Self-Dual Mixed-dimensional QED Transport Properties and  $\nu=1$  Bosonic Hall State WHH, Son PRB 2017, WHH, Son unpublished

WeiHan Hsiao Dam Thanh Son

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Mixed-dimensional
QED
Transport Properties
and u=1 Bosonic

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Discussion and ummary

### utility of dualities

- help understand the structure of phase
- help computation
  - $ightharpoonup T_c$  in 2+1 d classical Ising
  - ritical point and  $(\nu)$  in 1+1 d quantum Ising Kogut RMP (1979)
  - transport in AdS/CFT cf. Policastro, Son, Starinets PRL (2001), Herzog, Kovtun, Sachdev, Son PRD (2007)...

duality revisited modular invariance

- deeper understanding of the phases, emergent symmetries, etc.
- computation
  - thermal partition function at large N in level/rank cf. Minwalla, Yokoyama 1507.04546,...
  - $m \sigma$  and  $\Delta$  at large charge k in self-dual  $N_f=2$  QED <code>Cheng</code>, <code>Xu</code> PRB (2016)
- hint: strong-weak duality / self-dual

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#### outline

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a matter brane (on z = 0 plane) + a bulk photons

$$I = I_m[\Psi_p, A] + I_M[A; e]$$

Kovner, Rosenstein PRB (1990), Marino Nucl Phys B (1993)...

$$p = 1, \ I_m[\Psi_1, A] = \int d^3x \left( |D_A \phi|^2 - |\phi|^4 \right)$$

$$p = 2, \ I_m[\Psi_2, A] = \int d^3x \, \bar{\psi} i \not \!\! D_A \psi$$

$$I_M[A; e] = -\frac{1}{4e^2} \int d^4x \, F_{\mu\nu} F^{\mu\nu}$$

$$I_{BF}[a, A] = \frac{1}{2\pi} \int d^3x \, \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda$$

$$I_{CS}[A] = \frac{1}{4\pi} \int d^3x \, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

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### particle-vortex duality

#### using particle vortex duality

Peskin Ann Phys (1981), Dasgupta, Halperin PRL (1981)...,Son PRX (2015), Wang, Senthil PRB (2015), Metlitski, Vishwanath PRB(2015),...

$$I_m[\Psi_p,A] \leftrightarrow I_m[\tilde{\Psi}_p,a] + \frac{1}{p}I_{BF}[a,A]$$

duality mapping

$$J^{\mu} = \frac{1}{2\pi p} \epsilon^{\mu\nu\lambda} \partial_{\nu} a_{\lambda}$$
$$\tilde{J}^{\mu} = -\frac{1}{2\pi p} \epsilon^{\mu\nu\lambda} \partial_{\nu} A_{\lambda}$$

filling fraction  $\nu = 2\pi\rho/B$  mapping

$$\nu \tilde{\nu} = -p^{-2}$$

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strong-weak duality

### may A be dynamical

adding  $I_M[A]$  into the game

$$I_{m}[\Psi_{p}, A] + I_{M}[A; e]$$

$$\updownarrow$$

$$I_{m}[\tilde{\Psi}, a] + \frac{1}{2\pi p} \int a \, dA + I_{M}[A; e]$$

integrating out A on the dual side, it is equivalent to

$$I_m[\tilde{\Psi}_p, a] + I_M[a; \tilde{e}], \ \tilde{e} = \frac{4\pi p}{e}$$

 $\star$  strong-weak duality + self-dual

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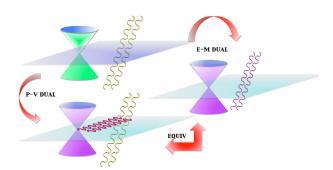


Figure:

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#### direct derivation

pseudo QED scheme Marino Nucl Phys B (1993)

$$I_M[A;e] \to I_{PQED}[A;e] = \frac{-i}{e^2} \int d^3x \, d^3x' F_{\mu\nu}(x) \frac{1}{\sqrt{-\partial^2}} F^{\mu\nu}(x')$$

integrating all off-plane components and A on dual side

$$I_m[\Psi_p,A] + I_{PQED}[A;e] \leftrightarrow I_m[\tilde{\Psi}_p,a] + I_{PQED}[a;\tilde{e}]$$

See also Mross, Alicea, Motrunich PRX (2017), Goldman, Fradkin 1801.04936

sensible in that particle-vortex duality  $\sim \mathsf{S}$  operation

Kapustin, Strassler JHEP (1999), Witten arXiv:hep-th/0307041

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#### EM-dual derivation i

lacktriangle Maxwell eqns via BCs; looking at even and odd under z 
ightarrow -z

$$A = A^{s} + A^{a}; A^{s}(-z) = A^{s}(z), A^{a}(-z) = -A^{a}(z)$$

dof separation

$$I[\Psi_p,A] = I[\Psi_p,A_{\mu}^s,A_{z}^a] + I_g[A_{\mu}^a,A_{z}^s], \ \mu = 0,1,2$$

• integrating out  $I_q \rightarrow$  orbifold BC

$$A_{\mu}(z) = A_{\mu}(-z)$$
$$A_{z}(z) = -A_{z}(-z)$$

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#### EM-dual derivation ii

BCs of particles

$$E_z(0^+) = \frac{e^2}{2}\rho = -\frac{e^2}{4\pi p}b$$

$$\mathbf{B}_{\parallel}(0^{+}) = \frac{e^{2}}{2}\mathbf{J} \times \hat{\mathbf{z}} = \frac{e^{2}}{4\pi p}\mathbf{e}$$

electromagnetic duality

$$\mathbf{E} = -\operatorname{sgn}(z)\tilde{\mathbf{B}}$$

$$\mathbf{B}=\mathrm{sgn}(z)\tilde{\mathbf{E}}$$

$$ilde{B}_z = rac{e^2}{4\pi p} b$$
  $ilde{\mathbf{E}}_{\parallel} = rac{e^2}{4\pi p} \mathbf{e}_{\parallel}$ 

$$\tilde{\mathbf{E}}_{\parallel} = \frac{e^2}{4\pi n} \mathbf{e}_{\parallel}$$

modular invariance

#### EM-dual derivation iii

► BCs of vortices

$$e_z(0^+) = \frac{4\pi p}{e^2} \tilde{E}_z(0^+) = \frac{\tilde{e}^2}{2} \tilde{\rho}$$
$$\mathbf{b}_{\parallel}(0^+) = \frac{4\pi p}{e^2} \tilde{\mathbf{B}}_{\parallel}(0^+) = \frac{\tilde{e}^2}{2} \tilde{\mathbf{J}} \times \hat{\mathbf{z}}$$

resulting theory

$$I_m[\tilde{\Psi}_p, a] + I_M[a, \tilde{e}]$$

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### transport coefficients under duality

- particle-vortex duality provides constraints on response functions Wen, Zee Int J Phys (1990), Fradkin, Kivelson Nuc Phys B (1996), Burgess, Dolan PRB (2001)...
- electrical and thermal transports:  $\sigma$ ,  $\alpha$ ,  $\kappa$  and  $\bar{\kappa}$

$$J^{i} = \sigma^{ij}E^{j} + \alpha^{ij}\partial_{j}T$$
$$q^{i} = -T\alpha^{ij}E^{j} - \bar{\kappa}^{ij}\partial_{j}T$$

using 
$$J^i=-\epsilon^{ij}e^j/(2\pi p)$$
 and  $\tilde{J}^i=\epsilon^{ij}E^j/(2\pi p)$  
$$\sigma=-\frac{1}{(2\pi p)^2}\epsilon(\tilde{\sigma})^{-1}\epsilon$$
 
$$\alpha=\frac{1}{2\pi p}\epsilon\tilde{\sigma}^{-1}\tilde{\alpha}$$
 
$$\tilde{\kappa}=\tilde{\kappa}-T\tilde{\alpha}\tilde{\sigma}^{-1}\tilde{\alpha}=\tilde{\kappa}$$

found in other contexts with S duality (bulk EM fields + brane matter)

Herzog et al PRD (2007), Hartnoll et al PRD (2007), Donos et al JHEP (2017)...

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# self-dual point $e^2=4\pi p$

special case at  $\nu=p^{-1}$  and at  $e^2=4\pi p$ 

$$\sigma^T = \tilde{\sigma}$$
, etc.

$$\begin{split} \sigma_{xx}^2 + \sigma_{xy}^2 &= \frac{1}{(2\pi p)^2} \\ \frac{\alpha_{xy}}{\alpha_{xx}} &= \tan\left(\frac{\pi}{4} + \frac{\theta_H}{2}\right) \\ \bar{\kappa}_{xy} &= -\kappa_{xy} = p\pi T(\alpha_{xx}^2 + \alpha_{xy}^2) \end{split}$$

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- ▶ may be a Fermi liquid Pasquier, Haldane Nuc Phys B (1998), Read PRB (1998)
- ▶ (possibly be) a theory without CS term Read PRB (1998)
- ▶ may be a Pfaffian state Regnault, Jolicoeur PRL (2003) PRB (2004)
- emergent particle-hole symmetry for 2-component bosons
   Mross, Alicea, Motrunich PRL (2016), Wang, Senthil PRB (2016), Geraedts, Repellin, Wang, Mong, Senthil,
   Regnault PRB (2017)

what about a single boson?

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#### fermionization

take a Wilson-Fisher boson  $\phi$ 

$$\begin{split} I_m[\phi,A] \leftrightarrow I_m[\psi,a] - \frac{1}{2}I_{CS}[a] - I_{BF}[a,A] - I_{CS}[A] \\ \updownarrow & \updownarrow \\ I_m[\tilde{\phi},\tilde{a}] + I_{BF}[\tilde{a},A] \leftrightarrow I_m[\chi,c] + \frac{1}{2}I_{CS}[c] - I_{BF}[c,A] + I_{CS}[A] \end{split}$$

- ► T symmetry? quantum symmetry for fermion Seiberg, Senthil, Wang, Witten (2016)
- ▶ particle-vortex duality ↔ field redefinition Mross, Alicea, Motrunich PRX (2017), Chen, Son, Wang, Raghu PRL (2018)

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# duality mapping

vary actions w.r.t. A, a,  $\tilde{a}$  and c

$$\begin{split} J^{\mu}_{\phi} &= -\frac{1}{2\pi} \epsilon^{\mu\nu\lambda} \partial_{\nu} a_{\lambda} - \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} \partial_{\nu} A_{\lambda} \\ &= \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} \partial_{\nu} \tilde{a}_{\lambda} = \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} \partial_{\nu} A_{\lambda} - \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} \partial_{\nu} c_{\lambda} \\ J^{\mu}_{\psi} &= \frac{1}{4\pi} \epsilon^{\mu\nu\lambda} \partial_{\nu} a_{\lambda} + \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} \partial_{\nu} A_{\lambda} \\ J^{\mu}_{\tilde{\phi}} &= -\frac{1}{2\pi} \epsilon^{\mu\nu\lambda} \partial_{\nu} A_{\lambda} \\ J^{\mu}_{\chi} &= -\frac{1}{4\pi} \epsilon^{\mu\nu\lambda} \partial_{\nu} c_{\lambda} + \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} \partial_{\nu} A_{\lambda} \end{split}$$

 $\star \nu_{\phi} = 1 \Rightarrow \langle b \rangle = 0, \ \rho_{\phi} = -\langle B \rangle/(2\pi)$ : a Fermi sea

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### phenomenology

Jain sequence

general mapping 
$$(\nu_{\phi}-1)(\nu_{\psi}+1/2)=-1$$
 
$$\nu_{\psi}=n+1/2\Rightarrow\nu_{\phi}=\frac{n}{n+1}$$

$$\nu_{\psi} = -(p+1/2) \Rightarrow \nu'_{\phi} = \frac{p+2}{p+1} = 2 - \nu_{\phi}$$

shift: let 
$$\psi$$
 fills  $n$  Landau levels on a sphere

$$\int d^2x \, \rho_{\psi} = \left(n + \frac{1}{2}\right) \int \frac{d^2x}{(2\pi)} b + n(n+1)$$

↓ duality mappings

$$\frac{n+1}{n}N_{\phi}-N_{\Phi_B}=\nu^{-1}N_{\phi}-N_{\Phi_B}=(n+1)=\mathcal{S}$$

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## particle-vortex dual to particle-vortex dual

▶ T operation of  $\psi$ ?

$$\nu_{\phi}=1 
ightarrow \psi$$
 Fermi sea  $\stackrel{\mathsf{T}}{
ightarrow} \chi$  Fermi sea?

however 
$$u_\phi = 1 
ightarrow 
u_{\tilde{\phi}} = -1 
ightarrow 
u_\chi = 0$$

Fermi sea  $\leftrightarrow$  charge neutral point in  $0^{\text{th}}$  Landau level

► T: if a and A transform differently;

 $\mathsf{CT}$ : if a and A transform the same way

• can check  $\nu_{\psi}\nu_{\Upsilon}=-1/4$ : particle-vortex duality

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### transport mappings

$$\epsilon \left(\sigma_b - \frac{1}{2\pi}\epsilon\right)\epsilon^{-1}\left(\sigma_f + \frac{1}{4\pi}\epsilon\right) = \frac{1}{(2\pi)^2}$$

$$\alpha_b = -\frac{\epsilon}{2\pi}\left(\sigma_f + \frac{1}{4\pi}\epsilon\right)^{-1}\alpha_f$$

$$\bar{\kappa}_b = \bar{\kappa}_f - \alpha_f\left(\sigma_f + \frac{1}{4\pi}\epsilon\right)^{-1}\alpha_f T$$

lacktriangle consistency check:  $\psi$  and  $\chi$  are particle-vortex dual

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# self-dual with $I_M[A;e]$

- writing  $\sigma_{ij}^{ij} = \sigma_{\psi} \delta^{ij}$ ,  $\alpha_{ib}^{ij} = \alpha_{\psi} \delta^{ij}$ ,  $\kappa_{ib}^{ij} = \kappa_{\psi} \delta^{ij}$ , ...
- identities

$$\det[\sigma_\phi]=rac{1}{(2\pi)^2},rac{lpha_\phi^{xy}}{lpha_\phi^{xx}}=\tan(rac{\pi}{4}+rac{ heta_H}{2})$$
, etc. are satisfied

▶ how do  $\sigma_{\psi}, \alpha_{\psi}, ...$  know about  $e^2$ ?

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### restoring T symmetry

- ► RPA versus full 1PI diagram
- $ightharpoonup \langle aa \rangle$ s in 1PI invalidate the assumption  $\delta^{ij}$
- direct integration

$$I_m[\psi, a] + \frac{1}{2} \left( \frac{e^2 - \tilde{e}^2}{e^2 + \tilde{e}^2} \right) I_{CS}[a] + I_{PQED}[a; g = \sqrt{e^2 + \tilde{e}^2}]$$

- $e^2 = \tilde{e}^2$  CS level = 0, no off diagonal transport
- $e^2 = \tilde{e}^2 = 4\pi$ ,  $g^2 = 8\pi \Rightarrow$  simultaneous self-dual See also Mross. Alicea. Motrunich PRX (2017)

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#### EM dual visited

performing the same EM dual operation,

$$\begin{pmatrix} \mathbf{e}_\chi \\ \mathbf{b}_\chi \end{pmatrix} = \frac{1}{e^2 + \tilde{e}^2} \begin{pmatrix} \tilde{e}^2 - e^2 & 8\pi \\ -8\pi & \tilde{e}^2 - e^2 \end{pmatrix} \begin{pmatrix} \mathbf{e}_\psi \\ \mathbf{b}_\psi \end{pmatrix}$$

right at  $e^2 = \tilde{e}^2$ 

$$\mathbf{e}_{\chi} = \mathbf{b}_{\psi}, \mathbf{b}_{\chi} = -\mathbf{e}_{\psi}$$

more generally

$$\begin{split} I_m[\phi,A] + I_M[A;e] &\leftrightarrow I_m[\psi,a] + I_M[a;g] + I_{\theta}[a;\Delta\theta] \\ \updownarrow & \updownarrow \\ I_m[\tilde{\phi},\tilde{A}] + I_M[\tilde{A},\tilde{e}] &\leftrightarrow I_m[\chi,a] + I_M][a;g] + I_{\theta}[a;-\Delta\theta] \end{split}$$

$$g^{2} = e^{2} + \tilde{e}^{2}$$
$$\Delta \theta = \pi \frac{e^{2} - \tilde{e}^{2}}{e^{2} + \tilde{e}^{2}}$$

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# action of modular group i

going back to Seiberg, Senthil, Wang, Witten Ann Phys (2016)

self-dual point?

boson 
$$\rightarrow$$
 boson:  $\tilde{\tau}_b = -1/\tau_b = S[\tau_b]$ 

$$\text{fermion} \rightarrow \text{fermion: } \tilde{\tau}_f = (\tau_f - 1)/(2\tau_f - 1) = \text{ST}^{-2}\text{ST}^{-1}[\tau_f]$$

▶ different values of self-dual point?

$$\tau_b = i \rightarrow e_b^2 = 2\pi$$
 
$$\tau_f = \frac{1}{2} + \frac{i}{2} \rightarrow e_f^2 = 4\pi$$

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# action of modular group ii

► simultaneous self-dual?

$$\tau_f = \frac{1}{1 - \tau_b}, \ \tau_b = i \leftrightarrow \tau_f = \frac{1}{2} + \frac{i}{2}$$

00

comparing to Mross, Alicea, Motrunich PRX (2017), Goldman, Fradkin 1801.04936

$$ightharpoonup z_b = au_b, \ z_f = rac{1}{1-2 au_f} \ (\mathsf{f} o \mathsf{dual} \ \mathsf{f})$$

$$z_f = \frac{z_b - 1}{z_b + 1}, \ z_b = i \leftrightarrow z_f = i$$

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- ightharpoonup particle-vortex dual + bulk EM field  $\rightarrow$  strong-weak and self-dual
- deep relation with electromagnetic duality
- constraints on transport at self-dual point
- \* a description for bosonic Hall states (Jain sequence, shifts, etc.)
- \* fermions are particle-vortex dual as well
- $\star \nu = 1 \leftrightarrow$  a Fermi sea, transport properties understood
- \* simultaneous self-dual points for both bosons and fermions

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- ightharpoonup is such model realizable?  $v_F \to c$
- ▶ is there more response function we can infer from self-duality
- $\star$  how would axion QED manifest on  $\phi$  side
- $\star$  is there more response function we can infer from self-duality
- \* what may give us via bosonizing the Fermi surface
- \* paring instability
- \* spontaneous mass generation in mixed dimension QED, etc...

Thanks for your attention and I appreciate any comment

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