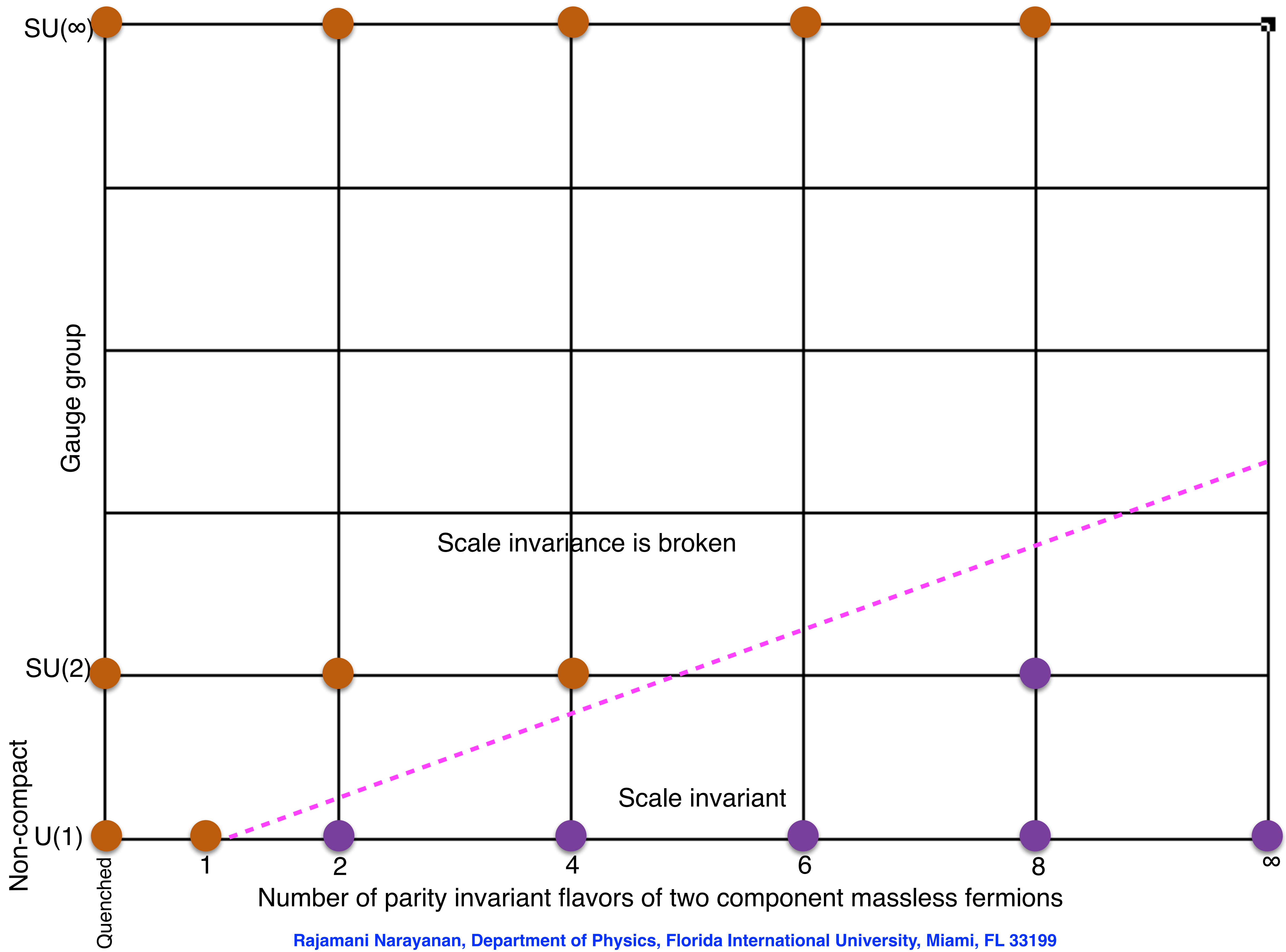
$$Z = \int [dC] e^{-\frac{\pi^2}{16M} \mathrm{Tr} C^2} \det{}^N C,$$

C is an M X M matrix and M is taken to infinity

C is hermitian for abelian theories
C is real symmetric for SU(2) theories

z_i are the eigenvalues of C

QED and QCD with massless fermions in three dimensions



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Abelian Gauge Fields with even number of two component massless fermions preserving parity

- N=2:
  - Does not have a non-zero bilinear condensate.
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive).
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus:
    - A free field like behavior at short distances ($x \to 0$).
    - Anomalous (scalar) and non-anomalous (vector) power law behavior at large distances ($x \to \infty$; $x/\ell \to 0$)
    - Finite volume behavior ($x \to \infty$, x/ℓ fixed at a non-zero value) where we extract a mass proportional to $1/\ell$
  - Flow of fermionic current central charge from ultra-violet to infra-red.
  - Flavor and topological current correlators — Extended O(4) symmetry supporting strong duality.

- N=4,6,8 theories show similar behavior of eigenvalues with a reduction in the anomalous dimension.
- The quenched theory (N=0) has a non-zero bilinear condensate.

Non-Abelian (SU(N_c)) Gauge Fields with even number of two component massless fermions preserving parity

- 't Hooft limit ($N_c \to \infty$, N finite) has a non-zero bilinear condensate.
- N_c=2, N=0, 2, 4 have a non-zero bilinear condensate.
- N_c=2, N=8,12 is consistent with being scale invariant.

Abelian Gauge Fields with a single two component massless fermion

- Parity is a symmetry in the continuum limit at any finite volume.
- Parity is spontaneously broken in the infinite volume limit.

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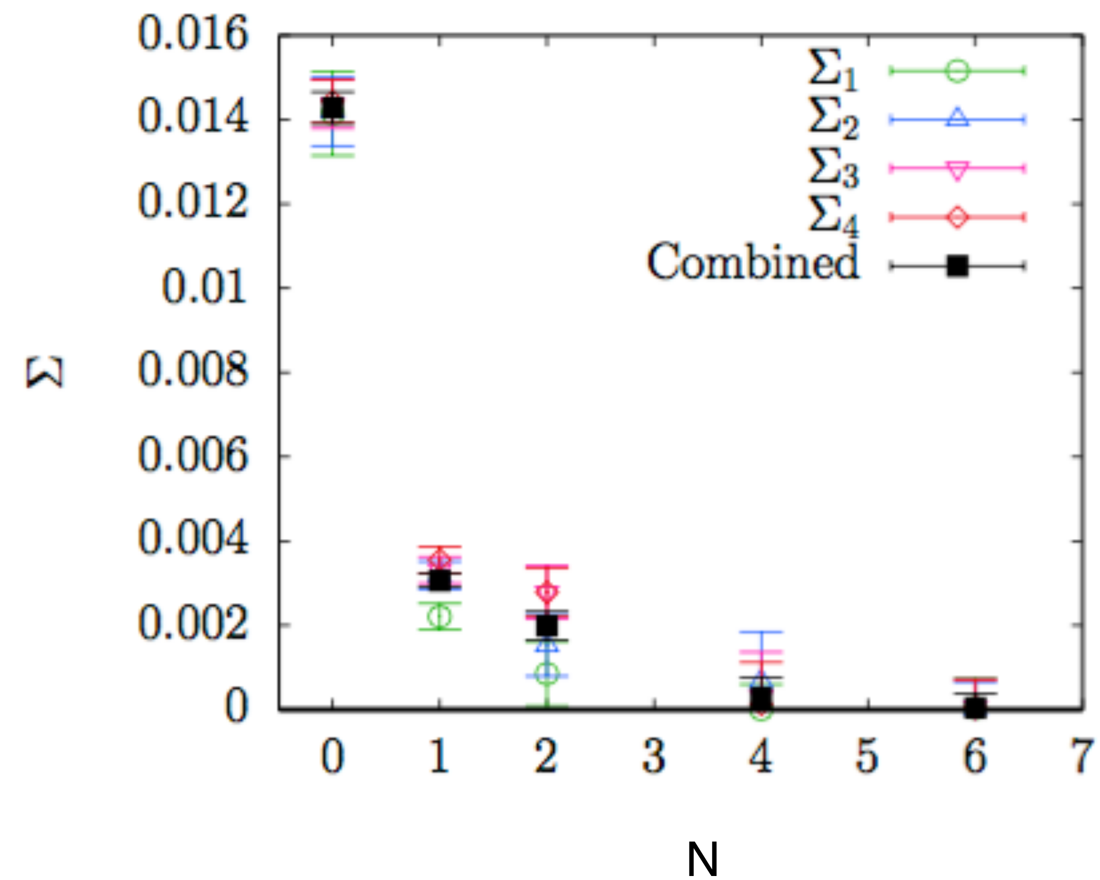
Behavior as a function of the number of flavors

U(1)

- Parity invariant QED with N > 0 are all scale invariant.
- γ_m = 1.0(2), 0.6(2), 0.37(6) and 0.28(6) for N = 2, 4, 6, 8.
- The numbers for the anomalous dimensions agree with an analytical calculation in 1/N assuming no bilinear condensate

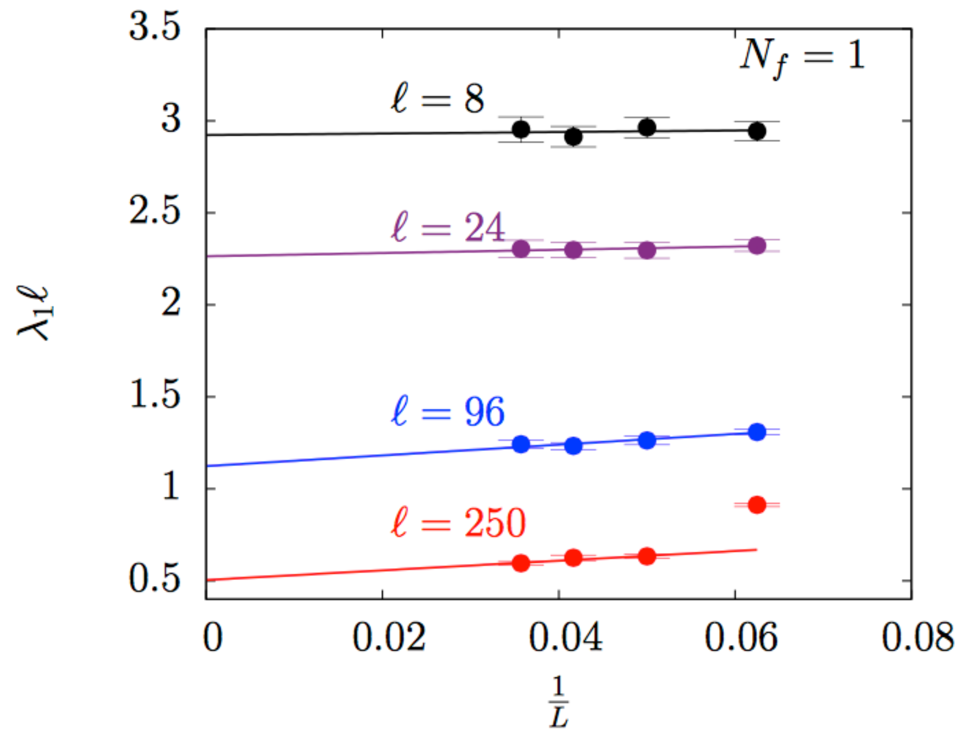
SU(2)



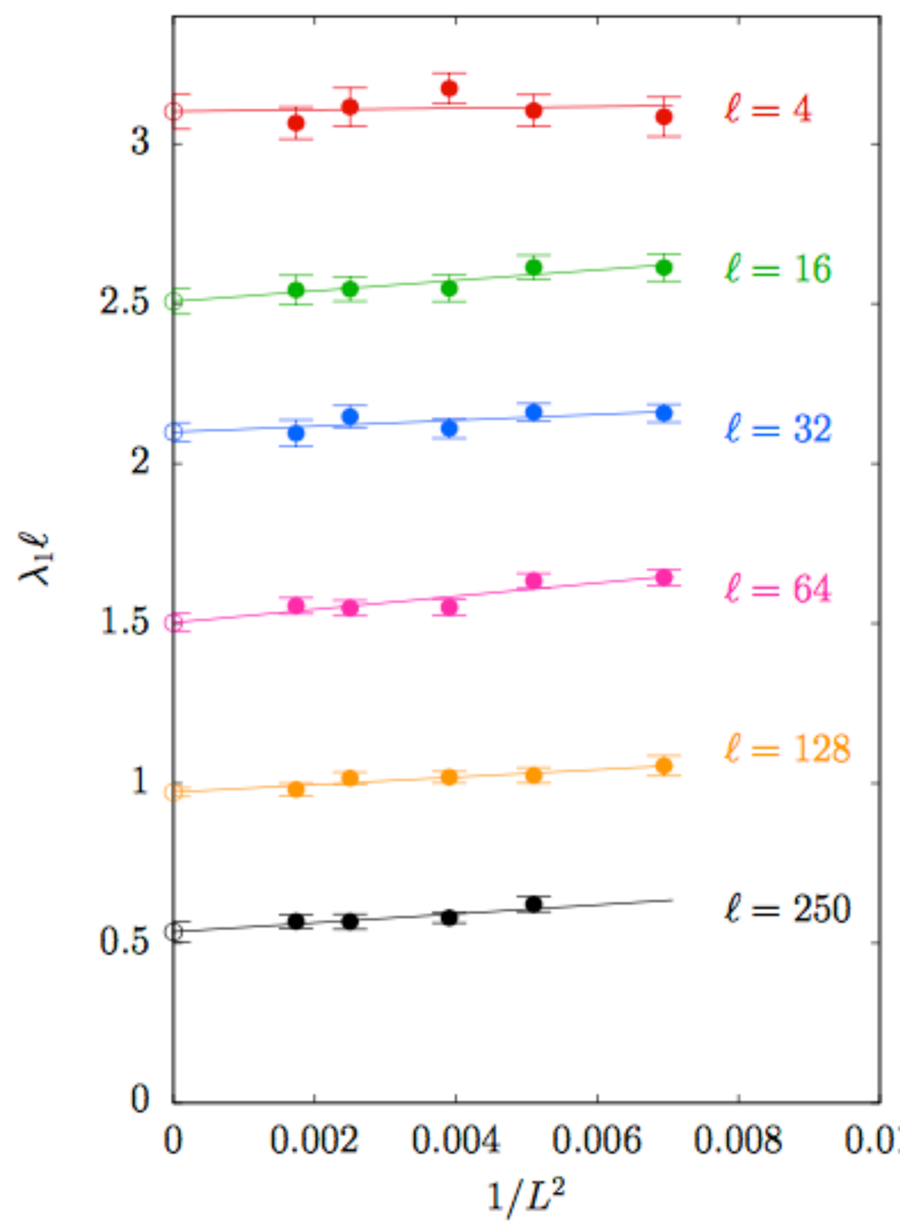
Evidence for the continuum limit of the low lying eigenvalues

N=2

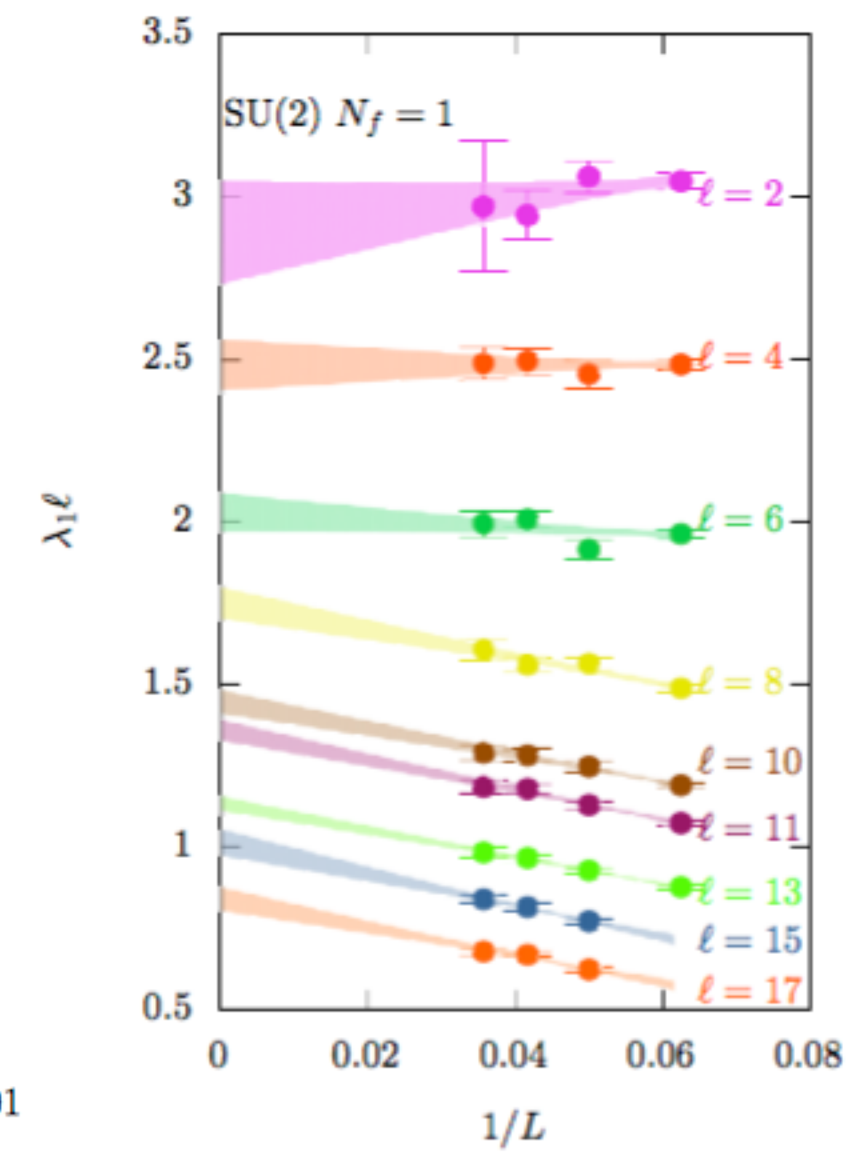
U(1), |, Wilson fermions



U(1), , overlap fermions



SU(2), Wilson fermions



Expectation values in the theory with dynamical fermions

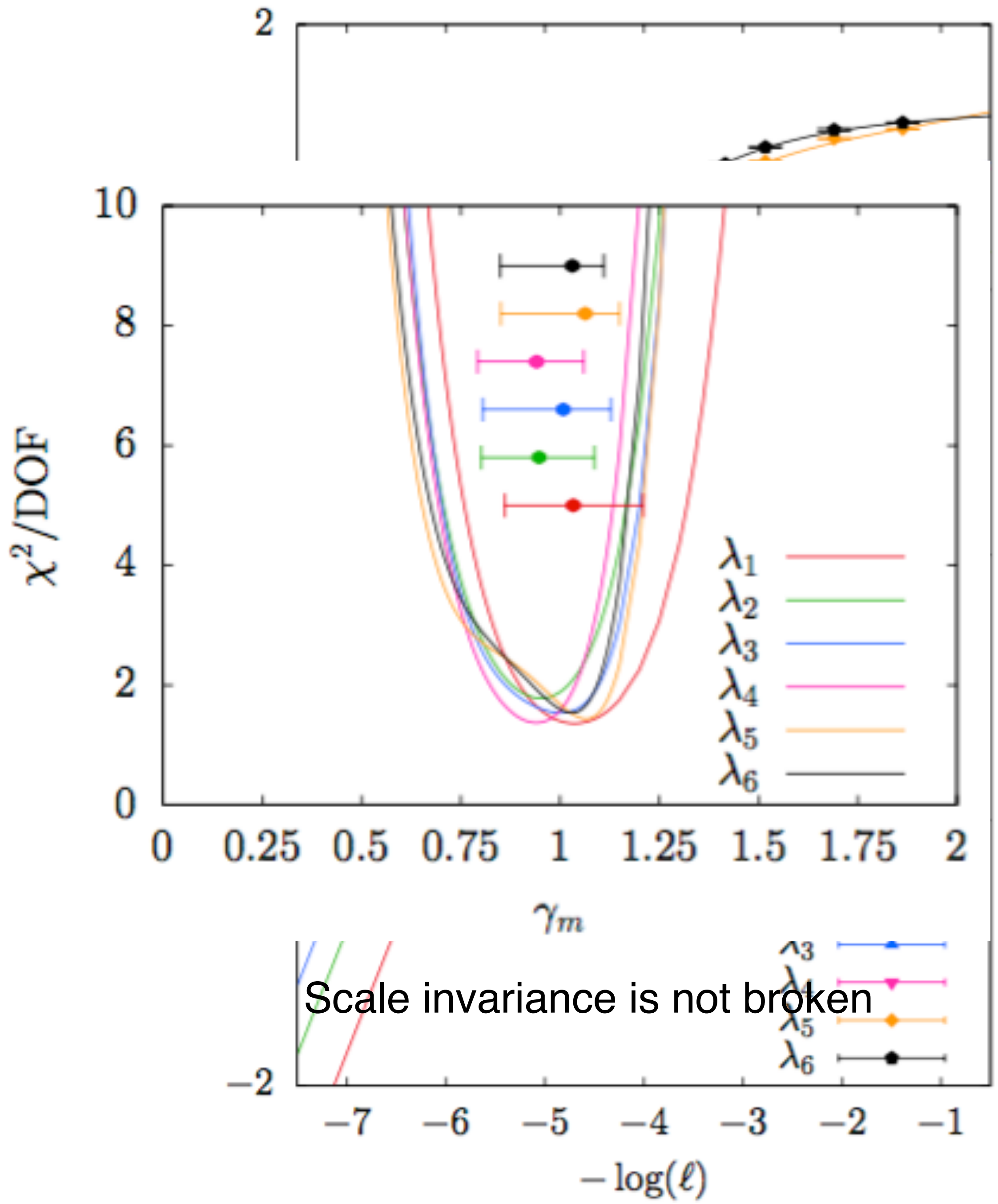
Behavior of the low lying eigenvalues

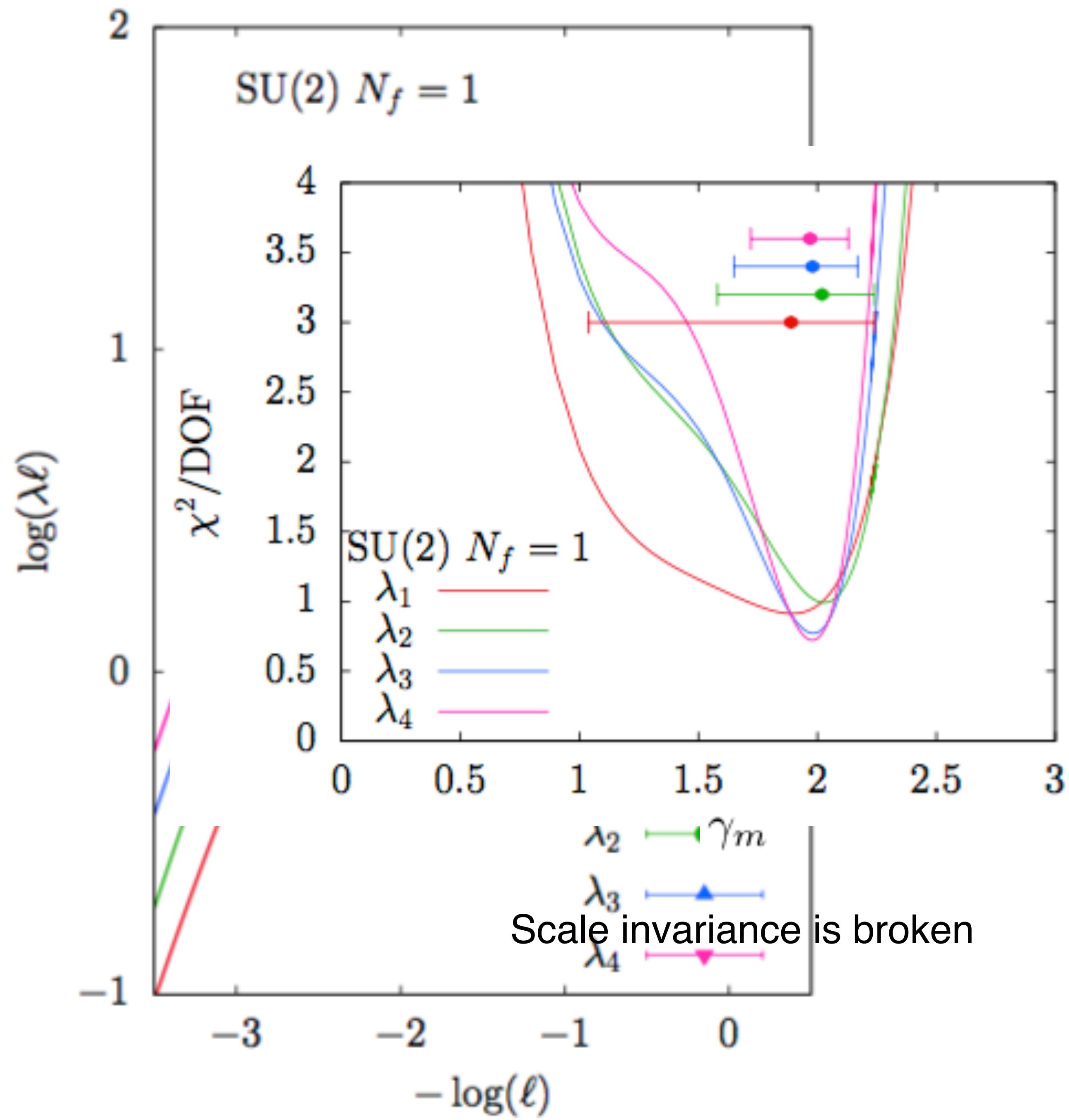
$$\lambda_i \sim \frac{1}{\ell^3}$$ There is a condensate in the theory and scale invariance is broken

$$\lambda_i \sim \frac{1}{\ell^{1+\gamma_m}} \qquad \gamma_m < 2$$ Scale invariance is not broken and γ_m is the mass anomalous dimension

U(1), N=2

SU(2), N=2





Approach to the infinite volume limit

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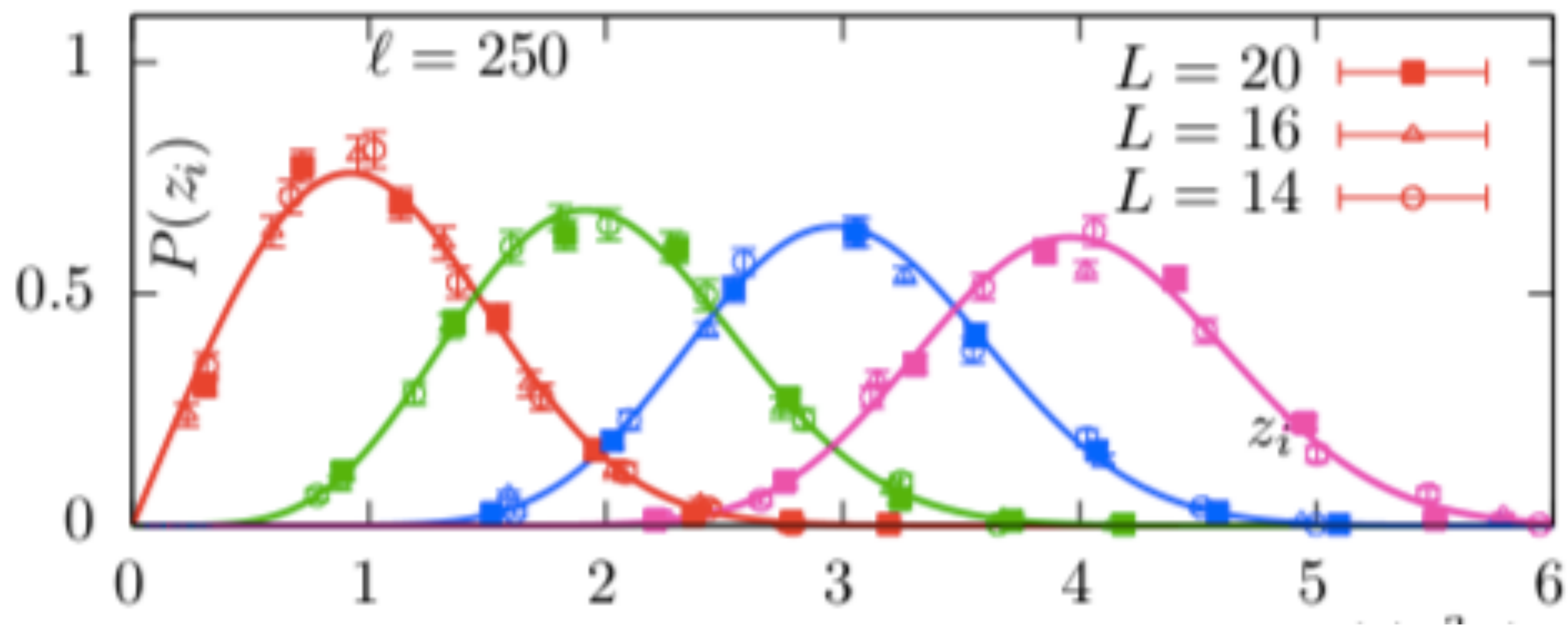
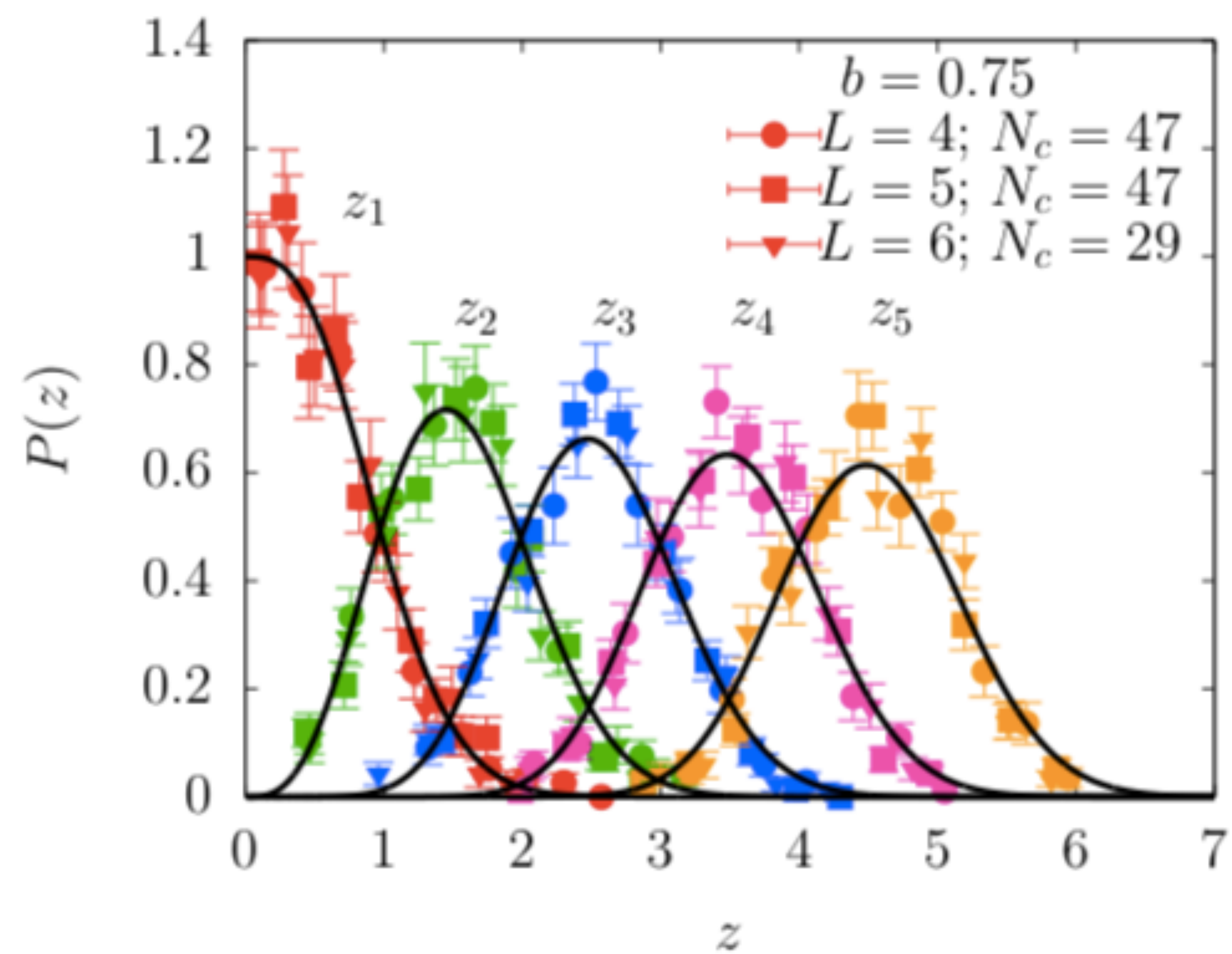
U(1) with a single two component fermion



Large N_c in the 't Hooft limit



Thank you for your attention