Quantum Entanglement
& Stability of Gapless Spin-Liquids

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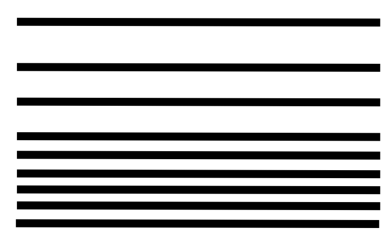
quantum entanglement is a Good Thing...

Holzhey, Larsen, Wilczek; Cardy Calabrese; Casini, Huerta

Characterize 1D & 3D CFTs

Gapless Phases



Detect Fermi Surfaces

Gioev, Klich; Wolf.

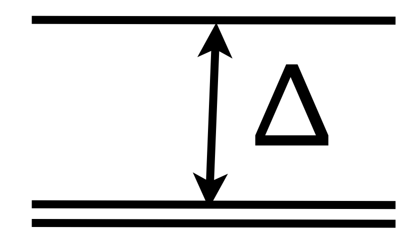
Characterize Symmetry-Broken Phases

Melko, Hastings, Singh; Metlitski, TG

Levin, Wen; Preskill, Kitaev; Zhang et al.

Detect and Characterize Topological Order

Gapped Phases



Detect Edge & Surface States

Li, Haldane; Qi, Ludwig, Katsura; Turner, Berg, Pollman; Fidkowski, Kitaev; Chen, Gu, Wen

- Entanglement can often detect universal properties of a phase, given only the ground state wavefunction.

  "Which phase is it?"

- This talk:

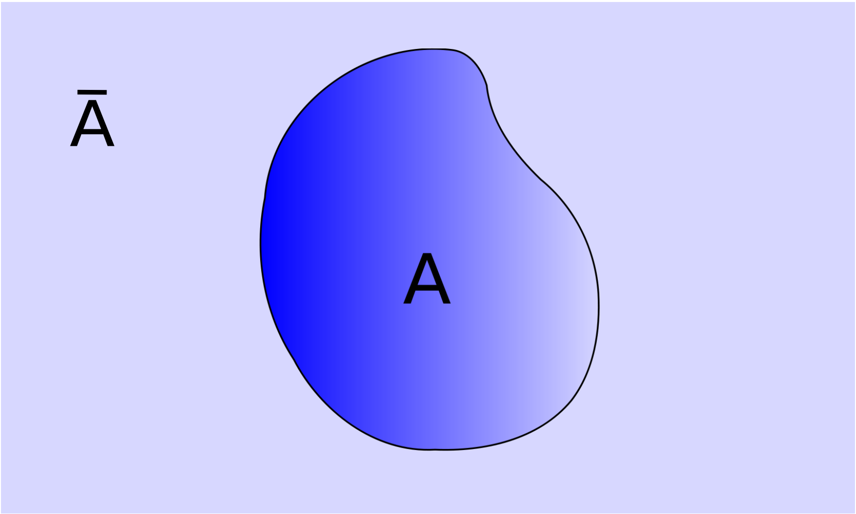
  RG Flows from Quantum Entanglement

  "Is the phase stable?"

  "If not, what are its instabilities?"

Entanglement Entropy

- Divide system into two parts...



Reduced density-matrix for A:

$$\rho_A = \text{Trace}_{\overline{A}} |\psi\rangle\langle\psi|$$

- von-Neumann entropy: $S = -\text{Trace}(\rho_A \log \rho_A)$
- Renyi entropies: $S_n = -\frac{1}{n-1} \log(\text{Trace}\rho_A^n)$
- Zero <u>if and only if</u> product state: $\psi = \phi_A \otimes \phi_{\overline{A}}$

$S = \log(2)$ for EPR singlet $|\psi\rangle = |\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle$

Entanglement & Universality

Entanglement = "Order parameter" for phases and phase transitions

Phase Transitions (Conformal Field Theories)

- 1D: $S \sim c \log(L) + O(1/L)$
- 2D: $S \sim L - \square + O(1/L)$
- 3D: $S \sim L^2 + a \log(L) + O(1/L)$

Phases

- Fermi Surface: $S \sim k\, L^{D-1} \log(L)$
- Topologically Ordered Phase: $S \sim L - \square$

Entanglement & Renormalization Group

Universal part of quantum entanglement for CFTs decreases under RG!

1D: $S_{line\ segement} \sim c \log(L)$

c-theorem (Zamolodchikov): "c" descreases under RG.

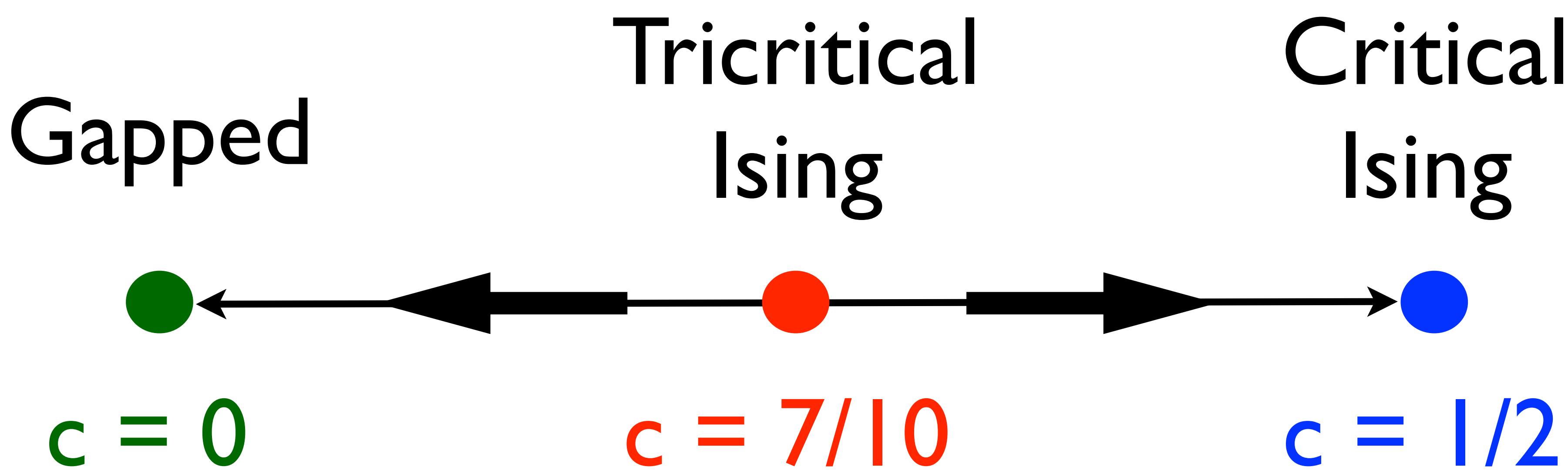
3D: $S_{sphere} \sim L^2 + a \log(L)$

a-theorem (Cardy): "a" decreases under RG.

2D: $S_{circle} \sim L$ - □

□-theorem (Casini, Huerta, Klebanov et al): "□" decreases under RG

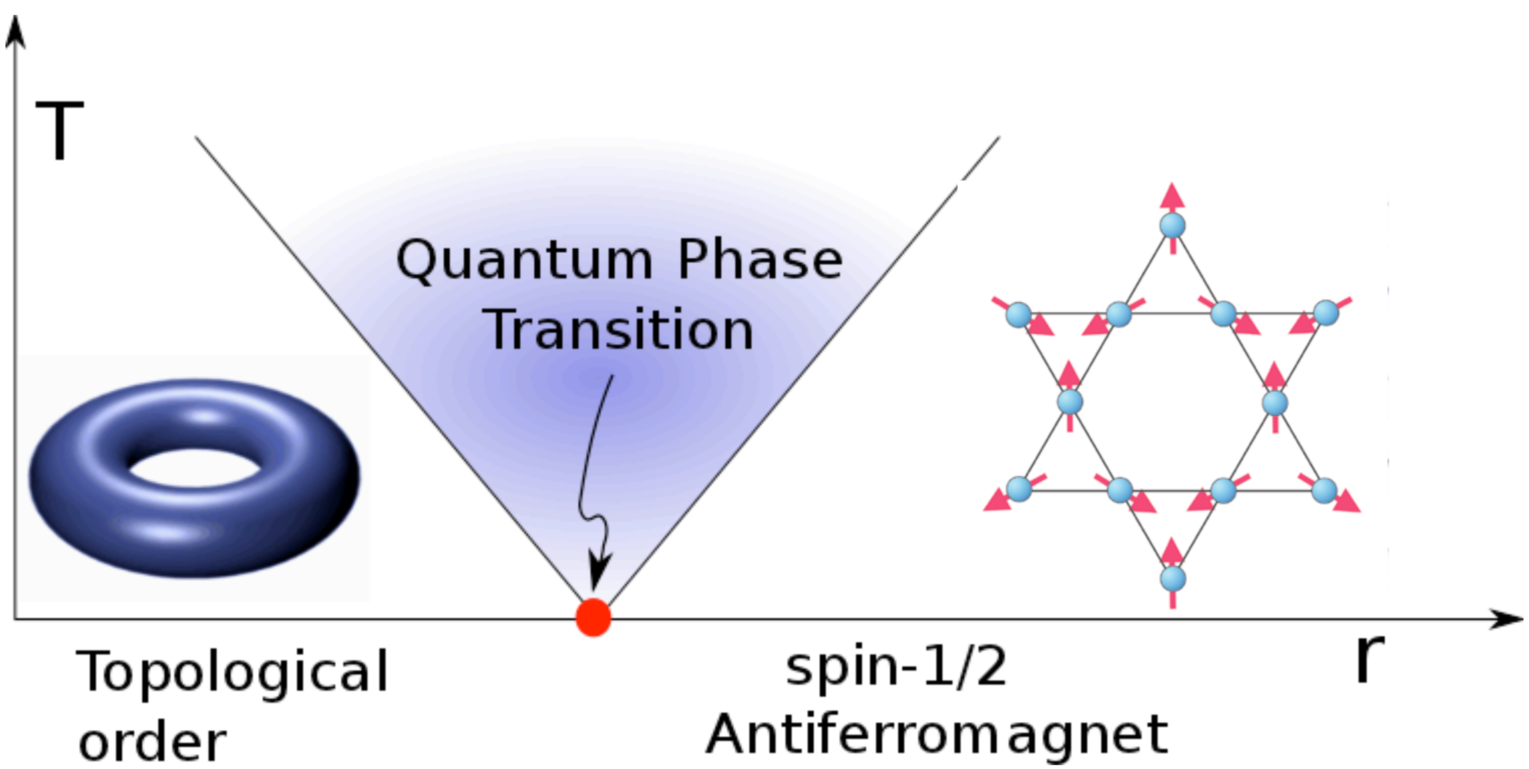
A 1D Example

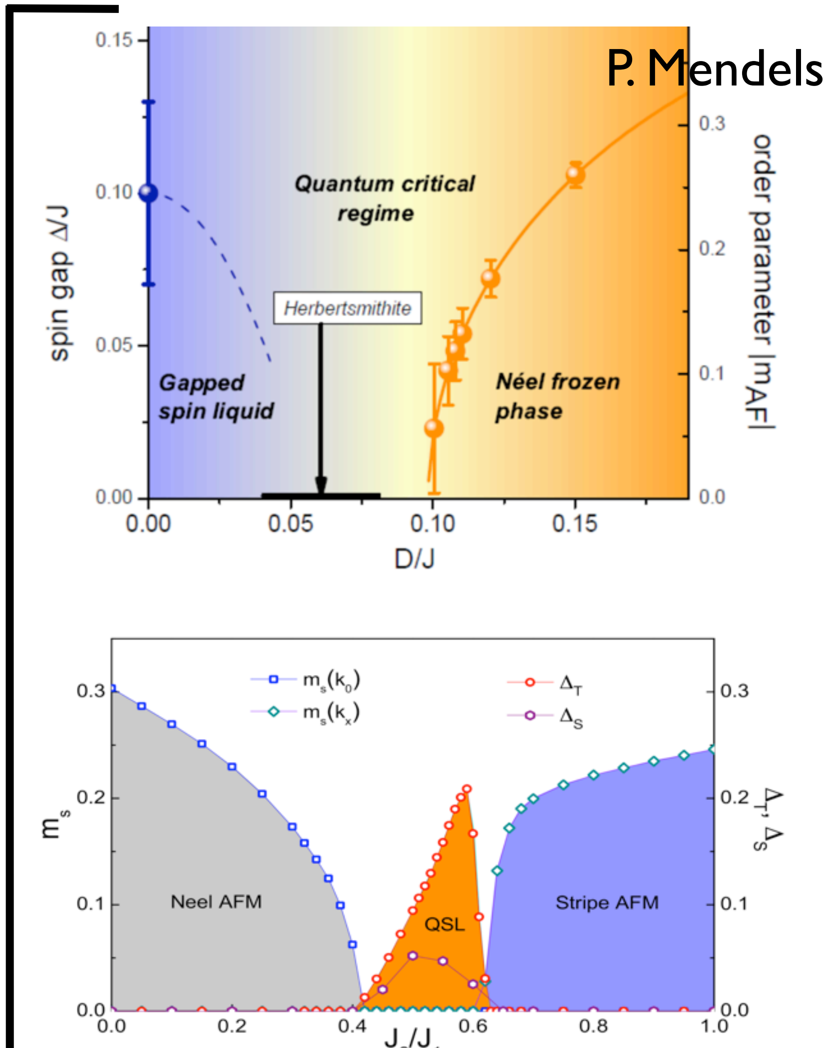


No other possibility!

Applications of entanglement monotonicity (“□ theorem”) to 2+1-d condensed matter systems?

Application I: Entanglement monotonicity & Quantum Phase Transitions





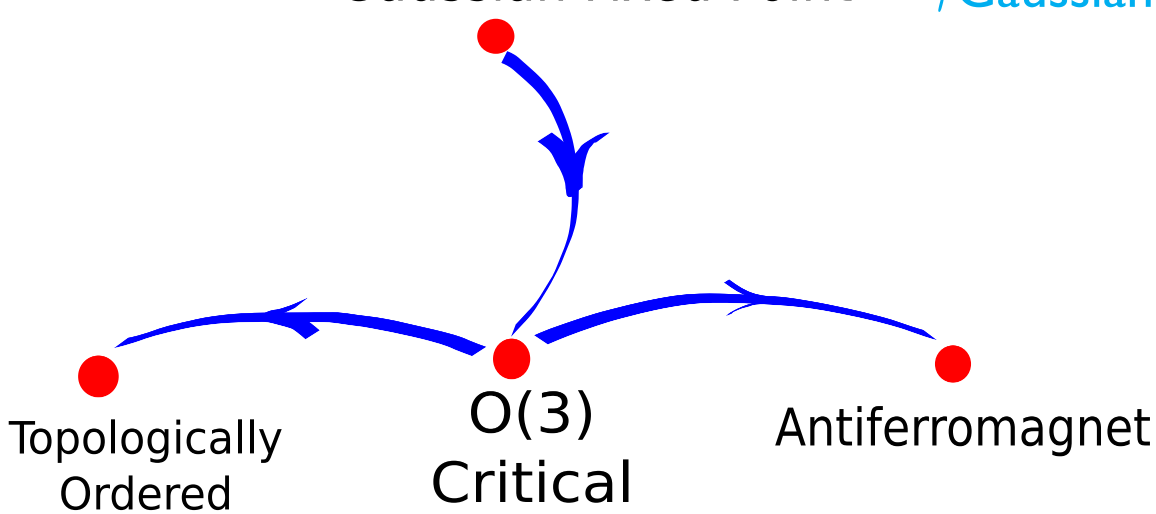
Balents et al 2011;
Wen et al 2011

Question: Nature of transition?

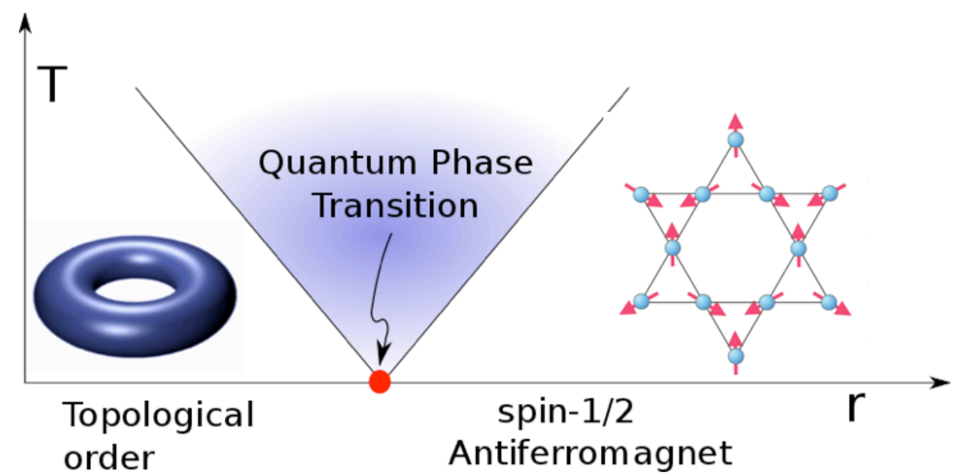
naive Landau-Ginzburg reasoning: O(3) Wilson-Fisher.

A No-Go Theorem for Quantum Phase Transitions

RG flow <u>assuming</u> O(3) transition



$$\gamma_{\text{Gaussian}} = 3 \times \gamma_{\text{Scalar}} = 0.18$$



$$\begin{aligned} \gamma_{\text{Topological}} &= \log(D) \\ &\geq \log(\sqrt{2}) \approx 0.35 \end{aligned}$$

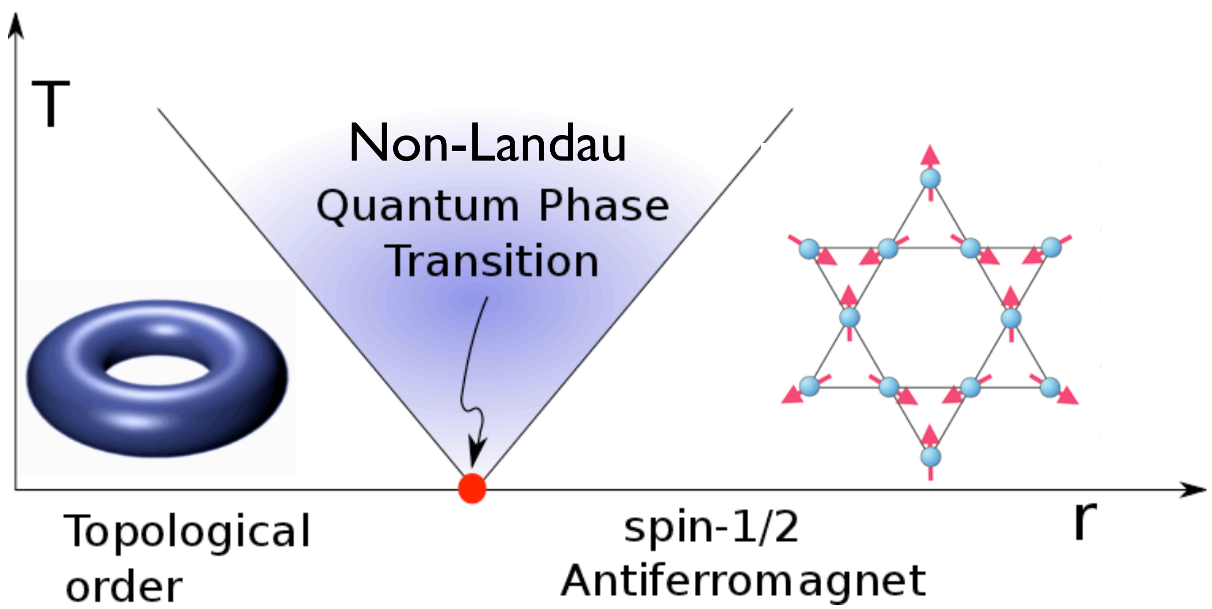
Contradiction with entanglement monotonicity!

$\Rightarrow$ **O(3) Transition impossible!**

Obvious generalizations (SF$\longleftrightarrow$FQH, Nematic $\longleftrightarrow \mathbb{Z}_2$ Spin liquid ...)

Lesson

Phase transitions out of topologically ordered phases *necessarily* lie beyond Landau-Ginzburg paradigm



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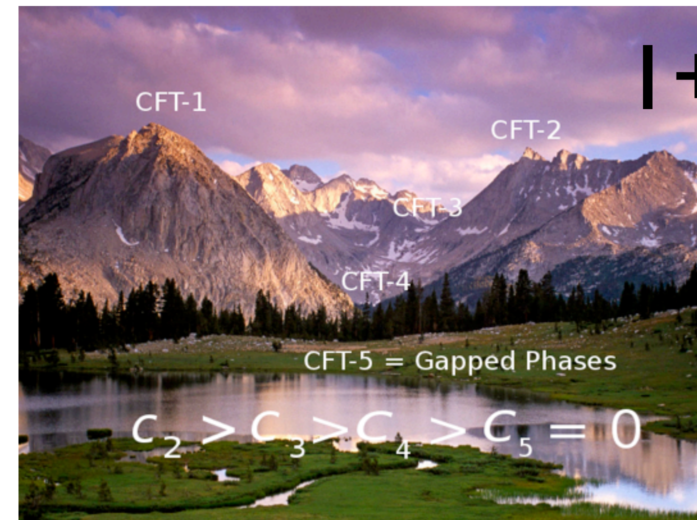
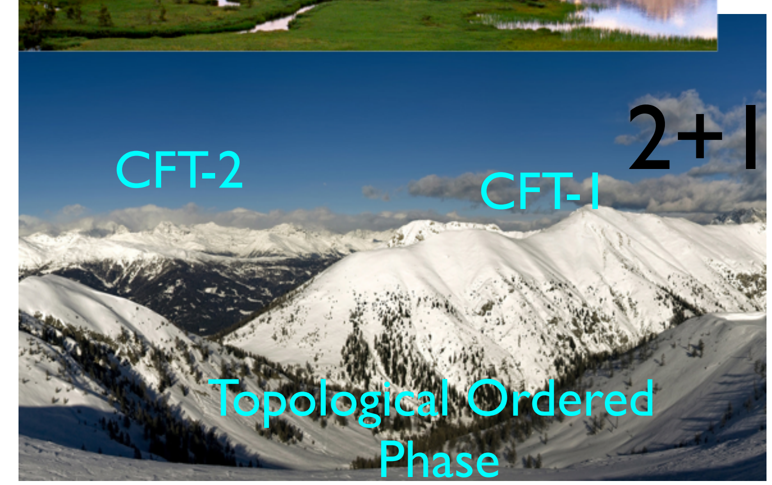
Contrast: 1D Vs 2D





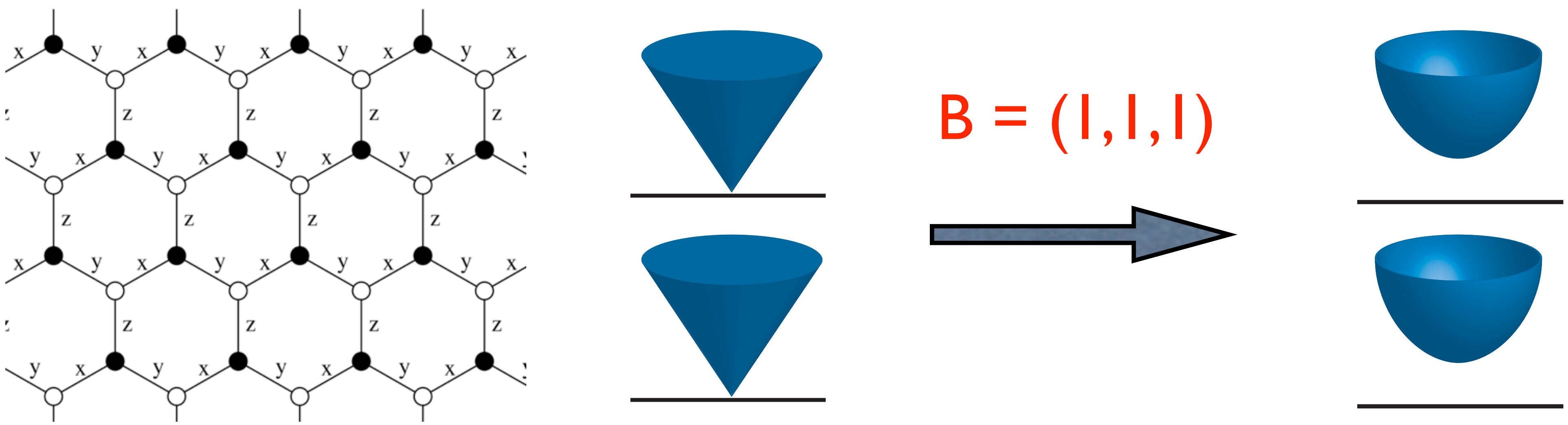
- In 1+1-D, with no symmetries, unique gapped CFT ($c = 0$)

- All c > 0 CFTs can be taken to this unique massive CFT.

- In 2+1-d, more than one gapped CFT.
  Distinct "Topological Ordered Phases".

F-theorem ⇒ A gapless theory may not be deformable to a given massive theory in 2+1-d!

Appetizer: Entanglement & Kitaev's Honeycomb Model

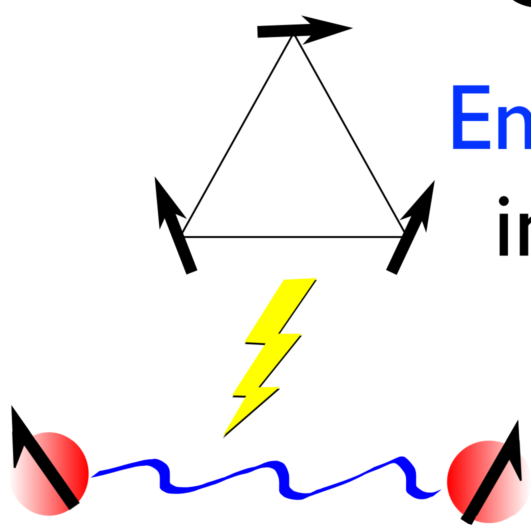


$$\gamma = \underbrace{\gamma_{\text{Dirac}}}_{0.22} + \underbrace{\gamma_{\mathbb{Z}_2\ \text{Topological}}}_{\log(2)} \longrightarrow \underbrace{\gamma_{\text{Ising Topological}}}_{\log(2)}$$

(Yao, Qi 2010)

Again consistent with entanglement monotonicity.

Application II: Entanglement Monotonicity & Stability of Spin-Liquids

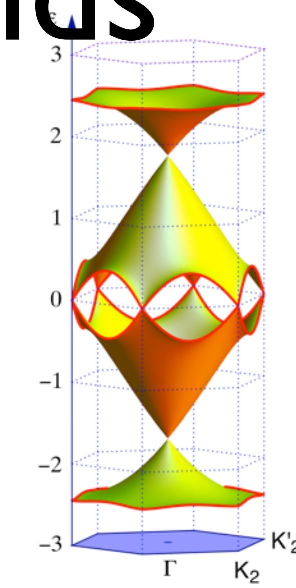


Emergent fermions and photons

in frustrated bosonic systems.

"Gapless Spin-liquids"

(Xiao-Gang Wen 2000)



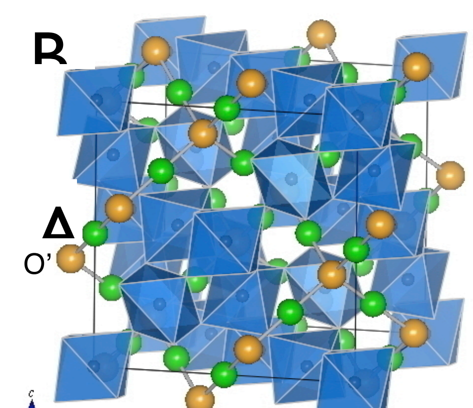
(Corboz et al 2012)

However, many instabilities in two dimensions!

Classic problem from 1970's:

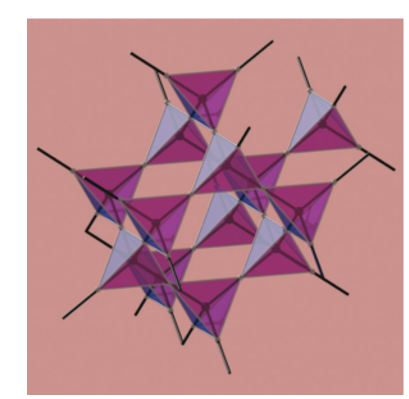
Stability of gauge theories against confinement or symmetry breaking?

3+1-d...



Quantum Spin Ice

Hermele, Balents, Fisher, Savary, Lee, Ross, Onoda, Gaulin.



Spin Ice.

Castelnovo, Moessner, Sondhi 2008.

Phase Diagram of Algebraic Spin-liquids

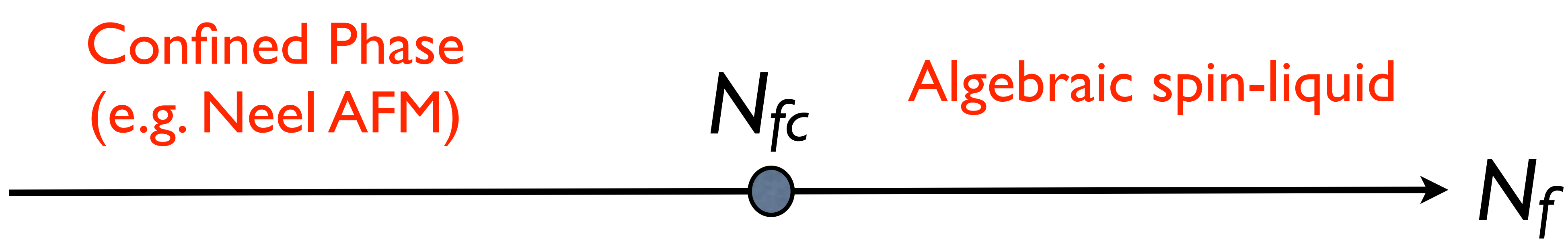
Low-energy description of Algebraic Spin-Liquids:

$$\mathcal{L}_{QED-3} = \sum_{a=1}^{N_f} \overline{\psi}_a \left[-i\gamma_\mu \left(\partial_\mu + i a_\mu\right)\right] \psi_a + \frac{1}{2g^2} F_{\mu\nu} F_{\mu\nu}$$

"QED-3"

ψ = spinon, a = emergent photon.

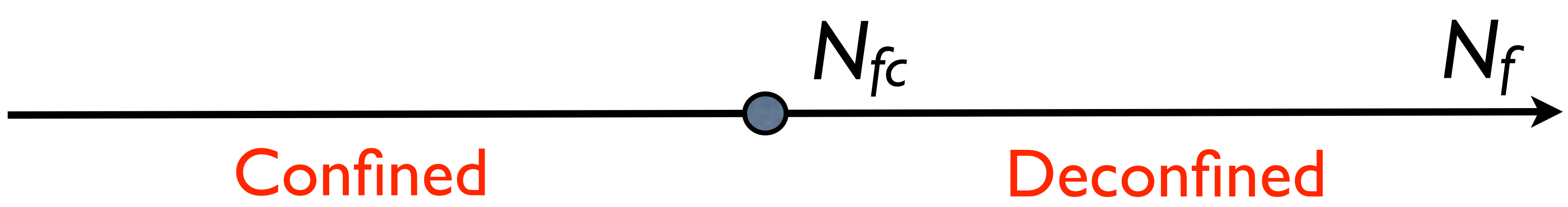
N_f determined by spinon band-structure.



Hermele et al 2005

Critical value of N_f above which ASL stable?

Entanglement Monotonicity & Stability of ASL



Vafa-Witten theorem in 2+1-d:

Massless particles in IR when $N_f > 6$!

What are these massless particles?

One guess:

Goldstone modes due to confinement.

Entanglement Monotonicity & Stability of ASL

Confinement generates N^2_f Goldstone modes

But, these are too many to satisfy entanglement monotonicity when
$N_f > N_{fc}$ where one can put exact upper bound on N_{fc}

$$\gamma_{QED-3} \propto N_f \quad \text{while} \quad \gamma_{\text{Goldstone Phase}} \propto N_f^2$$

Rough estimate: $N_{fc} \simeq 2 \times \frac{\gamma_{\text{Free Dirac Fermion}}}{\gamma_{\text{Free Real Scalar}}}$

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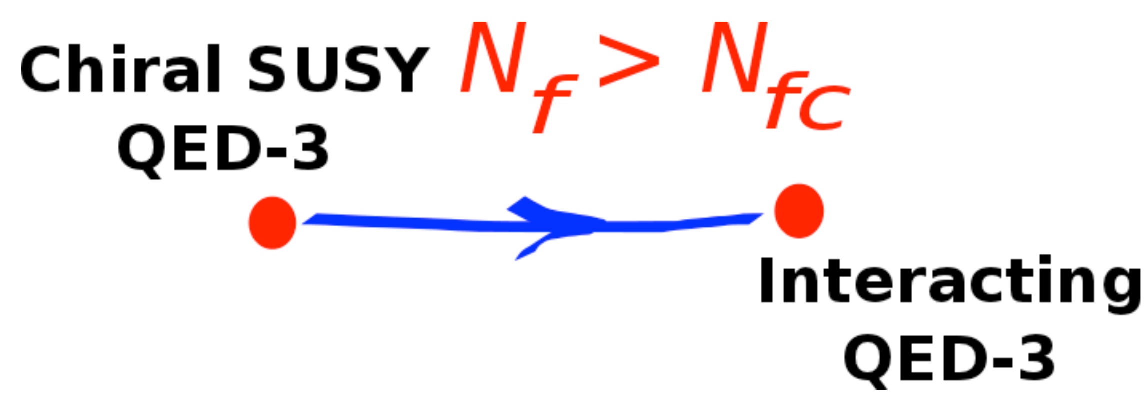
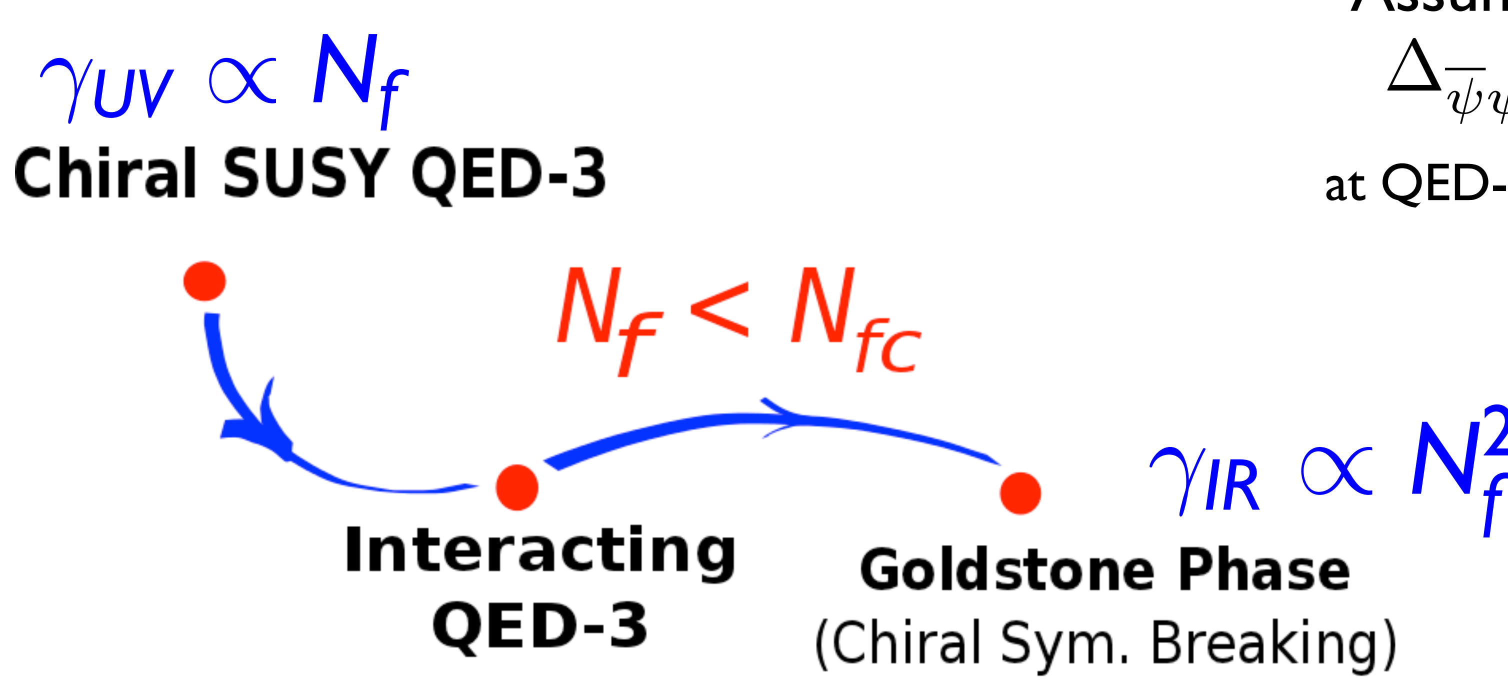
Rigorous bound: "Sandwich" Interacting theory between better-understood theories





Assumption:
$\Delta_{\overline{\psi}\psi} > 1$
at QED-3 fixed point

Rigorous bound: "Sandwich" Interacting theory between better-understood theories

$$\gamma_{\text{SQED-3}} > \gamma_{\text{Goldstone}}$$

Jafferis, Klebanov, Pufu, Safdi, Sachdev

$$\gamma_{SQED-3} = N_f \log(2) + \frac{1}{2}\log\left(\frac{N_f \pi}{2}\right) + \left(\frac{-1}{4} + \frac{10}{3\pi^2}\right)\frac{1}{N_f} + O(N_f^{-2})$$

$$\gamma_{\text{Goldstone}} = 2N_f^2 \gamma_{scalar} + \gamma_{scalar}$$

Deconfinement for $N_f > 13$

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Entanglement Monotonicity & Deconfinement in QED-3

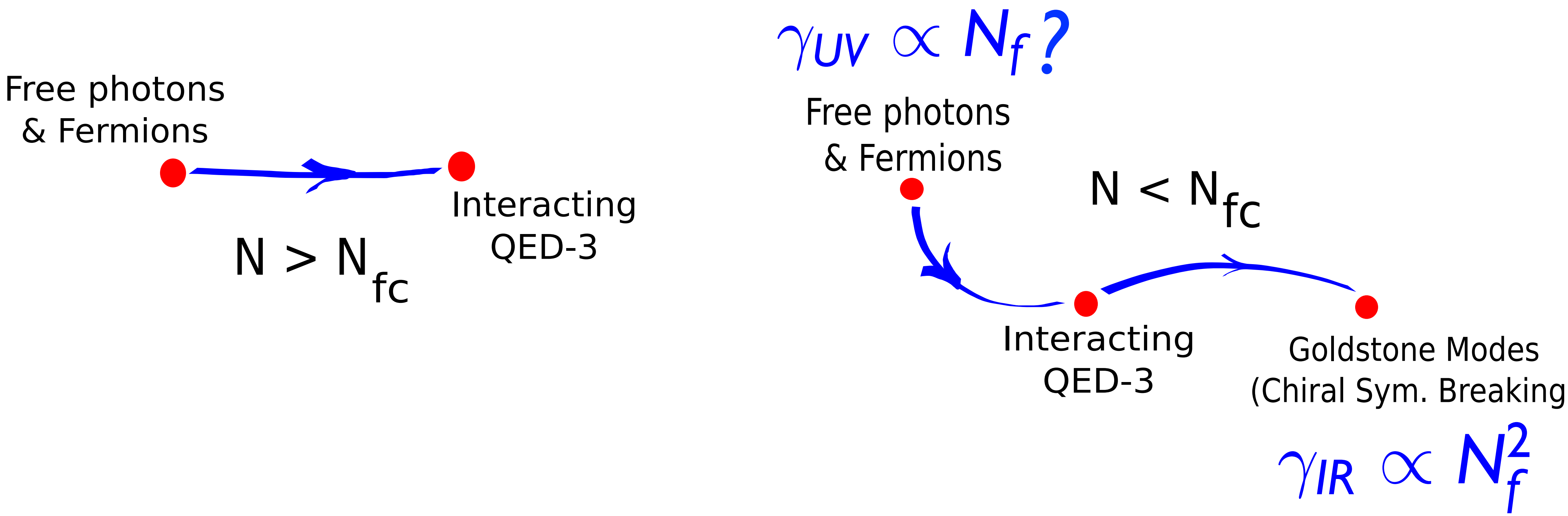
Four Possible Scenarios...

- Confinement without massless particles (not possible for $N_f > 6$, Vafa-Witten)
- Confinement with massless Goldstone modes (not possible for $N_f > 13$, Entanglement monotonicity)
- Deconfinement with mass gap (not possible for $N_f > 6$, Vafa-Witten)
- Deconfinement with massless fermions

Deconfinement with massless fermions for $N_f > 13$

“When you have eliminated the impossible, whatever remains, however improbable, must be the truth.”

A Better Bound?



Deconfinement for $N_f > 7$

Exciting Future Directions...

- Entanglement monotonicity for non-relativistic systems? Instabilities of Fermi and non-Fermi liquids?
- Constraining nature of phase transitions in symmetry protected topological phases?
- Derivation of entanglement monotonicity from wavefunction renormalization? (MERA, tensor-networks,...)?
- A whole new field to explore...

  "Ground State Ontology" (stability of phases via entanglement) as opposed to "Ground State Epistemology" (diagnosing quantum phases via entanglement).

Acknowledgements

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