Self-Dual Mixed-dimensional QED
Transport Properties and $\nu = 1$ Bosonic Hall State

WHH, Son PRB 2017, WHH, Son unpublished

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Aspen Winter conference 2018

March 23, 2018

utility of dualities

- help understand the structure of phase
- help computation
  - T_c in 2+1 d classical Ising
  - critical point and (ν) in 1+1 d quantum Ising Kogut RMP (1979)
  - transport in AdS/CFT cf. Policastro, Son, Starinets PRL (2001), Herzog, Kovtun, Sachdev, Son PRD (2007)...

IR dualities in recent years?

- deeper understanding of the phases, emergent symmetries, etc.
- computation
  - thermal partition function at large N in level/rank cf. Minwalla, Yokoyama 1507.04546,...
  - σ and Δ at large charge k in self-dual $N_f = 2$ QED Cheng, Xu PRB (2016)
- hint: strong-weak duality / self-dual

outline

the model and defs.

a matter brane (on $z = 0$ plane) + a bulk photons

$$I = I_m[\Psi_p, A] + I_M[A; e]$$

Kovner, Rosenstein PRB (1990), Marino Nucl Phys B (1993)...

$$p = 1,\ I_m[\Psi_1, A] = \int d^3x\, (|D_A\phi|^2 - |\phi|^4)$$

$$p = 2,\ I_m[\Psi_2, A] = \int d^3x\, \bar{\psi} i \not{D}_A \psi$$

$$I_M[A; e] = -\frac{1}{4e^2} \int d^4x\, F_{\mu\nu}F^{\mu\nu}$$

$$I_{BF}[a, A] = \frac{1}{2\pi} \int d^3x\, \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda$$

$$I_{CS}[A] = \frac{1}{4\pi} \int d^3x\, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

particle-vortex duality

using particle vortex duality

Peskin Ann Phys (1981), Dasgupta, Halperin PRL (1981)...,Son PRX (2015), Wang, Senthil PRB (2015), Metlitski, Vishwanath PRB(2015),...

$$I_m[\Psi_p, A] \leftrightarrow I_m[\tilde{\Psi}_p, a] + \frac{1}{p} I_{BF}[a, A]$$

duality mapping

$$J^\mu = \frac{1}{2\pi p}\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$$

$$\tilde{J}^\mu = -\frac{1}{2\pi p}\epsilon^{\mu\nu\lambda}\partial_\nu A_\lambda$$

filling fraction $\nu = 2\pi\rho/B$ mapping

$$\nu\tilde{\nu} = -p^{-2}$$

may A be dynamical

adding $I_M[A]$ into the game

$$I_m[\Psi_p, A] + I_M[A; e]$$
$$\updownarrow$$
$$I_m[\tilde{\Psi}, a] + \frac{1}{2\pi p}\int a\, dA + I_M[A; e]$$

integrating out A on the dual side, it is equivalent to

$$I_m[\tilde{\Psi}_p, a] + I_M[a; \tilde{e}], \;\; \tilde{e} = \frac{4\pi p}{e}$$

$\star$ strong-weak duality + self-dual

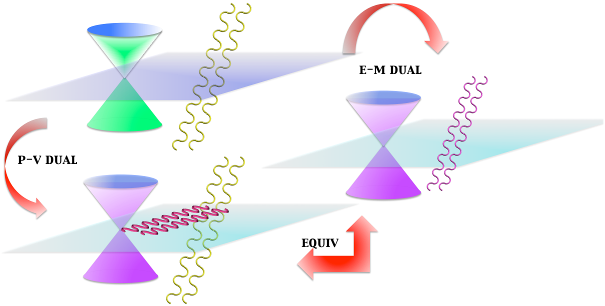

Figure:

direct derivation

pseudo QED scheme Marino Nucl Phys B (1993)

$$I_M[A;e] \to I_{PQED}[A;e] = \frac{-i}{e^2}\int d^3x\, d^3x'\, F_{\mu\nu}(x)\frac{1}{\sqrt{-\partial^2}}F^{\mu\nu}(x')$$

integrating all off-plane components and A on dual side

$$I_m[\Psi_p, A] + I_{PQED}[A;e] \leftrightarrow I_m[\tilde{\Psi}_p, a] + I_{PQED}[a;\tilde{e}]$$

See also Mross, Alicea, Motrunich PRX (2017), Goldman, Fradkin 1801.04936

sensible in that particle-vortex duality $\sim$ S operation

Kapustin, Strassler JHEP (1999), Witten arXiv:hep-th/0307041

EM-dual derivation i

- Maxwell eqns via BCs; looking at even and odd under $z \to -z$

$$A = A^s + A^a; A^s(-z) = A^s(z), A^a(-z) = -A^a(z)$$

- dof separation

$$I[\Psi_p, A] = I[\Psi_p, A^s_\mu, A^a_z] + I_g[A^a_\mu, A^s_z], \;\; \mu = 0, 1, 2$$

- integrating out $I_g \to$ orbifold BC

$$A_\mu(z) = A_\mu(-z)$$
$$A_z(z) = -A_z(-z)$$

EM-dual derivation ii

- BCs of particles

$$E_z(0^+) = \frac{e^2}{2}\rho = -\frac{e^2}{4\pi p}b$$

$$\mathbf{B}_\parallel(0^+) = \frac{e^2}{2}\mathbf{J}\times\hat{\mathbf{z}} = \frac{e^2}{4\pi p}\mathbf{e}$$

- electromagnetic duality

$$\mathbf{E} = -\mathrm{sgn}(z)\tilde{\mathbf{B}}$$

$$\mathbf{B} = \mathrm{sgn}(z)\tilde{\mathbf{E}}$$

-

$$\tilde{B}_z = \frac{e^2}{4\pi p}b$$

$$\tilde{\mathbf{E}}_\parallel = \frac{e^2}{4\pi p}\mathbf{e}_\parallel$$

EM-dual derivation iii

- BCs of vortices

$$e_z(0^+) = \frac{4\pi p}{e^2}\tilde{E}_z(0^+) = \frac{\tilde{e}^2}{2}\tilde{\rho}$$

$$\mathbf{b}_{\parallel}(0^+) = \frac{4\pi p}{e^2}\tilde{\mathbf{B}}_{\parallel}(0^+) = \frac{\tilde{e}^2}{2}\tilde{\mathbf{J}} \times \hat{\mathbf{z}}$$

- resulting theory

$$I_m[\tilde{\Psi}_p, a] + I_M[a, \tilde{e}]$$

transport coefficients under duality

- particle-vortex duality provides constraints on response functions
  Wen, Zee Int J Phys (1990), Fradkin, Kivelson Nuc Phys B (1996), Burgess, Dolan PRB (2001)...
- electrical and thermal transports: σ, α, κ and $\bar{\kappa}$

$$J^i = \sigma^{ij} E^j + \alpha^{ij} \partial_j T$$
$$q^i = -T\alpha^{ij} E^j - \bar{\kappa}^{ij} \partial_j T$$

using $J^i = -\epsilon^{ij} e^j/(2\pi p)$ and $\tilde{J}^i = \epsilon^{ij} E^j/(2\pi p)$

$$\sigma = -\frac{1}{(2\pi p)^2} \epsilon(\tilde{\sigma})^{-1} \epsilon$$
$$\alpha = \frac{1}{2\pi p} \epsilon \tilde{\sigma}^{-1} \tilde{\alpha}$$
$$\bar{\kappa} = \tilde{\bar{\kappa}} - T\tilde{\alpha}\tilde{\sigma}^{-1}\tilde{\alpha} = \tilde{\kappa}$$

found in other contexts with S duality (bulk EM fields + brane matter)

Herzog et al PRD (2007), Hartnoll et al PRD (2007), Donos et al JHEP (2017)...

self-dual point $e^2 = 4\pi p$

special case at $\nu = p^{-1}$ and at $e^2 = 4\pi p$

$\sigma^T = \tilde{\sigma}$, etc.

$$\sigma_{xx}^2 + \sigma_{xy}^2 = \frac{1}{(2\pi p)^2}$$
$$\frac{\alpha_{xy}}{\alpha_{xx}} = \tan\left(\frac{\pi}{4} + \frac{\theta_H}{2}\right)$$
$$\bar{\kappa}_{xy} = -\kappa_{xy} = p\pi T(\alpha_{xx}^2 + \alpha_{xy}^2)$$

$\nu = 1$ bosonic quantum hall state

- may be a Fermi liquid Pasquier, Haldane Nuc Phys B (1998), Read PRB (1998)
- (possibly be) a theory without CS term Read PRB (1998)
- may be a Pfaffian state Regnault, Jolicoeur PRL (2003) PRB (2004)
- emergent particle-hole symmetry for 2-component bosons
  Mross, Alicea, Motrunich PRL (2016), Wang, Senthil PRB (2016), Geraedts, Repellin, Wang, Mong, Senthil, Regnault PRB (2017)

what about a single boson?

fermionization

take a Wilson-Fisher boson ϕ

$$I_m[\phi, A] \leftrightarrow I_m[\psi, a] - \frac{1}{2} I_{CS}[a] - I_{BF}[a, A] - I_{CS}[A]$$

$$\updownarrow \qquad\qquad\qquad \updownarrow$$

$$I_m[\tilde{\phi}, \tilde{a}] + I_{BF}[\tilde{a}, A] \leftrightarrow I_m[\chi, c] + \frac{1}{2} I_{CS}[c] - I_{BF}[c, A] + I_{CS}[A]$$

- T symmetry? quantum symmetry for fermion Seiberg, Senthil, Wang, Witten (2016)
- particle-vortex duality $\leftrightarrow$ field redefinition Mross, Alicea, Motrunich PRX (2017), Chen, Son, Wang, Raghu PRL (2018)

duality mapping

vary actions w.r.t. A, a, $\tilde{a}$ and c

$$J^\mu_\phi = -\frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda - \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu A_\lambda$$
$$= \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu \tilde{a}_\lambda = \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu A_\lambda - \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu c_\lambda$$
$$J^\mu_\psi = \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda + \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu A_\lambda$$
$$J^\mu_{\tilde{\phi}} = -\frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu A_\lambda$$
$$J^\mu_\chi = -\frac{1}{4\pi}\epsilon^{\mu\nu\lambda}\partial_\nu c_\lambda + \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu A_\lambda$$

$\star$ $\nu_\phi = 1 \Rightarrow \langle b\rangle = 0$, $\rho_\phi = -\langle B\rangle/(2\pi)$: a Fermi sea

phenomenology

- Jain sequence

  general mapping $(\nu_\phi - 1)(\nu_\psi + 1/2) = -1$

  $\nu_\psi = n + 1/2 \Rightarrow \nu_\phi = \frac{n}{n+1}$

  $\nu_\psi = -(p + 1/2) \Rightarrow \nu'_\phi = \frac{p+2}{p+1} = 2 - \nu_\phi$

- shift: let ψ fills n Landau levels on a sphere

$$\int d^2x\, \rho_\psi = \left(n + \frac{1}{2}\right) \int \frac{d^2x}{(2\pi)} b + n(n+1)$$

$\downarrow$ duality mappings

$$\frac{n+1}{n} N_\phi - N_{\Phi_B} = \nu^{-1} N_\phi - N_{\Phi_B} = (n+1) = \mathcal{S}$$

particle-vortex dual to particle-vortex dual

- T operation of ψ?

  $\nu_\phi = 1 \to \psi$ Fermi sea $\xrightarrow{\text{T}} \chi$ Fermi sea?

  however $\nu_\phi = 1 \to \nu_{\tilde{\phi}} = -1 \to \nu_\chi = 0$

  Fermi sea $\leftrightarrow$ charge neutral point in 0^{th} Landau level

- T: if a and A transform differently;

  CT: if a and A transform the same way

- can check $\nu_\psi \nu_\chi = -1/4$: particle-vortex duality

transport mappings

- $$\epsilon\Big(\sigma_b - \frac{1}{2\pi}\epsilon\Big)\epsilon^{-1}\Big(\sigma_f + \frac{1}{4\pi}\epsilon\Big) = \frac{1}{(2\pi)^2}$$
$$\alpha_b = -\frac{\epsilon}{2\pi}\Big(\sigma_f + \frac{1}{4\pi}\epsilon\Big)^{-1}\alpha_f$$
$$\bar{\kappa}_b = \bar{\kappa}_f - \alpha_f\Big(\sigma_f + \frac{1}{4\pi}\Big)^{-1}\alpha_f T$$
- consistency check: ψ and χ are particle-vortex dual

self-dual with $I_M[A;e]$

- writing $\sigma_\psi^{ij} = \sigma_\psi \delta^{ij}, \alpha_\psi^{ij} = \alpha_\psi \delta^{ij}, \kappa_\psi^{ij} = \kappa_\psi \delta^{ij}, \ldots$
- identities

  $\det[\sigma_\phi] = \frac{1}{(2\pi)^2}, \frac{\alpha_\phi^{xy}}{\alpha_\phi^{xx}} = \tan(\frac{\pi}{4} + \frac{\theta_H}{2})$, etc. are satisfied
- how do $\sigma_\psi, \alpha_\psi, \ldots$ know about e^2?

restoring T symmetry

- RPA versus full 1PI diagram
- $\langle aa \rangle$s in 1PI invalidate the assumption δ^{ij}
- direct integration

$$I_m[\psi, a] + \frac{1}{2}\left(\frac{e^2 - \tilde{e}^2}{e^2 + \tilde{e}^2}\right) I_{CS}[a] + I_{PQED}[a; g = \sqrt{e^2 + \tilde{e}^2}]$$

- $e^2 = \tilde{e}^2$ CS level $= 0$, no off diagonal transport
- $e^2 = \tilde{e}^2 = 4\pi$, $g^2 = 8\pi \Rightarrow$ simultaneous self-dual

See also Mross, Alicea, Motrunich PRX (2017)

EM dual visited

performing the same EM dual operation,

$$\begin{pmatrix} \mathbf{e}_\chi \\ \mathbf{b}_\chi \end{pmatrix} = \frac{1}{e^2+\tilde{e}^2} \begin{pmatrix} \tilde{e}^2 - e^2 & 8\pi \\ -8\pi & \tilde{e}^2 - e^2 \end{pmatrix} \begin{pmatrix} \mathbf{e}_\psi \\ \mathbf{b}_\psi \end{pmatrix}$$

right at $e^2 = \tilde{e}^2$

$$\mathbf{e}_\chi = \mathbf{b}_\psi, \mathbf{b}_\chi = -\mathbf{e}_\psi$$

more generally

$$\begin{array}{c} I_m[\phi, A] + I_M[A; e] \leftrightarrow I_m[\psi, a] + I_M[a; g] + I_\theta[a; \Delta\theta] \\ \updownarrow \qquad\qquad\qquad\qquad \updownarrow \\ I_m[\tilde{\phi}, \tilde{A}] + I_M[\tilde{A}, \tilde{e}] \leftrightarrow I_m[\chi, a] + I_M][a; g] + I_\theta[a; -\Delta\theta] \end{array}$$

$$g^2 = e^2 + \tilde{e}^2$$

$$\Delta\theta = \pi \frac{e^2 - \tilde{e}^2}{e^2 + \tilde{e}^2}$$

action of modular group i

going back to Seiberg, Senthil, Wang, Witten Ann Phys (2016)

- self-dual point?

  boson → boson: $\tilde{\tau}_b = -1/\tau_b = \mathsf{S}[\tau_b]$

  fermion → fermion: $\tilde{\tau}_f = (\tau_f - 1)/(2\tau_f - 1) = \mathsf{ST}^{-2}\mathsf{ST}^{-1}[\tau_f]$

- different values of self-dual point?

$$\tau_b = i \to e_b^2 = 2\pi$$
$$\tau_f = \frac{1}{2} + \frac{i}{2} \to e_f^2 = 4\pi$$

action of modular group ii

- simultaneous self-dual?

$$\tau_f = \frac{1}{1-\tau_b}, \ \tau_b = i \leftrightarrow \tau_f = \frac{1}{2} + \frac{i}{2}$$

comparing to Mross, Alicea, Motrunich PRX (2017), Goldman, Fradkin 1801.04936

- $z_b = \tau_b$, $z_f = \frac{1}{1-2\tau_f}$ (f → dual f)

$$z_f = \frac{z_b - 1}{z_b + 1}, \ z_b = i \leftrightarrow z_f = i$$

summary

- particle-vortex dual + bulk EM field $\rightarrow$ strong-weak and self-dual
- deep relation with electromagnetic duality
- constraints on transport at self-dual point

$\star$ a description for bosonic Hall states (Jain sequence, shifts, etc.)

$\star$ fermions are particle-vortex dual as well

$\star$ $\nu = 1 \leftrightarrow$ a Fermi sea, transport properties understood

$\star$ simultaneous self-dual points for both bosons and fermions

outlook

- is such model realizable? $v_F \to c$
- is there more response function we can infer from self-duality

⋆ how would axion QED manifest on ϕ side

⋆ is there more response function we can infer from self-duality

⋆ what may give us via bosonizing the Fermi surface

⋆ paring instability

∗ spontaneous mass generation in mixed dimension QED, etc...

Thanks for your attention and I appreciate any comment