Conformal solids and holography

Angelo Esposito[1,⋆], Sebastian Garcia-Saenz[2], Alberto Nicolis[1] and Riccardo Penco[3]

[1]Department of Physics, Center for Theoretical Physics, Columbia University, 538W 120th Street, New York, NY, 10027, USA
[2]Sorbonne Universités, UPMC Univ. Paris 6 and CNRS, UMR 7095, Institut d'Astrophysique de Paris, GReCO, 98bis boulevard Arago, 75014 Paris, France
[3]Center for Particle Cosmology, Department of Physics and Astronomy, University of Pennsylvania 209 S. 33rd St., Philadelphia, PA 19104, USA

⋆ae2458@columbia.edu

COLUMBIA UNIVERSITY

1. Introduction

The application of the holographic duality to condensed matter has been extremely successful in both investigating the strongly coupled regime of known systems as well as identifying new exotic states of matter. Surprisingly, a very common state of matter has received little attention: ordinary solids. Drawing our inspiration from the effective field theory (EFT) approach, we provide the gravity dual of a solid and deduce its low energy spectrum from the bulk theory. The gravity dual is an $SO(d)$ magnetic monopole coupled to a scalar in the fundamental representation. We cannot resist the temptation to dub such configuration a *solidon*. We also study possible phase transitions and exhibit an example of "holographic melting".

2. EFT for flat solids

A volume element for a solid in d spacetime dimensions can be described in terms of its $d-1$ *comoving coordinates*, $\phi^I(x)$. In the equilibrium configuration they can be chosen to be aligned with the spatial coordinates, $\langle\phi^I\rangle = \alpha x^I$.

This background spontaneously breaks spatial rotations and translations (as well as boosts). To recover isotropy and homogeneity at long distances we postulate an internal $ISO(d-1)$ symmetry:

$$\phi^I \to R^I{}_J\phi^J + a^I. \tag{1}$$

From the low energy EFT viewpoint a homogeneous and isotropic solid is a system that *spontaneously* breaks $ISO(d-1)\times ISO(d-1)\to ISO(d-1)$ [1]. Its most generic action will have to feature this internal symmetry as well as Poincarè invariance.

Fluctuations around the equilibrium are nothing but the phonons, $\phi^I = \alpha(x^I+\pi^I)$. Their quadratic action is found to be

$$S^{(2)} \propto \int d^dx \left[\dot{\vec{\pi}}^2 - c_L^2\left(\partial_I\pi_L^I\right)^2 - c_T^2\left(\partial_I\pi_T^J\right)^2\right], \tag{2}$$

where c_L and c_T are the longitudinal and transverse sound speeds.

If the solid is *conformal* the two sound speeds are related by a universal relation [2]

$$\boxed{c_L^2 = \frac{1}{d-1} + 2\frac{d-2}{d-1}c_T^2\,.} \tag{3}$$

It follows that $0 \le c_T^2 \le 1/2$ and $1/(d-1) \le c_L^2 \le 1$. Conformal solids are always relativistic.

3. EFT for solids on a sphere

The Euclidean group is hard to gauge. For this reason, to find the gravity dual of a solid it is easier to start from the EFT for solids on a sphere (e.g. a thin spherical shell) [2]. In this case the Euclidean group must be replaced by an internal $SO(d)$, since this is the isometry group of a $(d-1)$-dimensional sphere.

The comoving coordinates are now angles, such that $\langle\phi^I\rangle = \theta^I$. Compared to the flat case no free parameter appears here. This is because, when one lives on a sphere, it impossible to compress or shear the solid without spoiling homogeneity.

The symmetry breaking pattern is now $SO(d)\times SO(d)\to SO(d)$. One way to realize it is to embed the $(d-1)$-dimensional sphere in a d-dimensional space and introduce a multiplet, $\vec{\Phi}$, in the fundamental of $SO(d)$. To realize the desired pattern we can align the vacuum expectation value (vev) of this field with the radial direction, e.g.

$$\langle\vec{\Phi}\rangle = \Phi_0 R(\theta)\cdot\hat{x}_d. \tag{4}$$

To describe the field out of equilibrium we just need to replace the angles θ^I with the comoving coordinates ϕ^I. The most general action for such solid will be Poincarè invariant and $SO(d)$ invariant. In particular, to lowest order in the energy expansion, it can be built out of the trace of products of the matrix $\mathcal{B}^{AB}(\phi) = \partial_\mu\Phi^A\partial^\mu\Phi^B$. It is given by

$$S = \mathcal{R}^{d-1}\int dt\, d\Omega_{d-1}\, F(X, Y_2, \dots, Y_{d-2}). \tag{5}$$

Here $X = \mathrm{tr}\mathcal{B}$ and $Y_n = \mathrm{tr}(\mathcal{B}^n)/X^n$, and $\mathcal{R}$ is the radius of the sphere.

To recover the EFT for flat solids one can focus on a small patch of the sphere, with size much smaller than the sphere radius $\mathcal{R}$. This amounts to replace $\theta^I\to x^I/\mathcal{R}$, for $|x^I|\ll\mathcal{R}$. An observer that has only access to momenta that are much bigger than the IR cutoff, $k\gg 1/\mathcal{R}$, will be unable to probe the global structure of the manifold. For her $\alpha\equiv 1/\mathcal{R}$ will effectively play the role of a free parameter, to be eventually determined from boundary conditions. This way the usual EFT for flat solids is recovered.
In the dual picture this will correspond to go from global coordinates to Poincarè patch.

4. Gravity dual

Setup — The holographic dictionary tells us to gauge the global symmetries of the boundary theory. From Section 3 our action in the bulk of AdS will describe an $SO(d)$ Yang-Mills theory with a real scalar in the adjoint representation

$$S = -\int d^{d+1}x\sqrt{-g}\left[\frac{1}{2}D_M\vec{\Phi}\cdot D^M\vec{\Phi} + V\left(|\vec{\Phi}|^2\right) + \frac{1}{8}F^{AB}_{MN}F^{AB\,MN}\right] + S_{\rm bdy}. \tag{6}$$

We will not need to specify the scalar potential, which we will take to be $V\left(|\vec{\Phi}|^2\right) = \frac{m^2}{2}|\vec{\Phi}|^2 + $ interactions. We are interested in the *zero temperature* case so will restrict ourselves to pure AdS with no backreaction (set $L=1$):

$$ds^2 = -(1+\rho^2)d\tau^2 + \frac{d\rho^2}{1+\rho^2} + \rho^2 d\Omega_{d-1}^2. \tag{7}$$

Also, inspired by Eq. (4) we choose our background fields to be [2]

$$\vec{\Phi} = \varphi(\rho)\hat{\rho}, \qquad \text{and} \qquad A_M^{AB} = \rho\,\psi(\rho)\,\partial_M\hat{\rho}\cdot T^{AB}\cdot\hat{\rho}. \tag{8}$$

The gauge field ansatz is a straightforward generalization of the 't Hooft-Polyakov monopole. We dub this configuration a *solidon*.

Boundary conditions — The asymptotic behavior of the fields is, as usual, $\varphi = \varphi_{(1)}\rho^{\Delta-d} + \varphi_{(2)}\rho^{-\Delta}+\dots$ and $\psi = \psi_{(1)}\rho^{-1} + \psi_{(2)}\rho^{1-d}+\dots$. Here Δ is the largest positive solution to the equation $\Delta(\Delta-d) = m^2$. Since the theory is nonabelian, the operator dual to the gauge field is now charged under the global $SO(d)$ of the boundary theory. In order to have spontaneous breaking we then have to set its source to zero, i.e. $\psi_{(1)} = 0$.
For the scalar field it is a trickier business. In absence of a temporal component for the gauge field (effective negative mass), the scalar field cannot condensate by itself. The only way we can achieve spontaneous breaking is then through *mixed boundary conditions*. This means that from now on we restrict our masses to $-d^2/4 < m^2 < 1 - d^2/4$ and impose $\varphi_{(2)} = f|\varphi_{(1)}|^{\nu-1}\varphi_{(1)}$, for some ν. The parameter f corresponds to the coupling of a multitrace deformation of the boundary theory.

Back to flat space — To go back to flat space we can now go from global coordinates to Poincarè patch. This is achieved by changing coordinates to $\rho = \mathcal{R}\,r$, $\tau = t/\mathcal{R}$, $\theta^i = \frac{\pi}{2} - x^i/\mathcal{R}$, and taking the large $\mathcal{R}$ limit. After this rescaling the first falloff of the gauge field is not zero anymore. This is in line with the appearance of a free parameter when going from the EFT of solids on a sphere to that for flat solids.

The phonon action — We now want to derive the action of the boundary phonons from the fields in the bulk. We parametrize the fluctuations as

$$\vec{\Phi} = (\varphi+\sigma)e^{-\pi^i T^{id}}\cdot\hat{x}_d, \qquad \text{and} \qquad A_M = e^{-\pi^i T^{id}}\cdot(\bar{A}_M + \alpha_M)\cdot e^{i\pi^i T^{id}}. \tag{9}$$

To find the boundary action for the phonons we now employ the techniques first developed in [3] and [4] for holographic fluids and superfluids. The plan of action is:

1. Write down the linearized equations for the fluctuations at *lowest order in boundary derivatives*. This corresponds to a low energy expansion on the boundary.
2. Solve the equations of motion for all the fluctuations, except for the angular ones, π^i.
3. Plug the solutions back into the quadratic action to recover the so called *partially on shell action*.

The power of our method is that the above procedure can be carried on even if the background profiles are not known. The solutions to the equations of motion can be written in terms of the background profiles even if their analytical expression is not available.

When *conformal symmetry* is preserved by the boundary conditions the partially on shell action reads [2]

$$\boxed{S^{(2)} = -\frac{d-2}{2}\frac{\psi_{(2)}}{q^2\psi_{(1)}}\int d^dx\left\{\dot{\vec{\pi}}^2 - \left[1+\lambda\frac{d-2}{d-1}\right]\left(\partial_j\pi_L^i\right)^2 - \frac{\lambda+1}{2}\left(\partial_j\pi_T^i\right)^2\right\},} \tag{10}$$

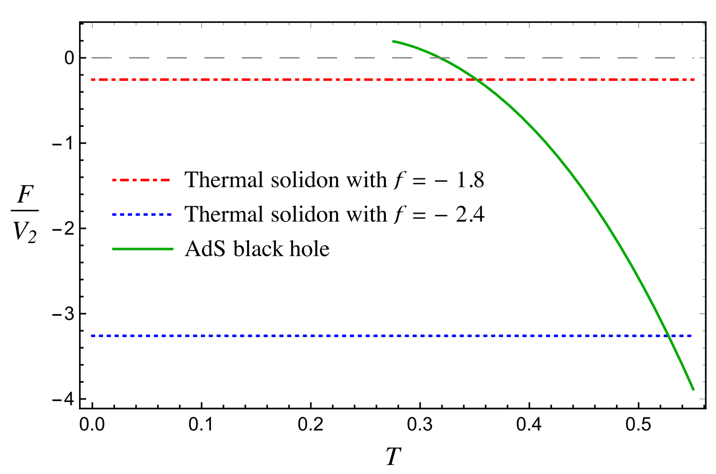
where λ is the only unknown integration constant. The longitudinal and transverse sounds speeds read

$$c_T^2 = \frac{\lambda+1}{2}, \qquad \text{and} \qquad c_L^2 = 1 + \lambda\frac{d-2}{d-1}, \tag{11}$$

and depend on the particular bulk theory under consideration. However, for *any* λ they are related by Eq. (3). This shows that we indeed found the gravity dual of a conformal solid.

6. Melting the solid

Does our holographic solid melt? To check that, we move back to global coordinates, reintroduce backreaction and compute the solidon free energy. We set finite temperature by simply compactifying the Euclidean time of our bulk theory (thermal soliton). We can compare our free energy with that of pure AdS as well as that of SAdS [2].



We specify $d=3$ and $\nu=1$, which corresponds to a relevant deformation, which should be present on naturalness grounds.

We can see that, when the temperature is raised, our solidon configuration becomes disfavored with respect to SAdS. The dual interpretation of this is that of a *solid-to-liquid* phase transition. This is also confirmed by the fact that the free energy has a discontinuity in its derivative, which is the hallmark of a first order phase transition.

It is also quite interesting to note that, when the temperature is lowered, the SAdS solution turns into our solidon *beofore* turning into pure AdS, i.e. before the Hawking-Page phase transition. This seems to suggest that there might be certain strongly coupled theories that, at low temperatures, undergo a transition to a solid state, rather than a confined one.

6. Conclusions

In this work [2] we have exhibited the gravity dual of a solid, a piece of the AdS/CMT program that was mostly lacking from the literature. We showed that the corresponding configuration on the gravity side is a solidon, i.e. an $SO(d)$ magnetic monopole coupled to a scalar field in the fundamental representation.

The low energy spectrum of our bulk theory exactly matches what expected for conformal solids from effective theories. We were able to compute that by employing the techniques developed in [4], regardless of the detailed knowledge of the background fields.

We also showed that, at finite temperature, our holographic solid undergoes a first order phase transition, i.e. a holographic melting. Surprisingly, this happens before the Hawking-Page temperature, which suggests that certain theories might prefer a solid state to a confined one at low temperatures.

It has also been shown (see e.g [5]) that a theory of massive gravity in the bulk of AdS closely resembles a solid on the boundary (although no actual proof was given). If would be interesting to investigate the connection between our dual and those theories of massive gravity.

7. References

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