Low Energy Effective Theories for non-Fermi liquids

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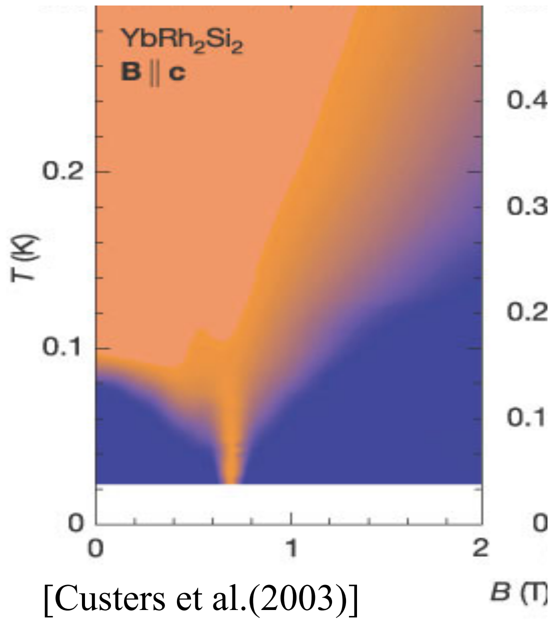
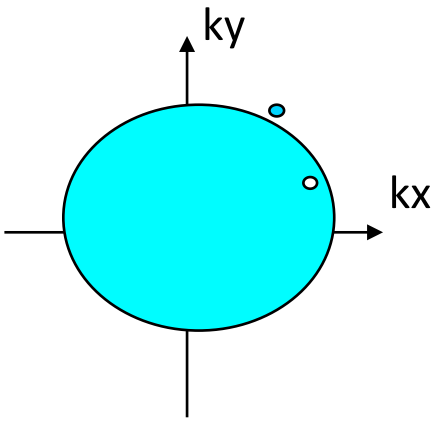
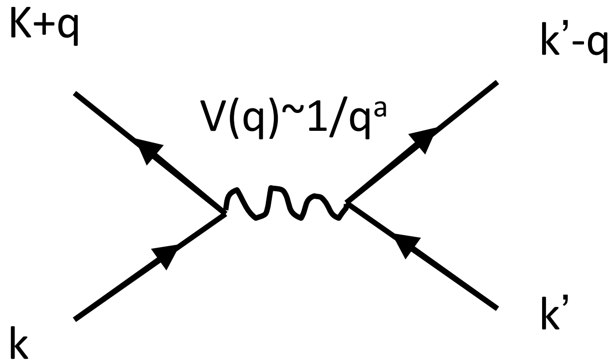
A route to **non-Fermi liquid** :

FS coupled with gapless collective mode



[Custers et al.(2003)]





Fermi surface

+ gapless boson

- QCP in metal (AF, Nematic, CDW, …)
- Bose metal (Quantum spin liquid with spinon FS, …)

- Quantitative theory that replaces FL theory needed

Field Theories for Non-Fermi liquids in 2+1D

- Coupling between fermion and boson grows strong at low energies [Reizer (89); Nagaosa, Lee (92); Halperin, Lee, Read (93), Polchinski(93); Althsuler,Ioffe,Millis(94); Kim, Furusaki, Lee, Wen (94), ….]
- Even in the large N limit, the saddle point approximation breaks down : all planar diagrams are important [SL(09)]
- In closed FS (two-patch theory), even non-planar diagrams become important [Metlitski and Sachdev (10)]
- In the presence of other small parameter (e.g., with modified boson dispersion ~ $|k|^{1+\varepsilon}$), one can approach the interacting theory perturbatively [Nayak, Wilczek(94); Mross, McGreevy, Liu, Senthil(10)]

Q. Can one obtain controlled NFL while keeping locality ?

PERTURBATIVE NON-FERMI LIQUIDS BASED ON A DIMENSIONAL REGULARIZATION

Denis Dalidovich
(Perimeter)

PRB (13)

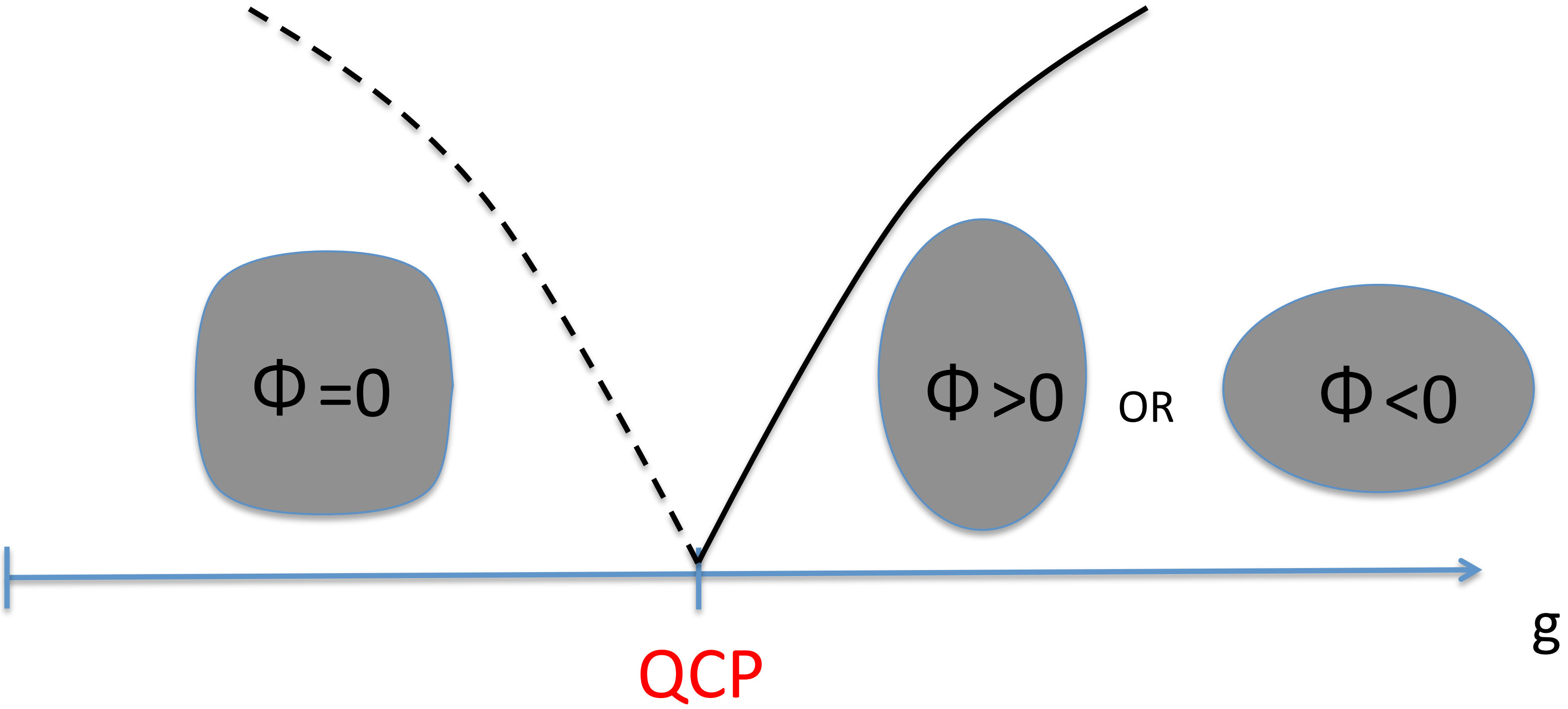
Dimension as tuning parameter

- Couplings become irrelevant above the upper critical dimension
- In dimensions slightly below the upper critical dimensions, couplings become slightly relevant and theories flow into a perturbative fixed point
- The choice of reg. scheme for systematic RG in relativistic QFT
  - Locality
  - Consistent with many symmetries

Earlier works : [Chakravarty, Norton, Syljuasen (95); Senthil, Shankar (09); Fitzpatrick, Kachru, Kaplan, Raghu (13)]

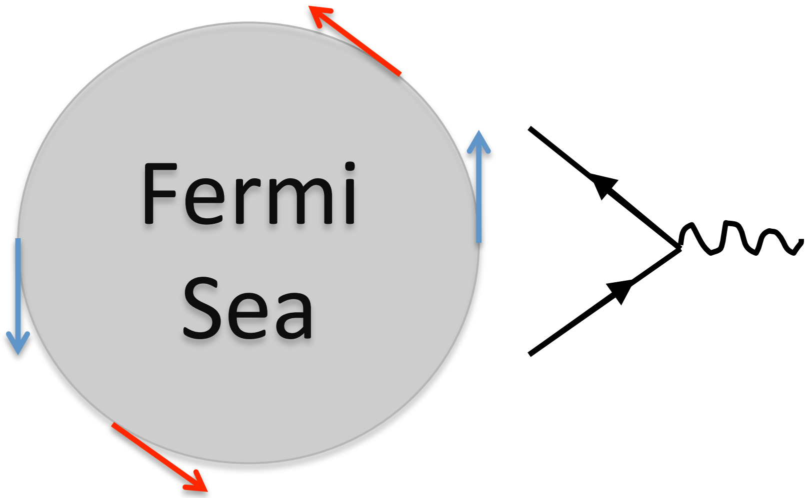
Nematic QPT

[Oganesyan, Kivelson, Fradkin(01); Lawler, Barci, Fernandez, Fradkin, Oxman(06); Metlitski, Sachdev (10), …]

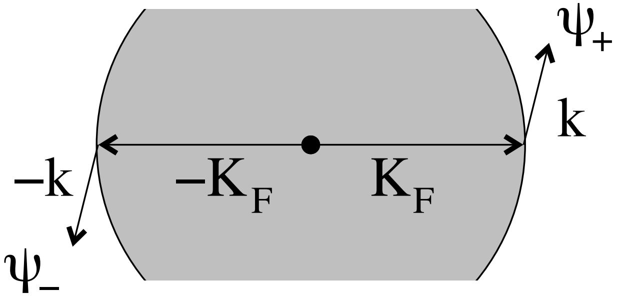


Spontaneous breaking of discrete rotational symmetry

Patch theory



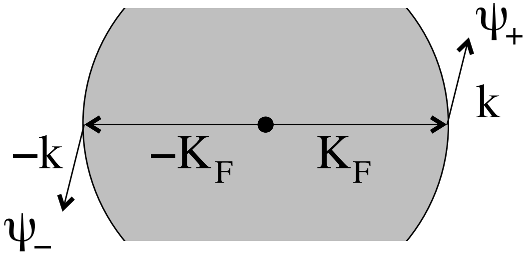
- Fermions are primarily scattered along the directions tangential to FS
- At low energies, fermions with different tangential vectors are decoupled from each other

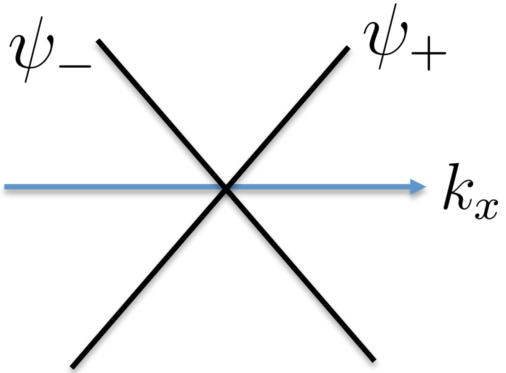


$$\begin{aligned}
S \;=\; & \sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^3k}{(2\pi)^3}\psi^{\dagger}_{s,j}(k)\left[ik_0 + sk_x + k_y^2\right]\psi_{s,j}(k) \\
& + \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left[k_0^2 + k_x^2 + k_y^2\right]\phi(-k)\phi(k) \\
& + \frac{e}{\sqrt{N}}\sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^3k d^3q}{(2\pi)^6}\;\phi(q)\;\psi^{\dagger}_{s,j}(k+q)\psi_{s,j}(k).
\end{aligned}$$

FS as a line of Dirac fermions

$$
\begin{aligned}
S \; = \; & \sum_j \int \frac{d^3k}{(2\pi)^3} \bar{\Psi}_j(k) \left[i k_0 \gamma_0 + i(k_x + k_y^2)\gamma_1 \right] \Psi_j(k) \\
+ \; & \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \left[k_0^2 + k_x^2 + k_y^2 \right] \phi(-k)\phi(k) \\
+ \; & \frac{e}{\sqrt{N}} \sum_j \int \frac{d^3k dq}{(2\pi)^6} \phi(q) \bar{\Psi}_j(k+q) \gamma_1 \Psi_j(k)
\end{aligned}
$$





$$
\Psi_j(k) = \begin{pmatrix} \psi_{+,j}(k) \\ \psi^\dagger_{-,j}(-k) \end{pmatrix}
$$

Theory in general dimensions

Add (d-2) new co-dimensions

$$
\begin{aligned}
S &= \sum_j \int \frac{d^3k}{(2\pi)^3} \bar{\Psi}_j(k) \left[i k_0 \gamma_0 + i(k_x + k_y^2)\gamma_1 \right] \Psi_j(k) \\
&+ \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \left[k_0^2 + k_1^2 + k_2^2 \right] \phi(-k)\phi(k) \\
&+ \frac{e}{\sqrt{N}} \sum_j \int \frac{d^3k dq}{(2\pi)^6} \phi(q) \bar{\Psi}_j(k+q) \gamma_1 \Psi_j(k)
\end{aligned}
$$

$$
\begin{aligned}
S &= \sum_j \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \bar{\Psi}_j(k) \left[i\vec{\Gamma} \cdot \vec{K} + i\gamma_{d-1}\delta_k \right] \Psi_j(k) \\
&+ \frac{1}{2} \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \left[|\vec{K}|^2 + k_{d-1}^2 + k_d^2 \right] \phi(-k)\phi(k) \\
&+ \frac{ie}{\sqrt{N}} \sqrt{d-1} \sum_j \int \frac{d^{d+1}k d^{d+1}q}{(2\pi)^{2d+2}} \phi(q) \bar{\Psi}_j(k+q) \gamma_{d-1} \Psi_j(k)
\end{aligned}
$$

$$k_0 \to \vec{K}$$

$$\vec{K} \equiv (k_0, k_1, ..., k_{d-2})$$

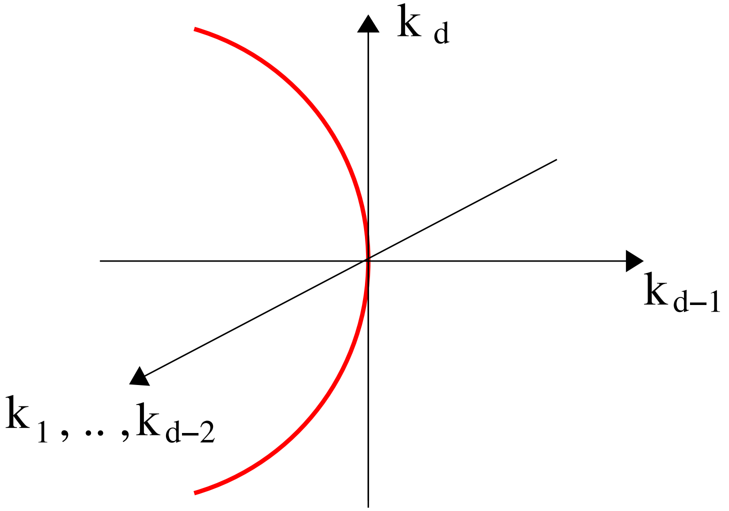
$$\gamma_0 \to \vec{\Gamma}$$

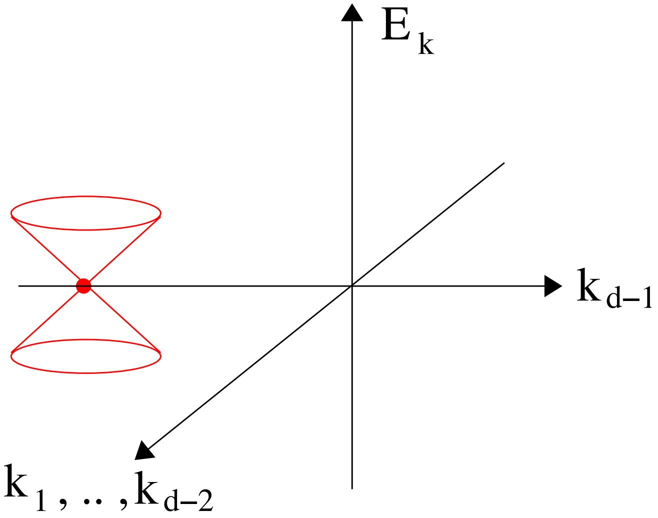
$$\vec{\Gamma} \equiv (\gamma_0, \gamma_1, ..., \gamma_{d-2})$$

$$\gamma_1(k_x + k_y^2) \to \gamma_{d-1}\delta_k$$

$$\delta_k \equiv (k_{d-1} + \sqrt{d-1}k_d^2)$$

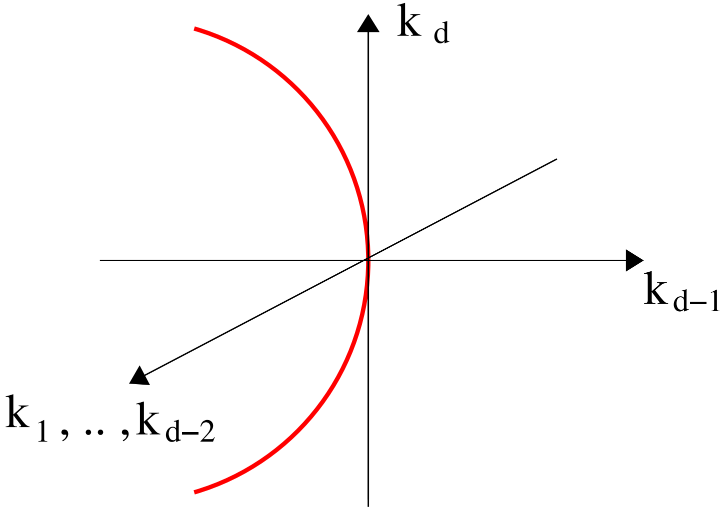
1-dimensional line of Dirac points embedded in d-dimensional k-space

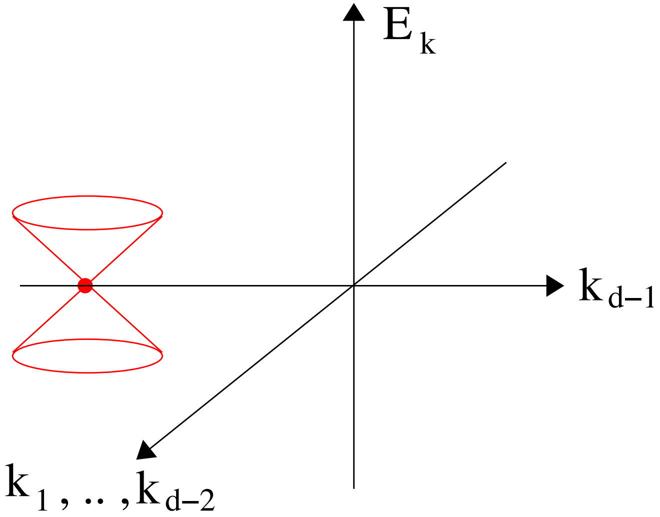




$$E_k = \pm\sqrt{k_1^2 + k_2^2 + .. + k_{d-2}^2 + \delta_k^2}$$

The theory at d = 3 describes a spin triplet p-wave SC

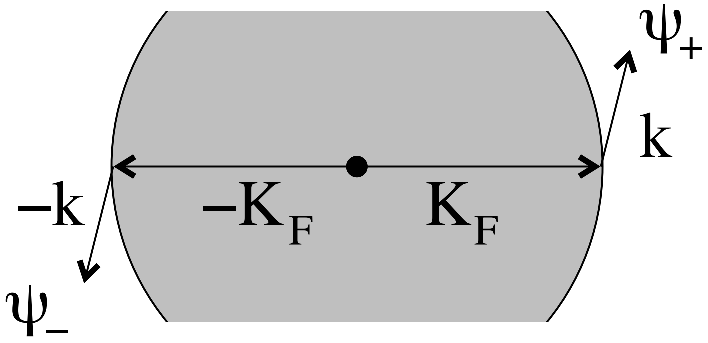


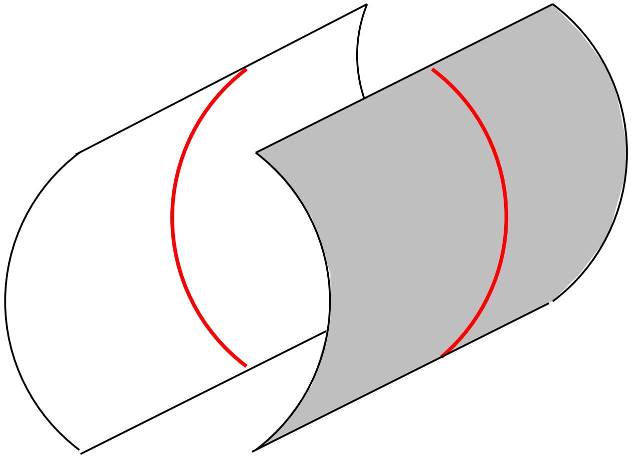


$$S = \int \frac{d^4k}{(2\pi)^3} \left\{ \sum_{s=\pm} \sum_{j=\uparrow,\downarrow} \psi^\dagger_{s,j}(k) \left(ik_0 + sk_2 + k_3^2 \right) \psi_{s,j}(k) \right.$$

$$\left. -k_1 \left(\psi^\dagger_{+,\uparrow}(k)\psi^\dagger_{-,\uparrow}(-k) + \psi^\dagger_{+,\downarrow}(k)\psi^\dagger_{-,\downarrow}(-k) + h.c. \right) \right\}$$

A continuous interpolation between 2d Fermi surface to 3d p-wave SC





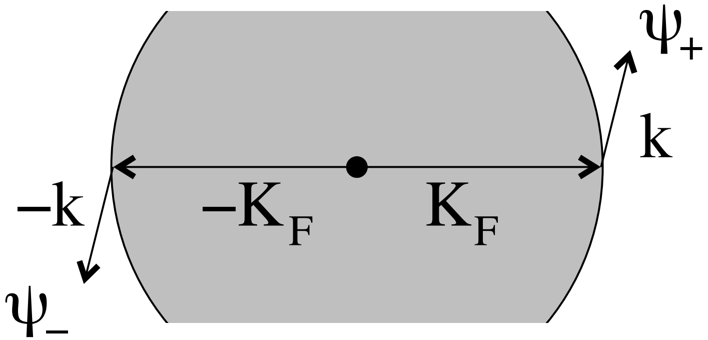
d = 2

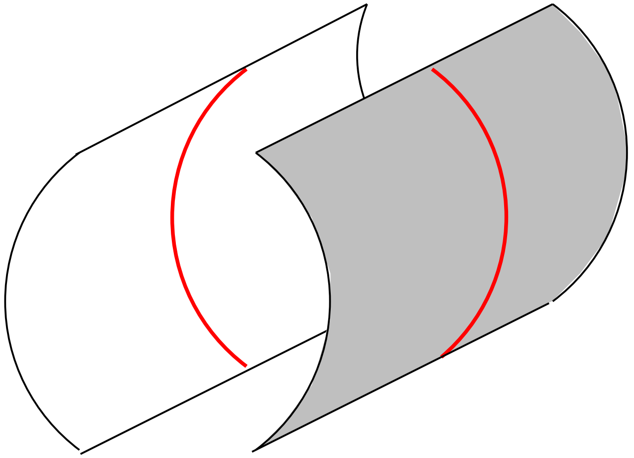
Strongly interaction
Non-Fermi liquid

d = 3

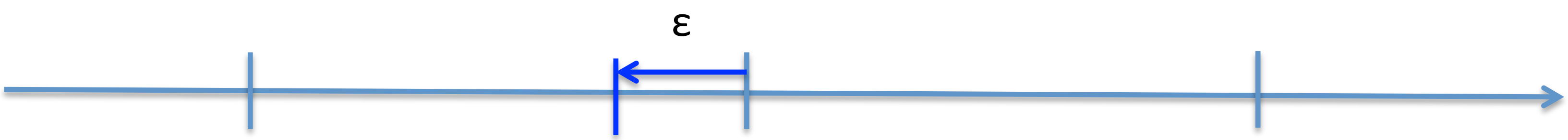
Fermi liquid
with decoupled boson

Perturbative non-Fermi liquid





Perturbative
Non-Fermi liquid



d = 2

Strongly interaction
Non-Fermi liquid

d = 5/2

Marginal
Fermi liquid

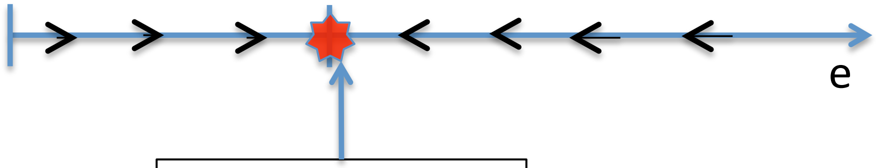
d = 3

Fermi liquid
with decoupled boson

Two-loop results

$$\frac{de}{dl} = \frac{\epsilon}{2}e - 0.02920\left(\frac{3}{2}-\epsilon\right)\frac{e^{7/3}}{N} + 0.01073\left(\frac{3}{2}-\epsilon\right)\frac{e^{11/3}}{N^2}$$

RG flow



Stable
Non-Fermi liquid
Fixed point

$$\boxed{\begin{aligned}\frac{e^{*4/3}}{N} &= 11.417\epsilon + 55.498\epsilon^2 \\ z &= \frac{3}{3-2\epsilon}\end{aligned}}$$

Physical properties

- Fermion Green fnt : $$G(k) = \frac{1}{|\delta_k|^{1-0.1508\epsilon^2}} \; g\left(\frac{|\vec{K}|^{1/z}}{\delta_k}\right)$$
- Boson Green fnt : $$D(k) = \frac{1}{k_d^2} \; f\left(\frac{|\vec{K}|^{1/z}}{k_d^2}\right)$$
- Specific heat : $$c \sim T^{(d-2)+\frac{1}{z}}$$
- Magnetic susceptibility : $$\chi_{ss} \sim T^{(d-1)-\frac{1}{z}}$$ $$\chi_{aa} \sim T^{(d-3)+\frac{1}{z}}$$

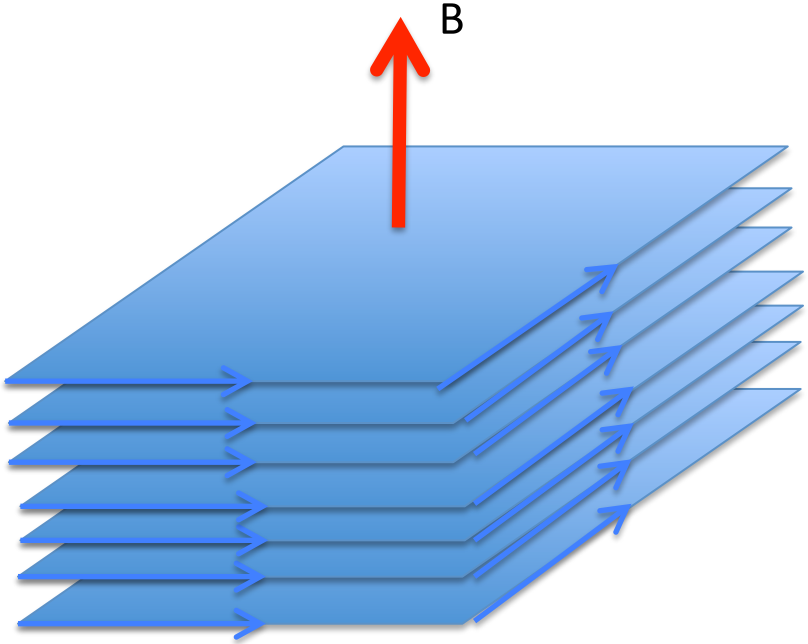
Q. Can one find a strongly interacting 2d non-Fermi liquid state that can be accessed non-perturbatively ?

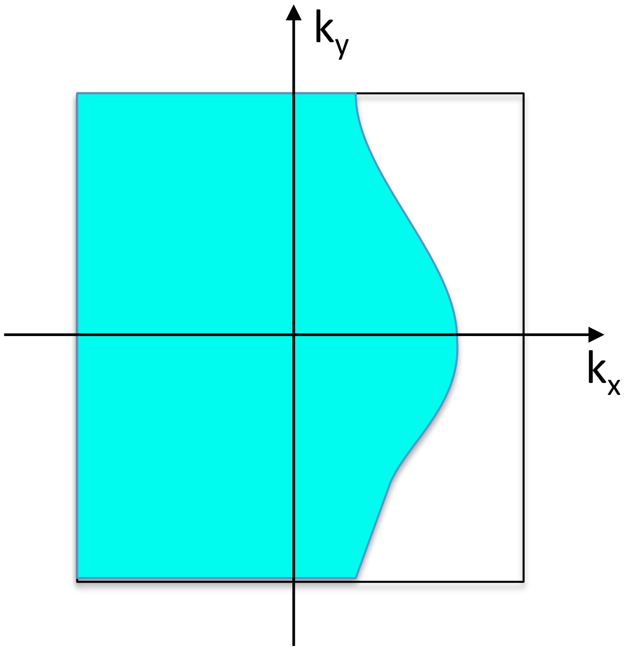
CHIRAL NON-FERMI LIQUID

Shouvik Sur
(McMaster)

arXiv:1310.7543

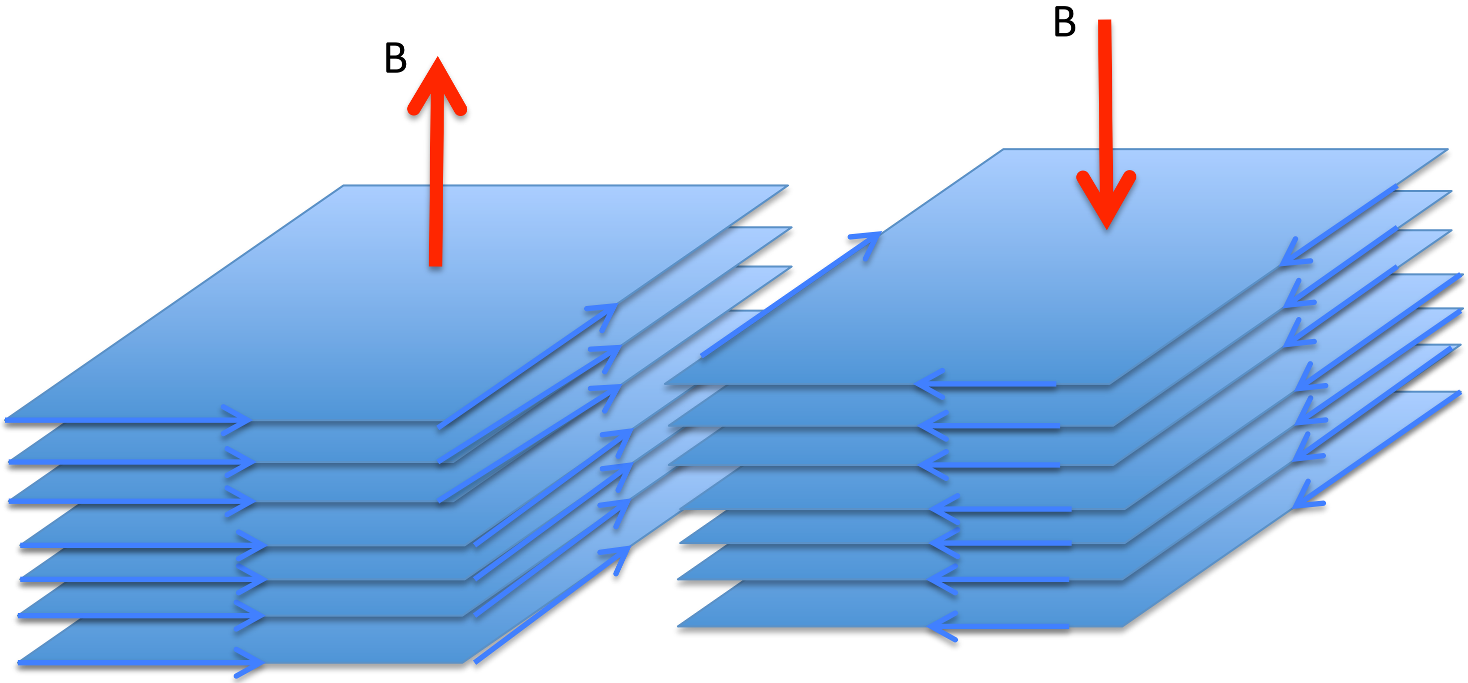
Chiral Metal





A stack of quantum Hall layers creates a two-dimensional chiral Fermi surface [Balents and Fisher (96)]

Chiral Metal with flavor

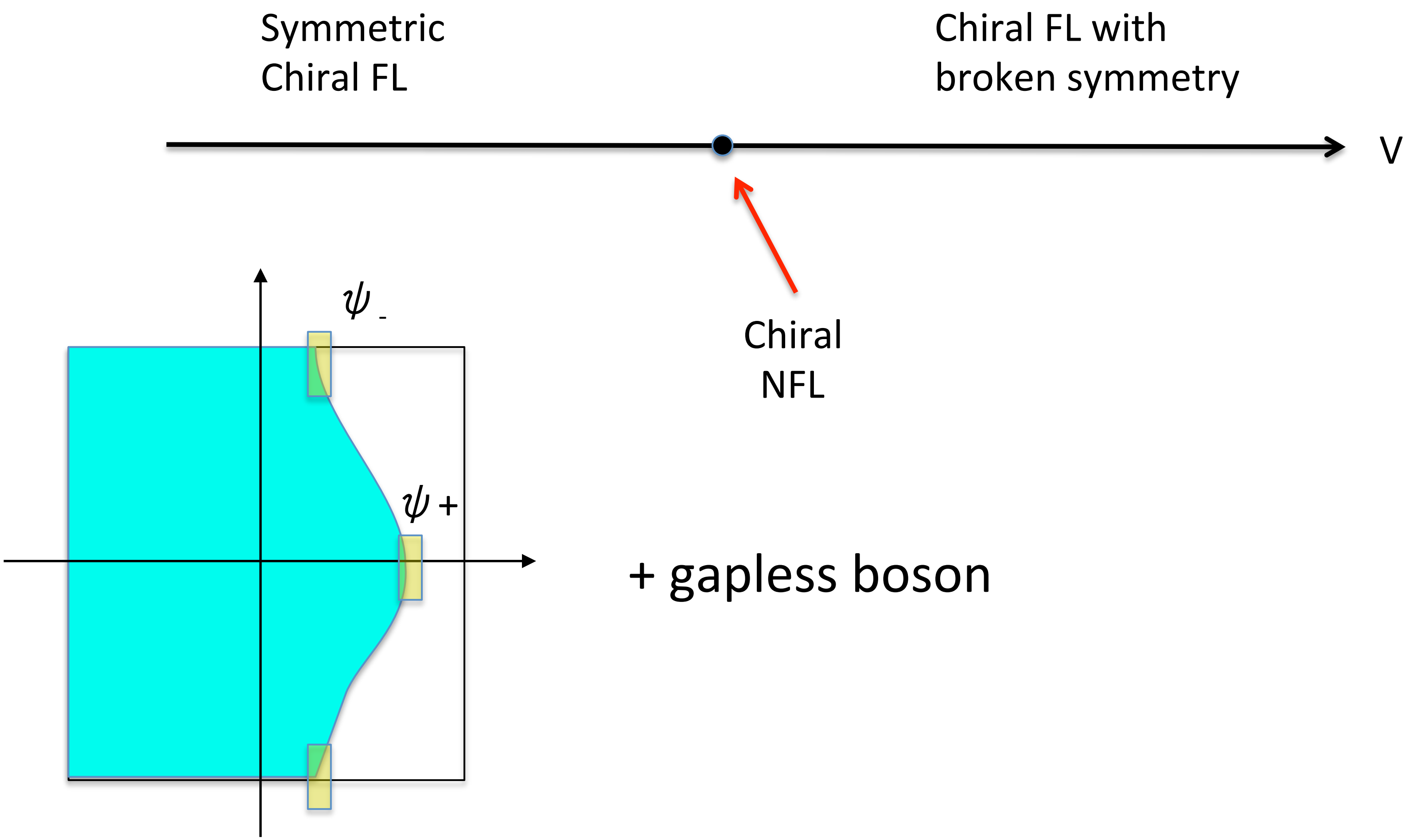


$$H = \int d^2k \; \epsilon_k \; \psi^\dagger_{jk}\psi_{jk} + V \int d^2k d^2k' d^2q \quad \psi^\dagger_{ik+q}\psi_{ik}\psi^\dagger_{jk'-q}\psi_{jk'}$$

The flavor symmetry can be broken spontaneously

e.g., exciton condensation : $\phi \sim < \psi_1^\dagger \psi_2 >$

Minimal theory for QPT in chiral metal : chiral patch theory



Symmetric
Chiral FL

Chiral FL with
broken symmetry

Chiral
NFL

+ gapless boson

Interaction driven scaling
(as opposed to the Gaussian scaling)

$$
\begin{aligned}
S \;=\; & \int dk\ \Big(i\boxed{\frac{k_0}{\Lambda^{1/2}}}^{\text{irrelevant}} + k_x + \gamma k_y^2\Big)\psi_j^*(k)\psi_j(k) \\
& + \int dk\ k_y^2\ \phi_\alpha(-k)\phi_\alpha(k) \\
& + \underset{\text{marginal}}{\boxed{g}} \int dk\ dq\ \phi_\alpha(k)\ \psi_i^*(k+q)\ T^\alpha_{ij}\ \psi_j(q)
\end{aligned}
$$

- The interaction is kept as a marginal term while one of the quadratic term is deemed irrelevant
- Irrelevant term enters as a scale

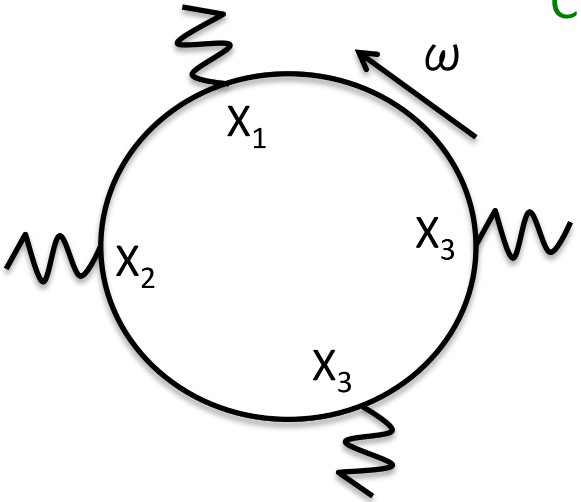
Wilsonian effective action with running length scale X$_0$

$$S_{X_0}(\underset{\substack{\nearrow \\ \text{dimensionful}}}{\Lambda}, \underset{\substack{\nwarrow \\ \text{dimensionless}}}{g})$$

- The Wilsonian effective action depends on all parameters of the theory
- In non-chiral case, divergence in $\Lambda \to \infty$ alter the naïve scaling
- In this case, thanks to chirality, the theory is UV finite : Λ can be dropped !

UV finiteness

$$G(x,\omega) \sim \Theta(-x\omega) e^{-\eta|x\omega|}$$



Chiral constraints : particles propagate only in one direction

$$\left.\begin{aligned} \omega(x_2 - x_1) &< 0 \\ \omega(x_3 - x_2) &< 0 \\ \omega(x_4 - x_3) &< 0 \end{aligned}\right\} \omega(x_4 - x_1) < 0$$

$$\omega(x_1 - x_4) < 0$$

When external frequencies are zero, the chiral constraints are mutually incompatible : virtual fluctuations of FS can not form a closed loop unless external energy is provided

➔ All internal frequencies are bounded by the external frequencies

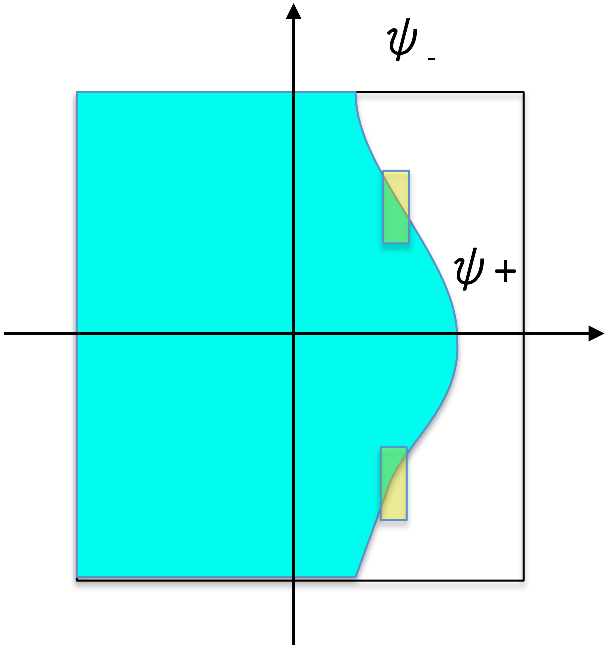
Stable fixed point

- UV/IR finiteness + absence of scale : the interaction driven scaling gives the exact scaling
- Exact Scaling form of the Green's function :

$$G^{-1}(k) = (k_x + k_y^2) g(|\omega|^{2/3}/(k_x + k_y^2))$$

- However, the full Green's function can not be computed perturbatively

Inflection points are described by QFT with different universality classes



$$S_0 = \int d^3k \left(ik_0 + k_x + s\gamma_n k_y^{\boxed{3}}\right)\psi_j^*(k)\psi_j(k) + \int d^3k|k|^2\phi^2 + \int d^3kd^3q\psi_i^*T_{ij}\psi_j\phi$$

$$z = \frac{4}{3}$$

- The general patch theory is classified by local curvature of FS, symmetry group and boson dispersion
- Full Fermi surface is described by a `direct sum’ of low energy effective theories in different universality classes

$$c \sim A\,T^{\frac{2}{3}} + B\,T^1$$

Summary & Outlook

- Perturbative non-Fermi liquid states based on dimensional regularization
  - Locality
  - Critical exponents computed perturbatively
  - Extension to SDW/CDW QCP [Shouvik Sur, SL, to appear]
- A class of strongly interacting non-Fermi liquid states without T,P symmetries
  - Chirality guarantees stability
  - Exact critical exponents
  - Multiple universality classes in one FS
  - 2d cousin of 1d chiral Luttinger
  - Ideal test ground for holographic NFL