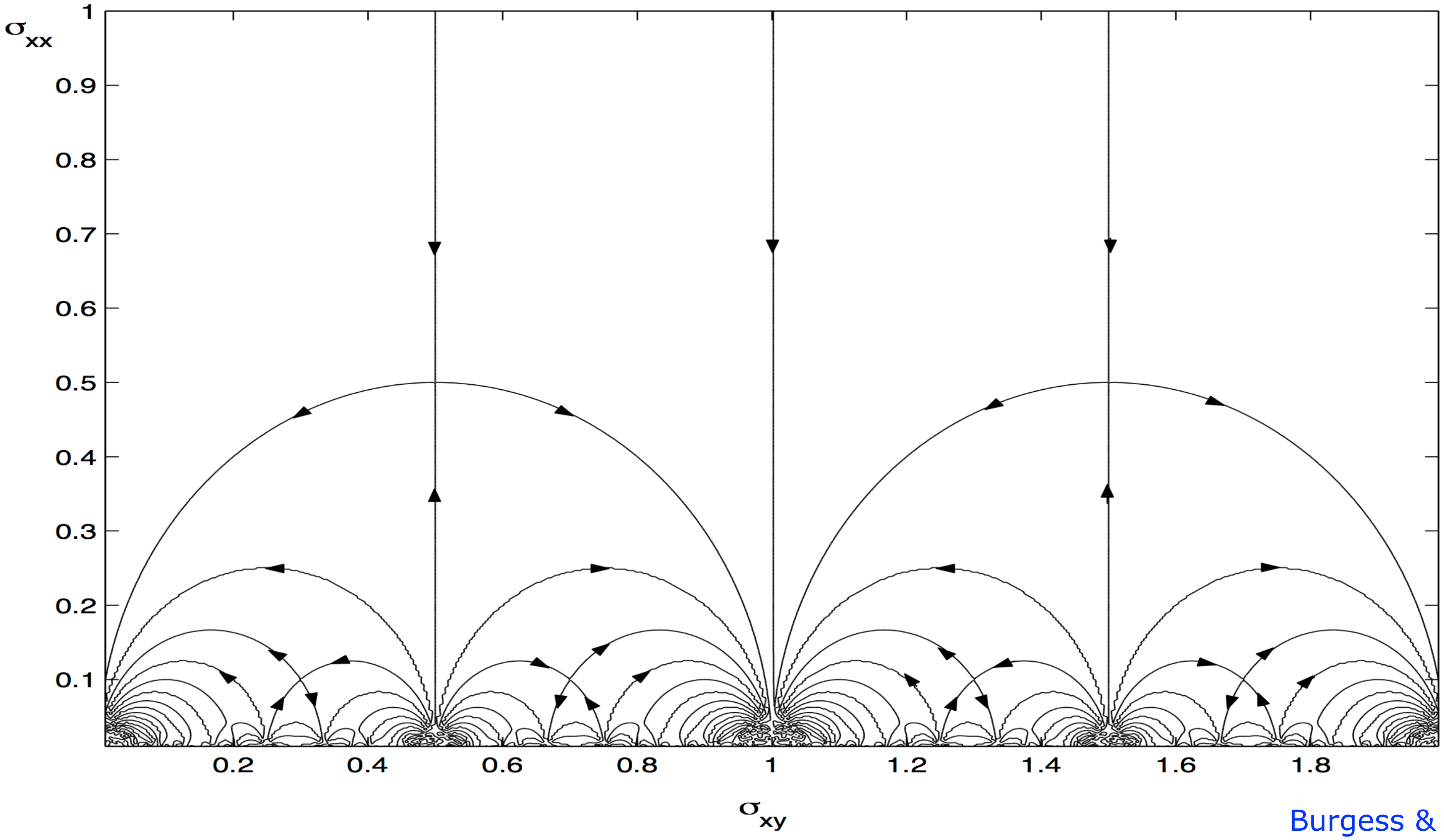
Superuniversality and non-Abelian bosonization in 2+1 dimensions



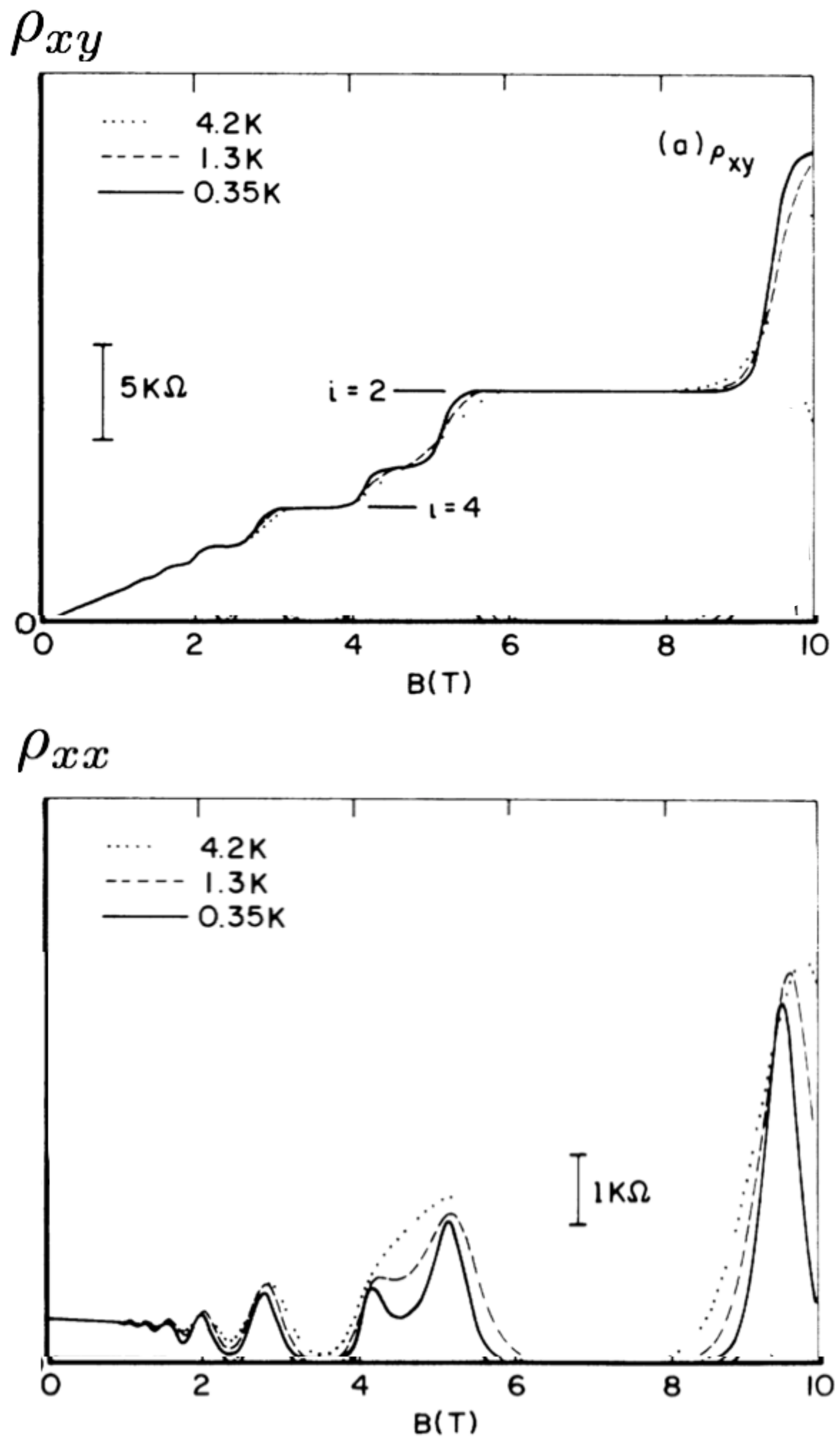
Burgess & Dolan

Michael Mulligan
UC Riverside

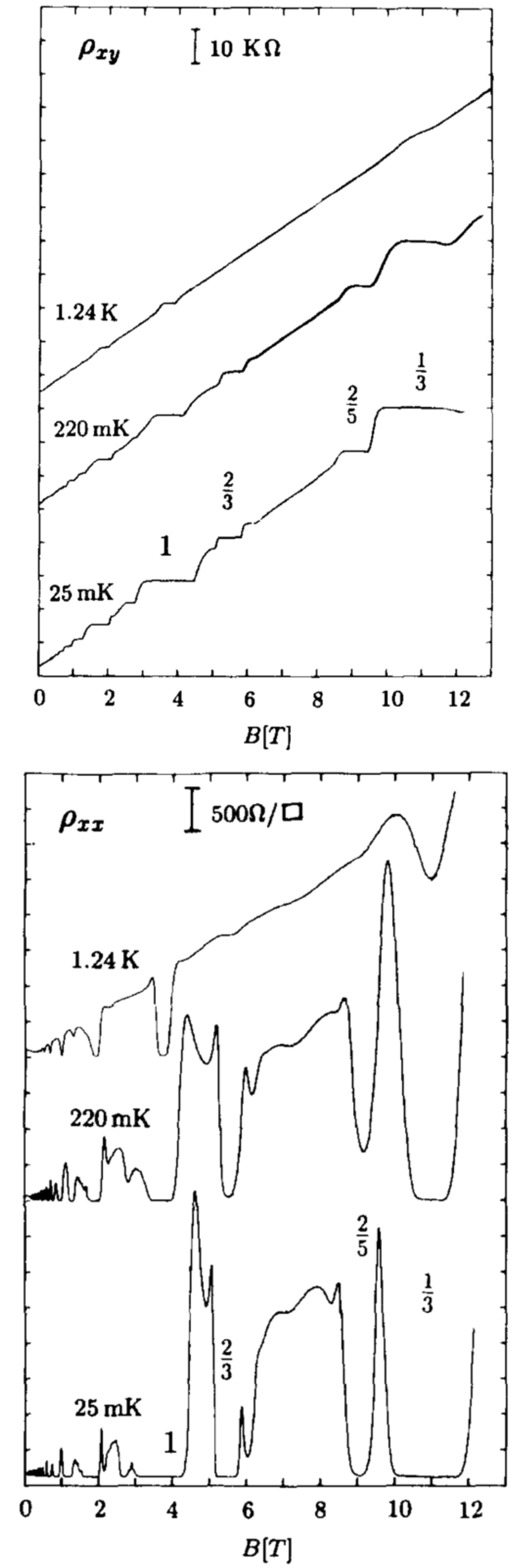
in collaboration with
Aaron Hui and Eun-Ah Kim (Cornell U.)

arXiv: 1712.04942 and 1710.11137

phase transitions between quantum Hall states represent some of the best examples of disordered quantum critical phenomena

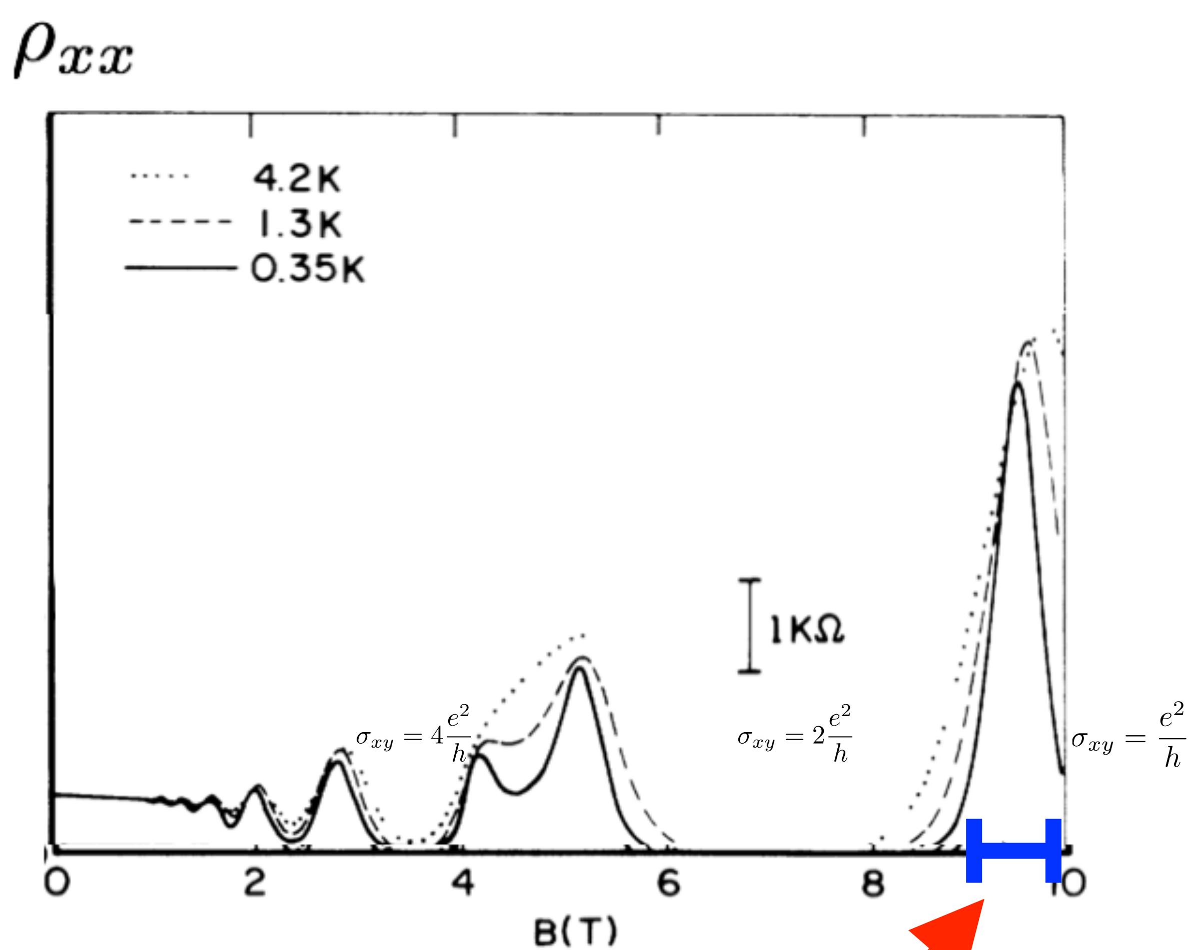


Wei, Tsui, Paalanen, & Pruisken

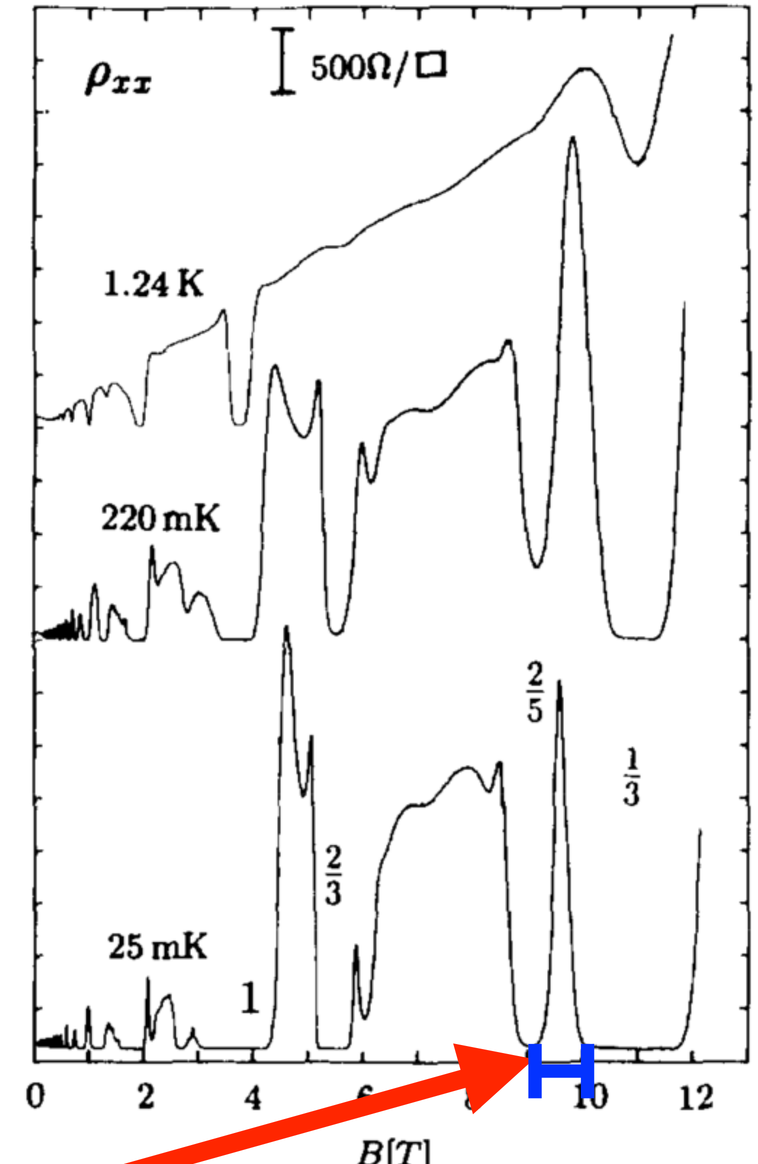


Engel, Wei, Tsui, & Shayegan

a puzzling feature of these phase transitions is their apparent similarity



Wei, Tsui, Paalanen, & Pruisken



Engel, Wei, Tsui, & Shayegan

$$\Delta B \sim T^{1/\nu z} \text{ where } \nu z \sim 7/3$$

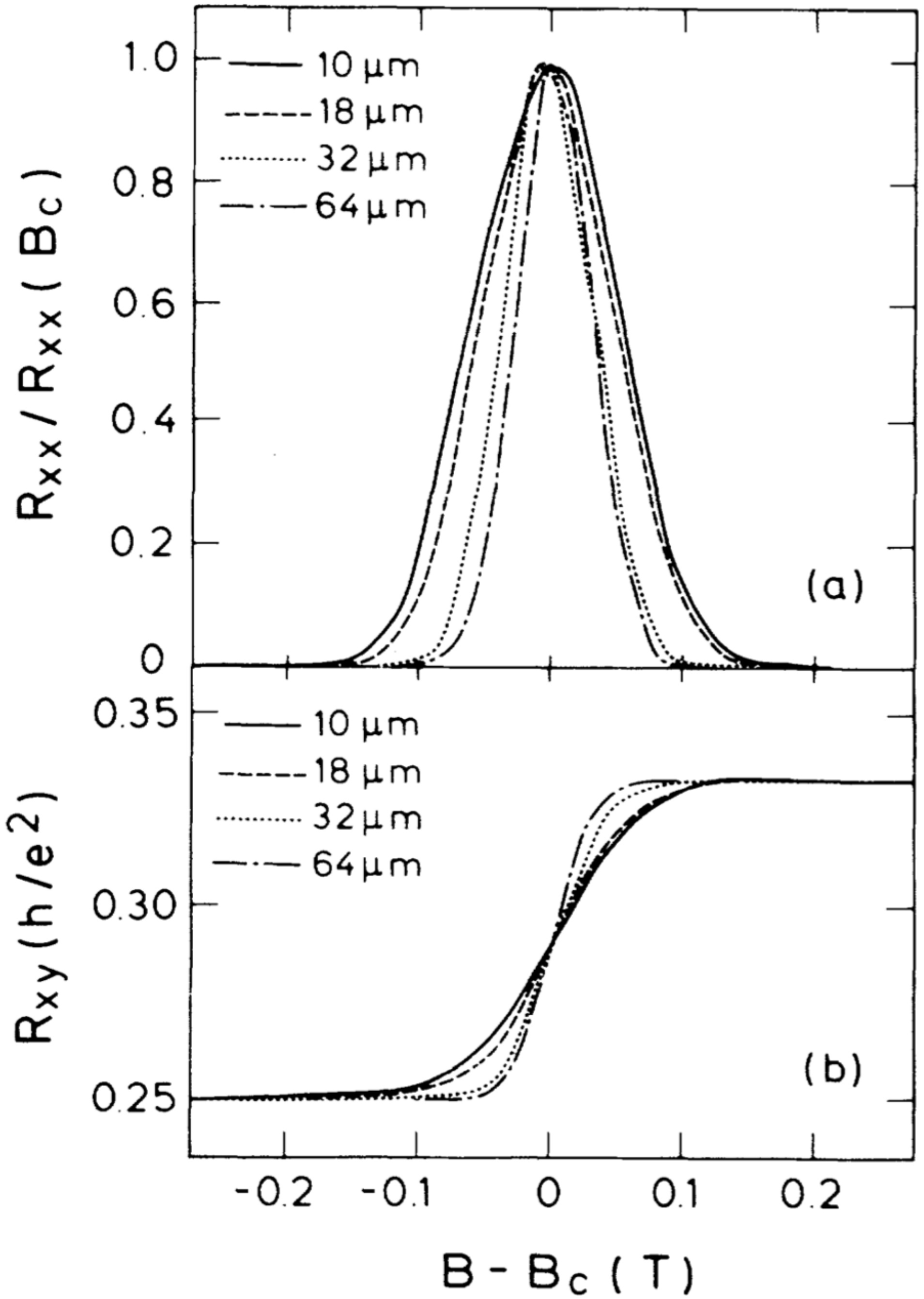
ν is the correlation length exponent: $\xi \sim (B - B_c)^{-\nu}$

z is the dynamical critical exponent: $\tau \sim \xi^z$

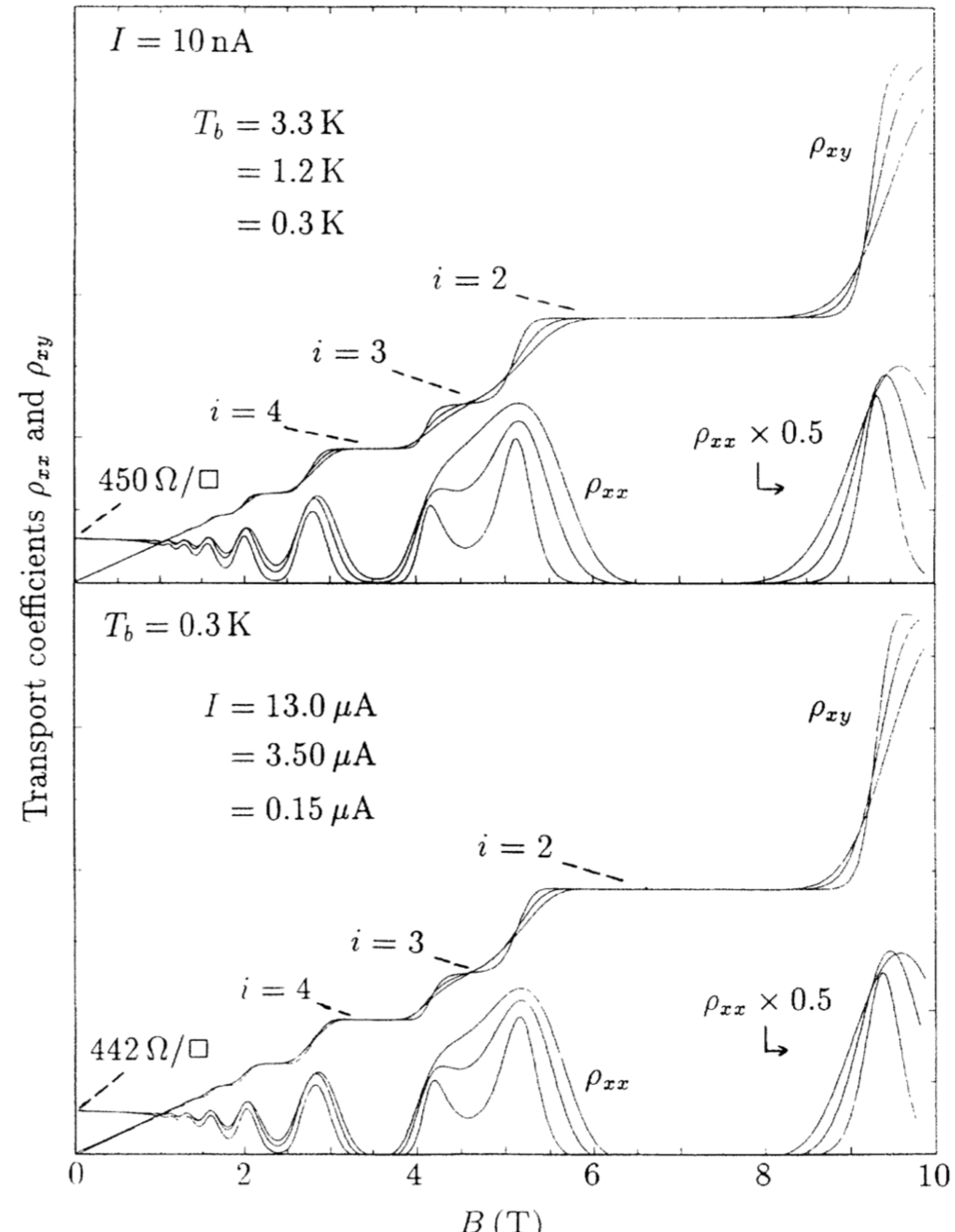
at integer quantum Hall plateau transitions, the product has been factorized

$$\nu \sim 7/3$$

$$z \sim 1$$



Koch, Haug, von Klitzing, & Ploog



Wei, Engel, & Tsui

$$\Delta B \sim L^{-1/\nu}$$

$$\Delta B \sim E^{1/\nu(z+1)}$$

scaling of the dc resistivity near these apparently continuous quantum phase transitions implies:

Sondhi, Girvin, Carini, & Shahar

$$\rho_{xx} = \frac{h}{e^2} f_{(a)}\left(\frac{B - B_c^{(a)}}{T^{1/\nu z}}\right)$$

$$\rho_{xy} = \frac{h}{e^2} g_{(a)}\left(\frac{B - B_c^{(a)}}{T^{1/\nu z}}\right)$$

(a) labels the particular phase transition, e.g., $1 \to 0$ or $1/3 \to 2/5$

In this talk, I will assume these measurements imply ν and z are the same at *all* phase transitions between Abelian quantum Hall states of spin-polarized electrons

Critical states are distinguished by their critical conductivities, i.e., $f_{(a)}(0)$ and $g_{(a)}(0)$

Shahar, Tsui, Shayegan, Bhatt, & Cunningham

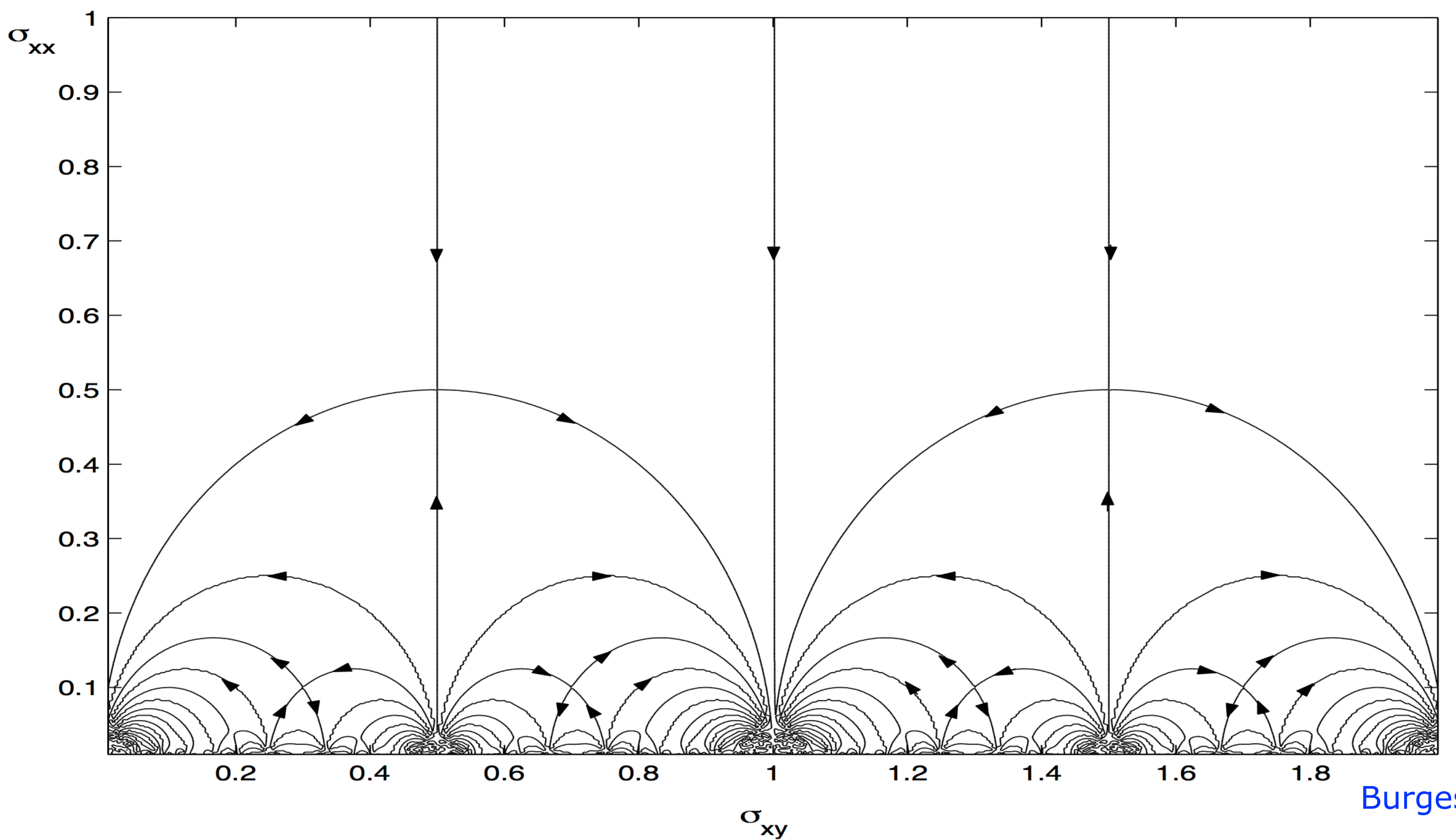
superuniversality is the sharing of critical indices among distinct critical points

Laughlin, Cohen, Kosterlitz, Levine, Libby, & Pruisken; Jain, Kivelson, & Trivedi; Kivelson, Lee, & Zhang; Lutken & Ross; Fradkin & Kivelson; Shimshoni, Sondhi, & Shahar; Burgess & Dolan; Geraedts & Motrunich; Goswami & Chakravarty; Goldman & Fradkin

such behavior is surprising:

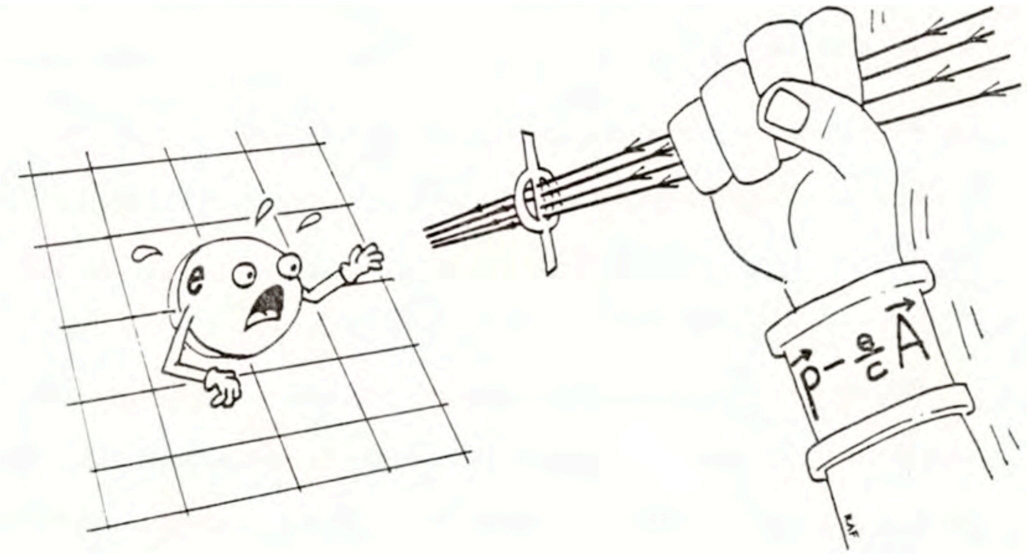
(i) "conventional" symmetry-breaking phase transitions are NOT superuniversal (below their critical dimensionality)

(ii) the basic theoretical framework for the integer and fractional quantum Hall effects are different: interactions are crucial to lifting the degeneracy of a partially filled Landau level



Burgess & Dolan

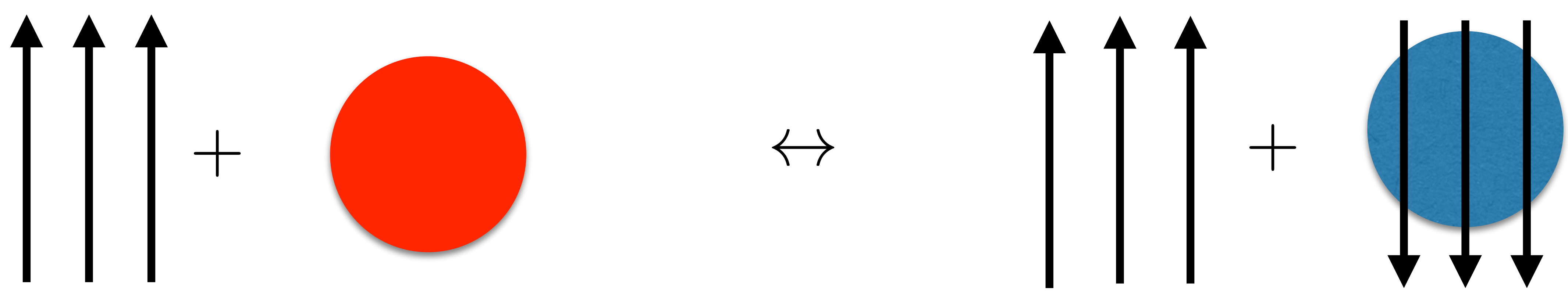
“composite bosons” (and “composite fermions”)



Girvin & MacDonald; Read;; Hansson, Kivelson, & Zhang; Lopez & Fradkin; Halperin, Lee, & Read; Kalmeyer & Zhang

from D. Arovas’ Ph.D thesis

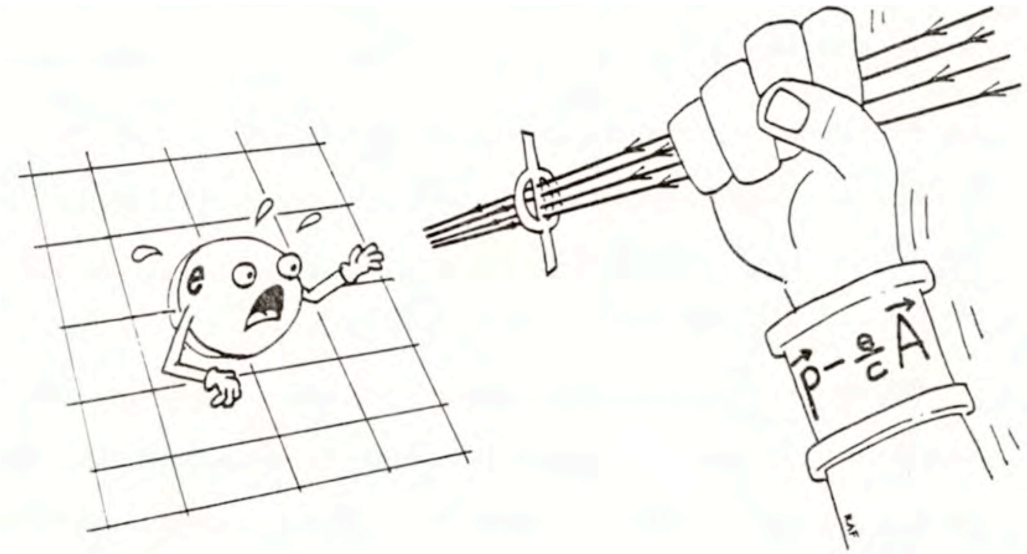
heuristic picture:
electrons at 1/3 filling = “composite bosons” in zero effective field



B electron B “composite boson”

$$B_{\text{eff}} = B - 6\pi n_e$$

“composite bosons” (and “composite fermions”)

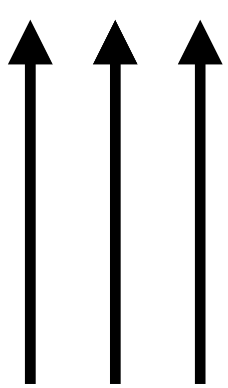


from D. Arovas’ Ph.D thesis

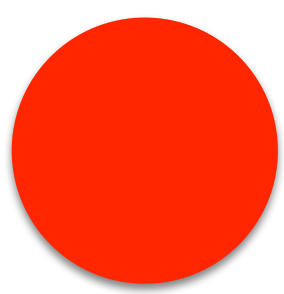
Girvin & MacDonald; Read;; Hansson, Kivelson, & Zhang;
Lopez & Fradkin; Halperin, Lee, & Read; Kalmeyer & Zhang

heuristic picture:

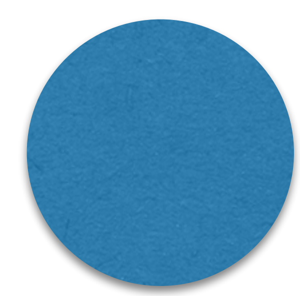
electrons at 1/3 filling = “composite bosons” in zero effective field



$+$



$\leftrightarrow$



B electron “composite boson”

$$B_{\text{eff}} = B - 6\pi n_e$$

"composite bosons" provide a useful theoretical picture that unites the integer and fractional quantum Hall phenomena

$$\mathcal{L}_{cb} = \varphi^* \left(i(\partial_t - i(\alpha_t + A_t) + \frac{1}{2m_e}(\partial_j - i(\alpha_j + A_j))^2 \right) \varphi - |\varphi|^4 + \frac{1}{m}\frac{1}{4\pi}\alpha d\alpha$$

$$\alpha d\alpha = \epsilon^{\mu\nu\rho}\alpha_\mu \partial_\nu \alpha_\rho$$

α : "statistical" gauge field

A : electromagnetic gauge field

m : number of flux quanta "attached" to φ

$m = 1$ gives IQHE;

$m = 3$ gives 1/3 Laughlin state

a quantum Hall transition is mapped to a magnetic field-tuned "superconductor" to "insulator" transition of composite bosons “composite bosons” provide a useful theoretical picture that unites the integer and fractional quantum Hall phenomena

$$\mathcal{L}_{cb} = \varphi^* \left(i(\partial_t - i(\alpha_t + A_t) + \frac{1}{2m_e}(\partial_j - i(\alpha_j + A_j))^2 \right) \varphi - |\varphi|^4 + \frac{1}{m}\frac{1}{4\pi}\alpha d\alpha$$

superuniversality obtains if exponents don’t depend on m

Kivelson, Lee, & Zhang

in mean-field theory, this occurs (obviously)

going beyond mean-field theory, prior field theoretic works, studying quantum Hall transitions tuned by a periodic potential, have computed exponents in a large flavor expansion, i.e.,

$$\nu = 1 - \frac{1}{N_f} F(m)$$

Fisher, Weichman, Grinstein, & Fisher;
Wen & Wu; Chen, Fisher, & Wu;
see also numerical works by Lee, Geraedts, & Motrunich

$$F(m) \sim \mathcal{O}(1) > 0 \text{ and depends strongly on } m$$

structure of the talk

I'll provide some theoretical optimism for superuniversality using new effective theories for a class of quantum Hall phase transitions between states whose quasiparticles have Abelian statistics

see also the recent work by
Geraedts & Motrunich and Goldman & Fradkin

these descriptions have an emergent U(N) gauge symmetry with N > 1

1. I'll provide a description for an integer quantum Hall transition

2. I'll use this description to generate transitions between a class of Abelian quantum Hall states using modular transformations

3. I'll show that correlation length exponents at distinct quantum Hall transitions are the same in a controlled 't Hooft large N limit

4. I'll argue that these results hold away from the controlled 't Hooft large N limit using non-Abelian bosonization conjectures

Note: I will not get realistic critical exponents for a GaAs 2DEG; additional physical ingredients are presumably necessary

the starting point

$$\mathcal{L}_{\rm IQHT}(A) = i\bar{\psi}\, \not{D}_a \psi - \frac{1}{2}\frac{1}{4\pi}{\rm Tr}\left[ada - \frac{2}{3}ia^3\right] - \frac{1}{2\pi}{\rm Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

U(1) -> U(N) generalization of the theory in
Seiberg, Senthil, Wang, & Witten

N: integer greater than 1

a: $U(N)$ Chern-Simons gauge field

b: $U(1)$ Chern-Simons gauge field

A: electromagnetic gauge field

ψ: Dirac fermion with 2 spinor components

in the fundamental rep of $U(N)$

quantization conditions

$$\mathcal{L}_{\rm IQHT}(A) = i\bar{\psi}\,\not{D}_a\psi - \frac{1}{2}\frac{1}{4\pi}{\rm Tr}\left[ada - \frac{2}{3}ia^3\right] - \frac{1}{2\pi}{\rm Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

in the absence of matter fields, like the Dirac fermion, only integral linear combinations of the terms below give well defined contributions to a 2+1D effective action

Deser, Jackiw, Templeton; Polychronakos

$$\frac{1}{4\pi}{\rm Tr}\left[ada - \frac{2}{3}ia^3\right],$$
$$\frac{1}{4\pi}{\rm Tr}[a]d{\rm Tr}[a],$$
$$\frac{1}{2\pi}{\rm Tr}[a]db,$$
$$\frac{1}{4\pi}bdb$$

the first two terms in $\mathcal{L}_{\rm IQHT}$ contribute well defined terms in the 1PI action

Niemi & Semenoff; Redlich; Witten

ultraviolet regularization: Yang-Mills term for a is implicitly assumed

$$\mathcal{L}_{\mathrm{IQHT}}(A) = i\bar{\psi}\,\not{D}_a\psi - \frac{1}{2}\frac{1}{4\pi}\mathrm{Tr}\Big[ada - \frac{2}{3}ia^3\Big] - \frac{1}{2\pi}\mathrm{Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

this means we augment,

$$\mathcal{L}_{\mathrm{IQHT}} \to \mathcal{L}_{\mathrm{IQHT}} - \frac{1}{4g^2}\mathrm{Tr}[F_a^2]$$

decomposing $U(N) \approx SU(N) \times U(1)$

the "classical" $SU(N)$ Chern-Simons level gets a one-loop exact shift:

Witten; Chen, Semenoff, & Wu

$$k_{SU(N)} = -\frac{1}{2} \mapsto -\frac{1}{2} - N$$

$\mathcal{L}_{\rm IQHT}$ realizes an integer quantum Hall phase transition

$$\mathcal{L}_{\rm IQHT}(A) = i\bar{\psi}\, \not{D}_a \psi - \frac{1}{2}\frac{1}{4\pi}\mathrm{Tr}\left[ada - \frac{2}{3}ia^3\right] - \frac{1}{2\pi}\mathrm{Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

tune the fermion mass $m_\psi \bar{\psi}\psi$ and integrate out ψ

$$\mathcal{L}_{\rm eff} = \frac{\mathrm{sign}(m_\psi) - 1}{2}\frac{1}{4\pi}\mathrm{Tr}\left[ada - \frac{2}{3}ia^3\right] - \frac{1}{2\pi}\mathrm{Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

$\mathcal{L}_{\text{IQHT}}$ realizes an integer quantum Hall phase transition

$$\mathcal{L}_{\text{IQHT}}(A) = i\bar{\psi}\,\not{D}_a\psi - \frac{1}{2}\frac{1}{4\pi}\text{Tr}\Big[ada - \frac{2}{3}ia^3\Big] - \frac{1}{2\pi}\text{Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

tune the fermion mass $m_\psi\bar{\psi}\psi$ and integrate out ψ

$$\mathcal{L}_{\text{eff}} = \frac{\text{sign}(m_\psi)-1}{2}\frac{1}{4\pi}\text{Tr}\Big[ada - \frac{2}{3}ia^3\Big] - \frac{1}{2\pi}\text{Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

$m_\psi < 0$: integer quantum Hall effect

$m_\psi > 0$: insulator

$\mathcal{L}_{\rm IQHT}$ realizes an integer quantum Hall phase transition

$$\mathcal{L}_{\rm IQHT}(A) = i\bar{\psi}\,\not{D}_a\psi - \frac{1}{2}\frac{1}{4\pi}{\rm Tr}\Big[ada - \frac{2}{3}ia^3\Big] - \frac{1}{2\pi}{\rm Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

tune the fermion mass $m_\psi\bar{\psi}\psi$ and integrate out ψ

$$\mathcal{L}_{\rm eff} = \frac{{\rm sign}(m_\psi)-1}{2}\frac{1}{4\pi}{\rm Tr}\Big[ada - \frac{2}{3}ia^3\Big] - \frac{1}{2\pi}{\rm Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

$m_\psi < 0:$

$$\mathcal{L}_{\rm eff} = -\frac{1}{4\pi}{\rm Tr}\Big[ada - \frac{2}{3}ia^3\Big] - \frac{1}{2\pi}{\rm Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

rank/level duality

Naculich & Schnitzer; Nakanishi & Tsuchiya; Hsin & Seiberg →

$$= \Big(\frac{N}{4\pi} - \frac{N+1}{4\pi}\Big)bdb - \frac{1}{2\pi}bdA$$

$$= -\frac{1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

$\mathcal{L}_{\text{IQHT}}$ realizes an integer quantum Hall phase transition

$$\mathcal{L}_{\text{IQHT}}(A) = i\bar{\psi}\, \not{D}_a \psi - \frac{1}{2}\frac{1}{4\pi}\text{Tr}\left[ada - \frac{2}{3}ia^3\right] - \frac{1}{2\pi}\text{Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

tune the fermion mass $m_\psi \bar{\psi}\psi$ and integrate out ψ

$$\mathcal{L}_{\text{eff}} = \frac{\text{sign}(m_\psi) - 1}{2}\frac{1}{4\pi}\text{Tr}\left[ada - \frac{2}{3}ia^3\right] - \frac{1}{2\pi}\text{Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

$$m_\psi > 0 :$$

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2\pi}\text{Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

$$\Longrightarrow$$

$$b = 0 \text{ and}$$

$$\mathcal{L}_{\text{eff}} = 0$$

$\mathcal{L}_{\mathrm{IQHT}}$ realizes an integer quantum Hall phase transition

$$\mathcal{L}_{\mathrm{IQHT}}(A) = i\bar{\psi}\,\not{D}_a\psi - \frac{1}{2}\frac{1}{4\pi}\mathrm{Tr}\left[ada - \frac{2}{3}ia^3\right] - \frac{1}{2\pi}\mathrm{Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

tune the fermion mass $m_\psi\bar{\psi}\psi$ and integrate out ψ

$$\mathcal{L}_{\mathrm{eff}} = \frac{\mathrm{sign}(m_\psi)-1}{2}\frac{1}{4\pi}\mathrm{Tr}\left[ada - \frac{2}{3}ia^3\right] - \frac{1}{2\pi}\mathrm{Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

$m_\psi < 0$: integer quantum Hall effect

$m_\psi > 0$: insulator

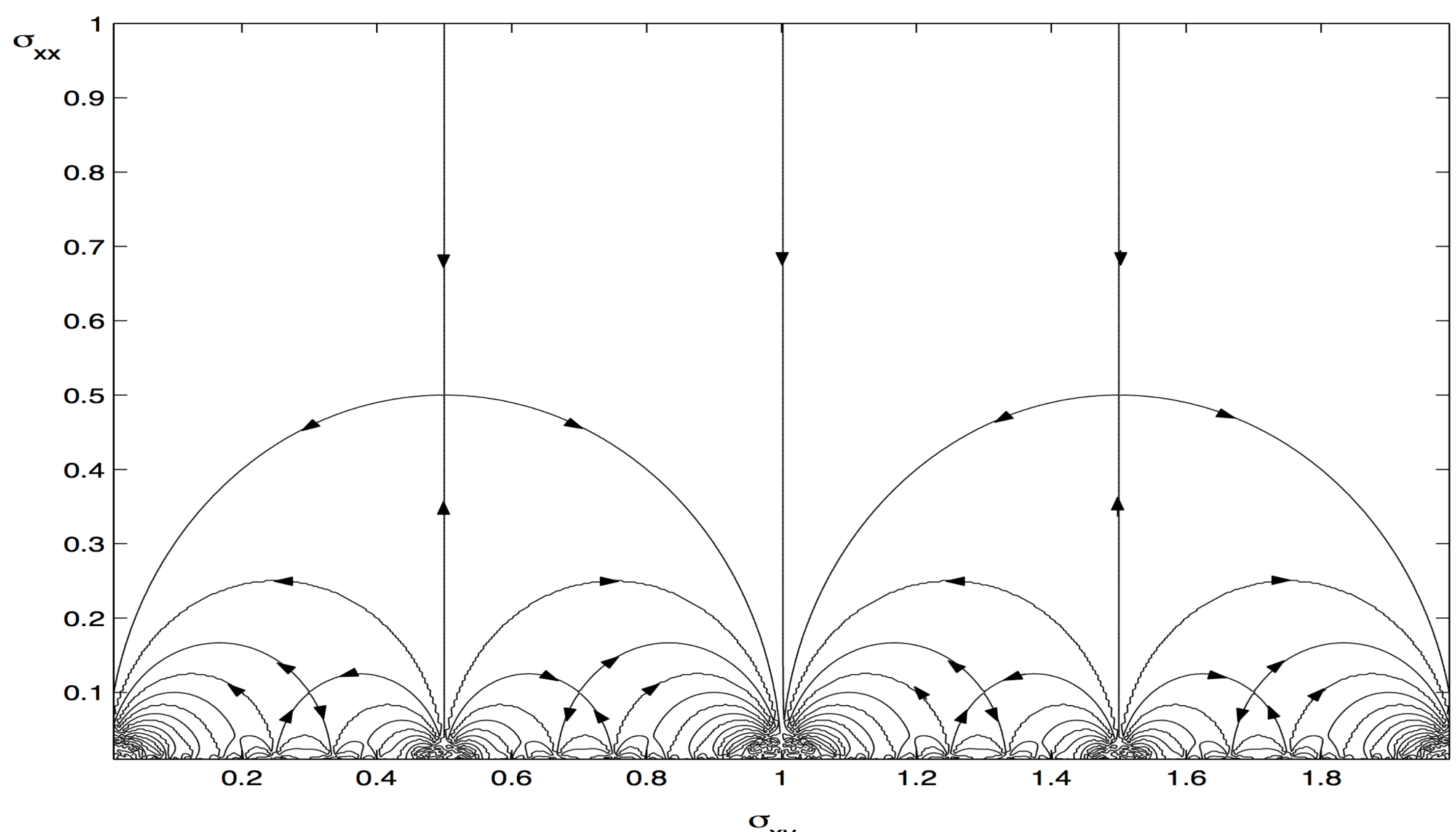
fractional quantum Hall transitions via modular transformations

modular group, $PSL(2,\mathbb{Z})$: group of 2×2 matrices with integer entries and unit determinant

complexified zero-temperature dc conductivity

$$\sigma = \sigma_{xy} + i\sigma_{xx}$$

$$\sigma \mapsto \frac{p\sigma + q}{r\sigma + s}, \text{ for } \begin{pmatrix} p & q \\ r & s \end{pmatrix} \in PSL(2,\mathbb{Z})$$



lifting the modular group to a Lagrangian $\mathcal{L}(\Phi, A)$

Witten; Leigh & Petkou

generators: $\mathcal{T} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $\mathcal{S} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$$\mathcal{T} : \mathcal{L}(\Phi, A) \mapsto \mathcal{L}(\Phi, A) + \frac{1}{4\pi} AdA,$$
$$\mathcal{S} : \mathcal{L}(\Phi, A) \mapsto \mathcal{L}(\Phi, c) - \frac{1}{2\pi} cdB$$

$$\mathcal{T} : \sigma \mapsto \sigma + 1$$
$$\mathcal{S} : \sigma \mapsto -1/\sigma$$

we can decompose a subset of modular transformations into two groups:

(i) addition of a Landau level: $\mathcal{T}$

and

(ii) attachment of m units of flux: $\mathcal{S}^{-1}\mathcal{T}^{-m}\mathcal{S}$

e.g.,

$\sigma = 1 \to 0$ transition

$\downarrow$

some modular transformation →

$$\sigma = \frac{1}{(m+1)} \to 0 \text{ transition}$$

via $\mathcal{S}^{-1}\mathcal{T}^{-m}\mathcal{S}$

from a theory for the $\sigma = 1 \to 0$ transition

we can generate a class of fractional quantum Hall transitions,

e.g., $\sigma = 1/(m+1) \to 0$

$$\mathcal{L}_{\text{IQHT}}(A)$$

some modular transformation →

$$\downarrow$$

$$\mathcal{L} = \mathcal{L}_{\text{IQHT}}(c) + \mathcal{L}_{\text{mod}}(A)$$

$$\mathcal{L}_{\text{IQHT}}(c) = i\bar{\psi}\, \not{D}_a \psi - \frac{1}{2}\frac{1}{4\pi}\text{Tr}\left[ada - \frac{2}{3}ia^3\right] - \frac{1}{2\pi}\text{Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdc$$

$$\mathcal{L}_{\text{mod}}(A) = -\frac{1}{2\pi}cdg - \frac{m}{4\pi}gdg - \frac{1}{2\pi}gdA$$

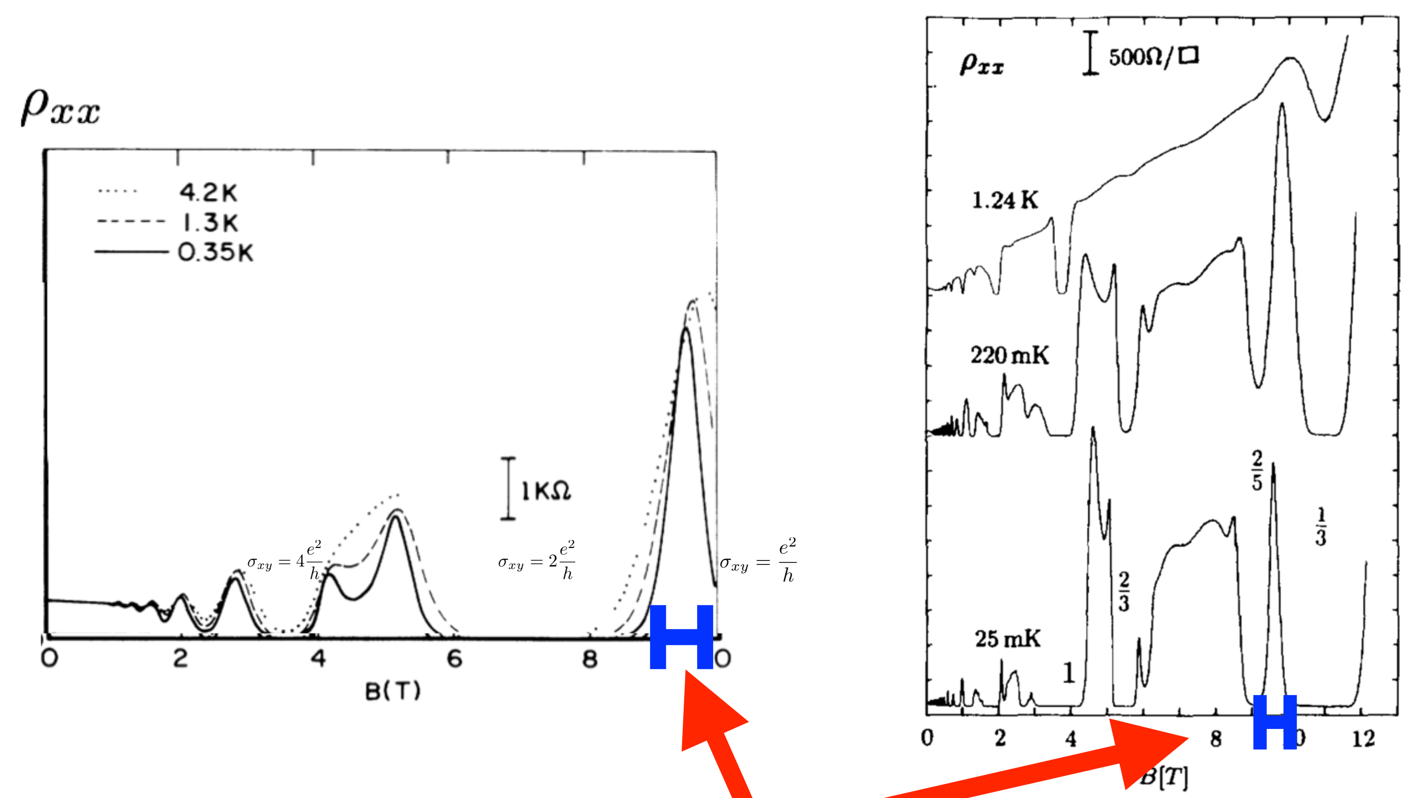
focusing on the $\sigma = 1/(m+1) \to 0$ transition,

we wish to calculate:

$\nu^{-1} = 1 - \gamma_{\bar{\psi}\psi}$.

$z = 1$ automatically, since theory is relativistic

I will argue that $\gamma_{\bar{\psi}\psi}$ is independent of m in the 't Hooft large N limit



$\Delta B \sim T^{1/\nu z}$

for this perturbative calculation, it's helpful to rewrite the Lagrangian in a less precise, but simpler form

$$\mathcal{L}_{\rm s} = i\bar{\psi}\, \not{D}_a \psi - \frac{1}{2}\frac{1}{4\pi}\mathrm{Tr}\Big[ada - \frac{2}{3}ia^3\Big] + \frac{1}{N+1+m}\frac{1}{4\pi}(\mathrm{Tr}[a] - A)d(\mathrm{Tr}[a] - A)$$

next, we set the background E&M field to zero and decompose:

$$a = \mathcal{A}_{SU(N)} + \mathcal{A}_{U(1)}\mathbb{I}$$

$$\mathcal{L}_{\rm s} = i\bar{\psi}\, \not{D}_a \psi + \frac{k_{SU(N)}}{4\pi}\mathrm{Tr}\Big[\mathcal{A}_{SU(N)}d\mathcal{A}_{SU(N)} - \frac{2}{3}i\mathcal{A}_{SU(N)}^3\Big] + \frac{k_{U(1)}}{4\pi}\mathcal{A}_{U(1)}d\mathcal{A}_{U(1)}$$

$$k_{SU(N)} = -\frac{1}{2} - N \text{ and } k_{U(1)} = \frac{N^2 - N - Nm}{2(N+1+m)}$$

some intuition

$$\mathcal{L}_{\rm s} = i\bar{\psi}\,\not{D}_a\psi + \frac{k_{SU(N)}}{4\pi}\mathrm{Tr}\Big[\mathcal{A}_{SU(N)}d\mathcal{A}_{SU(N)} - \frac{2}{3}i\mathcal{A}_{SU(N)}^3\Big] + \frac{k_{U(1)}}{4\pi}\mathcal{A}_{U(1)}d\mathcal{A}_{U(1)}$$

$$a = \mathcal{A}_{SU(N)} + \mathcal{A}_{U(1)}\mathbb{I}$$

$$k_{SU(N)} = -\frac{1}{2} - N \text{ and } k_{U(1)} = \frac{N^2 - N - Nm}{2(N+1+m)}$$

since $|k_{U(1)}| \propto N$ as $N \to \infty$

fluctuations of $\mathcal{A}_{U(1)}$ can be made arbitrarily weak,

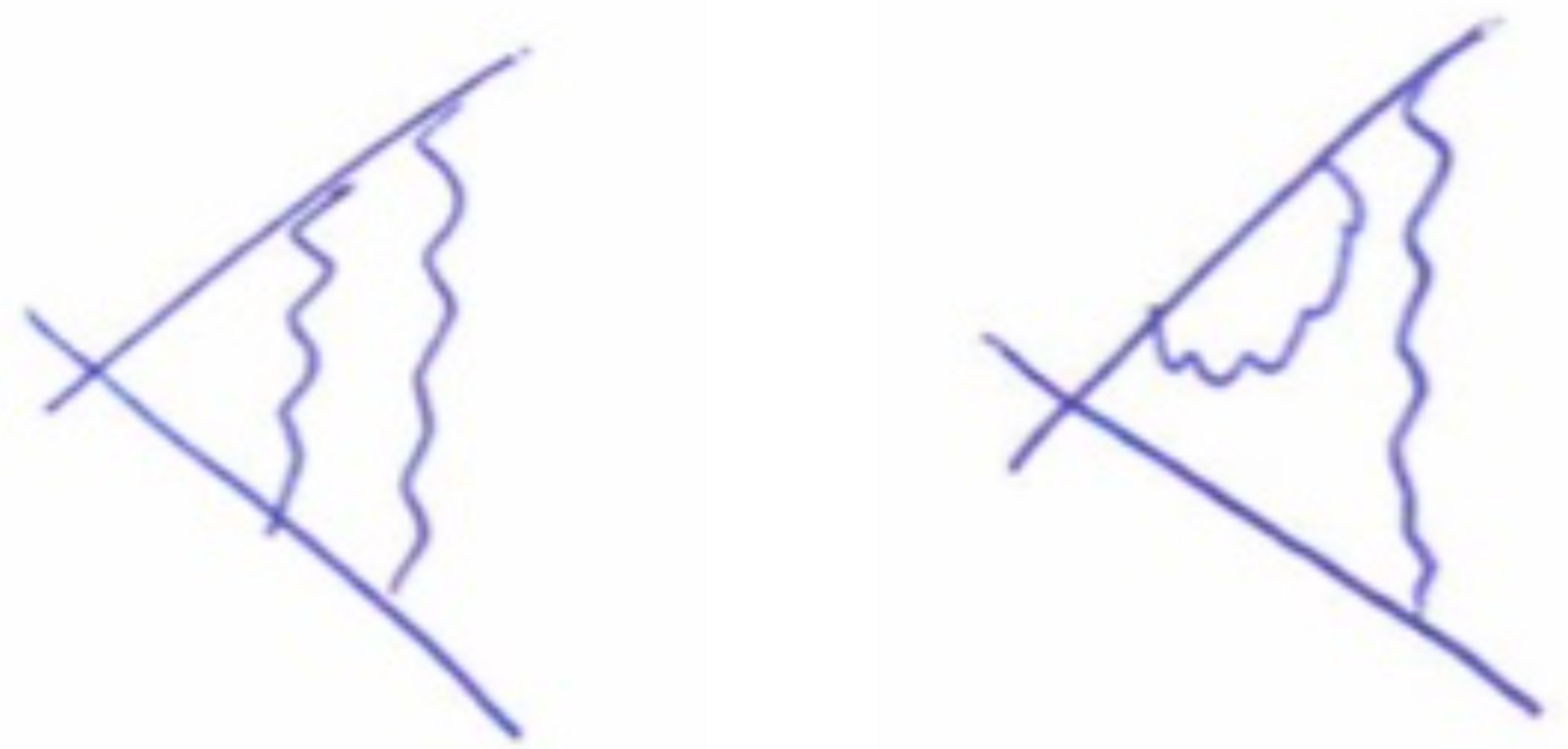
if $\mathcal{A}_{SU(N)}$ could be ignored

’t Hooft large N limit

$$N \to \infty$$
$$\text{with}$$
$$\frac{N}{k_{SU(N)}} \text{ and } \frac{N}{k_{U(1)}} \text{ finite}$$

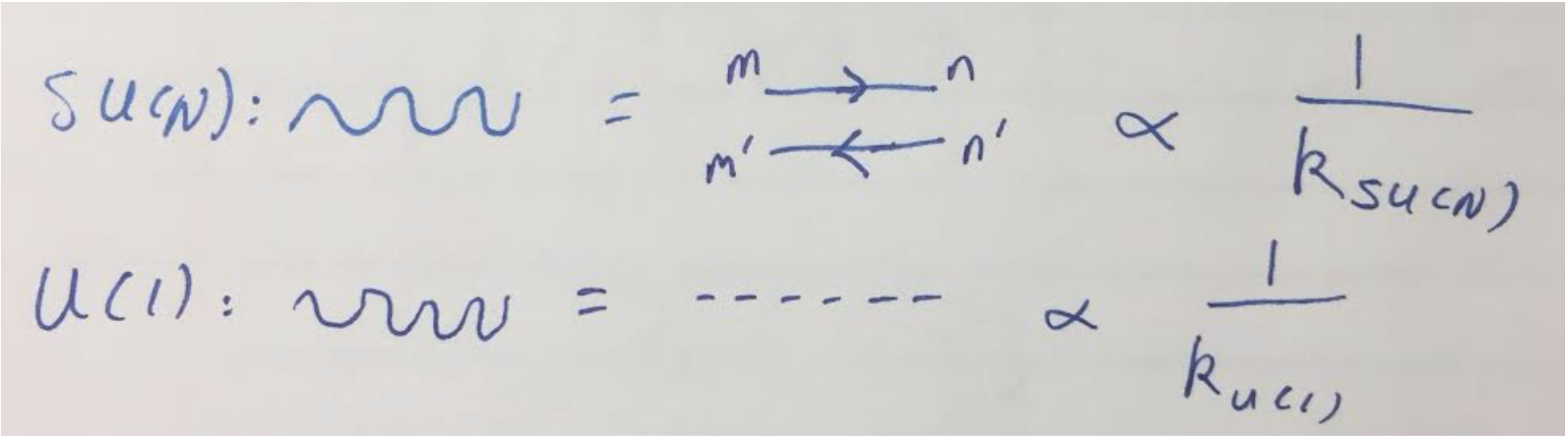
note: non-trivial even at infinite N!

in this limit, leading non-zero contributions to anomalous dimension are

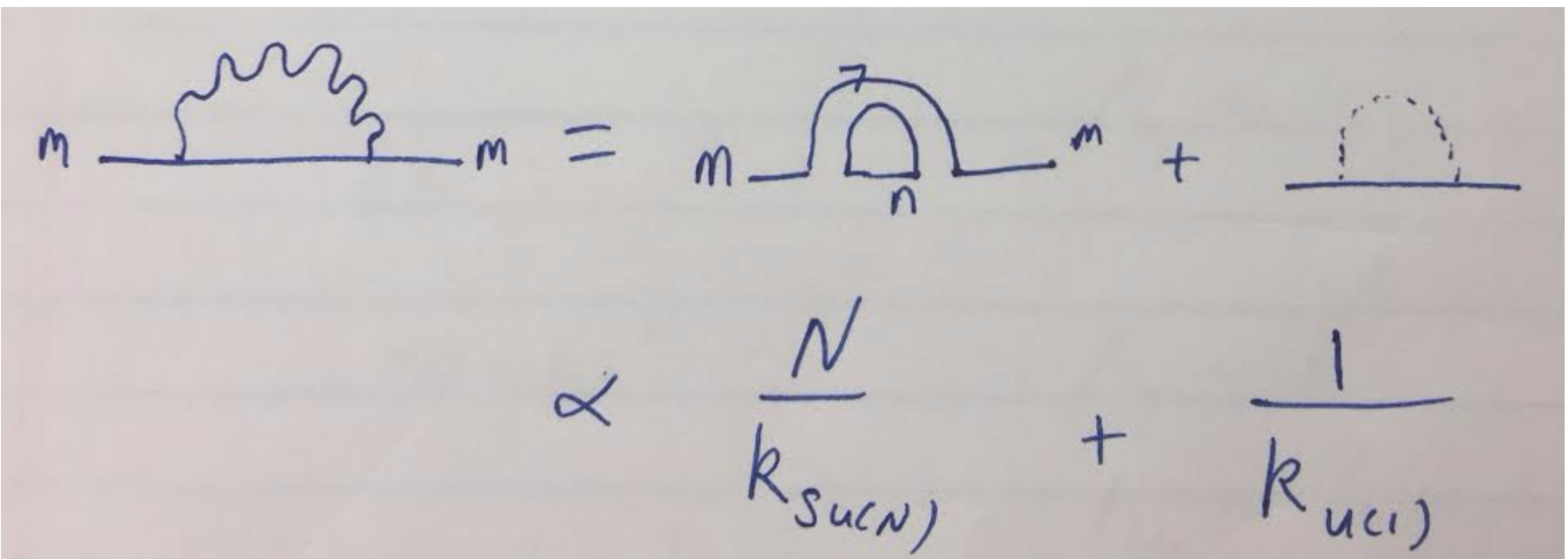


(1-loop vertex and 1- and 2-loop fermion self-energies are finite)

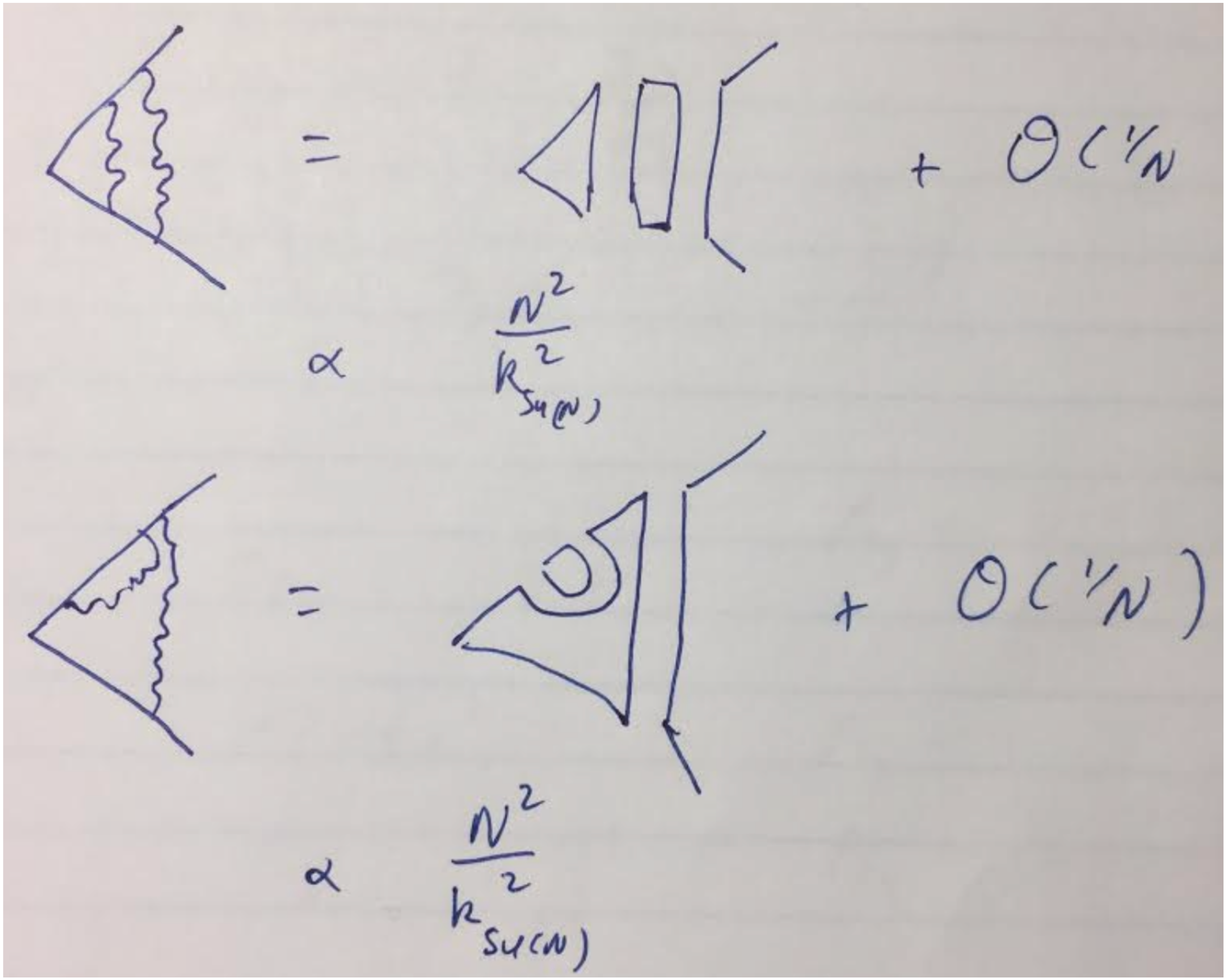
’t Hooft large N limit



example: fermion self-energy



’t Hooft large N limit



as long as $|k_{U(1)}| \sim N$

leading 't Hooft large N limits of $U(N)$ and $SU(N)$ are the same perturbatively

$$\gamma_{\bar{\psi}\psi} = c_1 \left(\frac{N}{k_{SU(N)}} \right)^2 + \mathcal{O}(1/N) f(m)$$

in perturbation theory, $\mathcal{A}_{U(1)}$ first contributes at $\mathcal{O}(1/N)$

i.e., dependence on m in $1/(m+1) \to 0$ transition occurs at $\mathcal{O}(1/N)$

this is superuniversality in the 't Hooft large N limit!

the value of the 2-loop planar contribution to the mass anomalous dimension is known:

Giombi, Minwalla, Prakash, Trivedi, Wadia, & Yin

$$\gamma_{\bar{\psi}\psi} = 0 + \mathcal{O}\Big((\frac{N}{k_{SU(N)}})^3 \Big)$$

or

$$\nu = 1$$

in perturbation theory, this result holds for all m

higher-order terms in perturbation theory may change the value for the anomalous dimension (or exponent), but will not invalidate the m independence

to what extent do these results hold away from the controlled 't Hooft large N limit?

consistency of various dualities implies that

$$\mathcal{L}_{\mathrm{IQHT}}(A) = i\bar{\psi}\, \not{D}_a \psi - \frac{1}{2}\frac{1}{4\pi}\mathrm{Tr}\left[ada - \frac{2}{3}ia^3\right] - \frac{1}{2\pi}\mathrm{Tr}[a]db - \frac{N+1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

is in the same universality class as the theory of a free Dirac fermion for any N > 0!

$$i\bar{\Psi}\, \not{D}_A \Psi + \frac{1}{8\pi}AdA$$

For N=1:Son; Senthil & Wang; Metlitski & Vishwanath; Seiberg, Senthil, Wang, & Witten; Karch & Tong; Kachru, MM, Torroba, & Wang; Geraedts, Zaletel, Mong, Metlitski, Vishwanath, & Motrunich; Shankar & Murthy; Mross, Alicea, & Motrunich; Balram & Jain

this means at m = 0

$$\gamma_{\bar{\psi}\psi} = 0$$

within a formal perturbative expansion, the planar contribution must vanish at large N

using this large N limit, this should likewise hold when m > 0, since m only enters at sub-planar order in perturbation theory!

the argument for the N-independent duality to a free fermion goes as follows:

Giombi, Minwalla, Prakash, Trivedi, Wadia, & Yin;
Aharony, Gur-Ari, & Yacoby; Aharony; Hsin & Seiberg;
Seiberg, Senthil, Wang, & Witten

$$|D_A\phi|^2 - |\phi|^4 + \frac{1}{4\pi}AdA$$

$$\updownarrow$$

$$\bar{\psi}\, \not{D}_a \psi - \frac{1}{8\pi}\mathrm{Tr}[ada - \frac{2}{3}ia^3] - \frac{1}{2\pi}\mathrm{Tr}[a]dA - \frac{N-1}{4\pi}AdA$$

the argument for the N-independent duality to a free fermion goes as follows:

Giombi, Minwalla, Prakash, Trivedi, Wadia, & Yin; Aharony, Gur-Ari, & Yacoby; Aharony; Hsin & Seiberg; Seiberg, Senthil, Wang, & Witten

$$|D_A\phi|^2 - |\phi|^4 + \frac{1}{4\pi}AdA$$

$$\updownarrow$$

$$\bar{\psi}\, \not{D}_a \psi - \frac{1}{8\pi}\mathrm{Tr}[ada - \frac{2}{3}ia^3] - \frac{1}{2\pi}\mathrm{Tr}[a]dA - \frac{N-1}{4\pi}AdA$$

applying $\mathcal{S}\mathcal{T}^{-2}$ to both sides:

$$|D_b\phi|^2 - |\phi|^4 - \frac{1}{4\pi}bdb - \frac{1}{2\pi}bdA$$

← this is dual to a free fermion

$$\updownarrow$$

$$\bar{\psi}\, \not{D}_a \psi - \frac{1}{8\pi}\mathrm{Tr}[ada - \frac{2}{3}ia^3] - \frac{1}{2\pi}\mathrm{Tr}[a]dc - \frac{N+1}{4\pi}cdc - \frac{1}{2\pi}cdA$$

the argument for the N-independent duality to a free fermion goes as follows:

Giombi, Minwalla, Prakash, Trivedi, Wadia, & Yin;
Aharony, Gur-Ari, & Yacoby; Aharony; Hsin & Seiberg;
Seiberg, Senthil, Wang, & Witten

$$|D_A \phi|^2 - |\phi|^4 + \frac{1}{4\pi} AdA$$

$$\updownarrow$$

$$\bar{\psi}\, \not{D}_a \psi - \frac{1}{8\pi}\mathrm{Tr}[ada - \frac{2}{3} i a^3] - \frac{1}{2\pi}\mathrm{Tr}[a]dA - \frac{N-1}{4\pi} AdA$$

applying $\mathcal{S}\mathcal{T}^{-2}$ to both sides:

$$i\bar{\Psi}\, \not{D}_A \Psi + \frac{1}{8\pi} AdA$$

$$\updownarrow$$

$$\bar{\psi}\, \not{D}_a \psi - \frac{1}{8\pi}\mathrm{Tr}[ada - \frac{2}{3} i a^3] - \frac{1}{2\pi}\mathrm{Tr}[a]dc - \frac{N+1}{4\pi} cdc$$

$$- \frac{1}{2\pi} cdA$$

things to do:

(i) understand nonperturbative corrections at m >0

(ii) compare large flavor and ’t Hooft expansions

(iii) calculate electrical and thermal conductivities

(iv) add disorder and the Coulomb interaction

(v) study possible applications to transitions tuned by a periodic potential in graphene

(vi) deform our models to more faithfully represent transitions in a GaAs 2D electron gas

Lee, Geraedts, & Motrunich

Lee, Wang, Zaletel, Vishwanath, & He; experiments by Young et al.