Symmetric Fermion Mass Generation in Lattice Field Theory

Shailesh Chandrasekharan

Duke University

Work done in collaboration with Venkitesh Ayyar
(PRD 91, 065035; PRD 93, 081701;JHEP 1610, 058; PRD96, 114506)

Aspen Winter Conference on "Dualities in Field Theories," Feb 23, 2016

US Department of Energy,
Office of Science, Nuclear Physics Division

Outline

- Traditional origin of fermion masses
- Symmetric Mass Generation
- Our Lattice Model
- Our Results in 4d,3d and 2d.
- Conclusions

Traditional Origin of fermion masses

Higgs-Yukawa models

$$S = \int d^d x \left\{ |\partial_\mu \phi(x)|^2 + m^2 |\phi(x)|^2 + \lambda |\phi(x)|^4 \right.$$

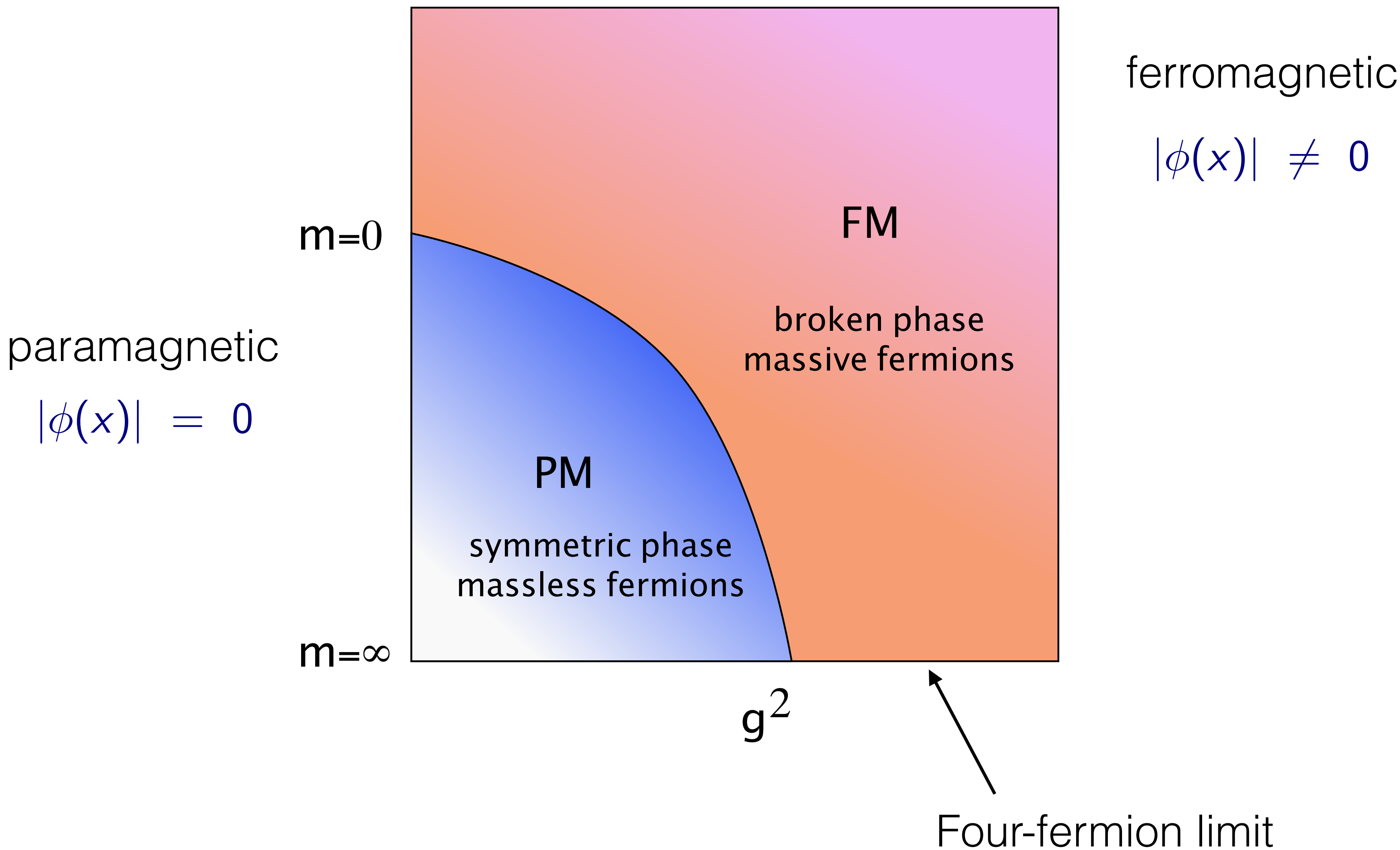
$$\left. + \overline{\psi}(x)\gamma_\mu \partial_\mu \psi(x) + g\ \phi(x)\overline{\psi}(x)\psi(x) \right\}$$

Four-fermion models

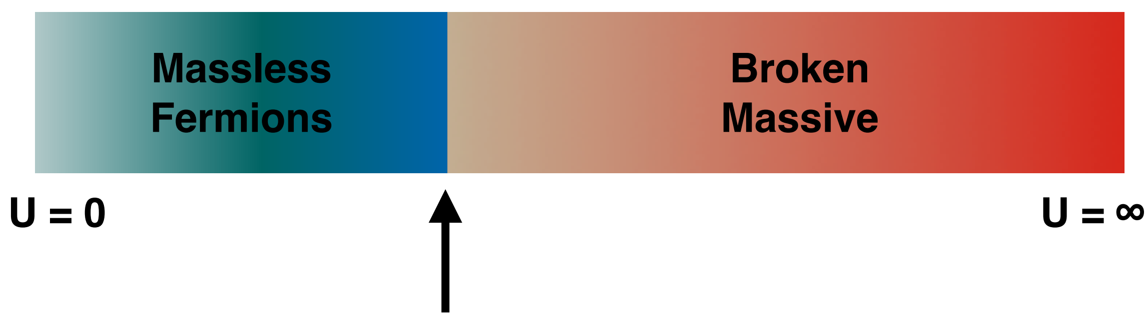
$$S = \int d^d x \left\{ \overline{\psi}(x)\gamma_\mu \partial_\mu \psi(x) - U \left(\overline{\psi}(x)\psi(x) \right)^2 \right\}$$

The two models are related through RG

Traditional Phase diagram of Higgs Yukawa models



Expected phase diagram of a lattice four-fermion model



U_c
(quantum critical point)

Can fermions become massive
without spontaneous symmetry breaking?

Symmetric Mass Generation

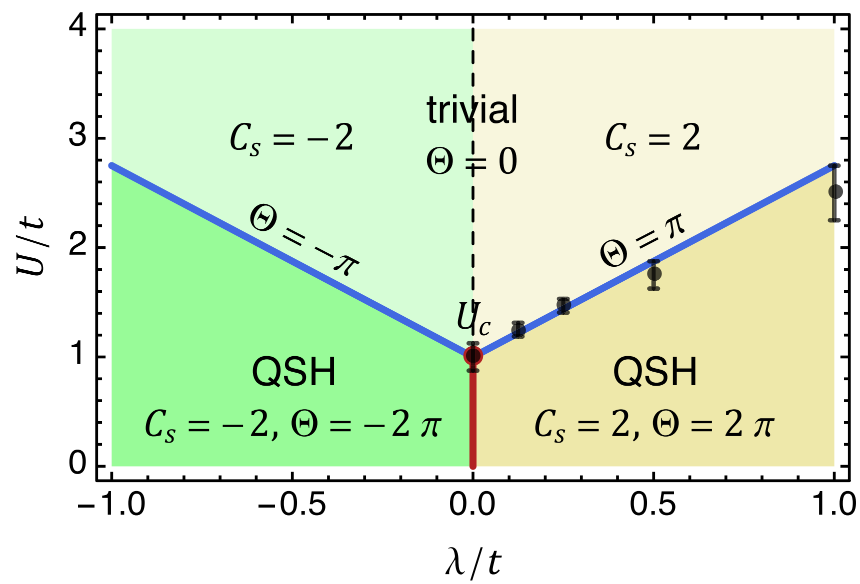
PHYSICAL REVIEW B **91**, 115121 (2015)

Exotic quantum phase transitions of strongly interacting topological insulators

Kevin Slagle, Yi-Zhuang You, and Cenke Xu

There can be a continuous phase transition between a massless fermion phase and a massive phase in 2+1d with 4 "Dirac" fermions, without spontaneous symmetry breaking!

"Dirac" = 4 components



model on honeycomb lattice with two species of spin-1/2 fermions

Motivation for such transitions in 4d

CHIRAL GAUGE THEORIES ON THE LATTICE*

Estia EICHTEN

*Fermilab,** P.O. Box 500, IL 60510, USA*

John PRESKILL[1]

California Institute of Technology, Pasadena, CA 91125, USA

Received 5 November 1985

· · · · · · Briefly, the basic idea of our approach is that the unwanted "mirror fermions" can acquire large masses consistent with gauge invariance by pairing up with *composite* fermion states with appropriate gauge quantum numbers. These composite fermions may be bound, not by the gauge interaction itself, but by an auxiliary interaction introduced for this explicit purpose. Thus, the mirror fermions can be forced to decouple as the continuum limit is approached.

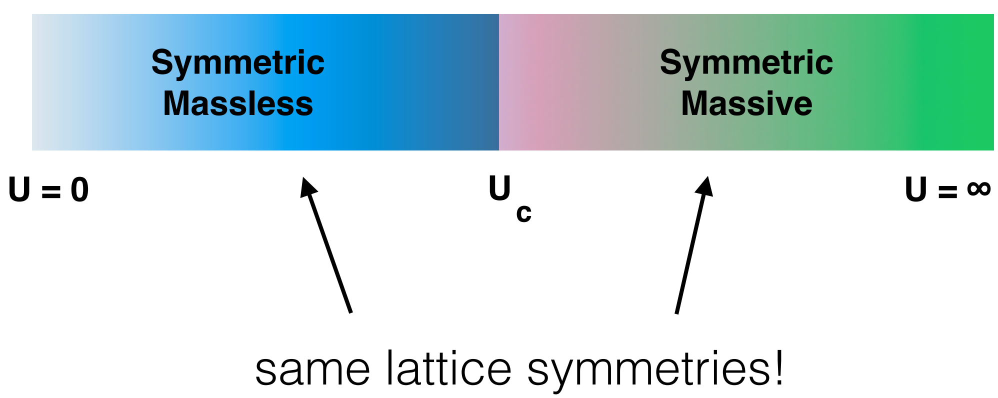
Absence of chiral fermions in the Eichten–Preskill model

Maarten F.L. Golterman [1], Donald N. Petcher [2] and Elena Rivas [3]
Department of Physics, Washington University, St. Louis, MO 63130-4899, USA

Received 10 June 1992

6. Conclusion

When Eichten and Preskill originally presented their model, a clear element of the scenario they envisaged for it to successfully produce a continuum theory of (asymptotically free) chiral fermions entailed the existence of a phase transition for which the fermion mass was an order parameter, and over which no symmetry breaking occurred.



"Search for the exotic transition!"

We have analyzed the model in several regions of the phase diagram, and all indications are that no such phase transition exists. Indeed, we do find phases with massive and massless fermions, but always a broken phase appears in between. In the symmetric phase with massive fermions (a paramagnetic phase in strong Yukawa coupling, or PMS phase), bound states are formed which pair up with the original chiral fields to form Dirac representations, all of which are massive (although one massless Dirac fermion can be arranged by tuning). The fermions remain massive across the symmetry breaking phase boundary to the broken phase (ferromagnetic or FM phase), and finally across the phase boundary to the second symmetric phase (paramagnetic phase at weak Yukawa coupling, or PMW phase), all fermions become massless, including the doublers. The crucial ingredient for the failure of the emergence of a chiral theory of fermions as originally imagined is the existence of the broken phase separating the two symmetric phases. Through

Unaware of the predictions by Slagle, Yu and Xu, and roughly around, the same time, we were exploring an interesting and puzzling phase transition in a lattice field theory that was studied earlier, but now using new and better fermion algorithms.

We later realized that this is the same phase transition that Slagle, Yu and Xu were talking about.

Several papers have been written on the subject of Symmetric Mass Generation (SMG)

He,Wu,You,Xu,Meng,Lu, PRB 94 241111 (2016)

You, He, Vishwanath, Xu PRX 8 011026 (2018)

You, He, Vishwanath, Xu PRB 97 125112 (2018)

This talk is about our results, from the lattice field theory perspective!

Our Lattice Model

Consider the continuum action

$$S = \int d^d x \left\{ \overline{\psi}(x)\ \gamma_\mu \partial_\mu\ \psi(x) \right.$$
$$\left. - U\Big(\psi^1(x)\psi^2(x)\psi^3(x)\psi^4(x)\ +\ \overline{\psi}^4(x)\overline{\psi}^3(x)\overline{\psi}^2(x)\overline{\psi}^1(x)\Big) \right\}$$

Partition Function

$$Z = \int [d\overline{\psi} d\psi]\ \mathrm{e}^{-S} = \int [d\overline{\psi} d\psi]\ \mathrm{e}^{-S_0} \prod_x \Big(1\ +\ U\ \psi_1(x)\psi_2(x)\psi_3(x)\psi_4(x)\Big)$$
$$\Big(1\ +\ U\ \overline{\psi}_1(x)\overline{\psi}_2(x)\overline{\psi}_3(x)\overline{\psi}_4(x)\Big)$$

At large U the theory seems to become massive, but without SSB, but due to a sufficiently large “four-fermion” condensate!

“Naively” the phase diagram of this model cannot be

It must be something like

We need a regularization of the UV physics to make sense out of all this!

Lattice Model:

Let us naively discretize the action on a Euclidean space-time lattice

continuum action

$$S = \int d^d x \left\{ \overline{\psi}(x)\, \gamma_\mu \partial_\mu\, \psi(x) \right.$$

$$\left. - U\Big(\psi^1(x)\psi^2(x)\psi^3(x)\psi^4(x) \; + \; \overline{\psi}^4(x)\overline{\psi}^3(x)\overline{\psi}^2(x)\overline{\psi}^1(x) \Big) \right\}$$

lattice action

$$S = \sum_x \left\{ \overline{\psi}(x)\, \gamma_\mu(\psi(x+\hat{\mu}) - \psi(x-\hat{\mu})) \right.$$

$$\left. - U\Big(\psi^1(x)\psi^2(x)\psi^3(x)\psi^4(x) \; + \; \overline{\psi}^4(x)\overline{\psi}^3(x)\overline{\psi}^2(x)\overline{\psi}^1(x) \Big) \right\}$$

Transform the fields: Spin diagonalization

$$\psi(x) \; = \; (\gamma_1)^{x_1}(\gamma_2)^{x_2}...(\gamma_d)^{x_d} \; \chi_x \qquad \overline{\psi}(x) \; = \; \overline{\chi}_x \; (\gamma_d)^{x_d}...(\gamma_2)^{x_2}(\gamma_1)^{x_1}$$

$$\psi^1(x)\psi^2(x)\psi^3(x)\psi^4(x) = \chi_x^1 \; \chi_x^2 \; \chi_x^3 \; \chi_x^4$$

$$\overline{\psi}^4(x)\overline{\psi}^3(x)\overline{\psi}^2(x)\overline{\psi}^1(x) = \overline{\chi}_x^4 \; \overline{\chi}_x^3 \; \overline{\chi}_x^2 \; \overline{\chi}_x^1$$

One obtains four-copies of “staggered fermions” coupled on each site

$$S \; = \; \sum_x \Big\{ \overline{\chi}_x \; \sum_\mu \eta_{\mu,x}(\chi(x+\hat{\mu}) - \chi(x-\hat{\mu})) \\ - U\Big(\chi_x^1 \; \chi_x^2 \; \chi_x^3 \; \chi_x^4 + \overline{\chi}_x^4 \; \overline{\chi}_x^3 \; \overline{\chi}_x^2 \overline{\chi}_x^1\Big) \; \Big\}$$

$$\eta_{1,x} \; = \; 1, \quad \eta_{\mu,x} = (-1)^{x_1+x_2+..+x_{\mu-1}} \quad \mu \; > \; 1$$

The action still contains two decoupled models

$$S \;=\; \sum_x \Big\{ \overline{\chi}_x \; \sum_\mu \eta_{\mu,x} (\chi(x+\hat{\mu}) - \chi(x-\hat{\mu})) \\ -U\Big(\chi^1_x \; \chi^2_x \; \chi^3_x \; \chi^4_x + \overline{\chi}^4_x \; \overline{\chi}^3_x \; \overline{\chi}^2_x \overline{\chi}^1_x \Big) \;\Big\}$$

$$\overline{\chi}_{x,odd} \longleftrightarrow \chi_{x,even} \qquad \Big| \qquad \overline{\chi}_{x,even} \;\cancel{\longleftrightarrow}\; \chi_{x,odd}$$

no coupling

Keeping only a single decoupled model we finally get our model:

$$S \;=\; \frac{1}{2} \sum_{x,y} \chi^T_x \; M_{x,y} \; \chi_y \;-\; U \sum_x \chi^4_x \; \chi^3_x \; \chi^2_x \; \chi^1_x$$

Our model has the usual “staggered fermion symmetries”:

Golterman and Smit, 1984

shift symmetry: $\chi^a_x \to \xi_{\rho,x}\chi^a_{x+\rho}$,

rotational symmetry: $\chi^a_x \to S_R(R^{-1}\ x)\chi^a_{R^{-1}\ x}$

axis reversal (parity): $x_\rho \to -x_\rho,\ x_\tau = x_\tau,\ \tau \neq \rho$

SU(4) global symmetry:

$$\chi^a_x \to (V)^{ab}\chi^b_x,\ x\ \in\ \text{even}$$

$$\chi^a_x \to (V^*)^{ab}\chi^b_x,\ x\ \in \text{odd}$$

$$V\ \in SU(4)$$

These symmetries forbid bilinear mass terms.

For example: consequence of the SU(4) symmetry $\langle\ \chi^a_x\ \chi^b_x\ \rangle = 0$

conventional wisdom: → massless fermions
→ massive fermions requires SSB

Fact:

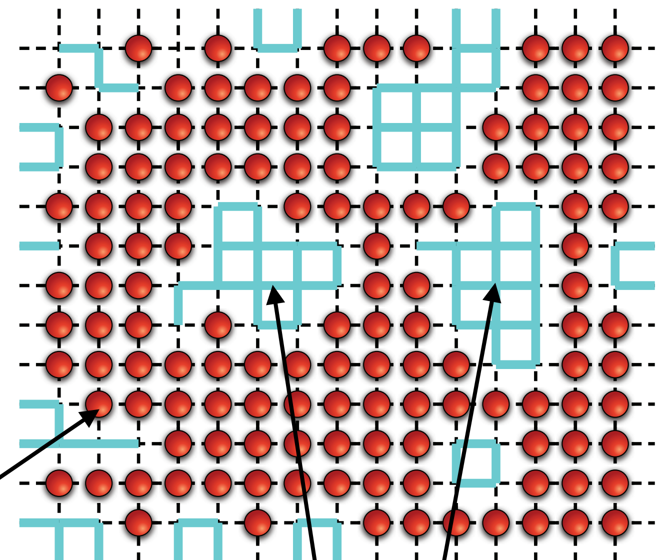
At sufficiently large U there is a fully massive phase without any SSB

Partition function

$$
\begin{aligned}
Z &= \int [d\chi]\ \mathrm{e}^{-\frac{1}{2}\chi^T M \chi + U \sum_x \chi^4_x \chi^3_x \chi^2_x \chi^1_x} \\
&= \int [d\chi]\ \mathrm{e}^{-\frac{1}{2}\chi^T M \chi} \prod_x (1 + U \chi^4_x \chi^3_x \chi^2_x \chi^1_x) \\
&= \sum_{[n]} U^k \int [d\chi]\ \mathrm{e}^{-\frac{1}{2}\chi^T W \chi} \\
&= \sum_{[n]} U^k \left(\mathrm{Pf}(W)\right)^4
\end{aligned}
$$

free femion bag weight

Illustration of configuration [n]



monomers

free fermion bag

k refers to the number of monomers

Using the fermion bag picture, it is easy to argue that all two point correlation functions with non-trivial quantum numbers decay exponentially at large U!

Examples:

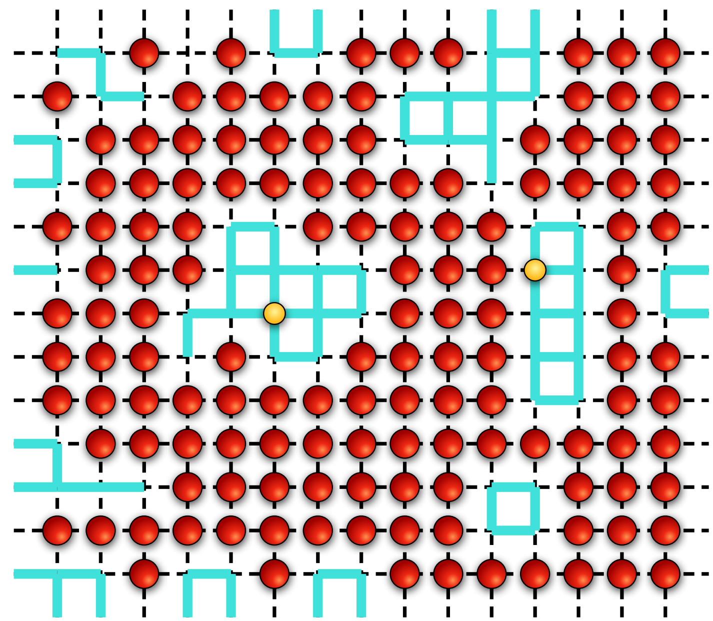
Fermion correlation function

$$F^a(0,x) = \left\langle \chi_0^a \;\; \chi_x^a \right\rangle$$

Fermion bilinear correlation functions

$$C_1(0,x) = \left\langle \chi_0^1 \chi_0^2 \;\; \chi_x^1 \chi_x^2 \right\rangle$$

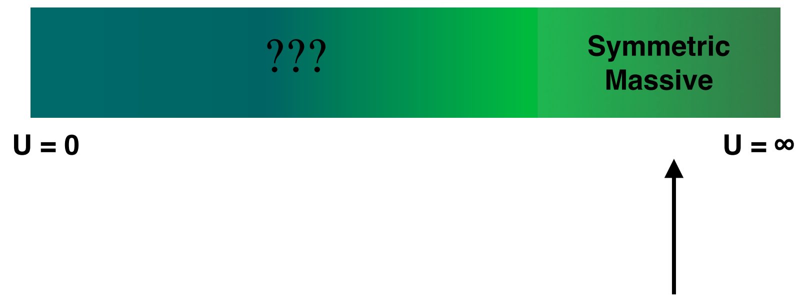
$$C_2(0,x) = \left\langle \chi_0^1 \chi_0^2 \;\; \chi_x^3 \chi_x^4 \right\rangle$$



Note that

$$\int_{\text{bag}} [d\chi] \;\; \chi_x^a \;\; = \;\; 0 \qquad\qquad \int_{\text{bag}} [d\chi] \;\; \chi_x^a \; \chi_x^b \;\; = \;\; 0$$

Hence the phase diagram of the lattice model should be



symmetric mass generation (SMG)

But from a quantum field theory perspective, is SMG simply a lattice artifact, or is there a continuum limit?

We can explore this question in various dimensions.

Note that our lattice model is well defined in all dimensions!

$$S = \frac{1}{2}\sum_{x,y} \chi_x^T \, M_{x,y} \, \chi_y - U \sum_{x} \chi_x^4 \, \chi_x^3 \, \chi_x^2 \, \chi_x^1$$

At U = 0

In d=2, the theory describes Nf=2 “Dirac” fermions

In d=3, the theory describes Nf=4 “Dirac” fermions

related to the model proposed by Slagle, You, Xu

In d=4, the theory describes Nf=8 “Dirac” fermions

In d=3, 4 the question of the continuum limit of the SMG phase was studied long ago! The conclusion was that it does not exist.

As far as we know the lattice theory was never studied in d=2!

Connections to Past Work

STUDY OF DIFFERENT LATTICE FORMULATIONS OF A YUKAWA MODEL WITH A REAL SCALAR FIELD

I-Hsiu LEE[1], Junko SHIGEMITSU[2] and Robert E. SHROCK[3]

[1] *Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*
[2] *Physics Department, Ohio State University, Columbus, OH 43210, USA*
[3] *Institute for Theoretical Physics, State University of New York, Stony Brook, NY 11794, USA*

Received 1 September 1989

Action

$$S = S_{\mathrm{B}} + S_{\mathrm{F}} + S_{\mathrm{Y}},$$

$$S_{\mathrm{B}} = \sum_n \phi_n^2 - 2\kappa \sum_{n,\mu} \phi_n \phi_{n+e_\mu} + \lambda \sum_n \left(\phi_n^2 - 1\right)^2,$$

$$S_{\mathrm{F}} = \tfrac{1}{2} \sum_{n,\mu,f} \bar{\chi}_{n,f} \eta_{n,\mu} \left(\chi_{n+e_\mu,f} - \chi_{n-e_\mu,f}\right) + \sum_{n,f} m_f \bar{\chi}_{n,f} \chi_{n,f},$$

Yukawa couplings

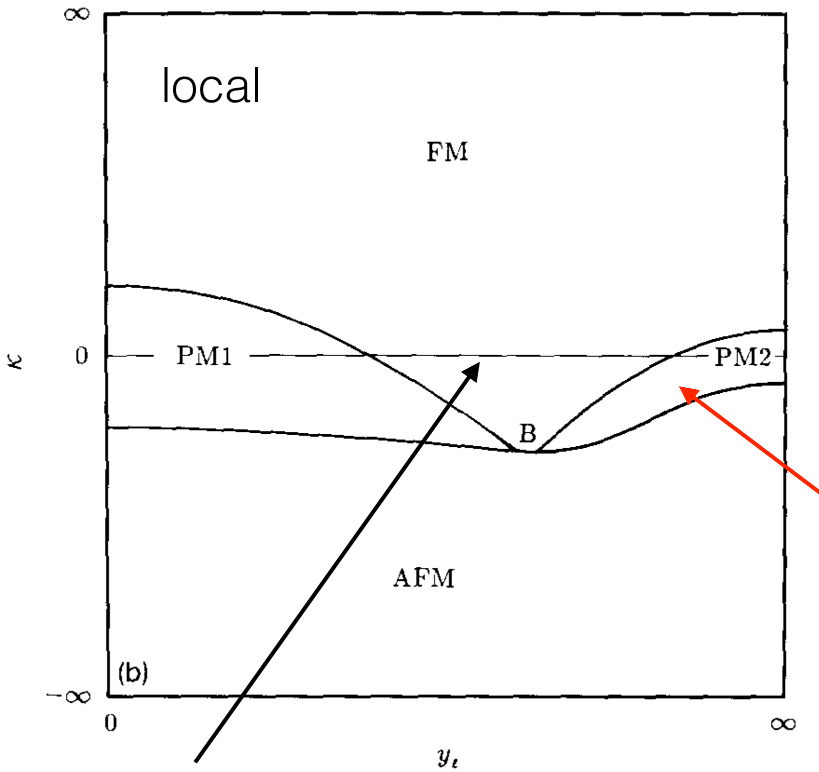
$$S_{\mathrm{Y}} = S_{\mathrm{Y},\ell} = y_\ell \sum_n \phi_n \sum_f \bar{\chi}_{n,f} \chi_{n,f},$$

local

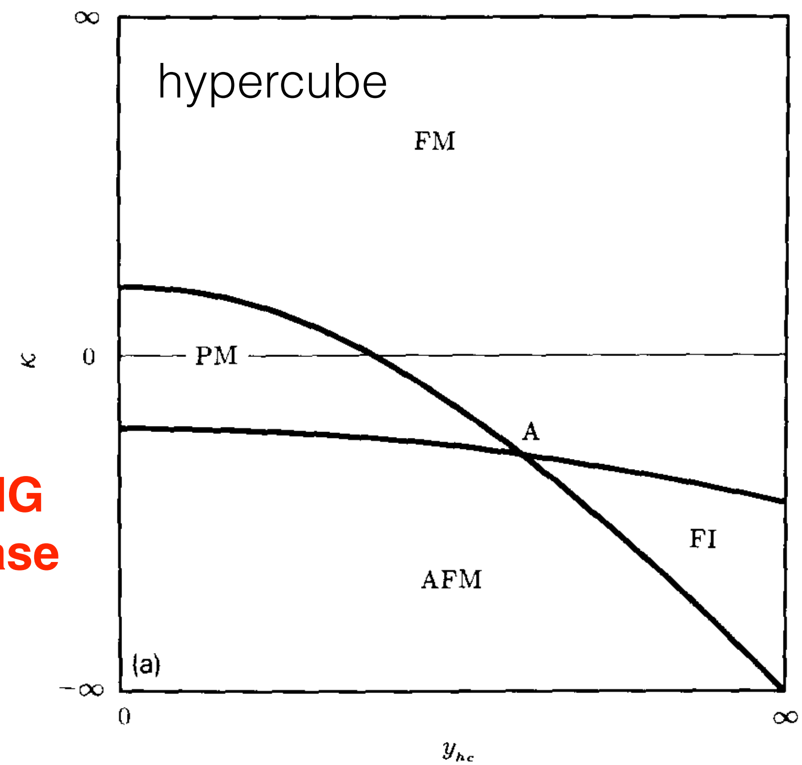
$$S_{\mathrm{Y}} = S_{\mathrm{Y,hc}} = 2^{-d} y_{\mathrm{hc}} \sum_n \phi_n \sum_{n' \in \mathrm{hc}(n);\, f} \bar{\chi}_{n',f} \chi_{n',f},$$

hyper cubic

Mean field phase diagram



SMG phase



note the large broken FM phase

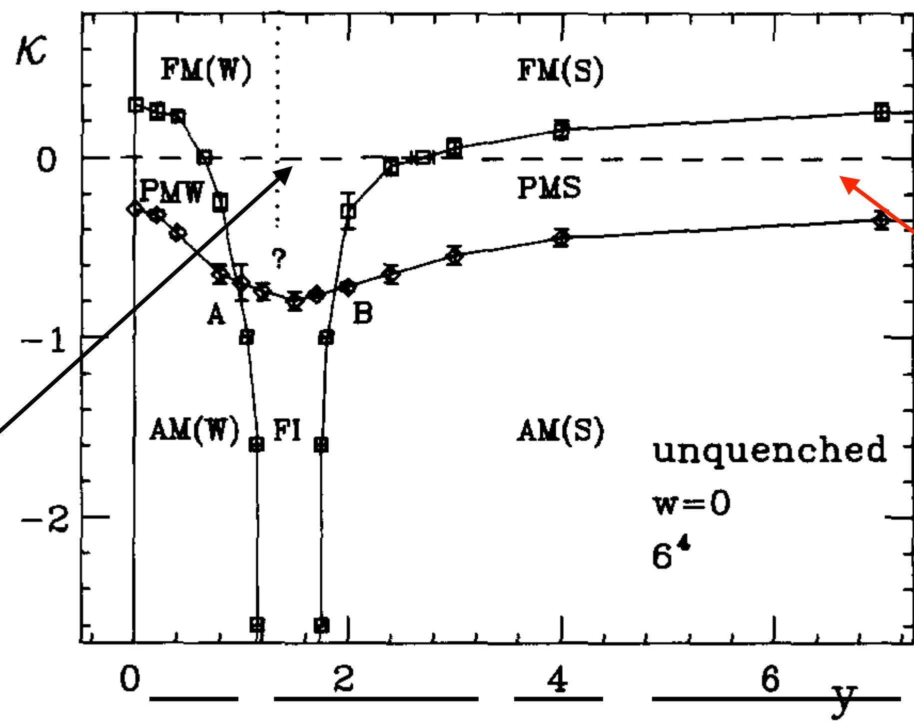
$\kappa = 0$ (four-fermion) limit

PHASE DIAGRAM OF A LATTICE SU(2) ⊗ SU(2) SCALAR-FERMION MODEL WITH NAIVE AND WILSON FERMIONS*

Wolfgang BOCK[1,2], Asit K. DE[1,2], Karl JANSEN[2], Jiří JERSÁK[1,2], Thomas NEUHAUS[3] and Jan SMIT[4]

[1] *Institut für Theoretische Physik E, RWTH Aachen, D-5100 Aachen, FRG*
[2] *HLRZ c/o KFA Jülich, P.O. Box 1913, D-5170 Jülich, FRG*
[3] *Fakultät für Physik, Universität Bielefeld, D-4800 Bielefeld, FRG*
[4] *Institute of Theoretical Physics, Valckenierstraat 65, NL-1018 XE Amsterdam, The Netherlands*

Received 26 March 1990

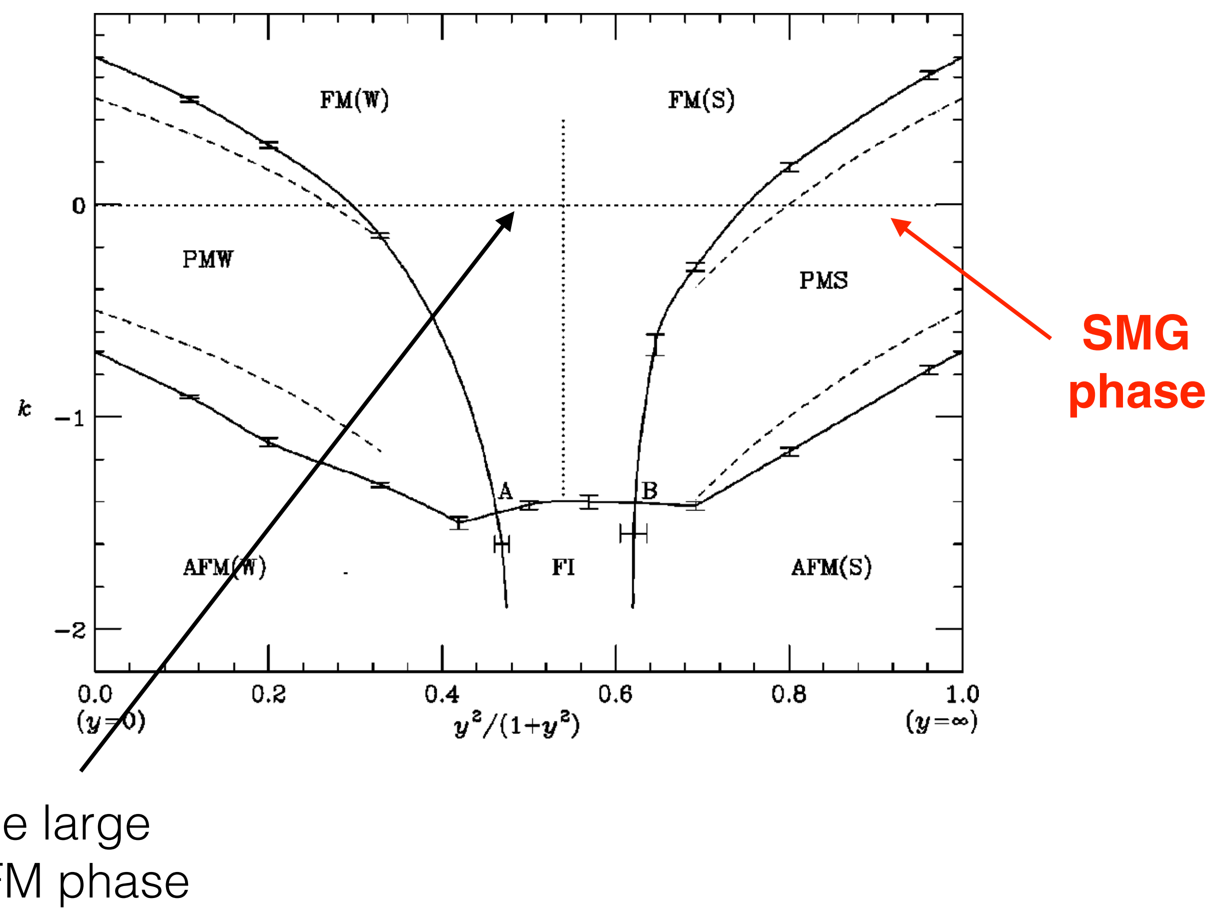


SMG phase

note the large broken FM phase

Phase diagram and quasiparticles of a lattice SU(2) scalar-fermion model in 2+1 dimensions

J. L. Alonso[1,*] Ph. Boucaud,[2,†] V. Martín-Mayor,[3,‡] and A. J. van der Sijs[4,§]
[1]*Departamento de Física Teórica, Universidad de Zaragoza, 50009 Zaragoza, Spain*
[2]*LPTHE, Université de Paris XI, 91405 Orsay Cedex, France*
[3]*Dipartimento di Fisica and Infn, Università di Roma La Sapienza, P. A. Moro 2, 00185 Roma, Italy*
[4]*Swiss Center for Scientific Computing, ETH-Zürich, ETH-Zentrum, CH-8092 Zürich, Switzerland*
(Received 21 July 1999; published 23 December 1999)



The presence of the broken phase between the usual massless fermion phase and the SMG phase made Golterman and Schmidt decide that the Eichten and Peskin Mechanism will not work!

But we have much better fermion algorithms today.
So we wanted to explore these phase transitions again!

Results in 4d

V. Ayyar and SC, JHEP 1610, 058 (2016)

Catterall and Schaich, PRD96, 034506 (2017)

Observables:

Monomer Density

$$\rho_m = U \frac{1}{L^d} \sum_x \left\langle \chi_x^4 \chi_x^3 \chi_x^2 \chi_x^1 \right\rangle$$

Correlation Function

$$C_1(0,x) = \left\langle \chi_0^1 \chi_0^2 \;\; \chi_x^1 \chi_x^2 \right\rangle$$

$$C_2(0,x) = \left\langle \chi_0^1 \chi_0^2 \;\; \chi_x^3 \chi_x^4 \right\rangle$$

Correlation Ratios

$$R_1 = C_1(0, L/2-1)/C_1(0,1)$$

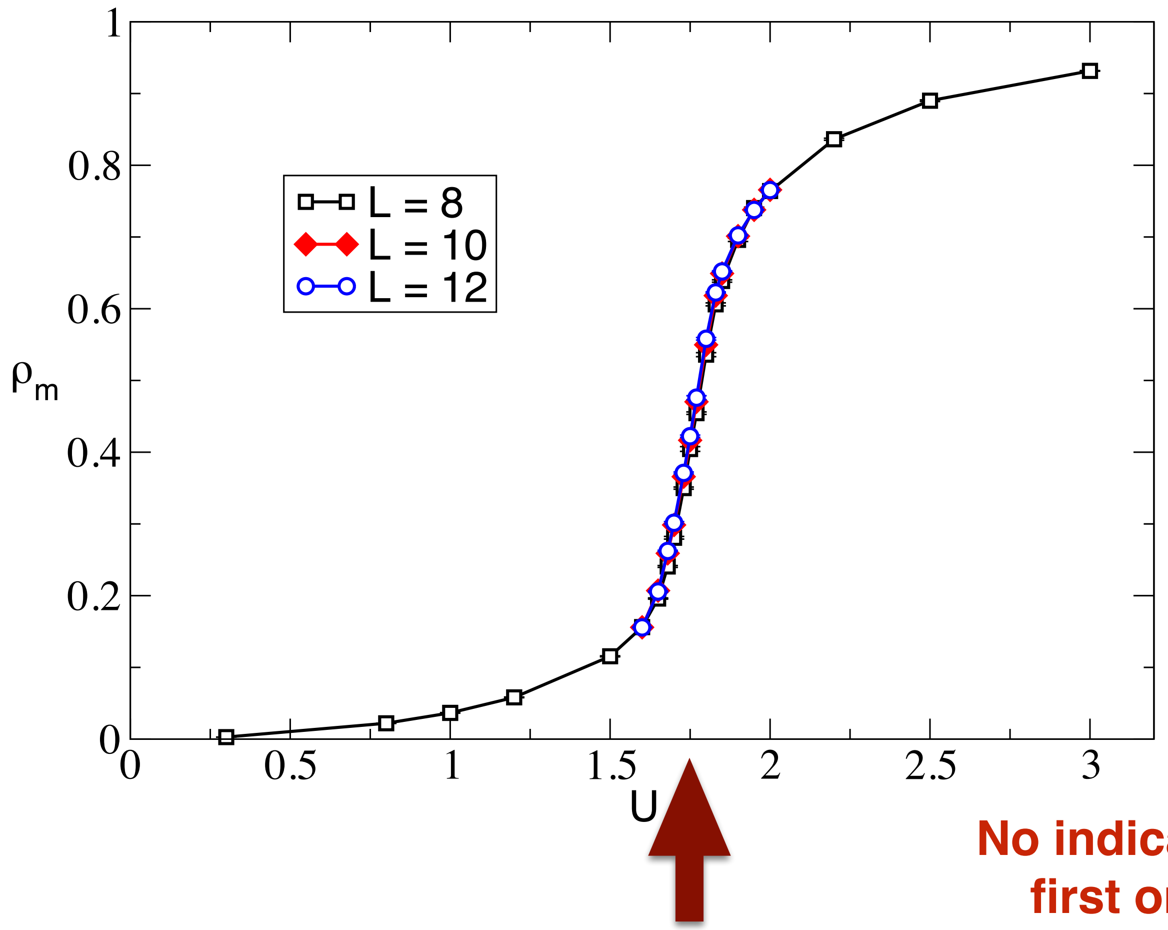
$$R_2 = C_2(0, L/2)/C_2(0,0)$$

Susceptibilities

$$\chi_a = \sum_x C_a(0,x)$$

$\chi_a \sim L^d$ (broken phase)

Monomer Density



No indication of a strong first order transition

Critical point ≈ 1.75?

Susceptibility

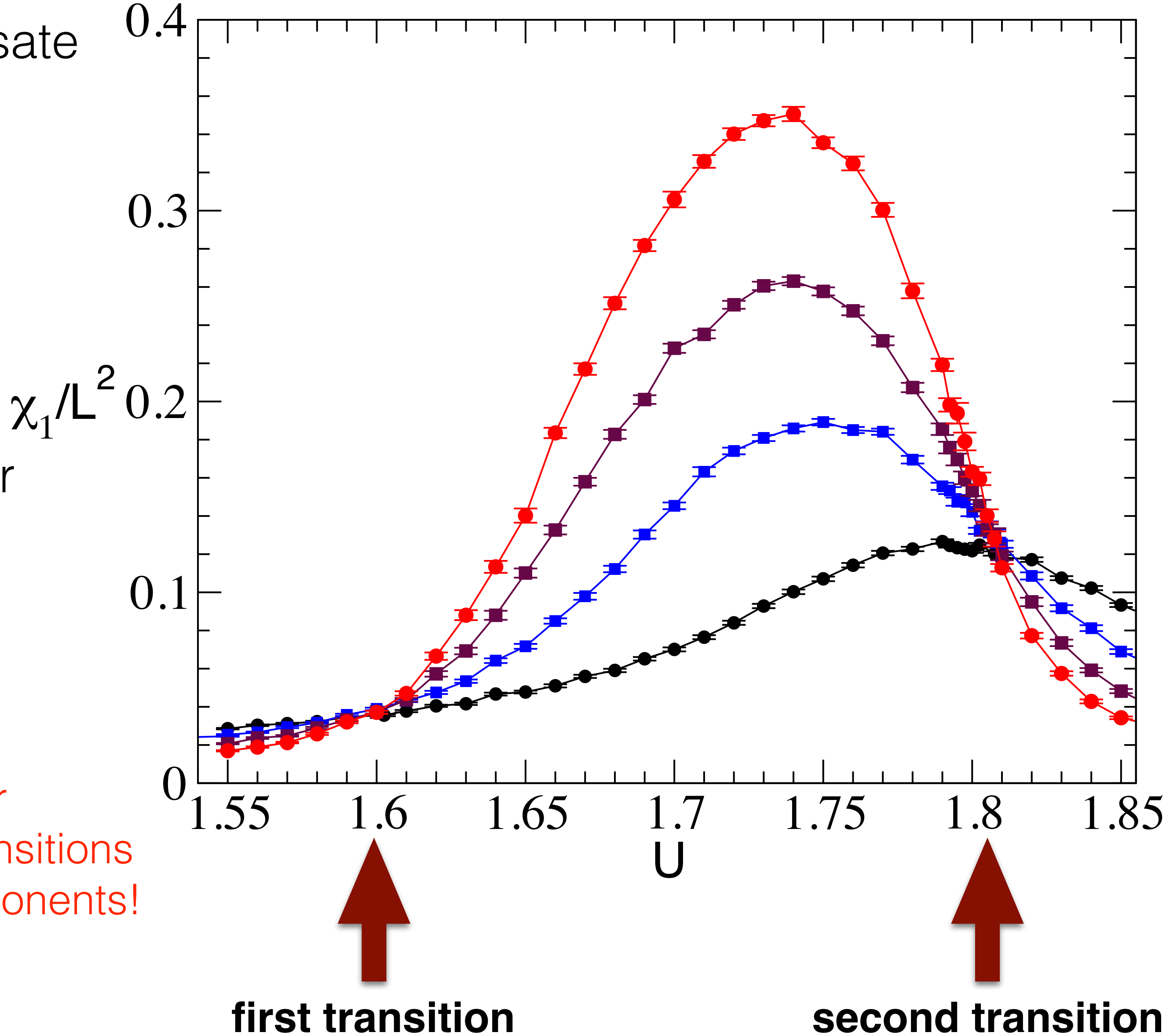
In the presence of a non-zero condensate we expect

$$\chi_1 \sim L^4$$

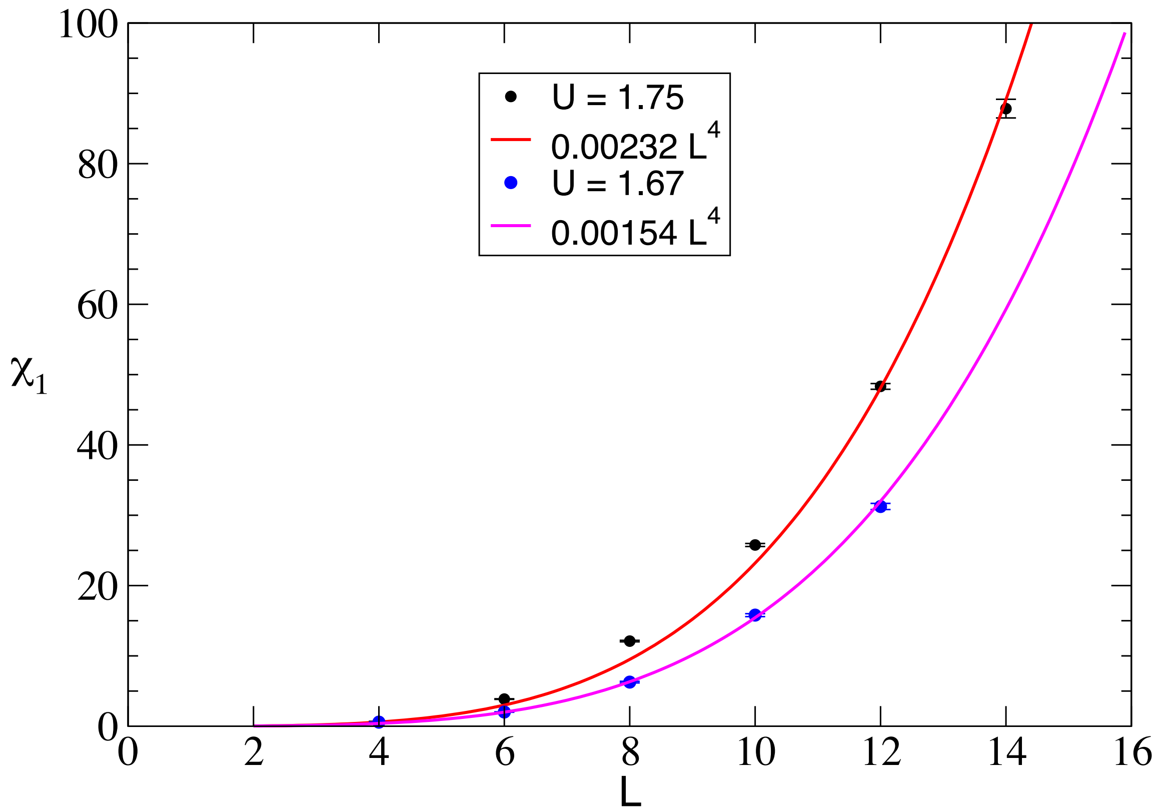
At a second order MF transition

$$\chi_1 \sim L^2$$

Evidence for two continuous transitions with mean field exponents!

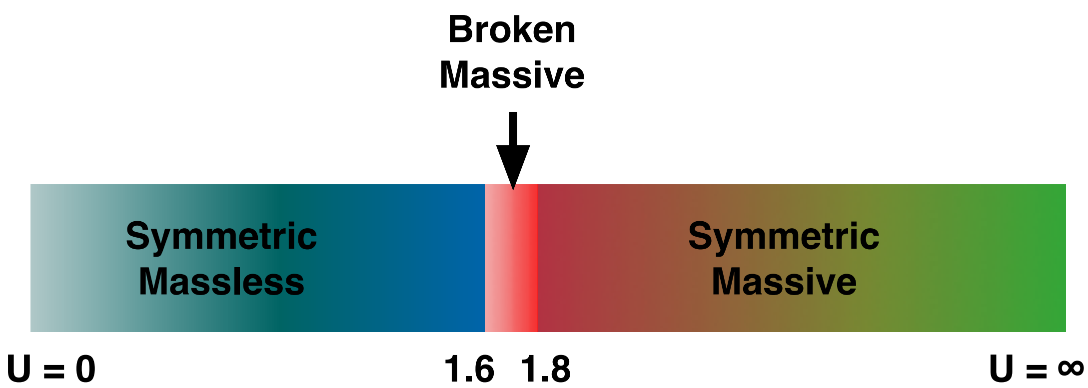


Susceptibility seems to grow as $\chi_1 \sim L^4$ in the intermediate region.



Evidence for an intermediate phase!

Our Phase Diagram in 4d



The broken phase is much narrower than what was observed earlier, but it does seem to exist!

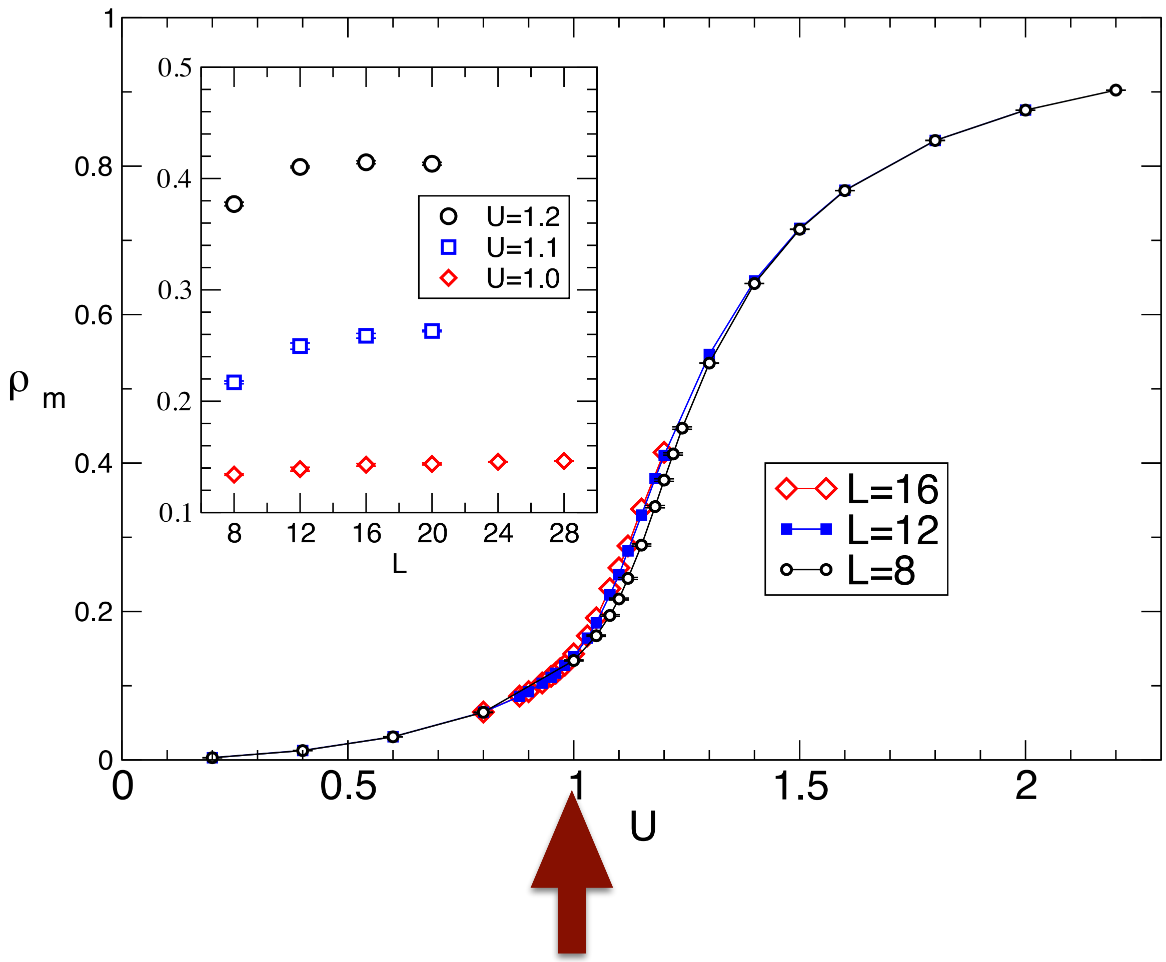
Results in 3D

V. Ayyar and SC, PRD 93, 081701 (2016)

V. Ayyar and SC, PRD 91, 065035 (2015)

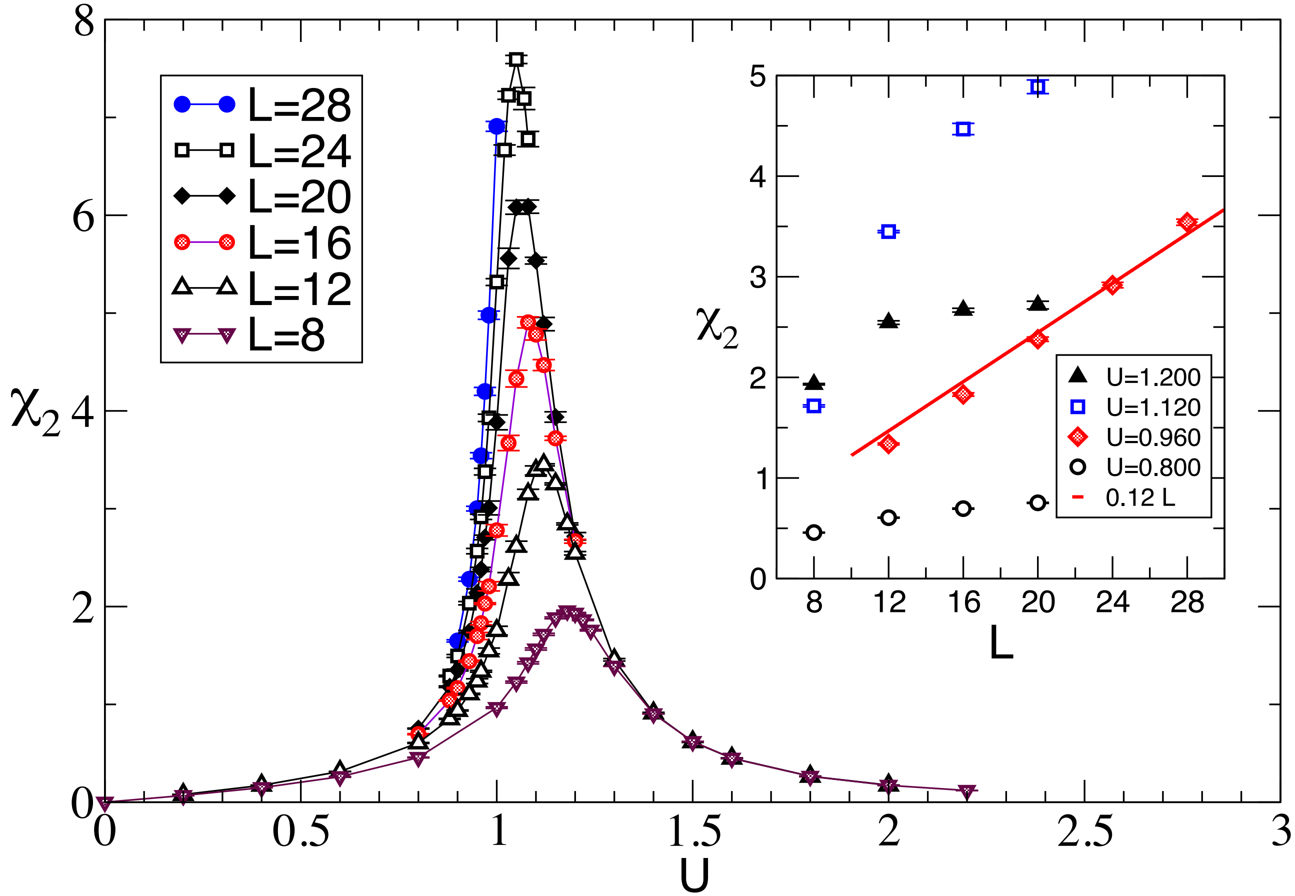
Catterall, JHEP 1601, 121 (2016)

Monomer density



critical point $\approx 1?$

Susceptibility



In the presence of a non-zero fermion bilinear condensate we expect $\chi_2 \sim L^3$.

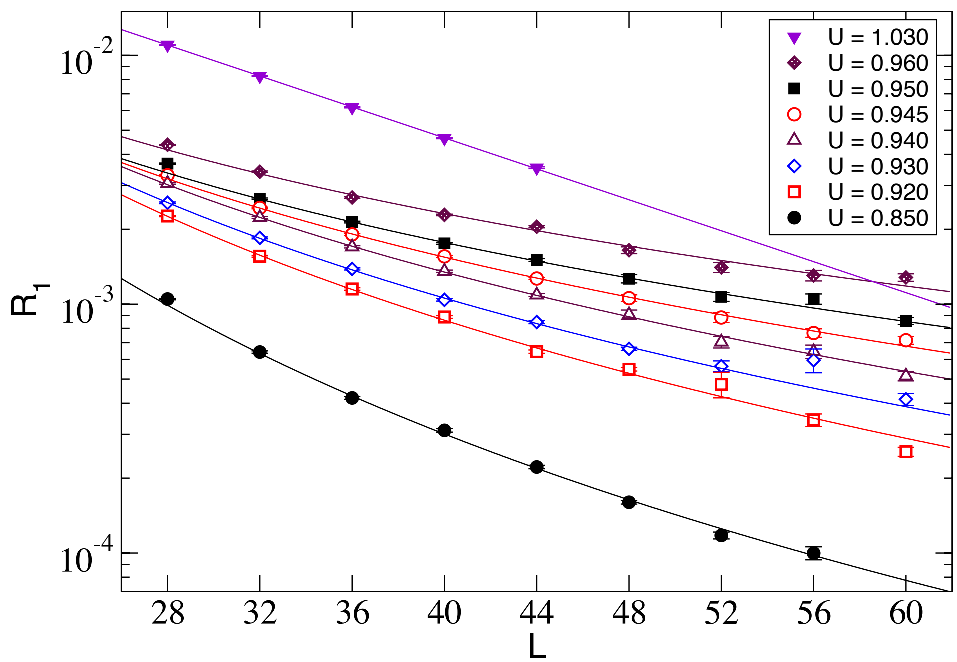
Correlation Ratios

$$R_1 = C_1(0, L/2-1)/C_1(0,1)$$

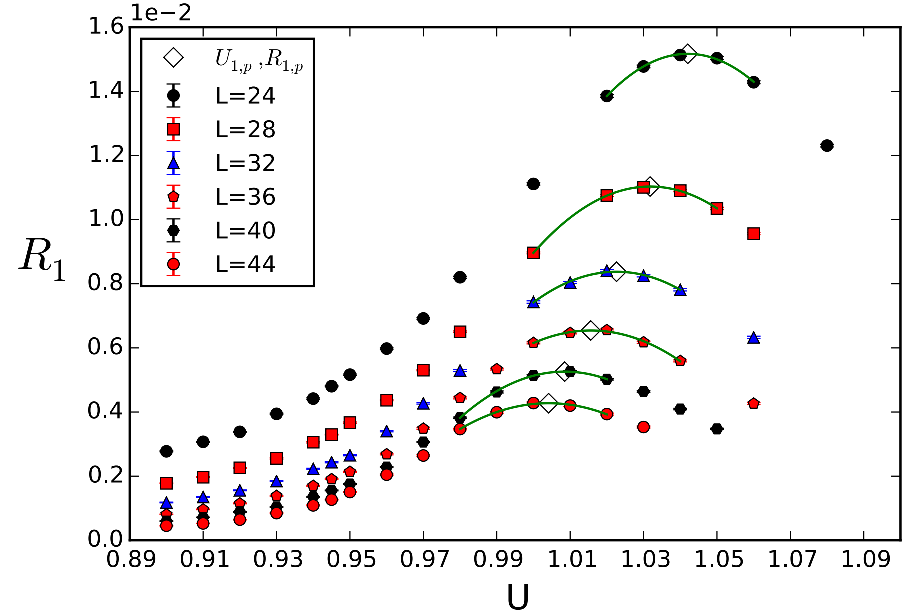
$$R_2 = C_2(0, L/2)/C_2(0,0)$$

$$R_1 \sim \begin{cases} 1/L^4 & U < U_c \\ 1/L^{1+\eta} & U = U_c \\ exp(-mL) & U > U_c \end{cases}$$

U	η	χ^2 /DOF	U	η	χ^2 /DOF
0.000	3	–	0.850	2.34(4)	2.5
0.920	1.64(5)	4.6	0.930	1.44(3)	1.9
0.940	1.22(2)	1.0	0.945	1.00(2)	0.7
0.950	0.77(2)	1.1	0.960	0.63(5)	6.4

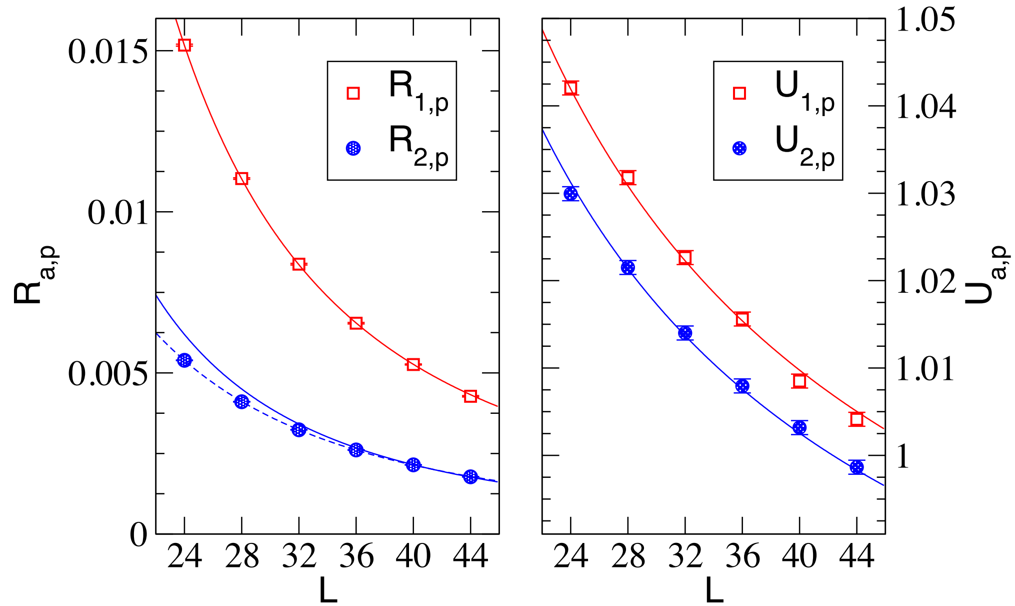


Scaling of the Peaks of Correlation Ratios



$$R_{a,\mathrm{peak}} = b_a / L^{1+\eta}$$

$$U_{a,\mathrm{peak}} = U_c + d_a L^{1/\nu}$$

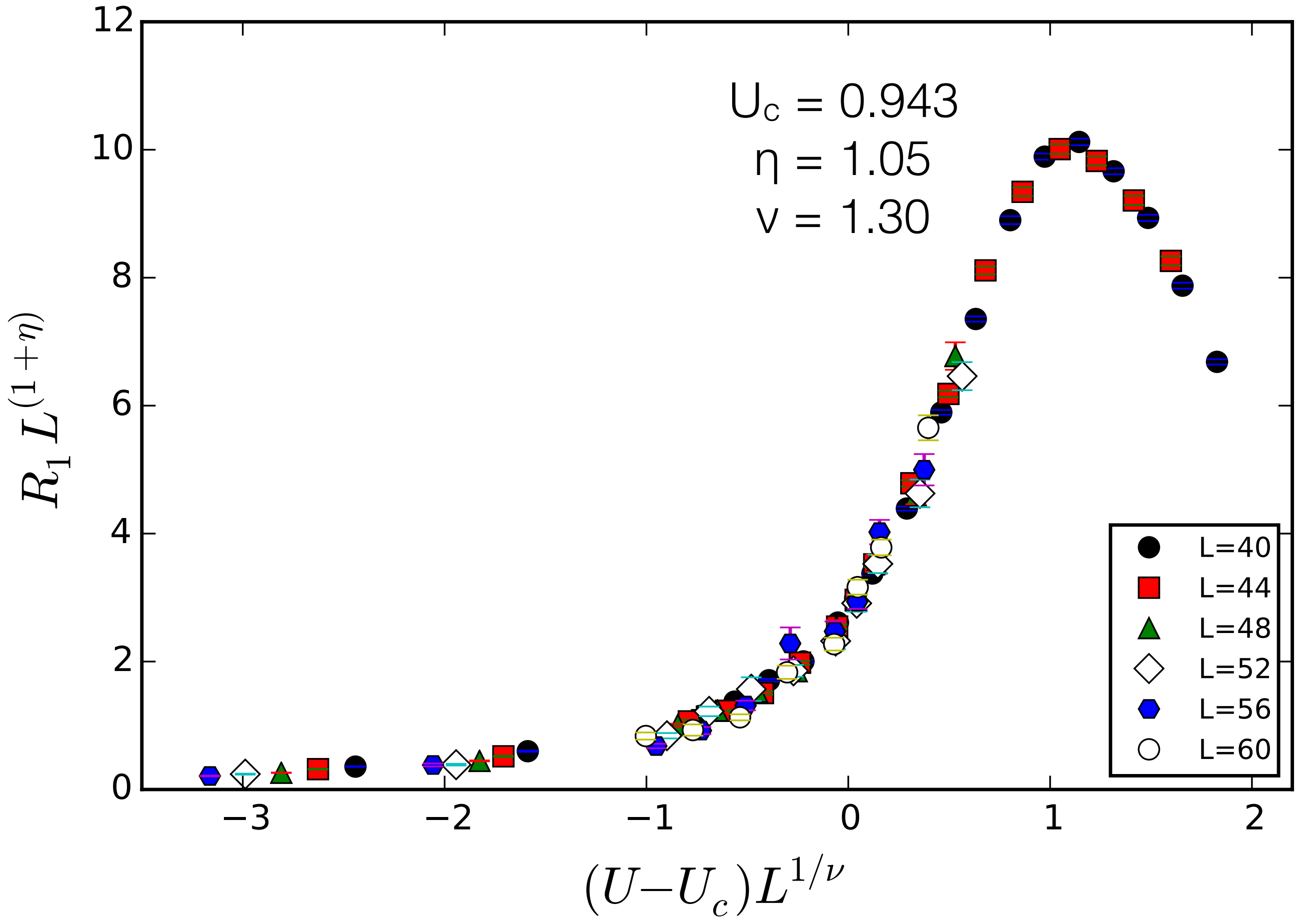


$$U_c = 0.943(2)$$

$$\eta = 1.05(5)$$

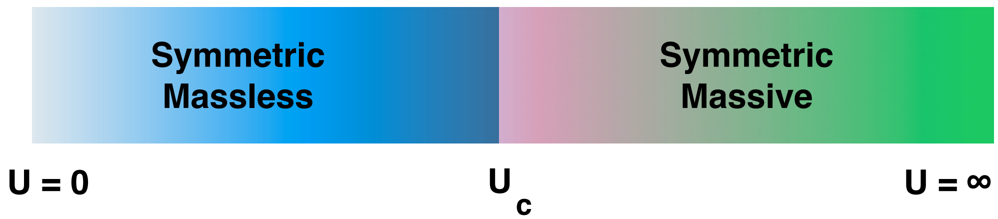
$$\nu = 1.30(7)$$

Critical Scaling



Strong evidence for the exotic transition!

Phase Diagram in 3d



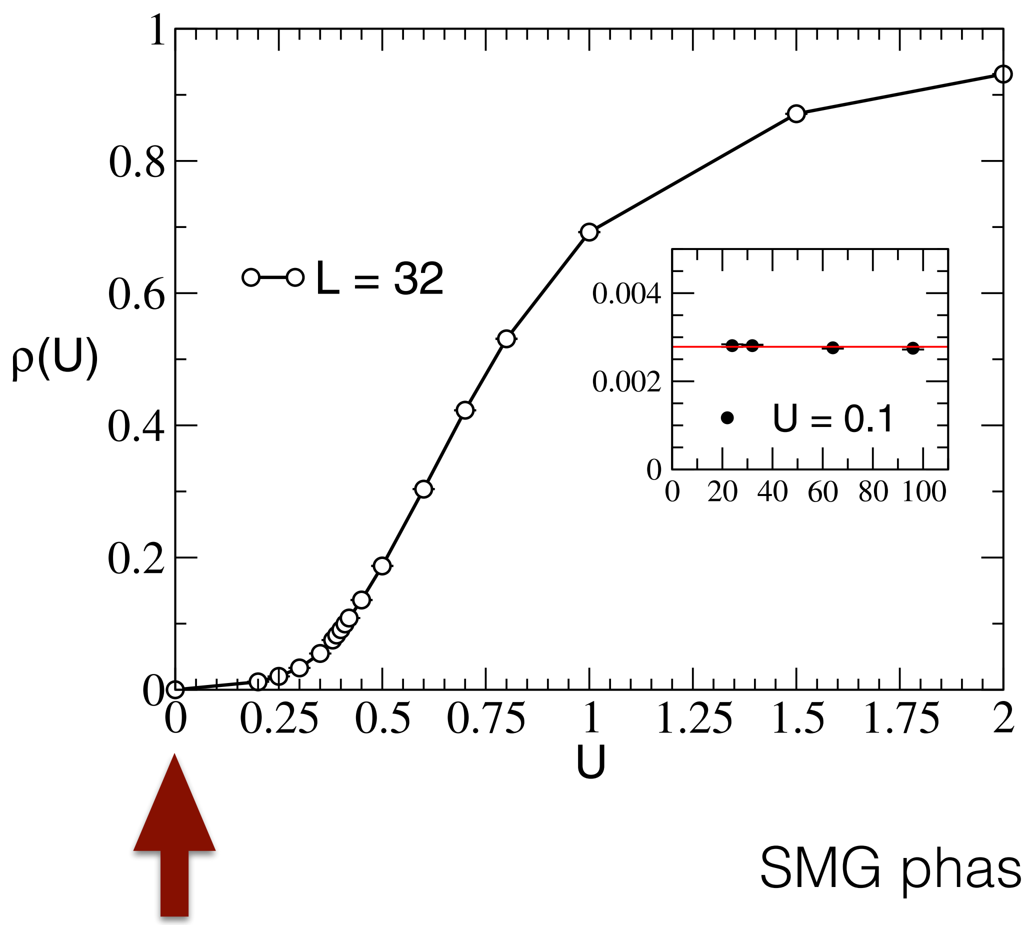
An exotic transition, should belong to the universality class predicted by Slagle, Yu, Xu.

He,Wu,You,Xu,Meng,Lu, PRB 94 241111 (2016)

Results in 2D

Four-fermion couplings are marginally relevant!

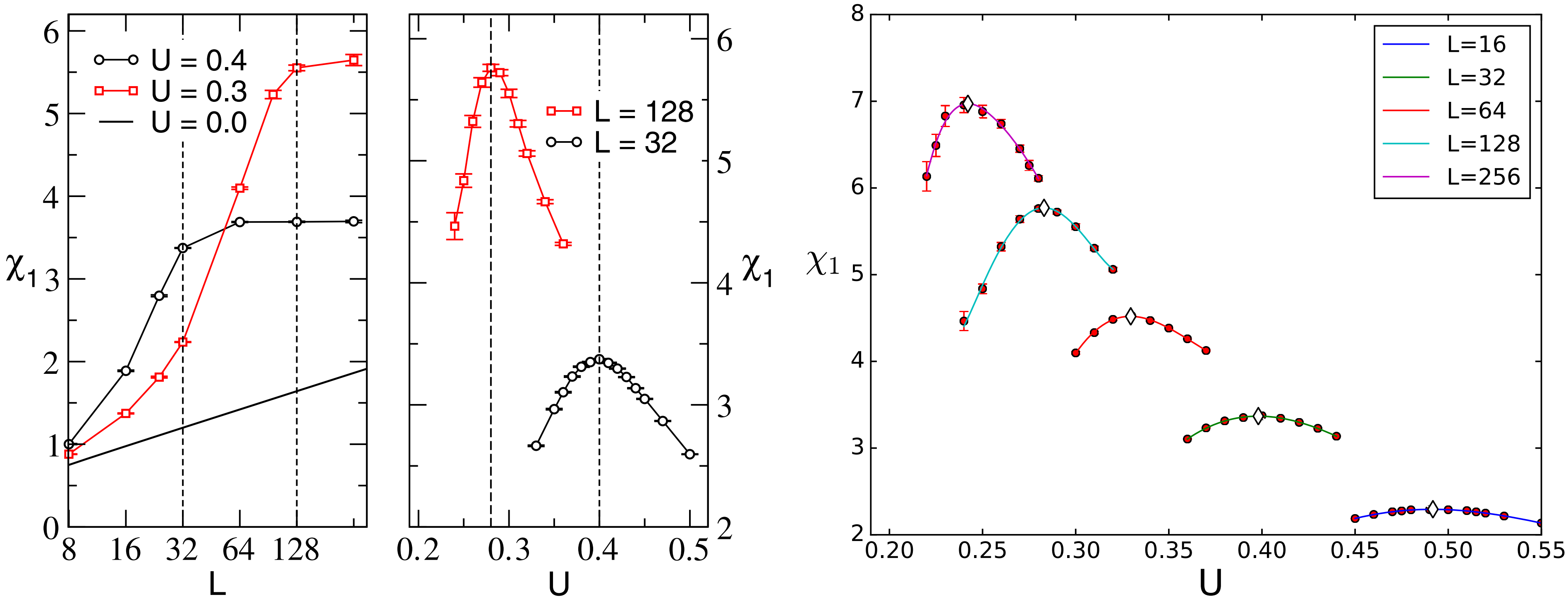
Monomer Density



SMG phase for all U?

U is marginally relevant,
so the critical point = 0

Susceptibility peak scaling:

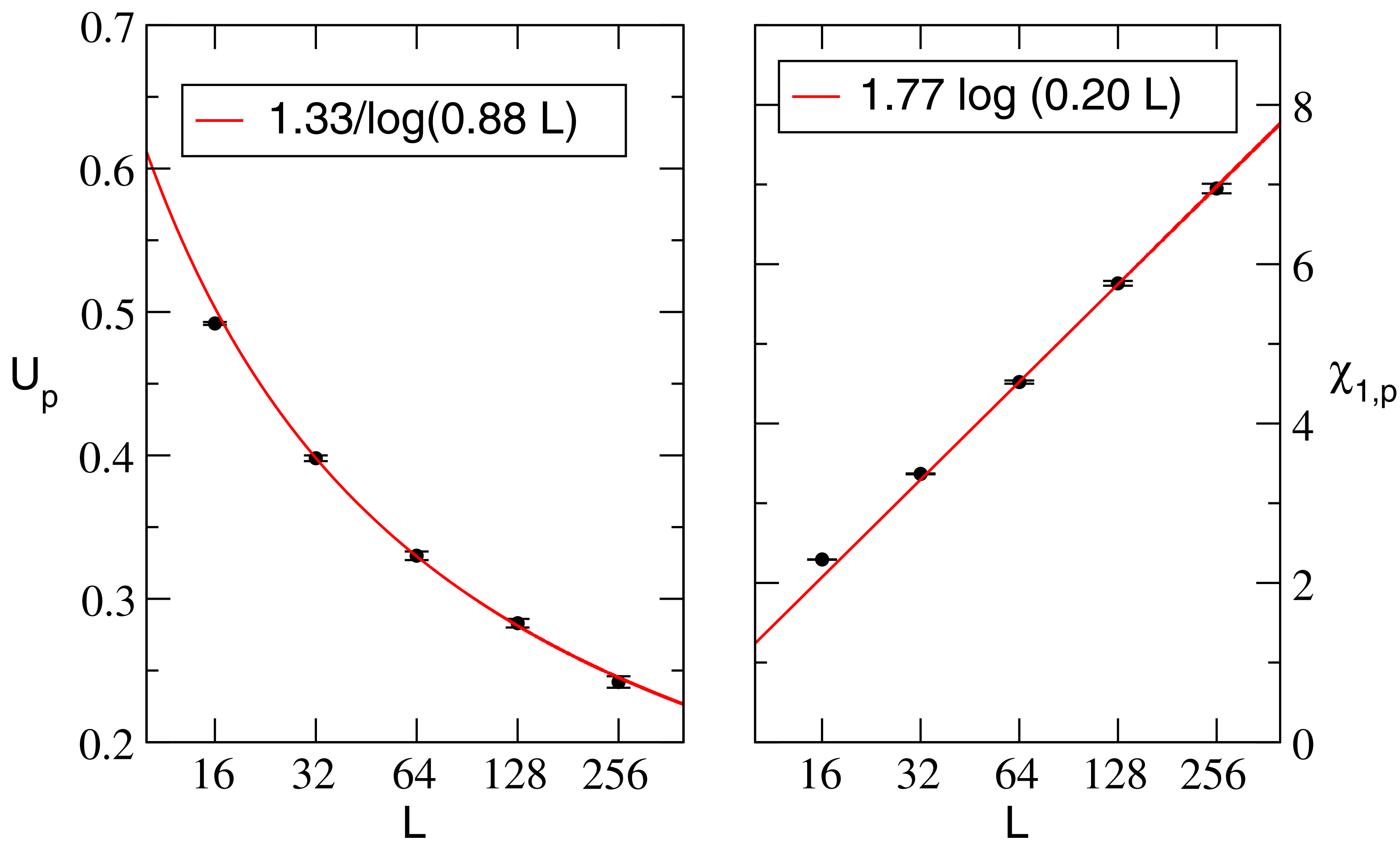


The location of the peak of the susceptibility, U_p at some fixed L

$$\chi_{1,\mathrm{p}} = \alpha \log(\Lambda_1 L), \qquad U_\mathrm{p} = \frac{\beta}{\log(\Lambda_2 L)}.$$

Our results are consistent with this expectation

$$U_{\mathrm{p}} = \frac{\beta}{\log(\Lambda_2 L)}. \qquad \chi_{1,\mathrm{p}} = \alpha \log(\Lambda_1 L),$$



Phase Diagram in 2d

Theory is asymtotically free (massive) theory.

Conclusions

- Fermion mass generation can be a dynamical phenomena not connected with spontaneous symmetry breaking in lattice field theory.
- This Symmetric Mass Generation (SMG) was known in lattice field theory, but it was assumed to be a lattice artifact.
- SMG seems to have a continuum limit in both 2d and 3d.
- In 2d the SMG phase is an asymptotically free. In 3D it was recently conjectured to be a gauge theory. **You, He, Vishwanath, Xu PRX 8 011026 (2018)**
- In 4d the search for the continuum limit continues….