Deconfined Quantum Critical Point on the Triangular Lattice

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A candidate theory for the “strange metal” (and perhaps pseudogap) phase in High-Tc, from SYK physics without randomness.
Chaoming Jian.

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Outline:

1, review of deconfined QCP on the square lattice;

2, global phase diagram for SU(2) invariant spin-1/2 system on the triangular lattice;

3, an effective field theory for order phases on the triangular lattice;

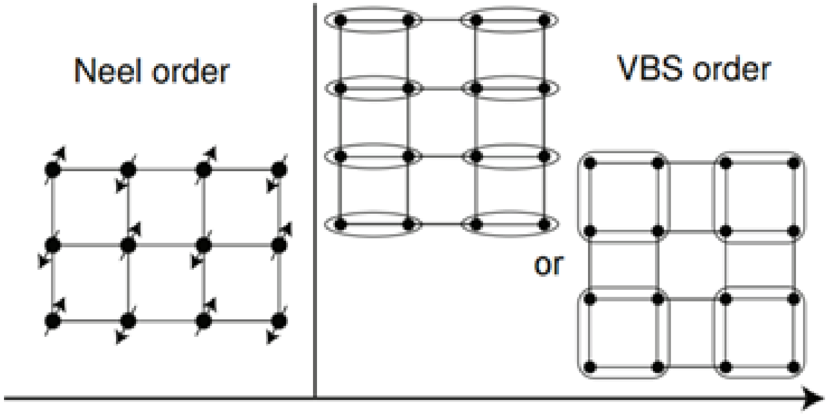
4, deconfined QCP on the triangular lattice, between 120° spiral spin state and VBS order.

5, deconfined QCP and boundary of 3d SPT state.

Reference:
arXiv:1710.04668, Jian, Thomson, Rasmussen, Bi, Xu

Review of dQCP on the square lattice:

Unconventional QCP:
direct 2nd order transition between two ordered phases that spontaneously break two totally different symmetries. (deconfined QCP, Senthil, Vishwanath, Balents, Sachdev, Fisher, 2004)



Ground state manifold: S^2 $\sim S^1$

Review of dQCP on the square lattice:

dQCP on the square lattice, in short, is a S^2-to-S^1 transition.
Or, its easy-plane version, is a S^1-to-S^1 transition.

Web of duality of the easy-plane dQCP on the square lattice:

$$\mathcal{L} = \sum_j |(\partial_\mu - i b_\mu) z_j|^2 + r|z_j|^2 + g|z_j|^4$$

$r < 0$ U(1) ordered   U(1) (VBS) order $r > 0$

$m < 0$ $m > 0$

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\psi}_j \gamma_\mu (\partial_\mu - i a_\mu) \psi_j + m \bar{\psi} \sigma^z \psi$$

Absolutely crucial ingredient of the theory: 2+1d U(1) order is dual to a photon phase.

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Review of dQCP on the square lattice:

Effective field theory of ordered phases around the dQCP on the square lattice (Senthil, Fisher, 2006):

The spin SU(2) invariant case:

$$S = \int d^2x d\tau \; \frac{1}{g}(\partial_\mu \vec{n})^2 + \int_0^1 du \; \frac{i2\pi}{\Omega_4} \epsilon_{abcde} n^a \partial_x n^b \partial_y n^c \partial_z n^d \partial_\tau n^e$$

It captures the physics that, the vortex of two of the five components (VBS), carry the spinor of the SO(3) symmetry of the other three components.

Challenge of generalizing the dQCP to the triangular (or other frustrated) lattice: the crucial ingredient, i.e. the (approximate) U(1) order is missing.

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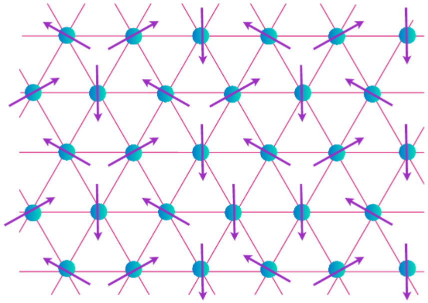
All familiar phases on the triangular lattice:



Z_2 spin liquid, with *e* and *m* both effective SU(2) fundamental, Sachdev 1992, Wang, Vishwanath, 2006, Lu 2015

$\sqrt{12} \times \sqrt{12}$ VBS

Vison condensate, fully ftrustrated, four minima in the BZ, (Moessener, Sondhi, 2001)

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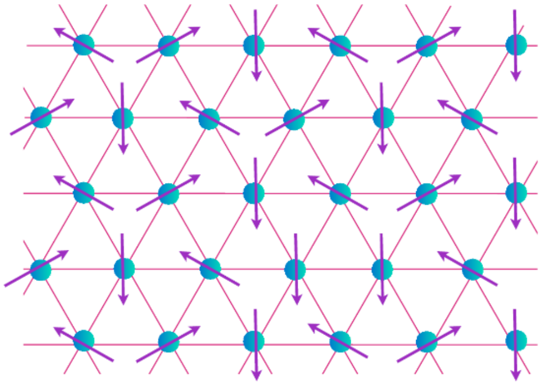
Closer look at the noncollinear AF order



The order parameter of a noncollinear AF order is three orthogonal vectors (a tetrad), with manifold SO(3). It breaks the SO(3) spin symmetry completely.

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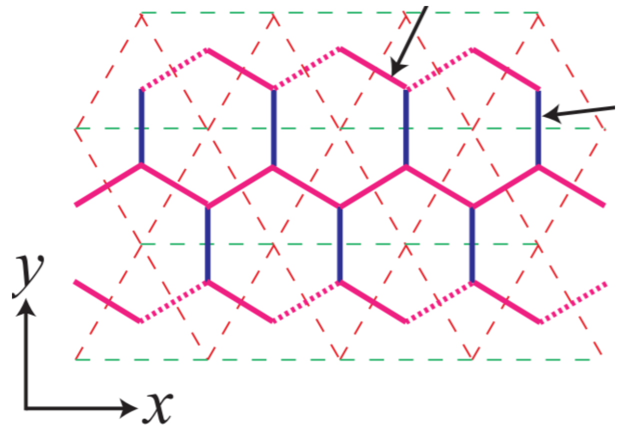
Closer look at the VBS state (Moessner, Sondhi 2001)



VBS phase is described by condensate of visons. The visons are real bosons hopping on the dual honeycomb lattice, with fully frustrated hoppings. With just nearest neighbor vison hoppings, there are four degenerate minima in the vison BZ, and an emergent SO(4) symmetry. The condensate corresponds $\sqrt{12} \times \sqrt{12}$ VBS

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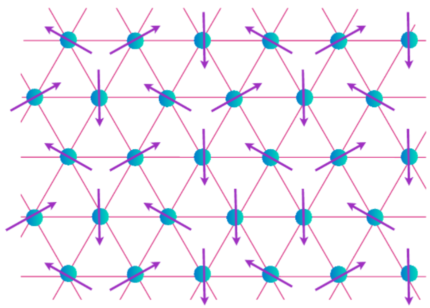
"Unified theory" and self-dual global phase diagram (Xu, Sachdev, 2008)



s_v

O(4)*

Z_2 spin liquid:

$$\mathcal{L} = \sum_{\alpha=1}^{2} \{|(\partial_\mu - i a_\mu) z_\alpha|^2 + s_z |z_\alpha|^2\} + \sum_{a=1}^{N_v} \{|(\partial_\mu - i b_\mu) v_a|^2 + s_v |v_a|^2\} + \frac{ik}{2\pi}\epsilon_{\mu\nu\lambda} a_\mu \partial_\nu b_\lambda$$

Ground state manifold: SO(3)

CP¹

O(4)*

s_z

$\sqrt{12} \times \sqrt{12}$ VBS

Ground state manifold: S^2 x S^2

Ground state manifold: SO(3)

CP¹

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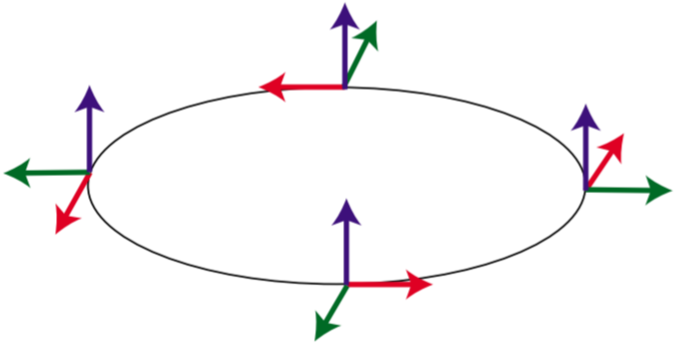
"intertwinement" between all the order parameters:
The Z_2 vortex of the one of the SO(3) GMS carries the spinor of the other SO(3).



What kind of topological term encapsulates the "intertwinement" between the order parameters, in particular, what kind of topological term will reduce to the O(5) WZW term when we break one of the SO(3) down to SO(2)?

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$$\mathcal{P} = \sum_{a,b=1}^{3} N_e^a N_m^b \sigma^{ab} + \sum_{a=1}^{3} M_e^a \sigma^{a0} + \sum_{b=1}^{3} M_m^b \sigma^{0b}$$

Some constraint: $\vec{N}_e \cdot \vec{M}_e = 0, \quad \vec{N}_m \cdot \vec{M}_m = 0, \quad \mathcal{P}^2 = 1_{4\times 4}$

$$\mathcal{P} \in \frac{U(4)}{U(2) \times U(2)}$$

$$\mathcal{L}_{wzw} \sim \int d^3x \int_0^1 du \ \mathrm{tr}[\mathcal{P}\partial_u \mathcal{P}\partial_\tau \mathcal{P}\partial_x \mathcal{P}\partial_y \mathcal{P}]$$

When we break one of the SO(3) down to SO(2): $\mathcal{P} = \vec{n} \cdot \vec{\Gamma}$
The WZW term of P precisely reduces to the O(5) WZW term.

Same theory as the boundary of a 3d bosonic SPT state with pSU(4) symmetry (Xu, 2012)

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What kind of the field theory will generate this topological term as its low energy effective field theory?

$$\mathcal{L} = \sum_{j=1}^{4} \bar{\psi}_j \gamma_\mu (\partial_\mu - i a_\mu) \psi_j + \bar{\psi}_i \psi_j \mathcal{P}_{ij}$$

$$\mathcal{S}_{eff}[a_\mu, \mathcal{P}] \ = \ -\ln \int D\bar{\psi} D\psi \exp\left[-\int d^3x \mathcal{L}(\psi, a_\mu, \mathcal{P}) \right]$$

Integrating out the fermions (they are massive), perform the standard expansion (Abanov, Wiegmann 2000), we can derive precisely the WZW term in the previous slide.

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what are the four fermion terms that break the emergent SU(4) symmetry of $N_f = 4$ QED down to the physical SO(3) x SO(3) symmetry?

Only two independent terms:

$$\mathcal{L}_1 = (\bar{\psi}\vec{\sigma}\psi) \cdot (\bar{\psi}\vec{\sigma}\psi)\,, \qquad \mathcal{L}_2 = (\bar{\psi}\vec{\mu}\psi) \cdot (\bar{\psi}\vec{\mu}\psi)$$

RG analysis (1/*N* expansion) shows that $L_1 + L_2$ is irrelevant, while $L_1 - L_2$ is relevant, both must be eigenvectors under RG flow, because $L_1 + L_2$ preserves the O(4) symmetry, while $L_1 - L_2$ breaks the O(4) down to SO(4).

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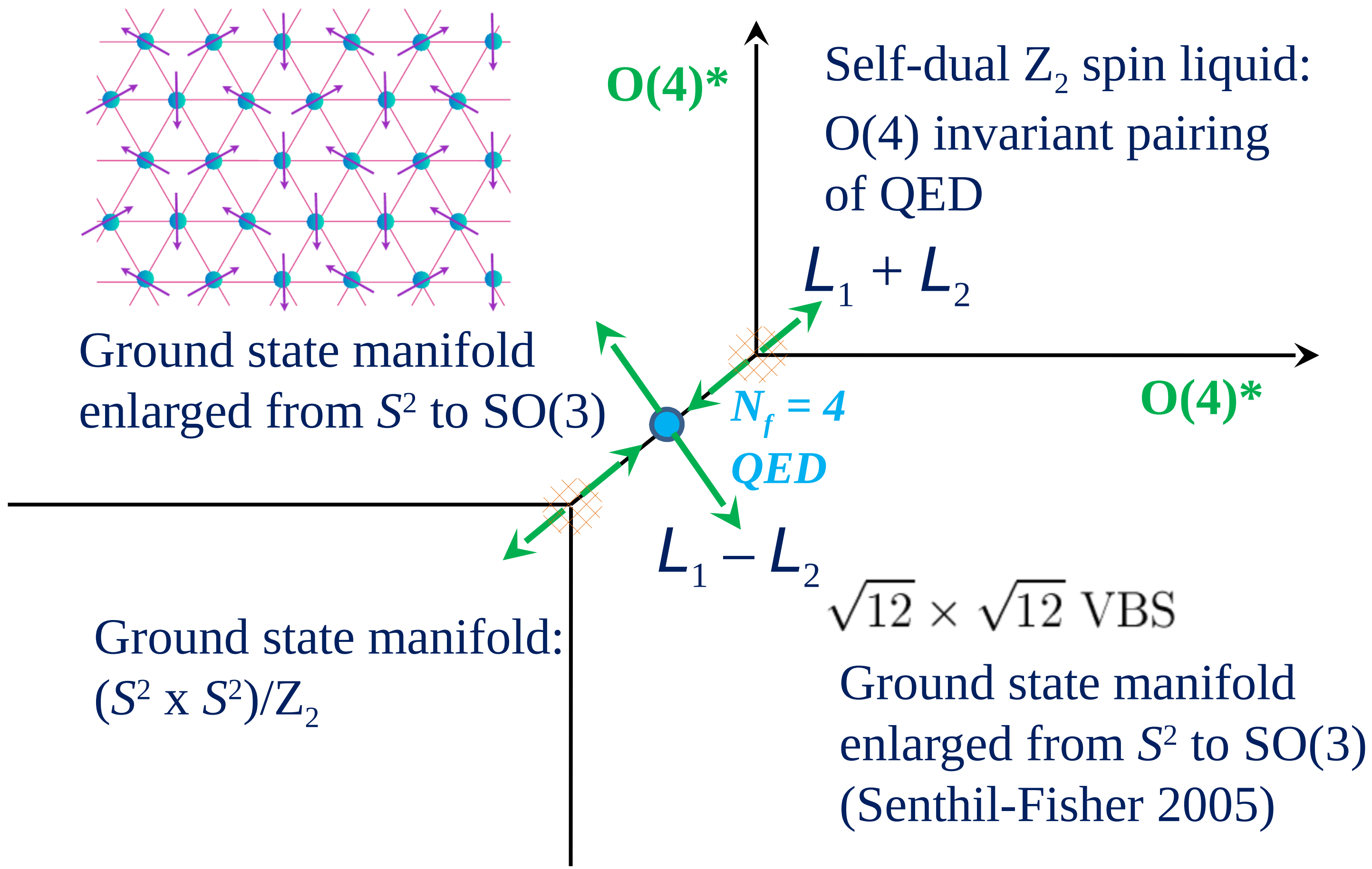
RG + Mean field calculation gives us a modified phase diagram



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The “Senthil-Fisher” mechanism that enlarges the GSM from S^2 to SO(3):

$$\mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - ia_\mu)\psi + m\bar{\psi}\boldsymbol{\sigma}\psi \cdot \boldsymbol{N}$$

Again, integrate out the fermions:

$$\mathcal{L}_{eff} = \frac{1}{g}(\partial_\mu \boldsymbol{N})^2 + i2\pi \mathrm{Hopf}[\boldsymbol{N}] + i2a_\mu J_\mu^T + \frac{1}{e^2} f_{\mu\nu}^2$$

Introduce the CP^1 field:

$$\boldsymbol{N} = z^\dagger \boldsymbol{\sigma} z \qquad\qquad z_\alpha = (z_1, z_2)^t = (n_1 + in_2, n_3 + in_4)^t$$

$$2ia_\mu J_\mu^T = \frac{i2}{2\pi}\epsilon_{\mu\nu\rho} a_\mu \partial_\nu \alpha_\mu$$

The four component unit vector *n* coupes to an effective Z2 gauge field. Its condensate has GSM SO(3) = S^3/Z_2 rather than S^2.

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Can we find a lattice model for this new dQCP?

Square lattice dQCP is realized in the so-called spin-1/2 J-Q model (Sandvik, et.al.), and 3d classical loop models (Nahum, et.al.).

It is a challenge to find a sign-problem-free spin-1/2 model on the triangular lattice to realize the desired dQCP.

But, sign-problem-free O(N) models on the triangular lattice with both spin nematic order (with GSM S^{N-1}/Z_2) and $\sqrt{12} \times \sqrt{12}$ VBS have been found (R. Kaul 2015).

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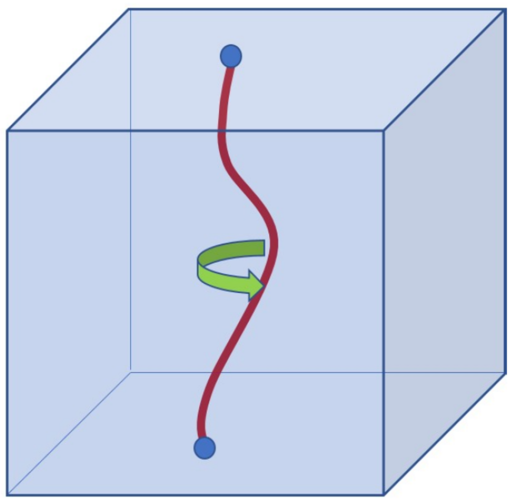
On the square lattice, if we view the square lattice symmetry as an onsite symmetry, the field theory of the dQCP is the same as the boundary of a 3d bosonic SPT state with SO(3) x U(1) symmetry.

Embed the VBS order parameter into a U(1) order parameter; then view the site centered rotation (90 degree) of the square lattice as the Z_4 subgroup of the U(1) symmetry.

The 3d SPT with SO(3)xU(1) symmetry, can be understood as a "decorated vortex loop" picture (Vishwanath, Senthil 2012): A SO(3) Haldane phase is decorated on each vortex loop of the U(1), then proliferate the vortex loop.



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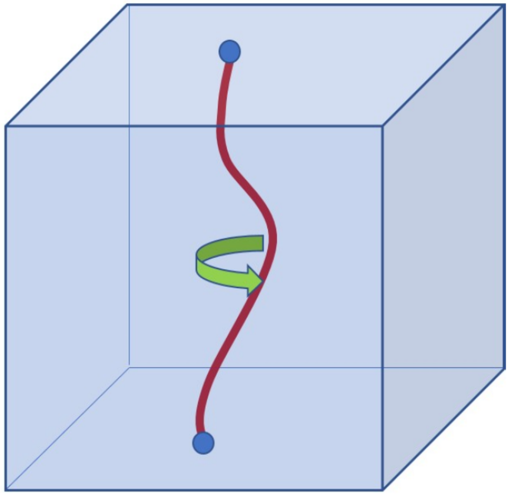
On the triangular lattice, we can view the physics around the dQCP as the boundary of a 3d bosonic SPT with $SO(3)_1 xSO(3)_2$ symmetry.

The physical picture of this 3d SPT is similar as before: consider a $SO(3)_2$ tetrad order parameter in the 3d bulk, and decorate the Z_2 vortex line ($\pi_1[SO(3)] = Z_2$) of the $SO(3)_2$ manifold with the Haldane phase of the $SO(3)_1$ symmetry, and vice versa. Then eventually proliferate the Z_2 vortex lines.

One possible 2d boundary of this 3d SPT is the self-dual Z_2 topological order whose *e* and *m* are spinor of $SO(3)_1 xSO(3)_2$ symmetry respectively. Analogue of the so called *eCmC* state.



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Summary: