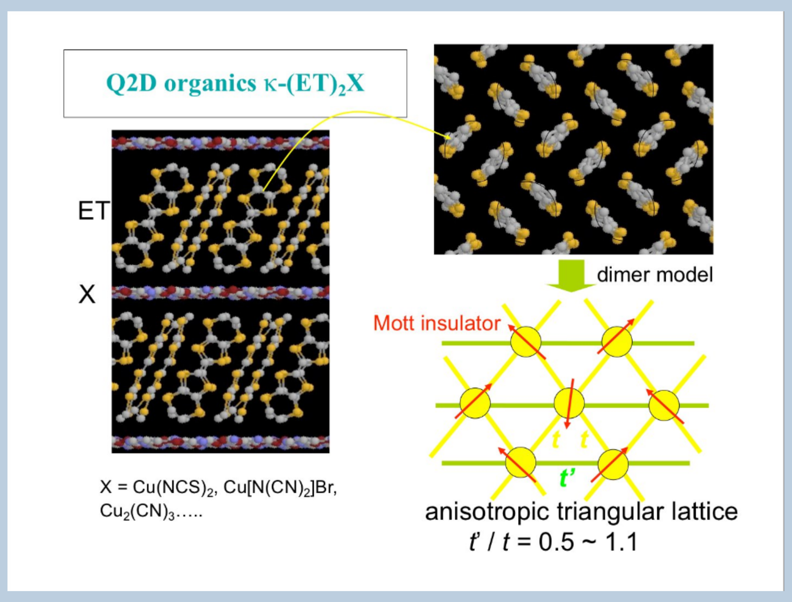
Fractionalized gapless vortex liquid

Chong Wang and T. Senthil
phchong@mit.edu

Massachusetts Institute of Technology

Problem

Theoretically constructing **gapless symmetric spin liquids** in **2D**?



- Usual states: symmetry-breaking or gapped
- Available tool: slave-particles with fermions in gapless mean-field states

$$S_i = \frac{1}{2} f_\alpha^\dagger \sigma_i^{\alpha\beta} f_\beta$$

- Can we go beyond it?
- States not described by slave particle & gauged mean field?

Answer

Fractionalize non-local objects!

- Simplest example: **vortex** in XY-spins

$$\Phi \sim \psi_1\psi_2$$

- Gapless ψ fermions $\rightarrow$ novel spin liquids!
- A natural construction on triangular lattice
- Interpretation: critical phases close to spin-nematic states

Background

Spin-vortex duality:

- Conserved $U(1)$ current (S_z) in $(2+1)$-D:

$$j^\mu = \frac{\epsilon^{\mu\nu\lambda}}{2\pi}\partial_\nu a_\lambda$$

- $U(1)$ gauge field a_μ: non-compact (flux conserved)
- Charge of a_μ: vorticity
- XY-spin model $\leftrightarrow$ bosons (vortices) coupled to a_μ

$$\sum J_{ij} S_i^+ S_j^- + U_{ij} S_i^z S_j^z + ...$$
$$\leftrightarrow \quad \sum t_{IJ} \Phi_I^\dagger e^{i a_{IJ}} \Phi_J + \cos(\nabla \times a) + ...$$

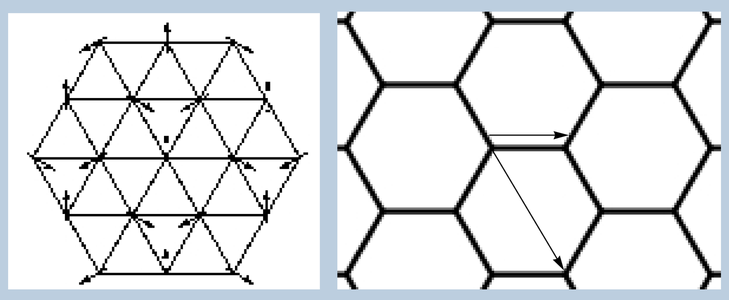
- Vortices gapped $\rightarrow$ gapless free a_μ $\rightarrow$ magnetic order
- Vortices condensed $\langle\Phi\rangle \neq 0$ $\rightarrow$ Higgsed a_μ $\rightarrow$ gapped paramagnet

Goal : a stable state with gapless vortices Φ

$\mathbb{Z}_2$ construction on triangular lattice

System : spin-1 XY anti-ferromagnet on triangular lattice

Dual theory : vortices(bosons) on honeycomb lattice



- **Frustration** $\rightarrow$ vortices at half-filling $\rightarrow$ treat vortex as hard-core boson
- Fractionalize vortex into two fermions:

$$\Phi = \psi_1\psi_2, \qquad N_\Phi = \frac{1}{2}\left(\psi_1^\dagger\psi_1 + \psi_2^\dagger\psi_2\right)$$

- Fractionalization $\rightarrow$ gauge structure $(SU(2), U(1), \mathbb{Z}_2...)$: $\psi_{i,\alpha} \rightarrow U_i^{\alpha\beta}\psi_{i,\beta}$
- Simplest example: $\mathbb{Z}_2$ mean-field ansatz $\rightarrow$ ψ fermions coupled with a $\mathbb{Z}_2$ gauge field
- Deconfined phase: $\mathbb{Z}_2$-flux (vison) as a gapped excitation
- ψ fermion carries $1/2$-vorticity $\rightarrow$ $1/2$-charge under a_μ

$$\mathcal{L}[\psi^\dagger, \psi, a_\mu] = \mathcal{L}[\psi^\dagger, \psi] + \mathcal{L}[a_\mu] + \frac{1}{2} a_\mu j_\psi^\mu$$

- **Gapless** ψ **fermion** $\rightarrow$ a_μ damped but not Higgsed $\rightarrow$ **gapless state, but no magnetic order**

Nature of the state

- A simple interpretation:
  - **Spin-nematic state** ($\langle (S^+)^2\rangle \neq 0$ but $\langle S^+\rangle = 0$): bosonic half-vortices
  - Bound state of half-vortex and single spin: fermion
  - Gapless vortex: critical state
- Interesting features:
  - Gapless, but single-spin excitations (S^+) are gapped
  - **Critical** nematic field $(S^+)^2$ (and many other fields)
- Relations to familiar states:
  - **Spin-nematic order**: ψ fermions gapped
  - $\mathbb{Z}_2$ **topological order** ($\sim$ toric code): $\langle\psi\psi\rangle \neq 0$
  - Magnetic order ($\langle S^+\rangle \neq 0$): vison condensed
  - Trivial paramagnet: fermion-vison bound state condensed

Details of the construction

- $\mathbb{Z}_2$ Mean-field ansatz:

$$H_{mean} = -\sum_{<ij>} \psi_i^\dagger(\eta\tau^0 + \lambda\tau^1)\psi_j - \sum_{<<<ij>>>} \psi_i^\dagger(\xi\tau^3)\psi_j$$

- Band structure gives $2\times 2 = 4$ Dirac cones: low energy theory described by QED_3 with $N_f = 4$ fermion flavors

$$\mathcal{L} = -i\bar{\psi}\gamma^\mu\nabla_\mu\psi + \frac{1}{2e^2} f_{\mu\nu}^2$$

- Fermion mass gap not allowed by global symmetries
- Flavor number $N_f = 4$: large enough to stabilize the theory against spontaneous mass generation
- Emergent symmetries: Lorentz invariance + SU(4) flavor symmetry
- Nematic order parameter $(S^+)^2 \sim$ monopole fields: critical