Quantum critical dynamics: Monte Carlo & holography

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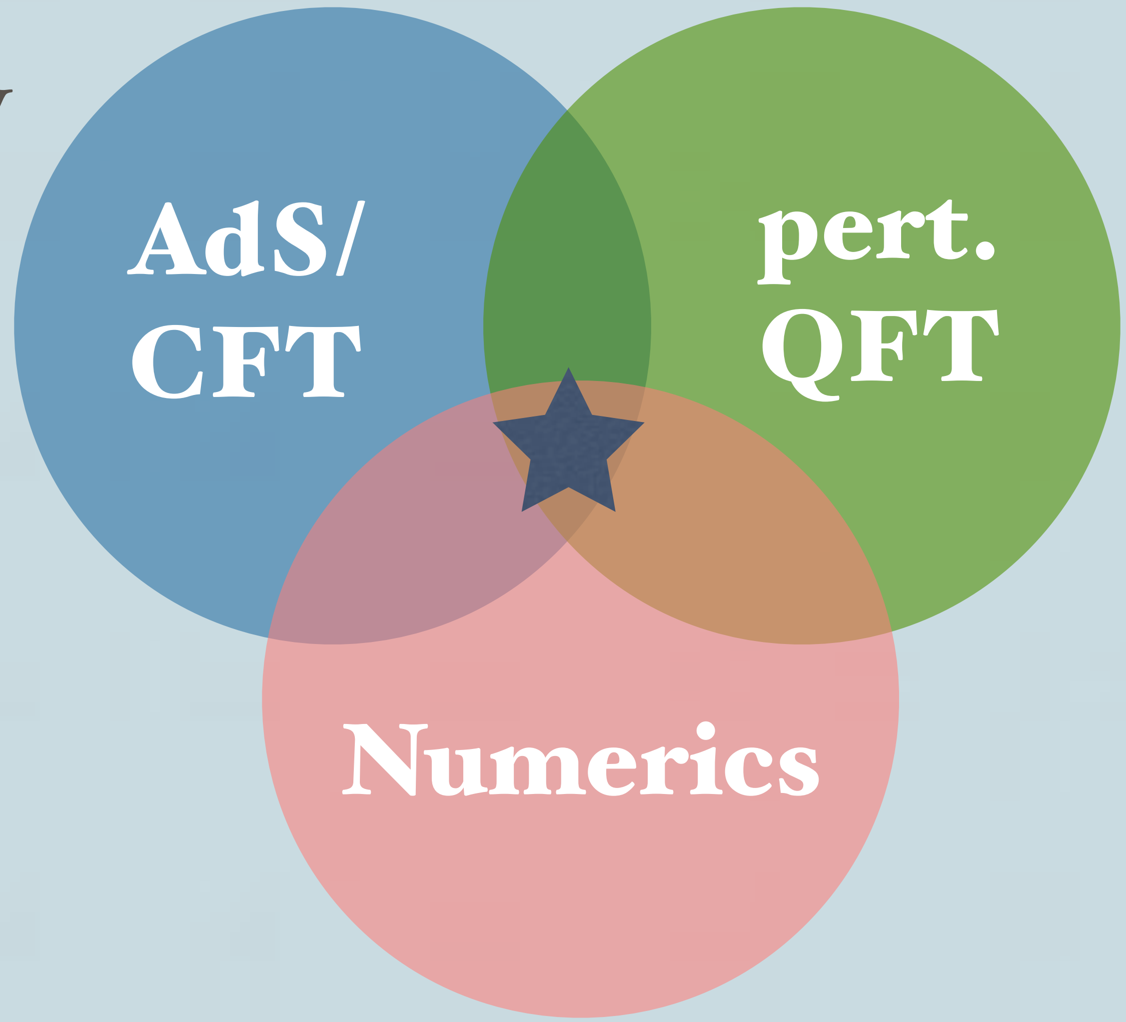
★Simulations, QFT and AdS/CFT to gain insight into **finite-*T* & real time** correlators near QCPs

★2+1D *QCPs with Lorentz invariance,* $z = 1$ *(often a CFT)*

- **Dynamics** of systems w/out quasiparticles...

S. Sachdev

T. Senthil,
P. Ghaemi,
Y.B. Kim,
S. Sachdev



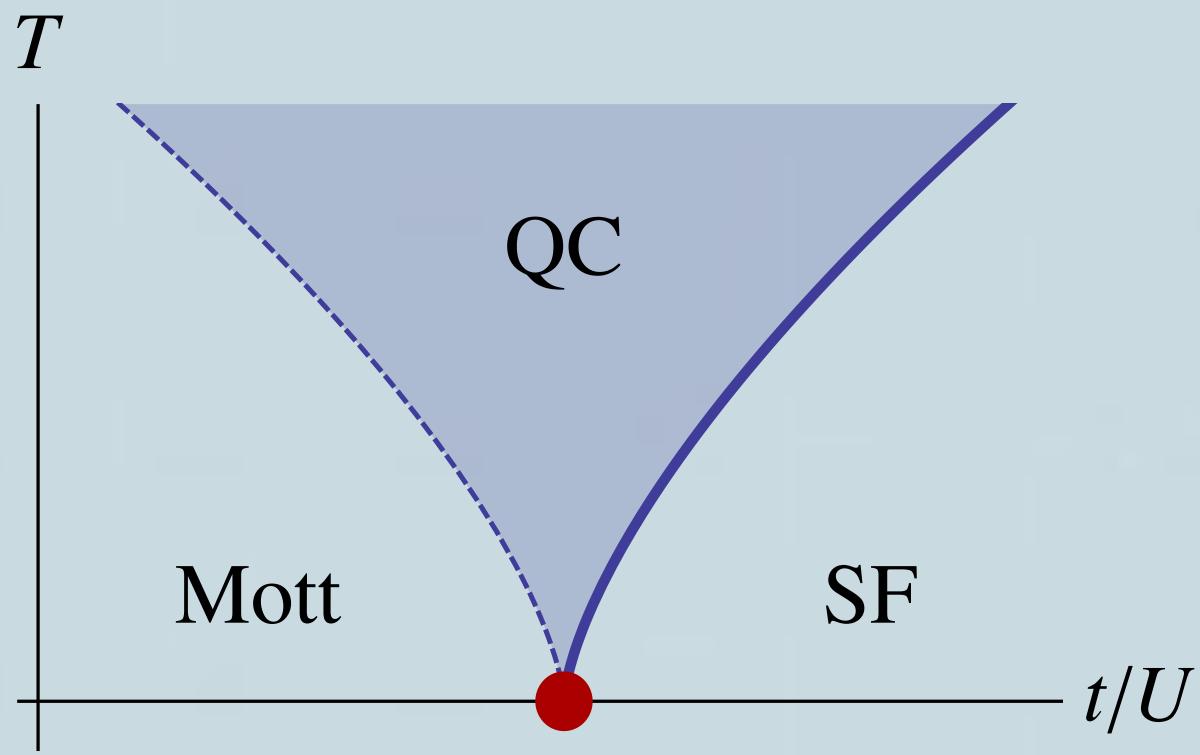
E. Sorensen
[McMaster]

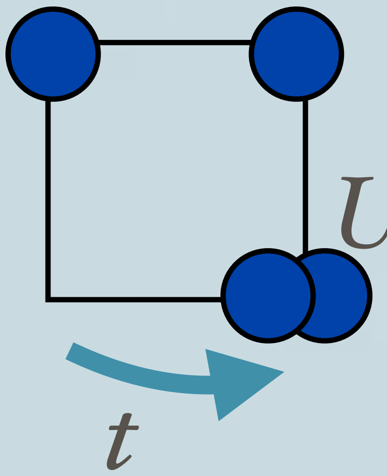
S. Hartnoll
C. Herzog
R. Myers
S.S. Lee

Bose-Hubbard in 2+1D

$$\boxed{H = -t \sum_{\langle i,j \rangle} b_i^\dagger b_j + U \sum_i n_i(n_i - 1) - \mu \sum_i n_i}$$

$\langle n_i \rangle = 1$





QCP = **CFT3**

❖ 2+1D O(2) universality: quantum rotors, XY spins etc

- SSB of O(2) order param.

O(N) model:

$$\mathcal{L} = \partial_\nu \vec{\phi} \cdot \partial^\nu \vec{\phi} + m^2 \vec{\phi}^2 + u(\vec{\phi}^2)^2$$

Flows to strong coupling
in 2+1D

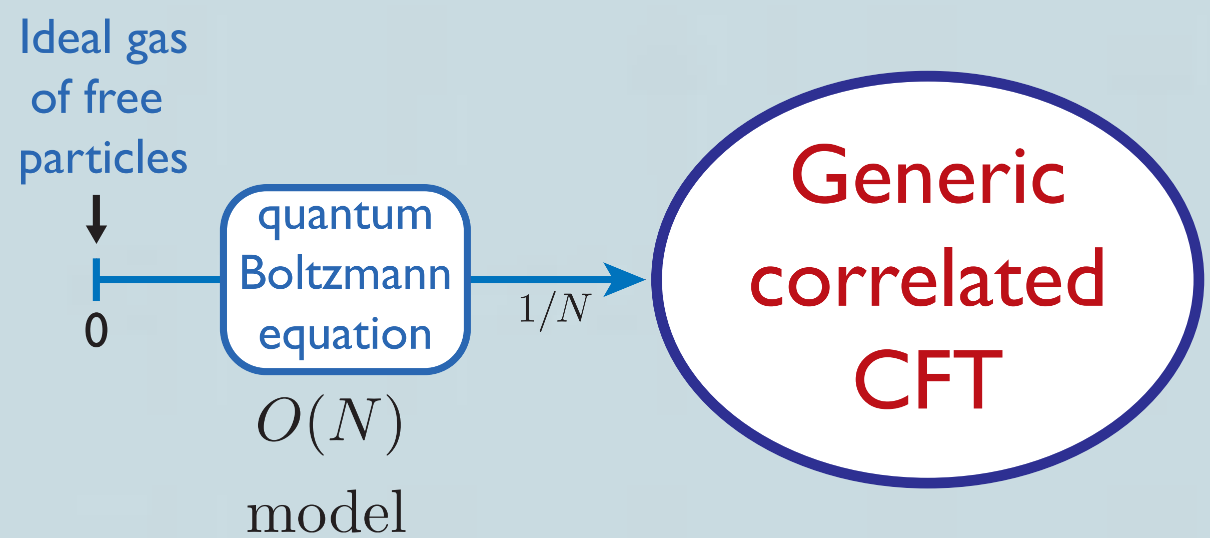
Current correlators

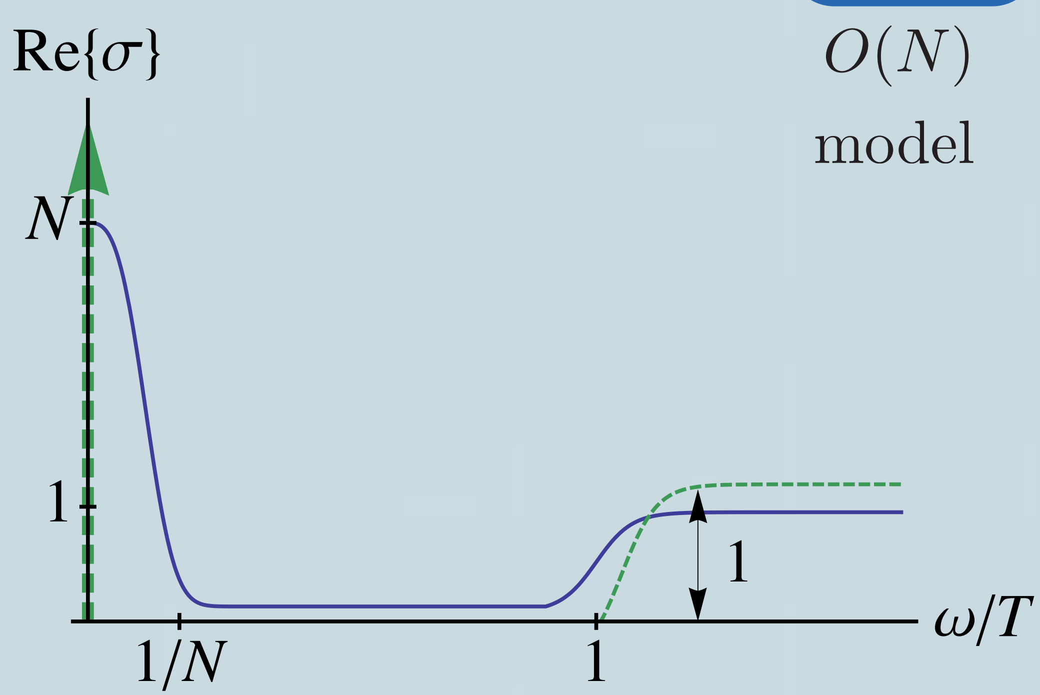
$$\partial_\mu J^\mu = 0$$

$$\sigma(\omega) = \frac{1}{\omega} \langle JJ \rangle \Big|_{\vec{q}=0}$$

Vector large-N

- O(2) → O(N) & take $N \gg 1$
  - N=∞ : **free**
- Conductivity σ(ω/T)





[Damle, Sachdev]

Small *N*,

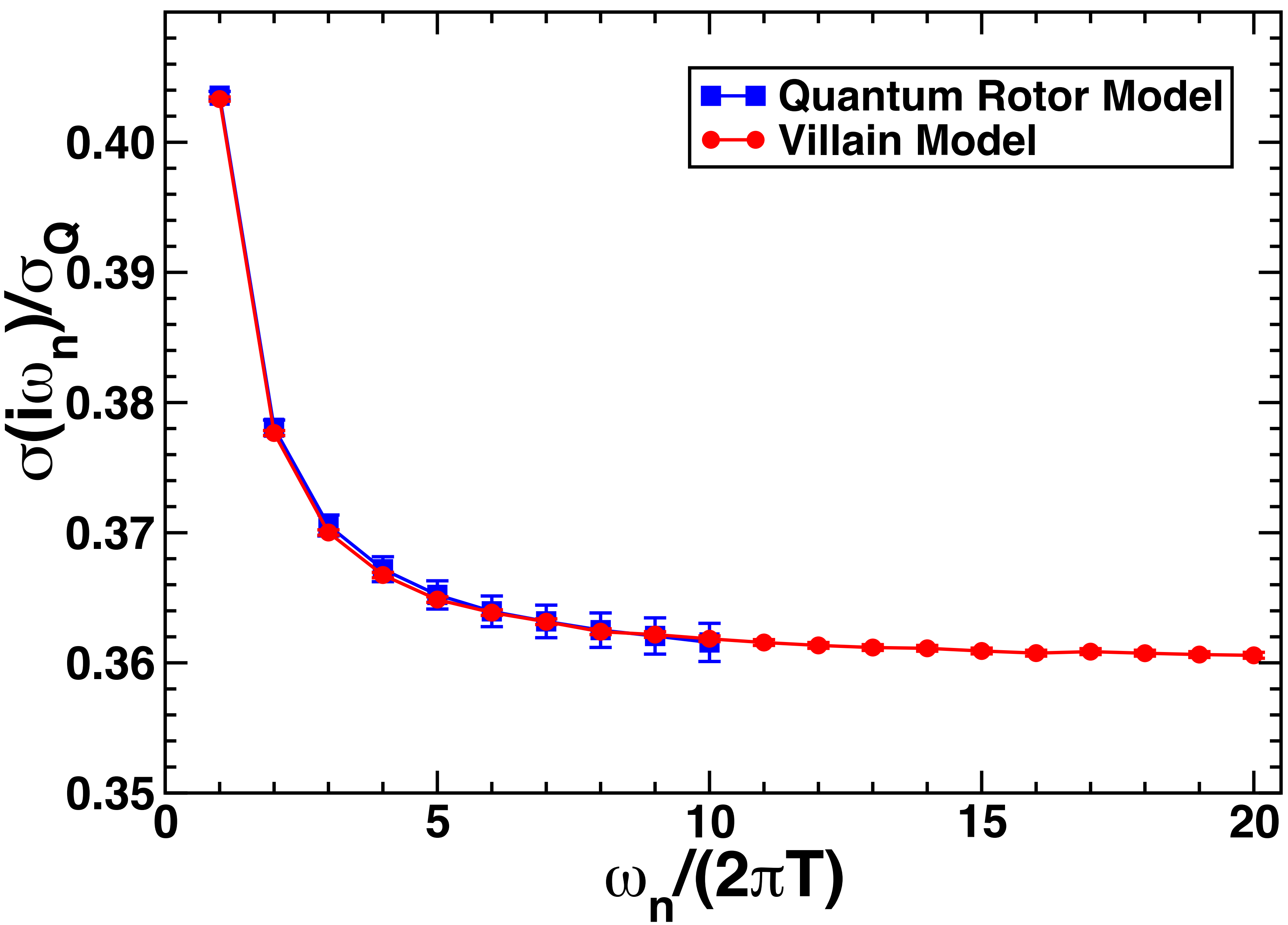
a.k.a. the real world

Quantum Monte Carlo

[W-K, **Sorensen**, Sachdev: 1309.2941]

- Simulate **O(2) QCP** (superfluid / insulator)
  - Loop-current model (Villain)
  - Quantum rotors
- Finite-T but **imaginary time**...
- Analytic continuation difficult!

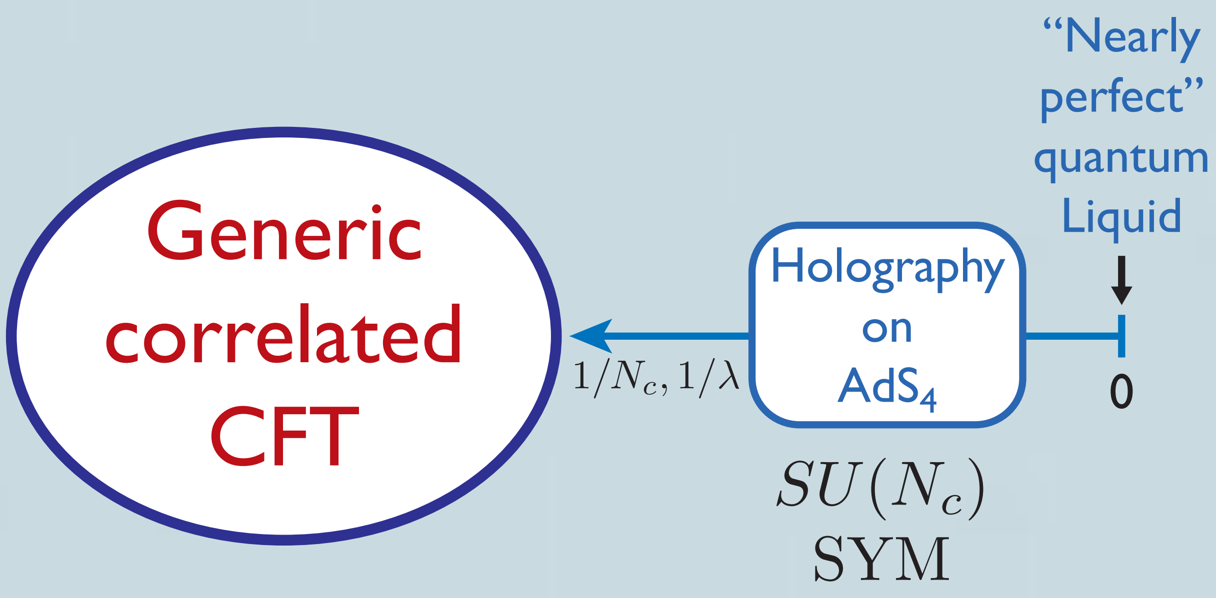
[W-K, **Sorensen**, Sachdev: 1309.2941]



[Prokof'ev *et al*; Gazit *et al*; Trivedi, Randeria *et al*]

Insights from a higher dimension

Matrix large-N



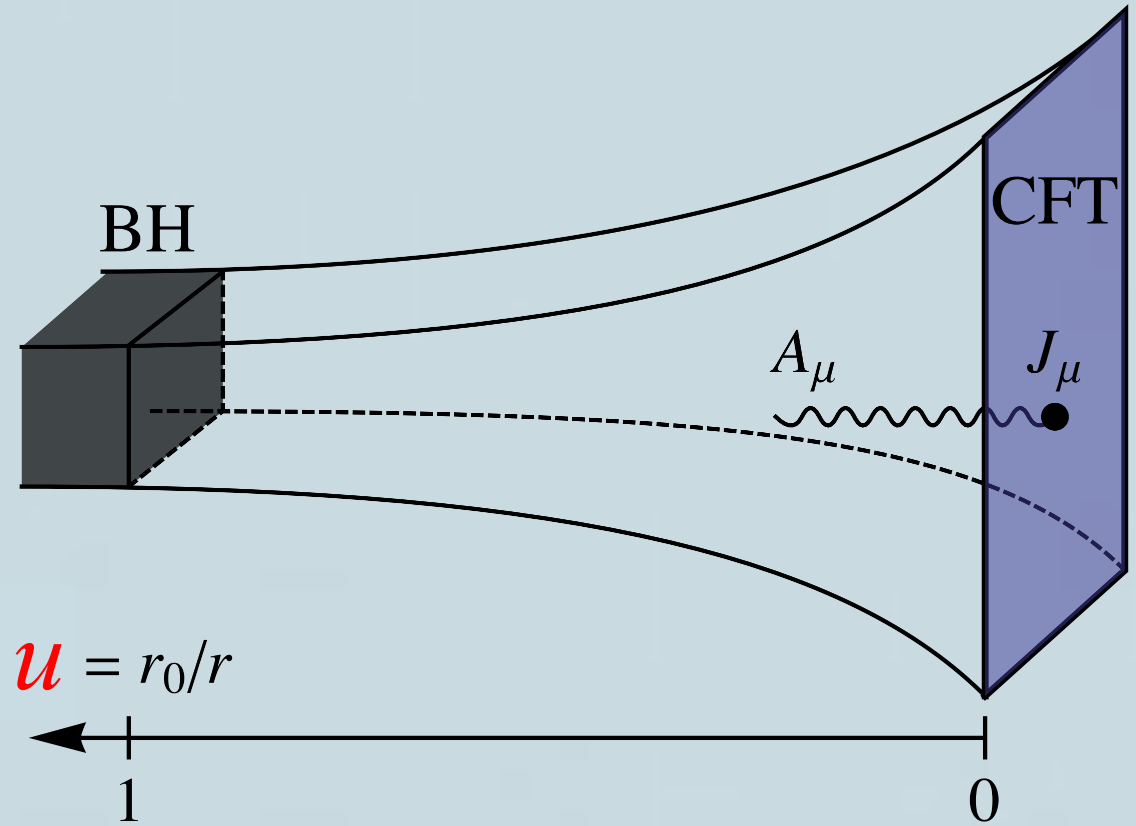
- Yang-Mills SU(N_c) with SUSY in large-N_c limit
  [Maldacena; Ahorony *et al*]

- *R*-charge conductivity via AdS/CFT (no qp's involved)
  [Herzog, Kovtun, Sachdev, Son]

σ via AdS/CFT

[Maldacena *etc*]

spacetime:
B-Hole in AdS₄



$$A_\mu(t, x, y; u) \leftrightarrow J_\mu^{\mathrm{CFT}}(t, x, y)$$

- Solve **classical** EoM for A $\longrightarrow$ get J-correlator in **QFT**

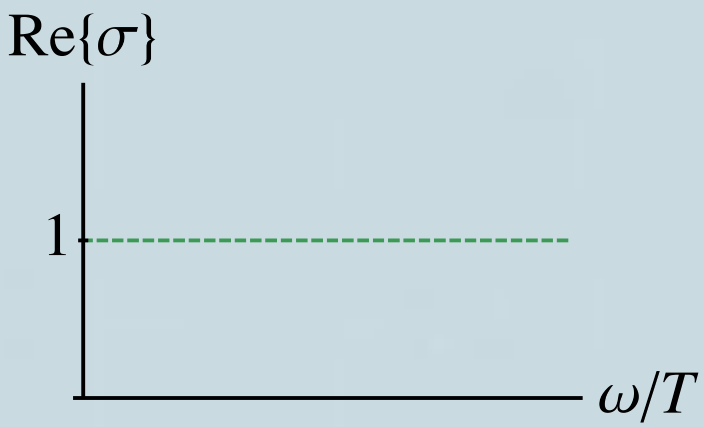
$$\sigma(\omega/T) = \frac{T}{\omega} \left. \frac{\partial_u A_y(\omega, \vec{0}; u)}{A_y(\omega, \vec{0}; u)} \right|_{u=0}$$

Maxwell & beyond

$$S_{\mathrm{bulk}}[A_\mu] = \int d^4x\sqrt{-g}\frac{1}{g_4^2}\left[F^2 + \gamma C^{abcd}F_{ab}F_{cd}\right]$$

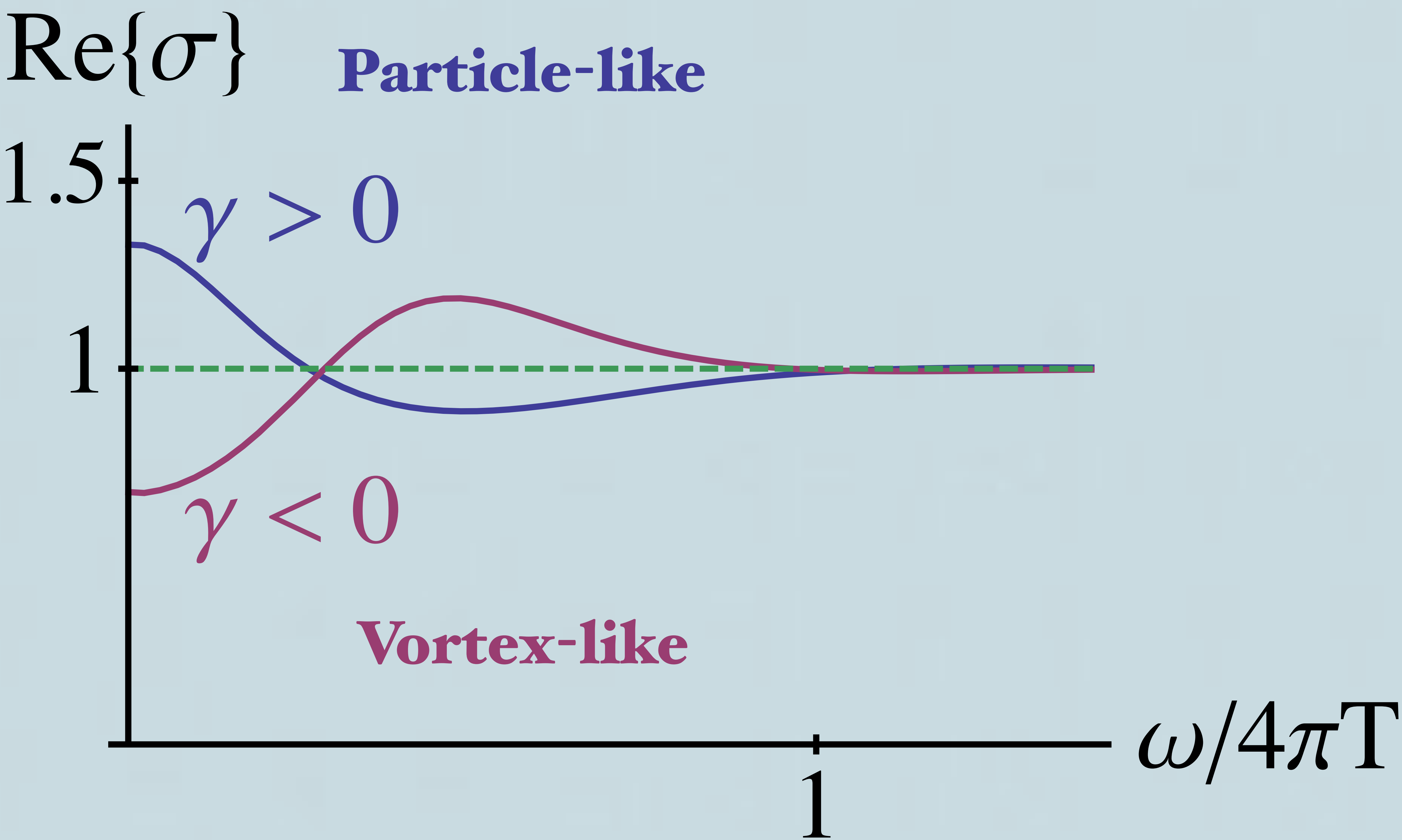
C = traceless part of Riemann

- SUSY Yang-Mills $\gamma = 0$
- Derivative expansion
- Consistency: $|\gamma| \leq 1/12$



[Herzog, Kovtun, Sachdev, Son; Ritz, Ward; Myers, Sachdev, Singh]

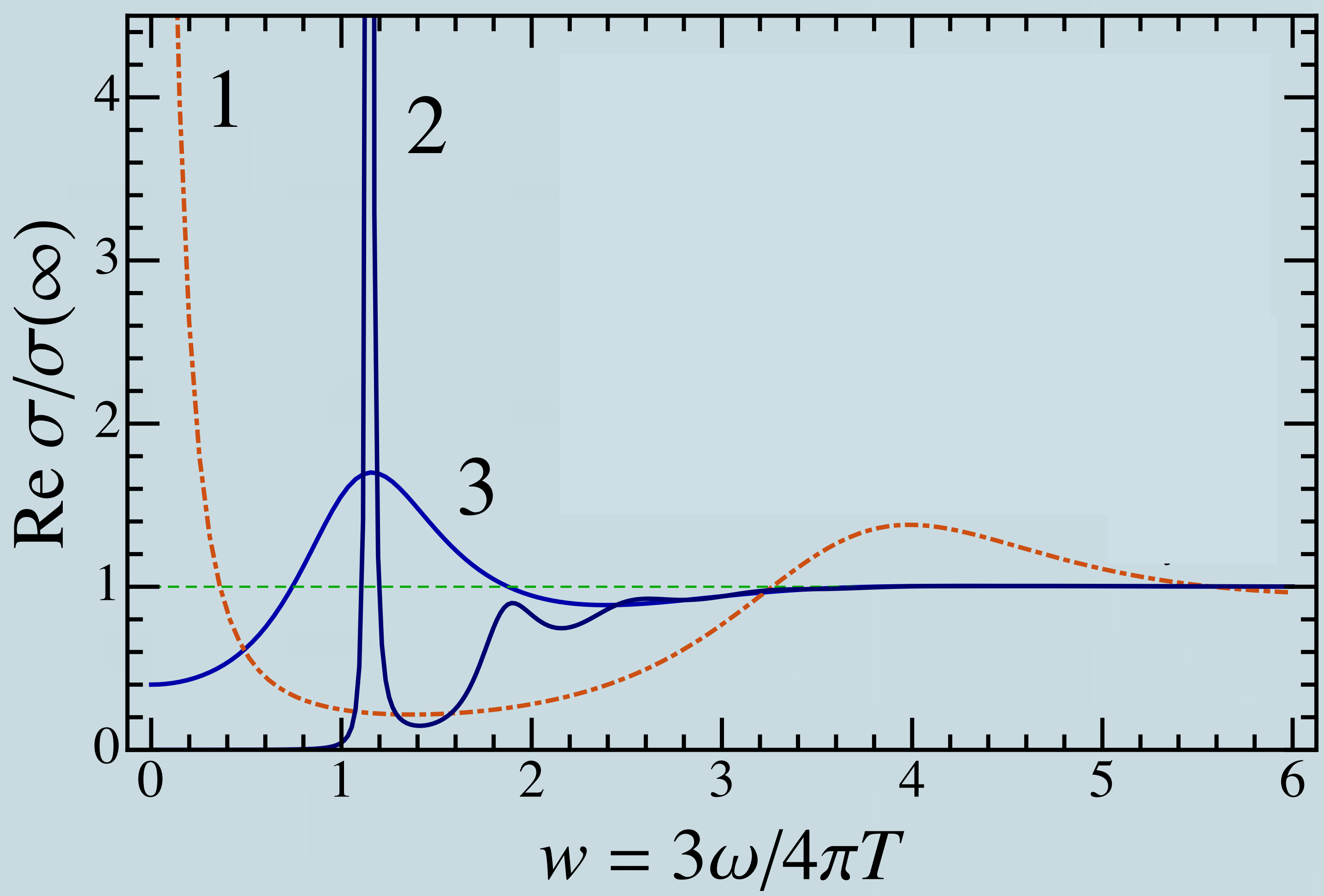
[Myers, Sachdev, Singh]



Higher derivative Pandora's box

[WK 1312:3334 ;
Bai, Pang 1312.3351]

- **Break all bounds** already at 6 derivatives



Quasi-normal modes

[Son, Starinets]

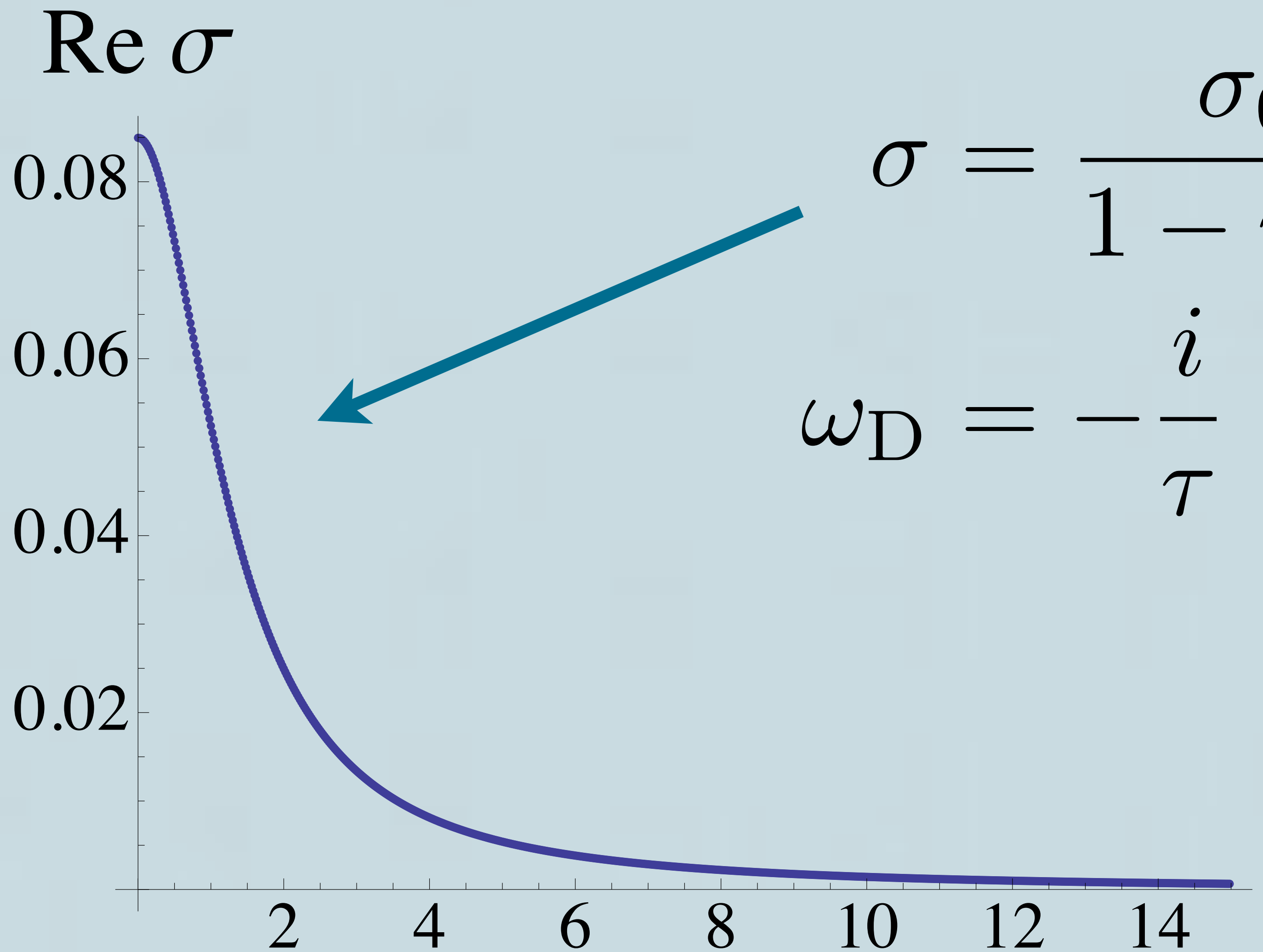
Poles & zeros of $\sigma(\omega/T)$ in the **complex**-frequency plane
$\rightarrow$ NOT qp's!

- **CFT**: Discrete excitations modes $\Rightarrow$ substitute for qp's
- **Gravity**: *Damped* eigenmodes of Black Hole

QNM on CFT side

[W-K, Ghaemi, Senthil, Kim]

O(N) model:



$$\sigma = \frac{\sigma_0}{1 - i\omega\tau}$$

$$\omega_{\mathrm{D}} = -\frac{i}{\tau} \propto -i\frac{T}{N}$$

QNM in graphene

[Fritz, Schmalian, Muller, Sachdev]

Exact solution to QBE to leading order in α :

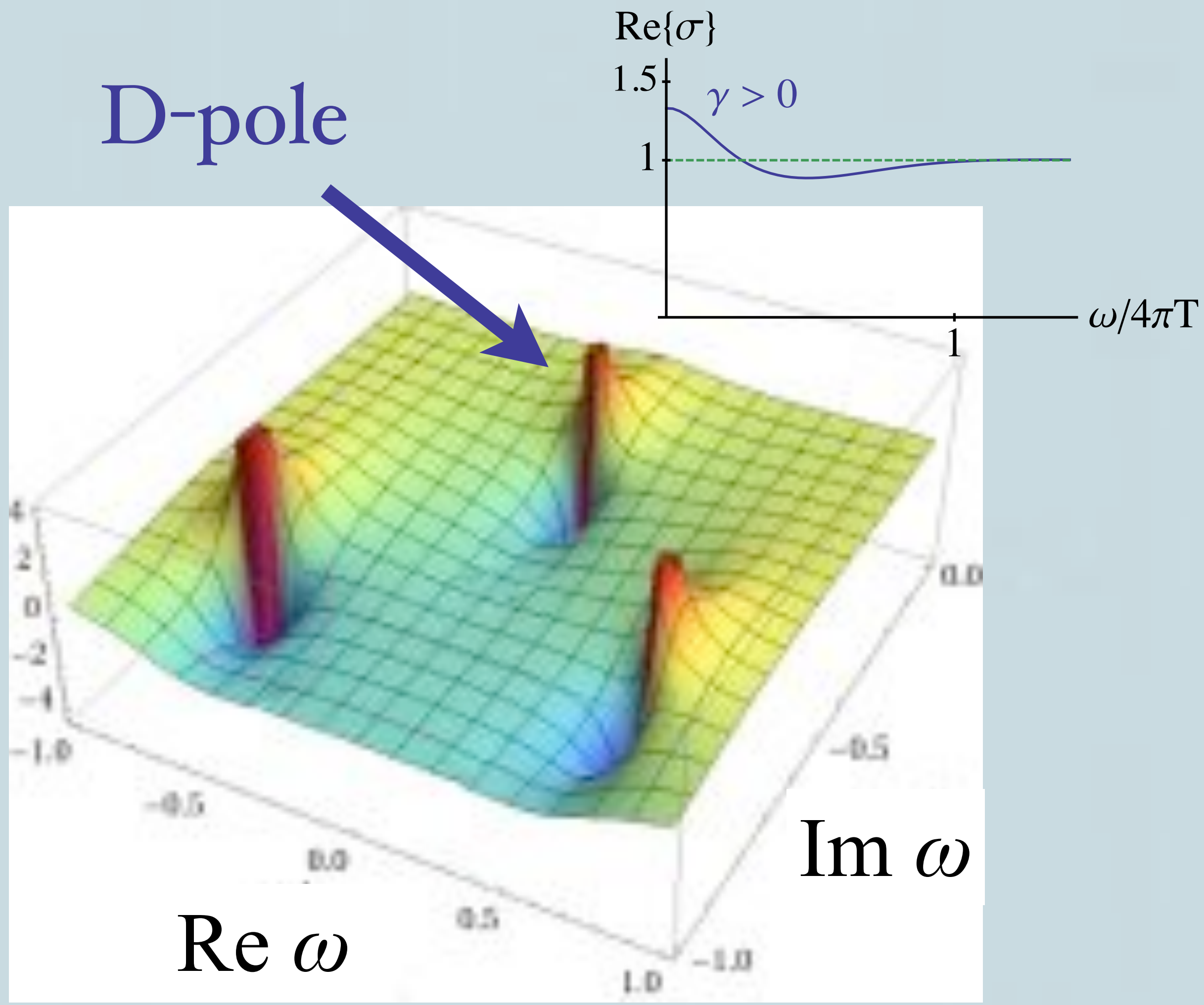
$$\sigma(\omega) = \frac{e^2}{h} \frac{T}{-i\omega + \alpha^2 T}$$

$$\alpha \sim \frac{1}{\ln(\Lambda/T)}$$

$$\boxed{\omega_D = -i\alpha^2 T}$$

Gravitational QNMs

- Behavior in LHP when $\gamma > 0$:



Re σ(ω/T)

“Particle-vortex” (S) duality

CFT: Gauge the U(1)

Gravity: Electric-Magnetic duality for A_μ

$$\sigma \to 1/\sigma$$

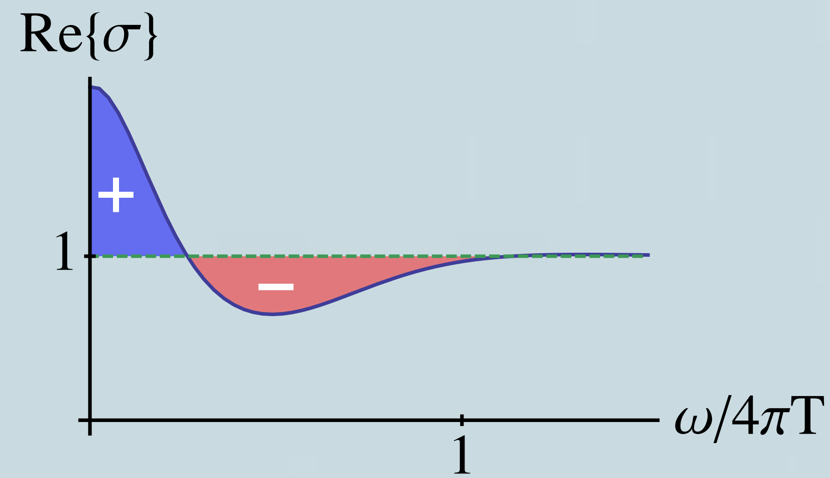
- QNMs: pole $\leftrightarrow$ zero
- $\gamma = 0$: self-dual!
- **Particle**-like σ: D-**pole** $(\gamma > 0)$
  Vortex-like σ: D-**zero** $(\gamma < 0)$

[Herzog *et al*; Myers, Sachdev, Singh; W-K & Sachdev]

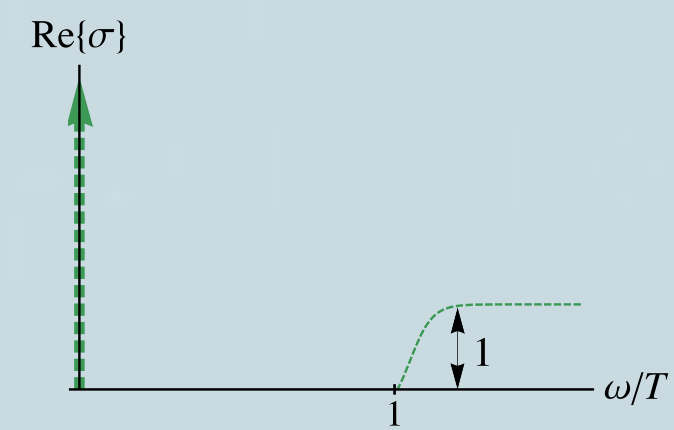
Sum rules

[W-K, Sachdev; Gulotta, Herzog, Kaminski]

$$\boxed{\int_0^\infty d\omega \, [\mathrm{Re}\, \sigma(\omega/T) - \sigma(\infty)] = 0}$$



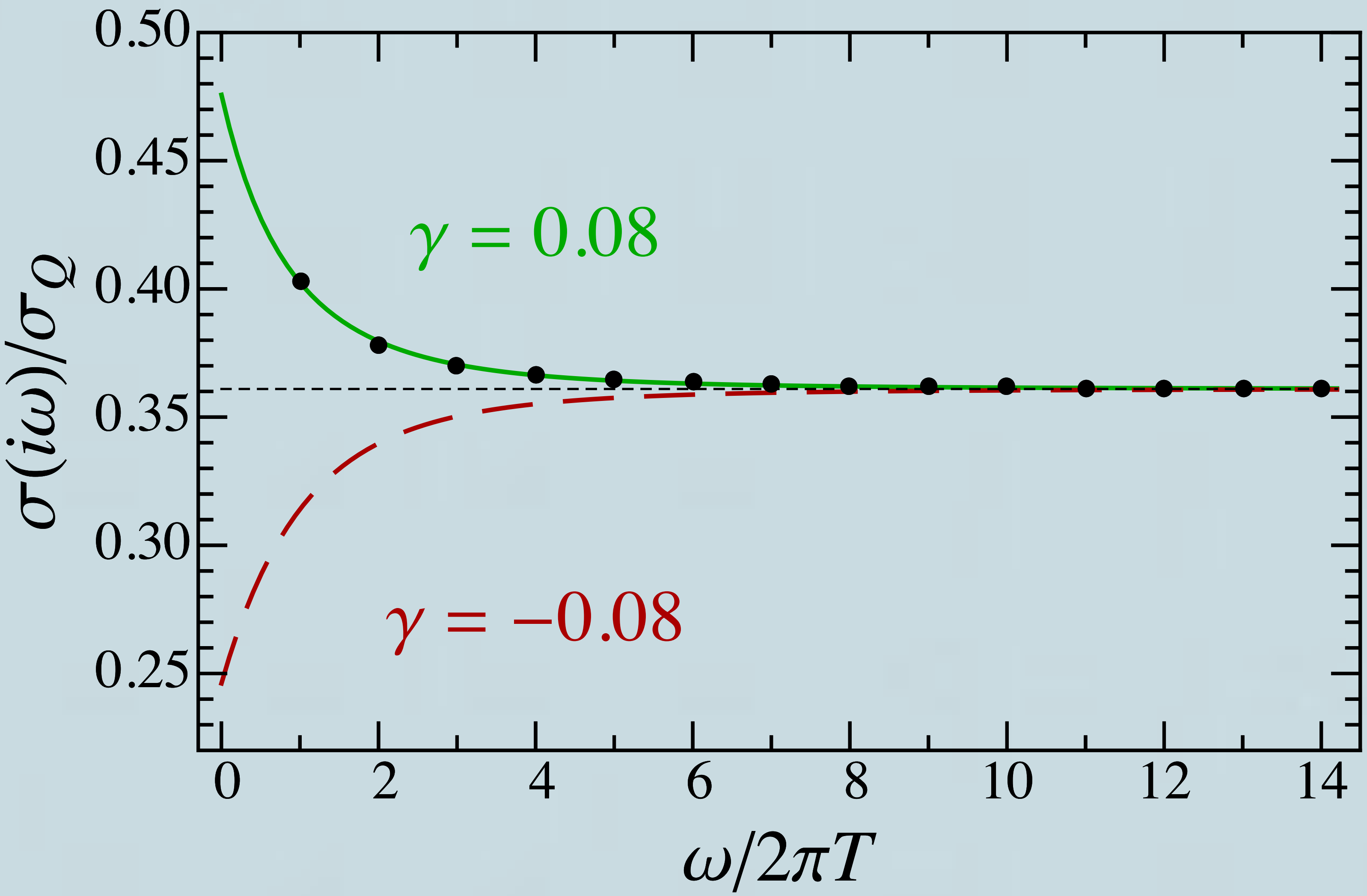
- ✔ super Yang-Mills
- ✔ O(*N*) model in *N*=∞ limit
- ✔ Free Dirac fermions (graphene)



❖ S-dual version:

$$\boxed{\int_0^\infty d\omega \left[\mathrm{Re}\left\{\frac{1}{\sigma(\omega/T)}\right\} - \frac{1}{\sigma(\infty)}\right] = 0}$$

Holographic fit

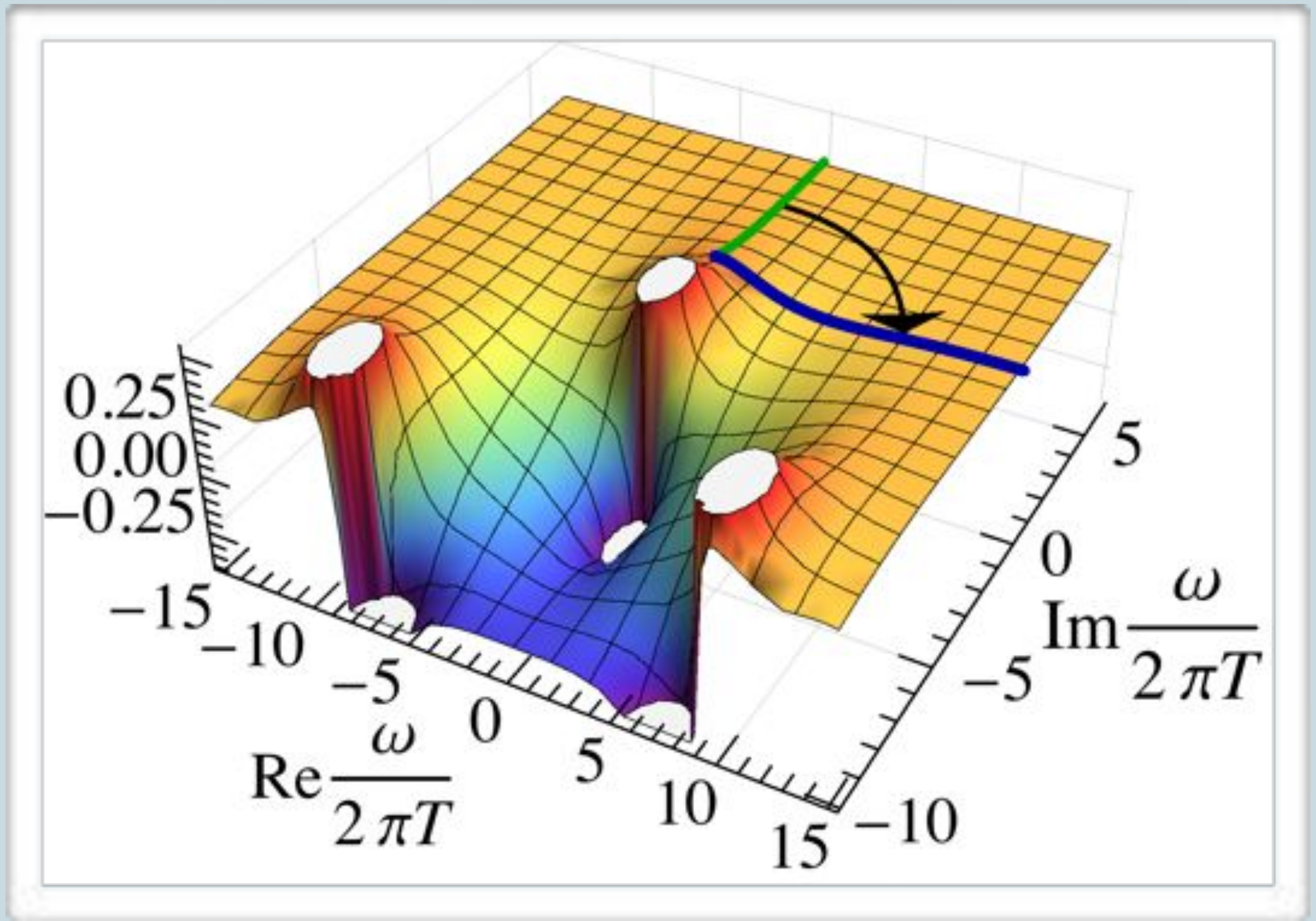


$\omega/T \to \alpha\ \omega/T$
$\alpha = 0.4$

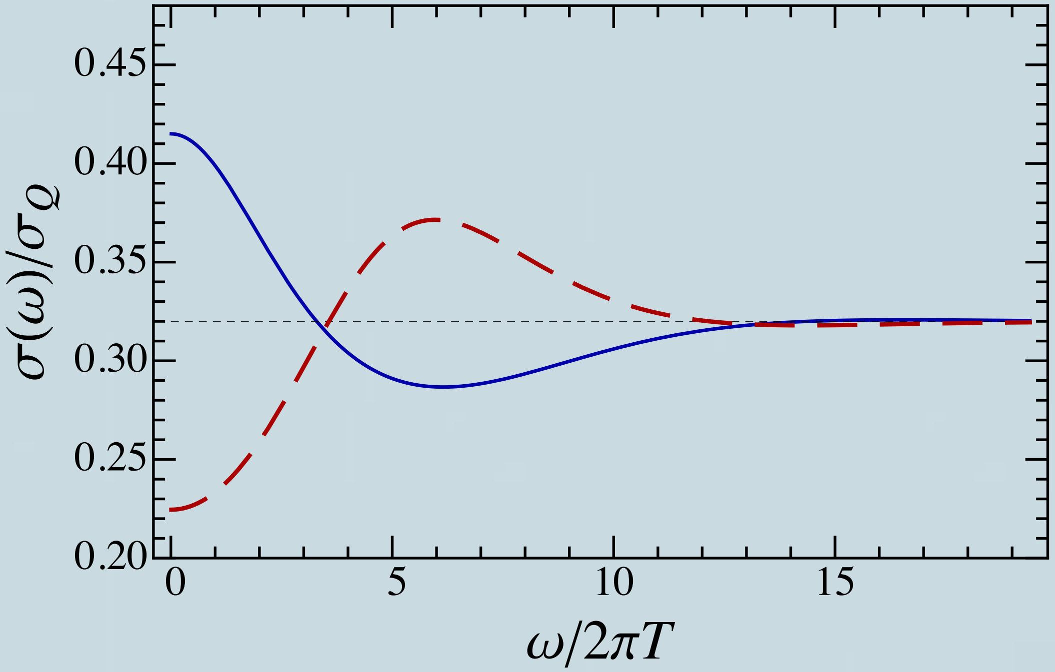
* Superfluid/insulator QCP has **particle-like** transport

* γ satisfies holographic bound

“Holographic continuation”



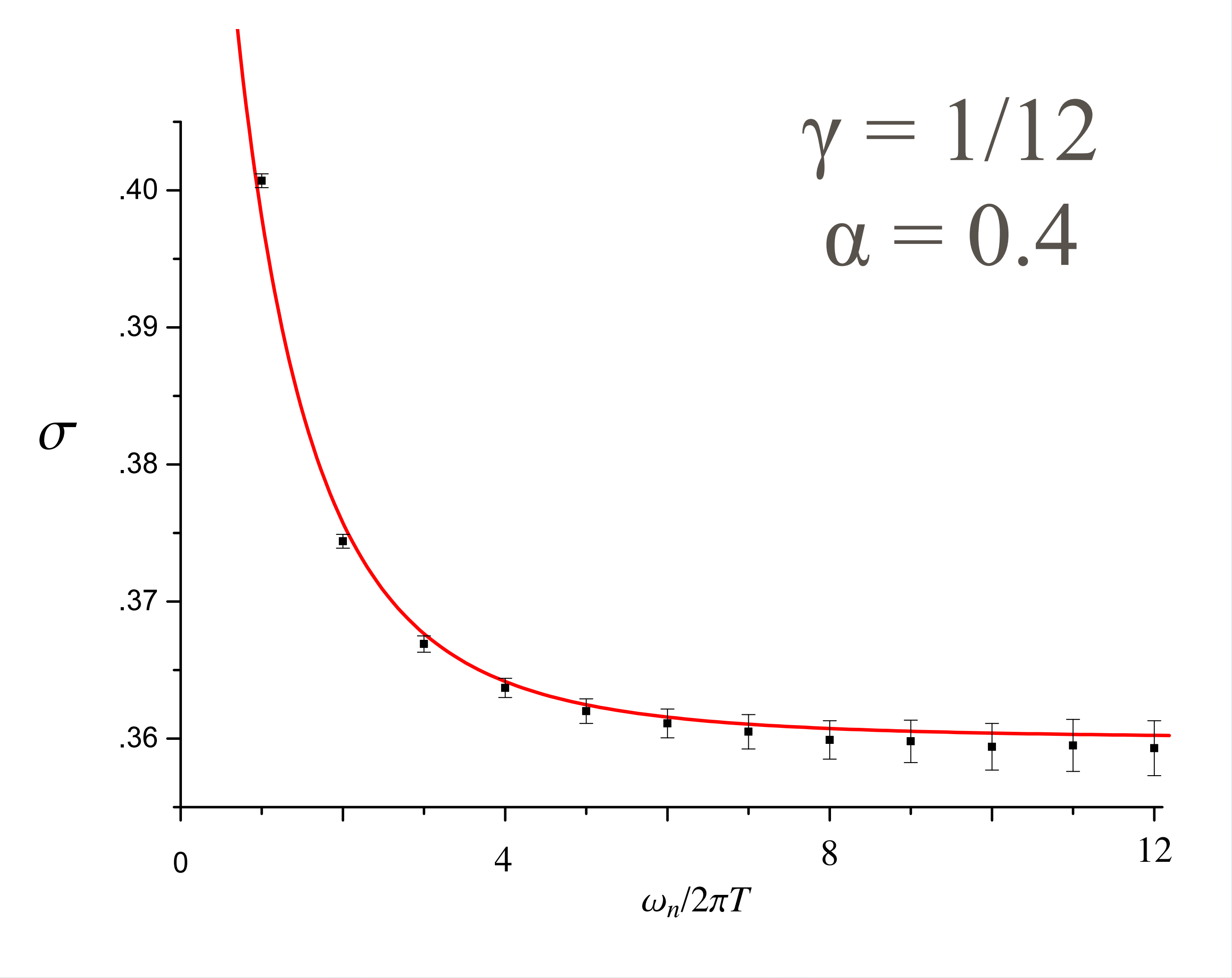
Physical conductivity



More support

[Chen, Liu, Deng, Pollet, Prokof'ev: 1309.5635]

Villain (shown) & Bose-Hubbard



More than continuation

- Fit imaginary frequency σ :
  - get γ
  - **full** $\langle J_\mu J_\nu \rangle$ correlator (including $\boldsymbol{k}$ dependence)

Conclusions

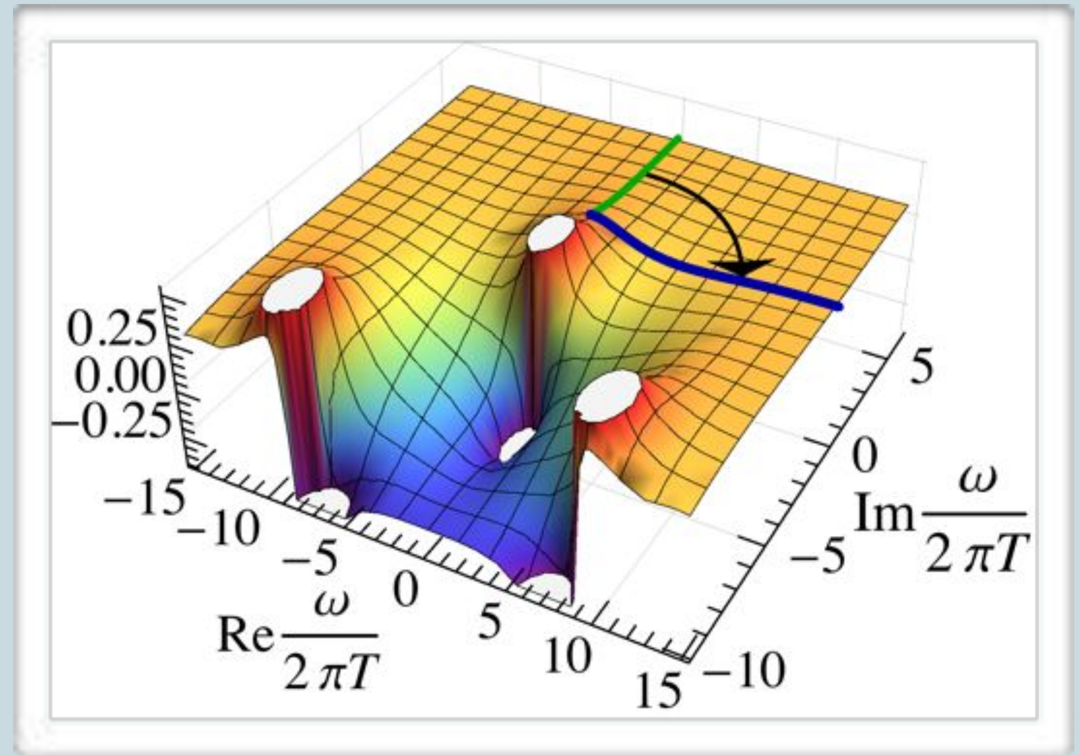
- Conductivity of CFT in 2+1D: $\sigma(\omega/T)$
- AdS/CFT provides non-perturbative insights:
  - **Quasi-normal modes**: poles/zero at complex ω
  - **Sum rules**

★ AdS/CFT useful to interpret **Quantum Monte Carlo** data => statements about superfluid-insulator QCP

Outlook

- Understand rescaling of ω/T
- Different correlation functions
- More simulations:
  O(N) for $N > 2$,   Abelian Higgs
- Experiments?
  - cold atoms? [tricky]

Thanks!



AdS/CFT

- WK, Sachdev, PRB 12 [arXiv:1210.4166]
- WK, Sachdev, PRB 13 [arXiv:1302.0847]
- WK, arXiv:1312:3334

QMC + AdS/CFT

- WK, Sorensen, Sachdev [arXiv:1309.2941]

QFT

- WK, Ghaemi, Senthil, Kim, PRB 12 [arXiv:1206.3309]