Fracton topological order from Higgs and partial confinement mechanisms of rank-two gauge theory

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Motivation

Fracton: They are fractional point-like excitations which cannot move freely in three dimensional space.

Fracton models: The gapped models having fractons as excitations are said to have fracton topological order. They have ground state degeneracy depending on lattice size on the 3-torus.

Higher rank U(1) gauge theories: A modified Gauss's law can lead to the conservation of not only the total charge, but also the conservation of dipole moment, quadrupole moment, etc, leading to constraint on the mobility of the gapped charge excitations.

Questions: Any relation between gapped fracton model and higher rank gauge theory with gapless modes?

Beyond exact solvable models, is there any other theoretical description for the fracton topological order?

Rank-2 Scalar Charge Theory

$$E_{\mu\nu}=\begin{bmatrix} E_{xx} & E_{xy} & E_{xz}\\ E_{yx} & E_{yy} & E_{yz}\\ E_{zx} & E_{zy} & E_{zz}\end{bmatrix}\qquad A_{\mu\nu}=\begin{bmatrix} A_{xx} & A_{xy} & A_{xz}\\ A_{yx} & A_{yy} & A_{yz}\\ A_{zx} & A_{zy} & A_{zz}\end{bmatrix}$$

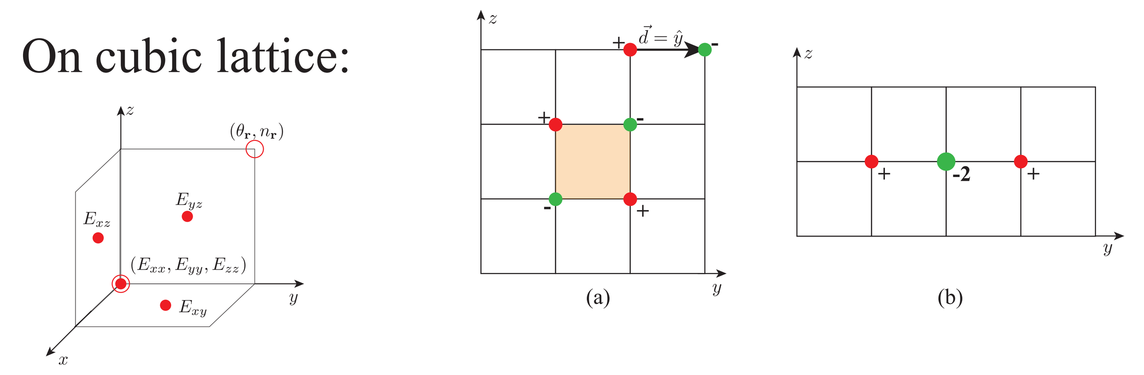
Gauss law: $\Delta_\mu\Delta_\nu E_{\mu\nu}=n_{\mathbf{r}}$

Hamiltonian: $H=U\sum_{\mathbf{r},\mu\le\nu}E_{\mu\nu}^2-K\sum_{\mathbf{r},\mu,\nu}\cos(B_{\mu\nu})+u\sum_{\mathbf{r}}n_{\mathbf{r}}^2-J\sum_{\mathbf{r},\mu\le\nu}\cos\left[\Delta_\mu\Delta_\nu\theta-A_{\mu\nu}\right]$

Gauge transformation: $A_{\mu\nu}\to A_{\mu\nu}+\Delta_\mu\Delta_\nu f\quad \theta_{\mathbf{r}}\to\theta_{\mathbf{r}}+f_{\mathbf{r}}$

On cubic lattice:

Charge configuration:



Charge Conservation

$$Q=\int_V \rho\, d^3\mathbf{r}=\text{const}$$

Dipole Conservation

$$\mathbf{d}=\int_V \mathbf{r}\rho\, d^3\mathbf{r}=\text{const}$$

Higgs Mechanism

$$H_{2e}=u_2\sum_{\mathbf{r}}N_{\mathbf{r}}^2-\Delta\sum_{\mathbf{r}}\cos\left[\Theta_{\mathbf{r}}-2\theta_{\mathbf{r}}\right]-J_2\sum_{\mathbf{r},\mu\le\nu}\cos\left[\Delta_\mu\Delta_\nu\Theta-2A_{\mu\nu}(\mathbf{r})\right]$$

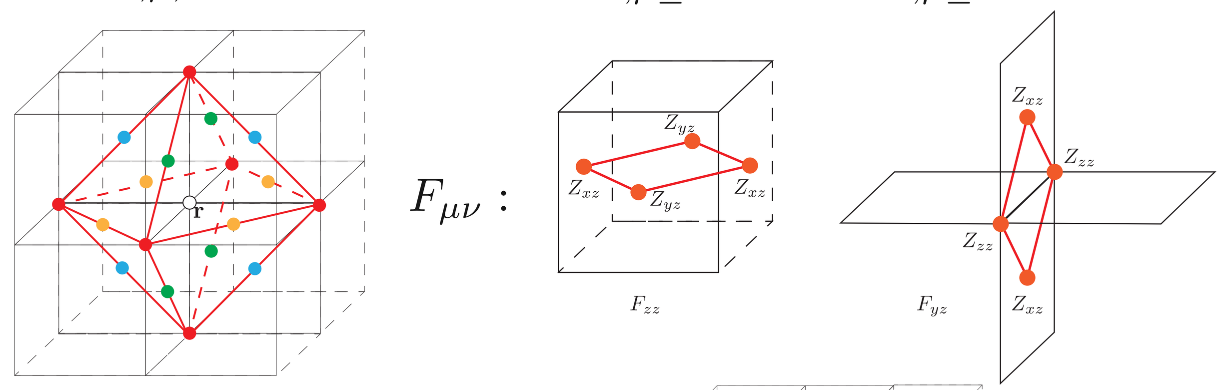
$J_2\to\infty$, $\Delta\to\infty$: **Taking this limit results in a condensation of the charge-2 field** $e^{i\Theta}$

Rank-2 $\mathbb{Z}_2$ Scalar Charge Theory

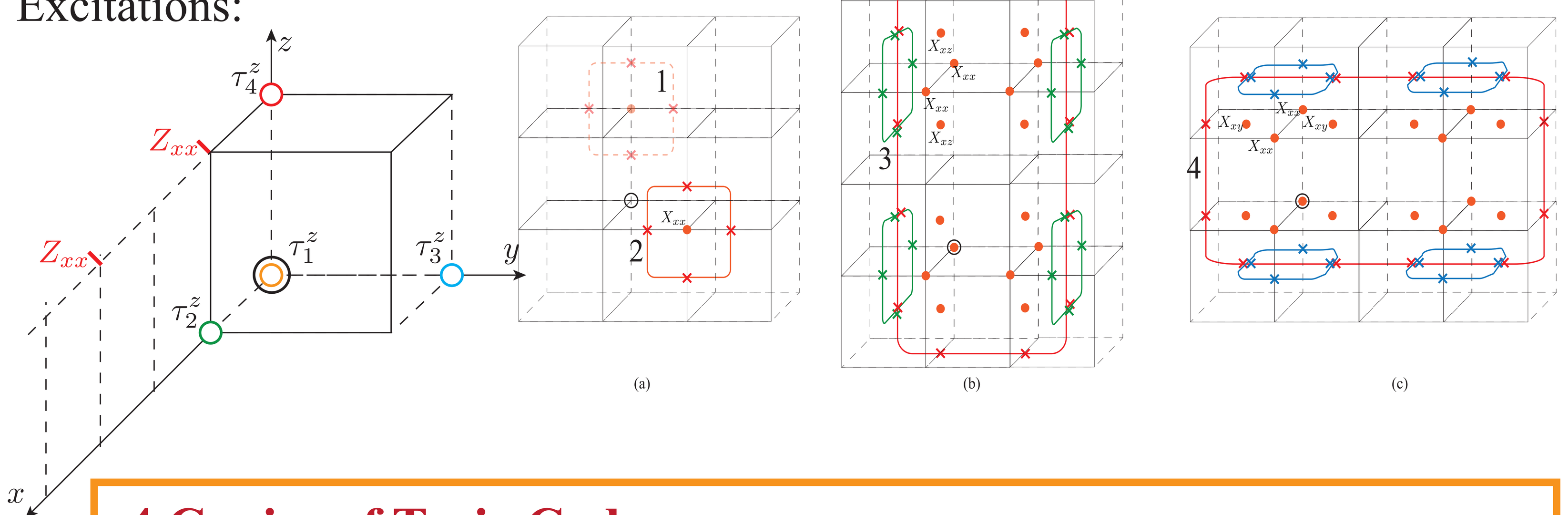
$J=U=0$

$$H_{\mathbb{Z}_2}=-K\sum_{\mathbf{r},\mu,\nu}F_{\mu\nu}-u\sum_{\mathbf{r}}G_{\mathbf{r}}-J\sum_{\mathbf{r},\mu\le\nu}Z_{\mu\nu}-U\sum_{\mathbf{r},\mu\le\nu}X_{\mu\nu}$$

$G_{\mathbf{r}}=$ $F_{\mu\nu}$:



Excitations:



4 Copies of Toric Codes: 4 point charges and 4 flux loops.

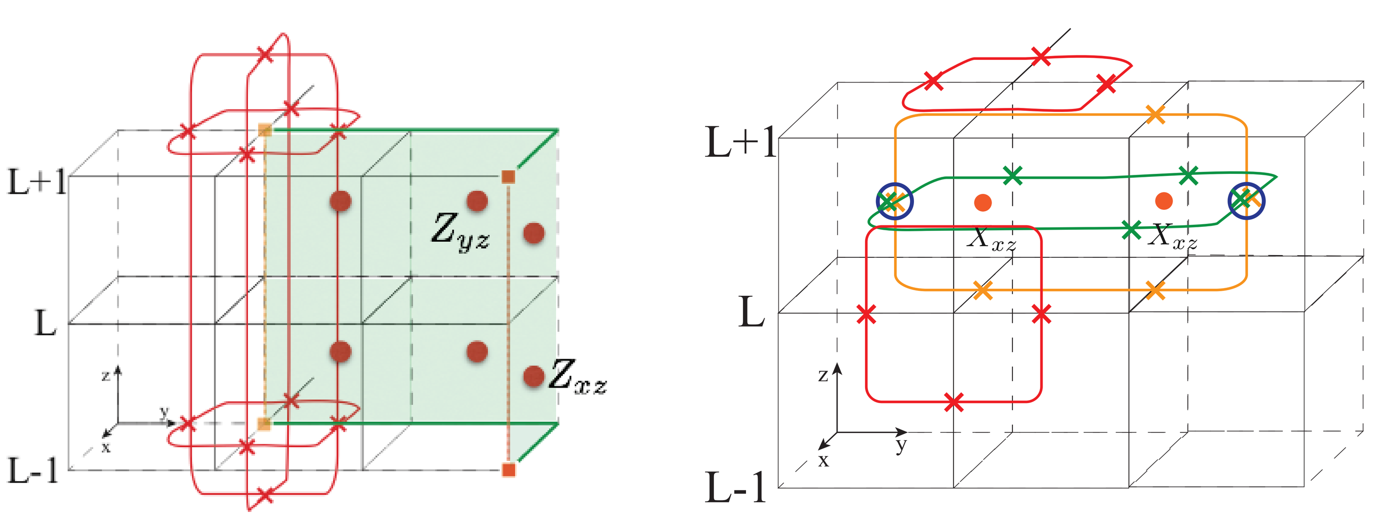
Higgsing the higher rank U(1) gauge theories **increases** the mobility of the charges.

Partial Confinement Transition

(Dimension Reduction -- flux loop condensation)

$$H_{4TC}=-K\sum_{\mathbf{r},\mu,\nu}F_{\mu\nu}-u\sum_{\mathbf{r}}G_{\mathbf{r}}-h\sum_{\mathbf{r}}\sum_{\mu=x,y,z}X_{\mu\mu}(\mathbf{r})\qquad h\to\infty$$

Red loops: condensate



1. The point charge excitations can no longer move.

2. bound states of two charges can propagate freely by string operators which is a product of off-diagonal Z operators.

3. Loop excitations reduce to two 1d excitations.

Partial Confinement Transition

(Monopole condensation)

$$E_{\mu\mu}=\frac{1}{2\pi}\epsilon_{\mu\lambda\sigma}\Delta_\lambda\alpha_{\sigma\mu}\ (\text{no sum over }\mu)\qquad E_{\mu\nu}=\frac{1}{2\pi}\left(\epsilon_{\mu\lambda\sigma}\Delta_\lambda\alpha_{\sigma\nu}+\epsilon_{\nu\lambda\sigma}\Delta_\lambda\alpha_{\sigma\mu}\right)\quad(\mu\neq\nu)$$

$$S_{\text{dual}}=\frac{1}{4\pi^2}\sum_{\tau,\mathbf{r}}\Big[u_d\sum_\mu(\epsilon_{\mu\lambda\sigma}\Delta_\lambda\alpha_{\sigma\mu})^2+u_{od}\sum_{\mu<\nu}(\epsilon_{\mu\lambda\sigma}\Delta_\lambda\alpha_{\sigma\nu}+\epsilon_{\nu\lambda\sigma}\Delta_\lambda\alpha_{\sigma\mu})^2\Big]$$
$$+2\pi^2k\beta_{\mu\nu}^2-i\beta_{\mu\nu}(\Delta_\tau\alpha_{\mu\nu}-\Delta_\mu\psi_\nu-\delta_{\mu\nu}\theta)$$
$$-t_m\sum_{\tau,\mathbf{r}}\sum_{\mu\neq\nu}\cos(\Delta_\mu\phi_\nu-\alpha_{\mu\nu})-t'_m\sum_{\tau,\mathbf{r}}\sum_{\mu,\nu}\cos(\Delta_\mu\phi_\mu-\Delta_\nu\phi_\nu-\alpha_{\mu\mu}+\alpha_{\nu\nu})$$
$$+\frac{1}{t''_m}n_\mu^2+in_\mu(\Delta_\tau\phi_\mu-\psi_\mu)$$

Vector magnetic charge n_μ created by $e^{i\phi_\mu}$ moving in 2d plane normal to μ

Integrate out ψ_μ and get the Gauss law for monopole: $\Delta_\mu\beta_{\mu\nu}=n_\nu$

$t_m,t''_m\to\infty\qquad \alpha_{\mu\nu}=0\quad(\text{for}\quad\mu\neq\nu)\ \Rightarrow E_{\mu\mu}=0$

Hollow U(1) Scalar Charge Theory

$$E_{\mu\nu}=\begin{bmatrix} 0 & E_{xy} & E_{xz}\\ E_{yx} & 0 & E_{yz}\\ E_{zx} & E_{zy} & 0\end{bmatrix}\qquad A_{\mu\nu}=\begin{bmatrix} 0 & A_{xy} & A_{xz}\\ A_{yx} & 0 & A_{yz}\\ A_{zx} & A_{zy} & 0\end{bmatrix}$$

$$S_{\text{dual}}^{\text{hollow}}=\frac{u_{od}}{4\pi^2}\sum_{\tau,\mathbf{r}}\left[(\Delta_zh_x-\Delta_xh_y)^2+(\Delta_xh_y-\Delta_yh_z)^2+(\Delta_yh_z-\Delta_yh_x)^2\right]$$
$$+\frac{1}{8\pi^2k}\sum_{\tau,\mathbf{r}}\sum_\mu(\Delta_\tau h_\mu-\theta)^2-t'_m\sum_{\tau,\mathbf{r}}\sum_{\mu<\nu}\cos(h_\mu-h_\nu)$$

$h_\mu=\alpha_{\mu\mu}$

Gauge transformation:

$h_\mu\to h_\mu+f$

$\theta\to\theta+\Delta_\tau f$

Confined

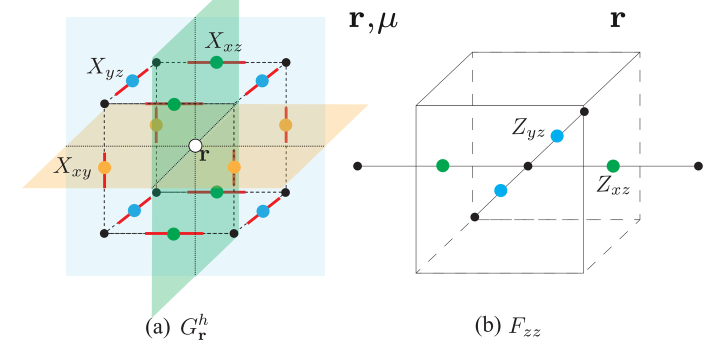
Higgs Mechanism

X-Cube Model

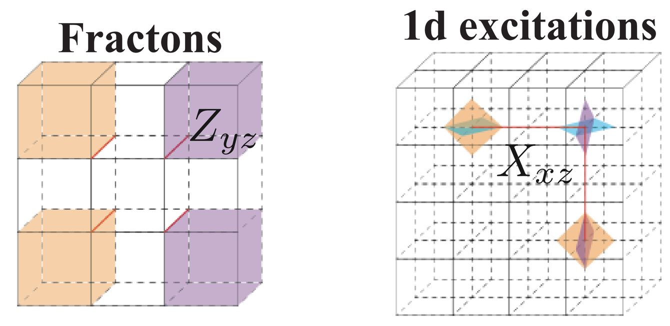
Described by rank-2 $\mathbb{Z}_2$ **hollow** scalar charge theory

The charge on every lattice plane is separately conserved

$$H_{\text{X-Cube}}=-K\sum_{\mathbf{r},\mu}F_{\mu\mu}-u\sum_{\mathbf{r}}G_{\mathbf{r}}^h$$



Fractons **1d excitations**



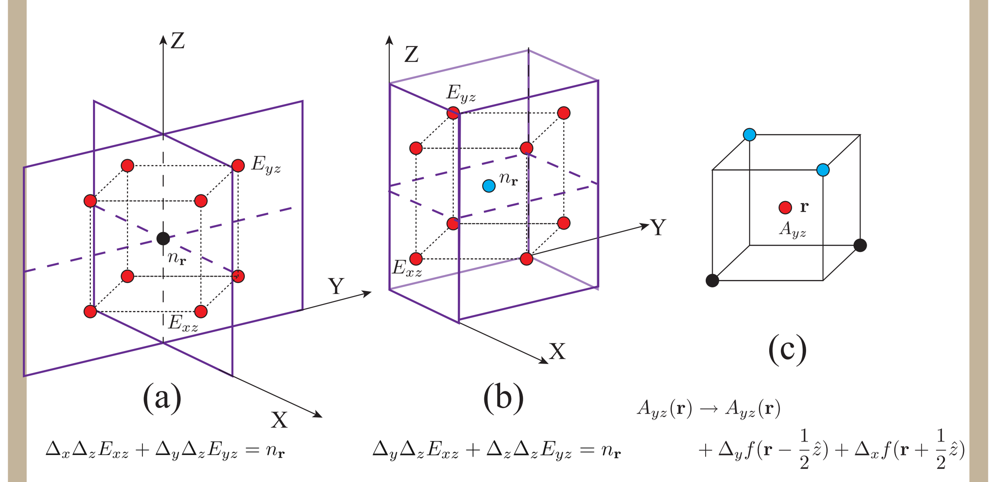
$\log_2 GSD=6L-3$

Checkerboard Model

$$H_{ckb}=-\sum_C\prod_{i\in C}X_i-\sum_C\prod_{i\in C}Z_i$$

$X_{\mu\nu}=\exp(i\pi E_{\mu\nu})\qquad Z_{\mu\nu}=\exp(i\pi A_{\mu\nu})$

$$E_{\mu\nu}=\begin{bmatrix} 0 & E_{xy} & E_{xz}\\ E_{yx} & 0 & 0\\ E_{zx} & 0 & 0\end{bmatrix}\qquad A_{\mu\nu}=\begin{bmatrix} 0 & A_{xy} & A_{xz}\\ A_{yx} & 0 & 0\\ A_{zx} & 0 & 0\end{bmatrix}$$



A rank-2 $\mathbb{Z}_2$ gauge theory with two different Gauss' laws at different vertices of the cubic lattice. **Non-uniform Gauss law in space!**

References

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[4] Cenke Xu and Congjun Wu, arXiv:0801.0744 (2008).
[5] Sagar Vijay, Jeongwan Haah, and Liang Fu, arXiv:1603.04442 (2016).

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