# Anyons in integer quantum Hall magnets

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A. Rahmani, R. Muniz, and I. Martin, Phys. Rev. X **3**, 031008 (2013)

## Motivation and background

Mechanisms for fractionalization:

1. Topological order: No local order parameter or Landau symmetry breaking. Example: FQHE.
2. Coupling of weakly interacting electrons to nontrivial classical (mean field) textures. Example: Polyacetylene.

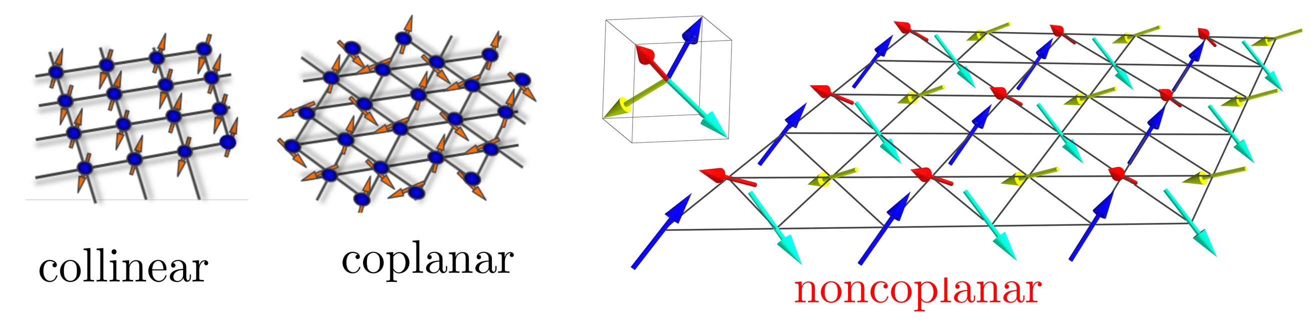
Quantum Hall systems, $J_x = \sigma_{xy} E_y$, $\sigma_{xy} = \nu e^2/h$, $q = e\nu$.

Question: Can single-layer Integer Quantum Hall systems exhibit fractionalization through the second mechanism above?

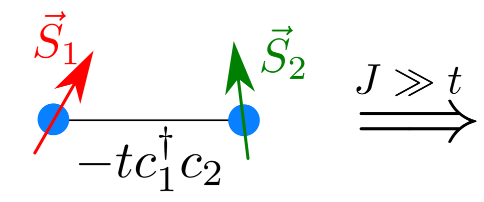
Answer: Yes

For spontaneous integer quantum Hall systems that emerge in itinerant magnetically ordered systems with noncoplanar ordering.

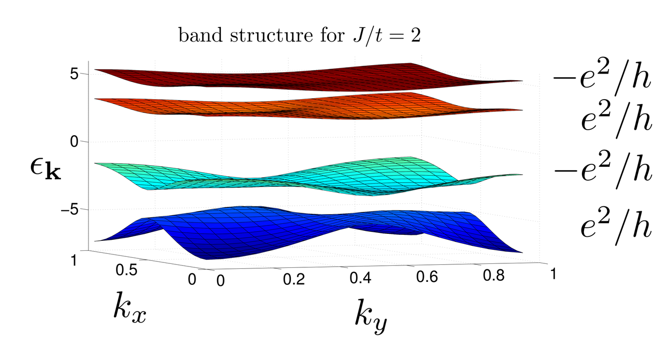
Types of magnetic ordering:

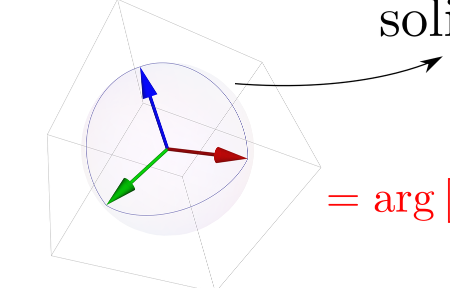



$$H = -t \sum_{\langle i,j\rangle} c^\dagger_{i\alpha} c_{j\alpha} + \text{h.c.} + J \sum_i \mathbf{S}_i \cdot c^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta}$$




$|\chi_i\rangle$ the spinor in the direction of $\vec{S}_i$.



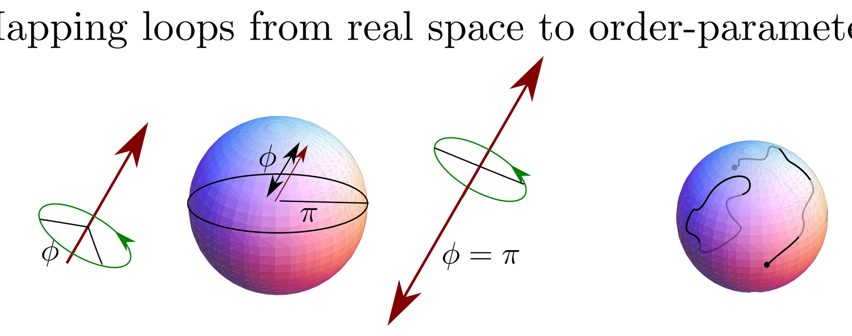




$= \arg\left[\langle\chi_1|\chi_2\rangle\langle\chi_2|\chi_3\rangle\langle\chi_3|\chi_1\rangle\right]$

## Topological defects and charge fractionalization

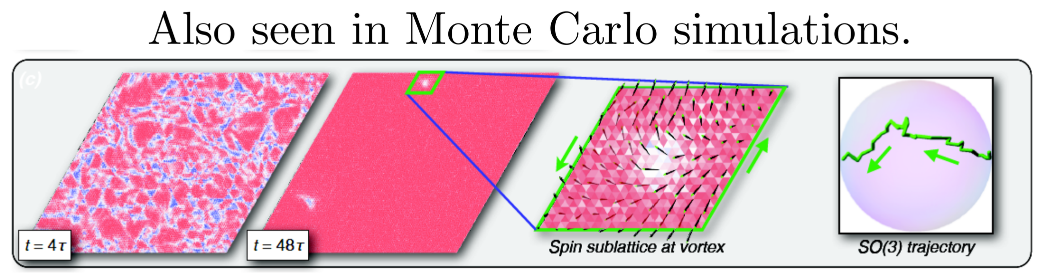
- Ordered media can have topologically stable defects.
- Homotopy theory classifies such defects.

Mapping loops from real space to order-parameter space:

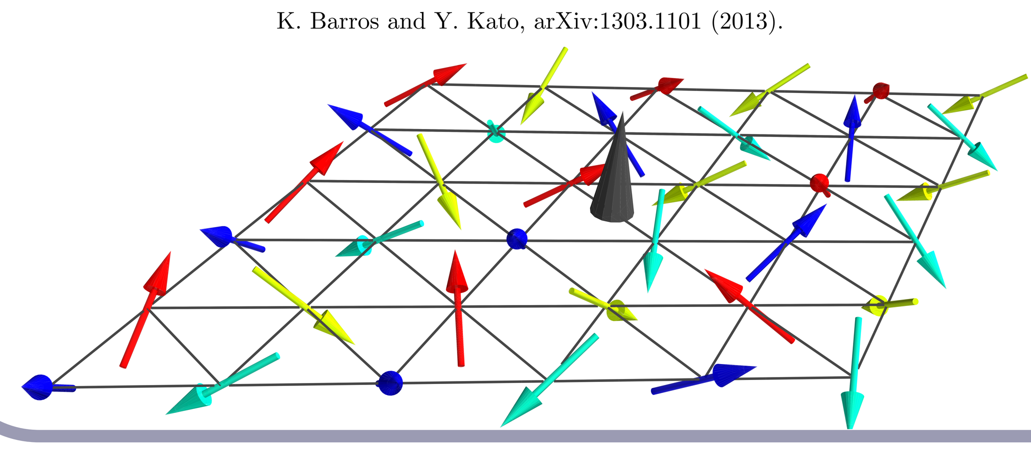



Noncontractible loops are stable $Z_2$ vortices.

Also seen in Monte Carlo simulations.

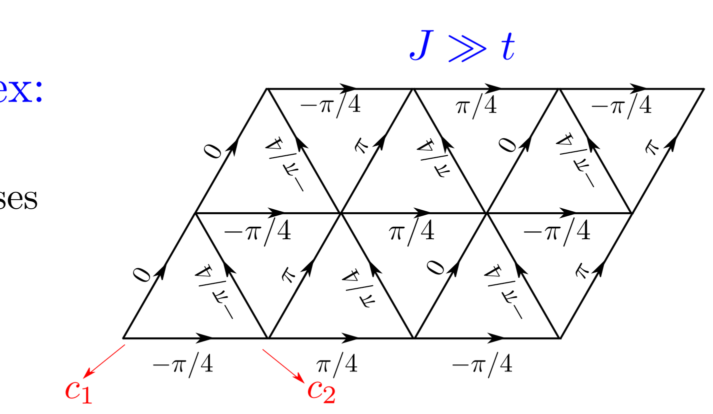


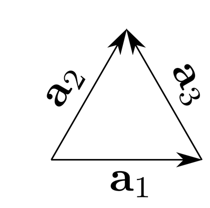

K. Barros and Y. Kato, arXiv:1303.1101 (2013).



Without the vortex:

Hopping amplitude phases



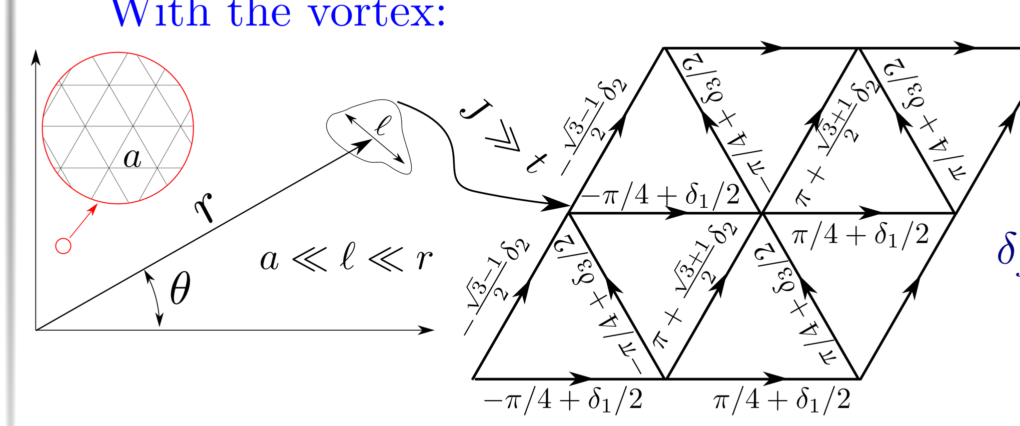




$\Psi^\dagger_{\mathbf{k}} = (c^\dagger_{1\mathbf{k}}, c^\dagger_{2\mathbf{k}})$

$H_0 = \Psi^\dagger_{\mathbf{k}} H_0(\mathbf{k}) \Psi_{\mathbf{k}}$

$$H_0(\mathbf{k}) = -2\begin{pmatrix} \cos(\mathbf{k}\cdot\mathbf{a}_2) & e^{i\pi/4}\cos(\mathbf{k}\cdot\mathbf{a}_1) + e^{-i\pi/4}\cos(\mathbf{k}\cdot\mathbf{a}_3) \\ e^{-i\pi/4}\cos(\mathbf{k}\cdot\mathbf{a}_1) + e^{i\pi/4}\cos(\mathbf{k}\cdot\mathbf{a}_3) & -\cos(\mathbf{k}\cdot\mathbf{a}_2)\end{pmatrix}$$

With the vortex:




$H = H_0 + V$

perturbation from vortex $= \Psi^\dagger_{\mathbf{k}} V(\mathbf{k}) \Psi_{\mathbf{k}}$

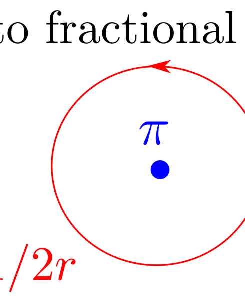
$\delta_j \equiv \delta_{\mathbf{r}+\mathbf{a}_j} - \delta_{\mathbf{r}} = \frac{1}{r^2}\mathbf{r}\times\mathbf{a}_j = \frac{1}{r}(\cos\theta, \sin\theta)\times\mathbf{a}_j$

$J_x(\mathbf{k}) = \frac{\partial H_0(\mathbf{k})}{\partial k_x}, \quad J_y(\mathbf{k}) = \frac{\partial H_0(\mathbf{k})}{\partial k_y}$

$$V(\mathbf{k}) = \sqrt{3}\sin(\mathbf{k}\cdot\mathbf{a}_2)\,\delta_2 \times I + \frac{\sin\theta}{2\,r} J_x(\mathbf{k}) - \frac{\cos\theta}{2\,r} J_y(\mathbf{k})$$

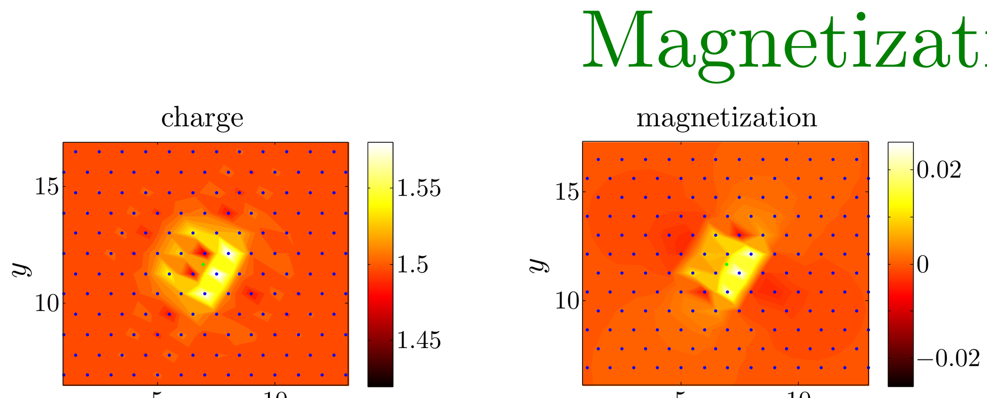
$J_\theta(\mathbf{k})/2r$

Similar to fractional flux!




$A_\theta = \pi/2\pi r = 1/2r$

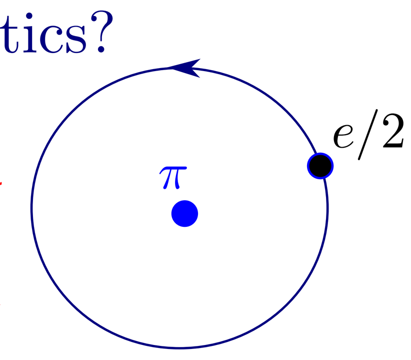
## Magnetization and exchange statistics




Spin-current operator has a nonvanishing expectation value.

What is the exchange statistics?

Naive answer: ~~$\frac{1}{2}\frac{\pi}{2} = \frac{\pi}{4}$~~




$$H = -t \sum_{\langle i,j\rangle} c^\dagger_{i\alpha} c_{j\alpha} + \text{h.c.} + J \sum_i \mathbf{S}_i \cdot c^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta}$$

rotated texture due to the vortex: $\mathbf{S}_i = \mathcal{R}_i \mathbf{S}^0_i$

$$c^\dagger_i \boldsymbol{\sigma}\cdot\mathbf{S}_i c_i = \psi^\dagger_i \boldsymbol{\sigma}\cdot\mathbf{S}^0_i \psi_i$$

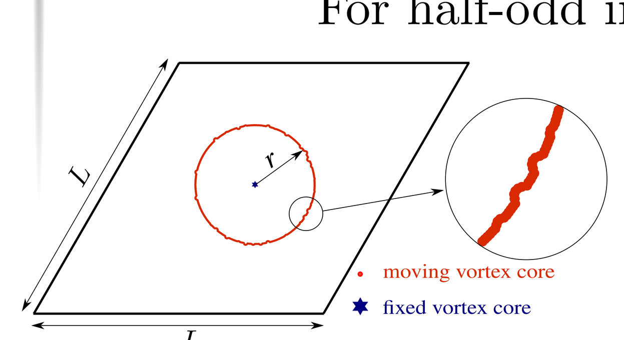
Write the Hamiltonian in terms of $\psi_i = U^\dagger_i c_i$

$H(c) = H_0(c, S) \quad H(\psi) = H_0(\psi, S_0) - J^\nu_a \mathcal{A}^\nu_a \qquad -iU^\dagger \partial^\nu U \equiv \mathcal{A}^\nu_a \sigma_a$

charge and spin $a$ current in the $\nu$ direction

non-Abelian flux $\mathbf{\Phi} = \oint \mathbf{A} d\mathbf{r} = (1 + 2m + \sigma_3)\pi \qquad \mathbf{n} = \begin{pmatrix} n_\uparrow \\ n_\downarrow \end{pmatrix}$

For half-odd integer charge, $\Theta = \text{tr}\left(\mathbf{\Phi}.\mathbf{n}\right)/2 = p\frac{\pi}{2} + \pi/4 + \pi m_z$




$\oint \langle\Psi|\partial_s|\Psi\rangle ds \to \Phi = \arg\left[\prod_{i=1}^N \langle\Psi_i|\Psi_{i+1}\rangle\right]$

| $L$ | $N$ | $q$ | $\overline{n_\uparrow - n_\downarrow}$ | $\phi = \pi(q + \overline{n_\uparrow - n_\downarrow})$ |
|---|---|---|---|---|
| 60 | 1796 | -1/2 | 0.28 | 5.59 |
| 60 | 1816 | +1/2 | 0.23 | 2.29 |

## Summary

Integer quantum Hall systems, which emerge due to the interplay of itinerant electrons and noncoplanar magnetic ordering, have intrinsic fractional excitations in addition to their integer quasiparticles. The existence, and topological stability, of these thermal excitations, i.e., $Z_2$ vortices, is guaranteed by the structure of the magnetic order parameter. The Berry phase for adiabatic exchange depends on magnetization.