Buyer-Seller Networks

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Trade is often restricted

- Traders in developing countries (e.g. Fafchamps and Minten, 1999)
- Garment industry (e.g. Uzzi, 1997)
- Fish markets (e.g. Kirman and Vriend, 2000)
- Japanese manufacturers and suppliers (e.g. Toshihiro, 1994)
- Financial markets (e.g. in’t Veld and Lelyveld, 2014)
- Lawyers (e.g. Lancaster and Uzzi, 2012)
Questions for today

- (When) do networked markets clear efficiently?
- How are the terms of trade affected?
- If investments are required to form relationships, will these be efficient?
Broad Set up

- Buyers have unit demand, sellers have unit supply.
- Investments are made to form the network.
- Then trade occurs.

_focus on a few papers—all can be fit into set up._

▶ Kranton and Minehart (2001).
▶ Elliott (2014).
▶ Manea (2011).
▶ Elliott and Nava (2016).
Broad Set up

- Buyers have unit demand, sellers have unit supply.
- Investments are made to form the network.
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Focus on a few papers—all can be fit into set up.

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- Elliott and Nava (2016).
Model: In Pictures

Parties

Buyers

\[ b_1 \]

\[ b_2 \]

\[ b_3 \]

Sellers

\[ s_1 \]

\[ s_2 \]

\[ s_3 \]
Model: In Pictures

Potential Gains from Trade

Formed Network

Bargaining Outcomes
Model: In Pictures

Formed Network

\begin{align*}
&\text{Buyers} \\
&s_1 & s_2 & s_3 \\
&b_1 \quad b_2 \quad b_3
\end{align*}

\begin{align*}
&\text{Parties} \\
&s_1 & s_2 & s_3 \\
&b_1 \quad b_2 \quad b_3
\end{align*}

\begin{align*}
&\text{Potential Gains from Trade} \\
&s_1 & s_2 & s_3 \\
&b_1 \quad b_2 \quad b_3
\end{align*}

\begin{align*}
&\text{Bargaining Outcomes} \\
&s_1 & s_2 & s_3 \\
&b_1 \quad b_2 \quad b_3
\end{align*}
Model: In Pictures

Bargaining Outcomes

u_1
b_1
s_1
v_1

10
17
15

u_2
b_2
s_2
v_2

14
10

u_3
b_3
s_3
v_3

10
Buyers $\mathcal{B}$, sellers $\mathcal{S}$.

Let surplus from $i \in \mathcal{B}$ matching with $j \in \mathcal{S}$ be $s_{ij}$.

Match is a function, $\mu : \mathcal{B} \cup \mathcal{S} \rightarrow \mathcal{B} \cup \mathcal{S}$ such that

- If $i \in \mathcal{B}$ then $\mu(i) \in \mathcal{S} \cup \{i\}$.
- If $j \in \mathcal{S}$ then $\mu(j) \in \mathcal{B} \cup \{j\}$.
- $\mu(i) = \mu(j)$ if and only if $\mu(j) = \mu(i)$.
1. Kranton and Minehart, 2001
2. Elliott, 2014
3. Corominas Bosch, 2004
4. Manea, 2011
5. Elliott and Nava, 2016
6. Experiments
**Stage 1:** Buyers simultaneously choose which sellers to link to at cost $c$.

**Stage 2:** Market clears.
Market clearing: Overdemanded goods

Suppose sellers have set prices $p$.

At these prices a set of goods is minimally overdemanded when:

(i) More buyers most preferred goods come only from this set than there are goods in the set.

(ii) No subset of these goods is also overdemanded.
Market clearing: Multi-unit auction

Multi-unit Auction Algorithm (Demange, Gale and Sotomayor, 1986)

0. Sellers prices are initialized to 0.

1. At current prices, buyers identify their most preferred goods.

2a. Price of minimally overdemanded goods are incremented. Return to step 1.

2b. Auction terminates if no goods are minimally overdemanded.
Example

\[ p_1 = 0 \quad p_2 = 0 \quad p_3 = 0 \]
Example

\[ p_1 = 0 \]
\[ p_2 = 0 \]
\[ p_3 = 2 \]
Example

\[ \begin{align*}
  p_1 &= 0 \\
p_2 &= 0 \\
p_3 &= 2.5
\end{align*} \]
Example

$p_1 = 0$
$p_2 = 0$
$p_3 = 4$
Example

\[ p_1 = 0 \]
\[ p_2 = 5 \]
\[ p_3 = 9 \]
Example

\[ p_1 = 0 \]
\[ p_2 = 5 \]
\[ p_3 = 9 \]
Equilibrium

Suppose buyers choose which products to demand in the multi-unit auction.
Equilibrium

Suppose buyers choose which products to demand in the multi-unit auction.

Proposition

In the market clearing stage there exists an efficient perfect equilibrium in which buyers bid truthfully.

- Holds for any network structure.
- So holds when buyers only know their own valuations.
Efficient Equilibrium Investment

Efficiency: Maximize surplus less investment costs.
Efficient Equilibrium Investment

Efficiency: Maximize surplus less investment costs.

Proposition
There exists an efficient perfect equilibrium of the two stage game.
Efficient Equilibrium Investment

Efficiency: Maximize surplus less investment costs.

Proposition

There exists an efficient perfect equilibrium of the two stage game.

To get intuition for these two results we’ll go on a detour.
Assignment Game

Auction proxies for decentralized market clearing.

Alternative: look for outcomes that are *stable*.

Pairwise-stability: No buyer-seller pair leave any gains from trade on the table.
Pairwise Stability

An outcome is a match $\mu$, and payoffs $(u, v)$.

An outcome $(\mu, u, v)$ is pairwise stable if and only if

(i) $u_i + v_j \geq s_{ij}$ for all $i \in B, j \in S$

(ii) $u_i \geq 0$ for all $i \in B$

(iii) $v_j \geq 0$ for all $j \in S$
An outcome is pairwise stable if and only robust to coalitional deviations (in the core).

The set of pairwise stable outcome (the core) is non-empty.

All pairwise stable (core) matches maximizes total surplus.

Payoffs form a complete lattice, partially ordered by buyer preferences.

There is a buyer (seller) optimal outcome in which all buyers (sellers) get their most preferred pairwise stable (core) payoff.
Buyer Optimal Core Outcomes
Buyer Optimal Core Outcomes
Buyer Optimal Core Outcomes
Buyer Optimal Core Outcomes
Buyer Optimal Core Outcomes

Diagram showing a network with nodes labeled as $b_1, b_2, b_3$, and $s_1, s_2, s_3$. The edges and their weights are as follows:
- $b_1$ to $s_1$: 10
- $b_1$ to $s_2$: 17
- $b_2$ to $s_1$: 14
- $b_2$ to $s_2$: 15
- $b_3$ to $s_1$: 0
- $b_3$ to $s_2$: 5
- $b_3$ to $s_3$: 10

Central paths highlighted in blue.
Buyer Optimal Core Outcomes
Buyer Optimal Core Outcomes

\[ \begin{array}{c}
\text{b}_1 & 10 \\
\text{s}_1 & 0 \\
\text{b}_2 & 5 \\
\text{s}_2 & 5 \\
\text{b}_3 & 9 \\
\end{array} \]
Buyer Optimal Core Outcomes

\[
\begin{align*}
&b_1 \quad 10 \\
&s_1 \quad 0 \\
&b_2 \quad 5 \\
&s_2 \quad 5 \\
&b_3 \quad 1 \\
&s_3 \quad 9
\end{align*}
\]
Tieing the approaches together

The multi-unit auction algorithm terminates at the buyer optimal core outcome.
Tieing the approaches together

The multi-unit auction algorithm terminates at the buyer optimal core outcome.

At the buyer (seller) optimal core outcome, buyers (sellers) receive payoffs equal to their marginal contributions to total surplus (Leonard, 1983).
Marginal Contributions to Total Surplus

Total Surplus: 30

Marginal contribution: 10

Total Surplus: 25
Total Surplus without b1: 29
Marginal contribution: 1
Marginal Contributions to Total Surplus

Total Surplus: 20

- $s_1$
- $b_2$
- $b_3$

Marginal contributions:

- $s_1$: 1
- $s_2$: 10
- $s_3$: 10
Marginal Contributions to Total Surplus

Total Surplus : 30

Total Surplus without $b_1$ : 20

Marginal contribution : 10
Marginal Contributions to Total Surplus

Total Surplus: 30

Total Surplus without $b_1$: 29
Marginal contribution: 1

Total Surplus without $b_2$: 25
Marginal contribution: 5

Total Surplus without $b_3$: 20
Marginal contribution: 10
Marginal Contributions to Total Surplus

Total Surplus: 25

Marginal contribution: 1
Marginal Contributions to Total Surplus

Total Surplus : 30

Total Surplus without $b_1$ : 25

Marginal contribution : 5
Marginal Contributions to Total Surplus

Total Surplus: 30

Diagram showing contributions from points $s_1$, $s_2$, and $s_3$ to $b_1$, $b_2$, and $b_3$ with contributions 10, 17, 15, 14, and 15.
Marginal Contributions to Total Surplus

Total Surplus: 29

\[ \text{Total Surplus: } 30 \]
\[ \text{Total Surplus without } b_1: 20 \]
\[ \text{Marginal contribution: } 10 \]

\[ \text{Total Surplus: } 30 \]
\[ \text{Total Surplus without } b_1: 25 \]
\[ \text{Marginal contribution: } 5 \]

\[ \text{Total Surplus: } 30 \]
\[ \text{Total Surplus without } b_1: 29 \]
\[ \text{Marginal contribution: } 1 \]
Marginal Contributions to Total Surplus

Total Surplus : 30

Total Surplus without $b_1$: 29

Marginal contribution : 1
Key Points so Far

- Multi-unit auction terminates at a pairwise stable outcome.

- And not just any pairwise stable outcome, the one most preferred by all buyers.

- At these outcomes, buyer receive their marginal contributions to total surplus.
In the market clearing stage there exists an efficient perfect equilibrium in which buyers bid truthfully.

Suppose everyone else bids truthfully.

- Algorithm cannot terminate when anyone else has a profitable pairwise deviation.
- Unprofitable to let it terminate when you have a profitable deviation.
- So bidding truthfully guarantees best possible outcome.
Proposition
There exists an efficient perfect equilibrium of the two stage game.

Suppose everyone but $i$ invests efficiently.
- Buyer $i$ gets paid her marginal contribution to total surplus.
- So wants to form links to maximize the increase in surplus.
- Does so by forming the efficient network.
Some Limitations

- Decentralized markets might clear at some other prices.
- Market is thin so competitive forces don’t yield buyer-optimal payoffs.
- Efficiency result relies on only buyers investing.
Outline

1. Kranton and Minehart, 2001
2. Elliott, 2014
3. Corominas Bosch, 2004
4. Manea, 2011
5. Elliott and Nava, 2016
6. Experiments
Generalizing the outcomes

Simple Formed Network

Proposed Bargaining Outcome

\[ v_2 = 5(1 - \beta) \]
\[ u_1 = 10 + 5\beta \]
\[ v_1 = 0 \]

\( \beta \): Buyer bargaining power
Generalizing the outcomes

Proposed Bargaining Outcome

\[ u_1 = 10 + 5\beta \]

\[ v_2 = 5(1-\beta) \]

\[ v_1 = 0 \]

\( \beta \): Buyer bargaining power
Generalizing the market outcomes

Let $u_i$ be $i$’s minimum core payoff.

Let $u_j$ be $j$’s minimum core payoff.

If buyer $i$ trades with seller $j$

$$u_i = u_i + \beta (s_{ij} - u_i - v_j)$$

$$v_j = v_j + (1 - \beta) (s_{ij} - u_i - v_j)$$

$\beta$ : buyers’ bargaining power relative to sellers.
Example
Example

\begin{equation}
0 + (1 - \beta) + (1 - \beta) + (1 - \beta) + \beta + \beta + \beta
\end{equation}
Example

\[ ( 1 - \beta ) b_1 + ( 1 - \beta ) b_2 + ( 1 - \beta ) b_3 + \beta b_1 + \beta b_2 + \beta b_3 \]
Example

\[ \beta \]

\[
\begin{align*}
10 & \quad 17 \\
15 & \quad 14 \\
10 & \quad 10 \\
0 & \quad 0
\end{align*}
\]
Example

\[
\begin{align*}
\text{Example} \\
\begin{array}{c}
s_1 \\
\end{array} & \begin{array}{c}
b_1 \\
10 \\
\end{array} & \begin{array}{c}
b_2 \\
4 \\
17 \\
\end{array} & \begin{array}{c}
b_3 \\
0 \\
\end{array}
\end{array}
\end{align*}
\]

\[
\begin{align*}
0 & + (1 - \beta) \\
5 & + (1 - \beta) \\
9 & + (1 - \beta)
\end{align*}
\]

\[
\begin{align*}
\beta & + 4 \\
\beta & + 9 \\
\beta & + 10
\end{align*}
\]
Example

\[ (1 - \beta)5 + (1 - \beta)9 + (1 - \beta)0 + \beta 4 + \beta 9 + \beta \]
Example

![Graph](image)

The graph shows a network with nodes labeled $s_1$, $s_2$, and $s_3$. The edges and weights are as follows:

- $s_1$ to $b_1$ with weight 10
- $s_1$ to $b_2$ with weight 10
- $s_2$ to $b_2$ with weight 15
- $s_2$ to $b_3$ with weight 14
- $s_3$ to $b_3$ with weight 10

Weights:

- $x_{12} = 10$
- $x_{13} = 15$
- $x_{23} = 14$
- $x_{24} = 10$
- $x_{34} = 10$

Additionally, there are expressions involving $\beta$:

- $\beta(1-\beta)$ for each step.
Example

\[ \begin{align*}
0 &+ (1-\beta) 5 + (1-\beta) 9 + (1-\beta) 0 + \beta 4 + \beta 9 + \beta \\
10 & \quad 10 & \quad 10
\end{align*} \]
Example

\[ \begin{align*}
4 &= 14 - 10 \\
\end{align*} \]
Example

\[ 9 = 15 - 10 + 14 - 10 \]

\[ 4 = 14 - 10 \]

\[ 0 \]
Example

10
b_1

10
s_1

17
15

14

5
b_2

10
s_2

1
b_3

10
s_3

0
1
5
9

\[(1 - \beta) + (1 - \beta) + (1 - \beta) + \beta + \beta + \beta\]
Example

\begin{align*}
\text{s}_1 & \quad 10 \quad \text{b}_1 \\
\text{s}_2 & \quad 5 \quad \text{b}_2 \\
\text{s}_3 & \quad 1 \quad \text{b}_3
\end{align*}

\begin{align*}
5 &= 15 - 10 \\
9 &= 14 - 10 + 15 - 10
\end{align*}
Example

\[0 + (1-\beta) 5 + (1-\beta) 9 + (1-\beta) 4 + \beta 9 + \beta\]
Example

\[ 9 + \beta \]

\[ b_1 \]

\[ 10 \]

\[ s_1 \]

\[ 0 + (1-\beta) \]

\[ 4 + \beta \]

\[ b_2 \]

\[ 17 \]

\[ 15 \]

\[ 10 \]

\[ s_2 \]

\[ 5 + (1-\beta) \]

\[ 0 + \beta \]

\[ b_3 \]

\[ 10 \]

\[ s_3 \]

\[ 9 + (1-\beta) \]

\[(1-\beta)\]

\[(1-\beta)\]

\[(1-\beta)\]
Incentives to form links

\[ u_{b1} = u_{b1} + \beta (s_{b1s1} - u_{b1} - v_{s1}) \]

\[ = (1 - \beta)u_{b1} + \beta (s_{b1s1} - v_{s1}) \]

\[ = (1 - \beta)(s_{b1s2} - s_{b2s2} + s_{b2s3} - s_{b3s3}) + \beta s_{b1s1} \]
Incentives to form links

\[ u_{b1} = u_{b1} + \beta (s_{b1s1} - u_{b1} - v_{s1}) \]

\[ = (1 - \beta)u_{b1} + \beta (s_{b1s1} - v_{s1}) \]

\[ = (1 - \beta)(s_{b1s2} - s_{b2s2} + s_{b2s3} - s_{b3s3}) + \beta s_{b1s1} \]

Independent of \( s_{b1s3} \), so no incentives to form this link.
Network Decomposition

\[
\begin{align*}
9 + \beta &\quad b_1 \\
10 &\quad s_1 \\

4 + \beta &\quad b_2 \\
15 &\quad s_2 \\

0 + \beta &\quad b_3 \\
10 &\quad s_3
\end{align*}
\]

\[0 + (1-\beta) + (1-\beta) + (1-\beta) + \beta + \beta + \beta + \beta \]
Inefficiencies

Might have over-investment to establish an “outside option.”

Might have under-investment because of a hold up problem.
Two sided investment

Buyers pay $\gamma c$, sellers pay $(1 - \gamma)c$.

Refine Nash equilibria using pairwise stability.
Two sided investment

Buyers pay $\gamma c$, sellers pay $(1 - \gamma)c$.

Refine Nash equilibria using pairwise stability.

**Proposition**

1. For all $\gamma \neq \beta$, there exist potential match surpluses such that all gains from trade are lost to underinvestment on all stable networks.

2. For all potential match surpluses, the surplus lost from overinvestment is less than 50% of potential gains from trade on all stable networks.
Bound for Overinvestment

Can characterize a tight bound on overinvestment inefficiency.

(10 buyers and 10 sellers)
Negotiated Investments

Suppose a buyers and seller can form a link whenever there exists a \( \gamma \in [0, 1] \) that makes it profitable for both.

Proposition

1. For all \( \beta \), there exist potential match surpluses such that all gains from trade are lost to overinvestment on all stable networks.

2. For all potential match surpluses, no surplus is ever lost to underinvestment on all stable networks.
Some limitations

- Outcomes can be bad, but they can also be good. What should we expect?

- Institutions might develop to mitigate inefficiencies.

- Approach to market clearing is “reduced form.” What bargaining game yields these outcomes?

- Are bargaining outcomes really always efficient?
Outline

1. Kranton and Minehart, 2001
2. Elliott, 2014
3. Corominas Bosch, 2004
4. Manea, 2011
5. Elliott and Nava, 2016
6. Experiments
Model

Surpluses are zero or one \((s_{ij} \in \{0, 1\})\).

One side of the market posts prices at a time.

Discount factor \(\delta\). Let \(\delta \rightarrow 1\).

Solution concept: Perfect equilibria
Protocol

(i) Buyers selected to be the proposers with probability \( p \); sellers with probability \( 1 - p \).

(ii) Proposers simultaneously post prices.

(iii) Responders simultaneously accept a price, or reject all prices.

Proposer \( i \) is matched to responder \( j \) only if \( s_{ij} = 1 \) and \( i \) accepts \( j \)’s price.

When multiple responders accept the same price, maximal possible match is implemented.

(iv) If \( i \) is matched to \( j \), they exit. Otherwise continue.
Example
Example

\[
\begin{array}{ccc}
 s_1 & b_1 & b_2 & b_3 \\
 0.75 & 0.5 & 1 & \\
\end{array}
\]
Example

\[
\begin{align*}
& b_1 & & b_2 & & b_3 \\
& s_1 & & s_2 & & s_3 \\
& 0.75 & & 0.5 & & 1
\end{align*}
\]

\text{reject}
Example

\[ s_1 \quad s_2 \quad s_3 \]

\[ b_1 \quad b_2 \quad b_3 \]

\[ \text{reject} \]

\[ 0.75 \quad 0.5 \quad 1 \]

\[ \text{reject} \]

\[ 0.9 \quad 0 \quad 0.9 \]

\[ \text{reject} \]

\[ 0.9 \quad 0 \quad 0.9 \]

\[ 0.9 \quad 0 \quad 0.1 \]

\[ 0 \quad 1 \quad 0 \]
Example

\[ s_1 \rightarrow b_1 \rightarrow s_2 \rightarrow b_2 \rightarrow s_3 \]

\[ s_3 \rightarrow b_3 \rightarrow s_2 \rightarrow b_2 \rightarrow s_3 \]

\[ s_1 \rightarrow b_1 \rightarrow s_2 \rightarrow b_2 \rightarrow s_3 \]

\[ s_3 \rightarrow b_3 \rightarrow s_2 \rightarrow b_2 \rightarrow s_3 \]

\[ s_1 \rightarrow b_1 \rightarrow s_2 \rightarrow b_2 \rightarrow s_3 \]
Example

\begin{align*}
0.9 &\quad b_1 \\
0 &\quad b_2 \\
b_3 &\quad b_3
\end{align*}

\begin{align*}
s_1 &\quad \text{reject} \\
s_2 &\quad 0.9 \\
s_3 &\quad 0
\end{align*}
Example

0.9

\(s_1\)
\(b_1\)
reject

0

\(s_2\)
\(b_2\)
0.9

\(s_3\)
\(b_3\)
Example

\begin{align*}
0.9 & \quad b_1 \\
0.1 & \quad b_2 \\
0 & \quad b_3 \\
\end{align*}

\begin{align*}
0.9 & \quad s_1 \\
0.1 & \quad s_2 \\
1 & \quad s_3 \\
\end{align*}

reject

1

0
Redundant Links?

Are there links agreement is never reached on?
Redundant Links?

Are there links agreement is never reached on?
Redundant Links?
Are there links agreement is never reached on?

\[ b_1 \quad b_2 \quad b_3 \]
\[ s_1 \quad s_2 \quad s_3 \]
Redundant Links?
Are there links agreement is never reached on?

\[ b_1 \to s_1 \to b_2 \to s_2 \to b_3 \]
Redundant Links?
Are there links agreement is never reached on?
Redundant Links?

Are there links agreement is never reached on?
Redundant Links?
Are there links agreement is never reached on?
Redundant Links?
Are there links agreement is never reached on?
Redundant Links?
Are there links agreement is never reached on?

b_1
s_1

b_2
s_2

b_3
s_3

b_4
s_4
Edmonds-Gallai Structure Theorem

Partition nodes into three types.

**C**: Those who are unmatched in at least one maximal matchings.

**A**: Those remaining agents with a link to at least one agent in C.

**B**: Other agents.

Theorem

(i) In every maximal matching if $i \in A$, then $\mu(i) \in C$.

(ii) In every maximal matching if $i \in B$, then $\mu(i) \in B$.
Decomposition

If bargaining always results in a maximal match, then

(i) Can remove all links connecting an agent in $A$ with another agent in $A$.

(ii) Can remove all links connecting an agent in $A$ with another agent in $B$. 
Decomposition

If bargaining always results in a maximal match, then

(i) Can remove all links connecting an agent in $A$ with another agent in $A$.

(ii) Can remove all links connecting an agent in $A$ with another agent in $B$.

Then have three types of components.

$\mathcal{G}_B$: Those components with fewer buyers than sellers

$\mathcal{G}_S$: Those components with fewer sellers than buyers

$\mathcal{G}_E$: Those components with equal numbers of sellers and buyers
Decomposition

\[ b_1 \quad b_2 \quad b_3 \]

\[ s_1 \quad s_2 \quad s_3 \]

- Type A
- Type B
- Type C
Decomposition

\[ b_1 \quad \text{Type C} \quad b_2 \quad \text{Type B} \quad b_3 \quad \text{Type A} \]

\[ s_1 \quad s_2 \quad s_3 \]

\[ \text{Type A} \quad \text{Type B} \quad \text{Type C} \]
Decomposition

- Type A
- Type B
- Type C
Decomposition

\[s_1 \rightarrow b_1 \rightarrow s_2 \rightarrow b_2 \rightarrow s_3 \rightarrow b_3\]

- \(s_1\): Type A
- \(s_2\): Type B
- \(s_3\): Type C
Decomposition

\[ \begin{align*}
\text{Type A} & \quad \text{(Green)} \\
\text{Type B} & \quad \text{(Yellow)} \\
\text{Type C} & \quad \text{(Red)}
\end{align*} \]

Diagram:

- \( b_1 \) connected to \( s_1 \) and \( b_2 \)
- \( b_2 \) connected to \( s_2 \) and \( b_3 \)
- \( s_1 \) and \( s_2 \) are connected to each other

Node labels:

- \( b_1 \)
- \( b_2 \)
- \( b_3 \)
- \( s_1 \)
- \( s_2 \)
Decomposition

\[ b_1 \rightarrow s_1 \rightarrow b_2 \rightarrow s_2 \rightarrow b_3 \]

- Type A: Green
- Type B: Yellow
- Type C: Red
Decomposition

\[ b_1 \]

\[ s_1 \]

\[ b_2 \]

\[ s_2 \]

\[ b_3 \]

- Green node: Type A
- Yellow node: Type B
- Red node: Type C
Decomposition

b₁ → s₁ → b₂ → s₂ → b₃

- b₁: Type A
- s₁: Type B
- b₂: Type B
- s₂: Type A
- b₃: Type C
Decomposition

- $b_1$: Type B
- $b_2$: Type B
- $b_3$: Type B
- $s_1$: Type A
- $s_2$: Type A
Decomposition

- Type A
- Type B
- Type C
Decomposition

\[ s_1 s_2 s_3 \]
\[ b_3 b_2 b_1 \]

: Type C
: Type B
: Type A

\[ s_2 s_4 \]
\[ b_4 b_2 b_1 b_3 \]
\[ s_1 s_3 \]

: Type C
: Type B
: Type A

\[ s_4 \]
Decomposition

- $s_1$, $s_2$, $s_3$, $s_4$
- $b_1$, $b_2$, $b_3$, $b_4$

- $s_1$: Type A
- $s_2$: Type B
- $s_3$: Type A
- $s_4$: Type C

- $b_1$: Type C
- $b_2$: Type B
- $b_3$: Type A
- $b_4$: Type B

Diagram:

- Nodes $s_1$, $s_2$, $s_3$, $s_4$ connected to $b_1$, $b_2$, $b_3$, $b_4$.
Decomposition

\(s_1\) \(s_2\) \(s_3\)
\(b_3\) \(b_2\) \(b_1\)

: Type C
: Type B
: Type A
Decomposition

\[ b_1 \quad b_2 \quad b_3 \quad b_4 \]

\[ s_1 \quad s_2 \quad s_3 \quad s_4 \]

- \( s_1 \): Type A
- \( s_2 \): Type B
- \( s_4 \): Type C
- \( s_3 \): Type B

\[
\begin{align*}
&b_1 \\ &\downarrow \\ &s_1 \\
&b_2 \\ &\downarrow \\ &s_2 \\
&b_3 \\ &\downarrow \\ &s_3 \\
&b_4 \\ &\downarrow \\ &s_4
\end{align*}
\]
Equilibrium

Proposition

There exists a perfect equilibrium in which in all components

(i) $G_B$, buyers get limit payoffs of 1 and sellers get limit payoffs of 0;

(ii) $G_S$, sellers get limit payoffs of 1 and buyers get limit payoffs of 0;

(iii) $G_E$, buyers get limit payoffs of $p$ and sellers get limit payoffs of $1 - p$. 

These payoffs correspond to $p = \beta$ from Elliott (2014).
Equilibrium

Proposition

There exists a perfect equilibrium in which in all components

(i) $\mathcal{G}_B$, buyers get limit payoffs of 1 and sellers get limit payoffs of 0;

(ii) $\mathcal{G}_S$, sellers get limit payoffs of 1 and buyers get limit payoffs of 0;

(iii) $\mathcal{G}_E$, buyers get limit payoffs of $p$ and sellers get limit payoffs of $1 - p$.

These payoffs correspond to $p = \beta$ from Elliott (2014).
Outline

1. Kranton and Minehart, 2001
2. Elliott, 2014
3. Corominas Bosch, 2004
4. Manea, 2011
5. Elliott and Nava, 2016
6. Experiments
Model

Surpluses are zero or one \((s_{ij} \in \{0, 1\})\).

Exiting agents are replaced by replicas.

Discount factor \(\delta\).

Solution concept: Perfect equilibria
Protocol

(i) A link is selected uniformly at random.

(ii) Assigned to be proposer and responder with equal probability.

(iii) Proposer makes an offer.

(iv) Responder accepts or rejects.

If reject move to next round.

If accept exit, but replaced with replicas for next round.
Protocol
Protocol

b₁

b₂

b₃

s₁

s₂

1/2

3/4

Rejected

Accepted

1/4
Protocol

Diagram:

- $b_1$ connected to $s_1$ and $b_3$
- $b_2$ connected to $s_2$
- $b_3$ connected to $s_2$

Status:

- $s_1$: 1/2
- $s_2$: 1/2

- $s_1$: 1/4
- $s_2$: 3/4

- Accepted

- Rejected
Protocol

\[ \frac{1}{2} \quad b_1 \quad s_1 \quad b_2 \quad s_2 \quad b_3 \]

\[ \frac{1}{2} \quad \frac{3}{4} \quad \text{Accepted} \]

\[ \frac{1}{4} \quad \frac{3}{4} \quad \text{Rejected} \]
Protocol

$1/2$

$b_1$

$s_1$

$b_2$

$s_2$

$b_3$

$1/2$

Rejected

Accepted
Protocol

\[ \begin{align*}
\mathbf{b}_1 &\rightarrow \mathbf{s}_1 \\
\mathbf{b}_1 &\rightarrow \mathbf{b}_2 \\
\mathbf{b}_2 &\rightarrow \mathbf{s}_2 \\
\mathbf{b}_2 &\rightarrow \mathbf{b}_3 \\
\mathbf{b}_3 &\rightarrow \mathbf{s}_2 \\
\end{align*} \]
Protocol

\[
\begin{align*}
&b_1 & b_2 & b_3 \\
&s_1 & & s_2 \\
&\text{Rejected} & & \text{Accepted}
\end{align*}
\]
Protocol

\[
\begin{align*}
  b_1 & \quad s_1 \\
  b_2 & \quad s_2 \\
  b_3 &
\end{align*}
\]
Protocol

\[ b_1 \rightarrow s_1, b_2 \rightarrow s_2, b_3 \rightarrow s_2 \]

\[ s_2 \rightarrow 1/4, \quad s_2 \rightarrow 3/4 \]

\[ b_2 \rightarrow 1/4, \quad b_2 \rightarrow 3/4 \]

\[ s_1 \rightarrow b_1, s_2 \rightarrow b_2, s_3 \rightarrow b_3 \]

\[ \text{Rejected} \]

\[ \text{Accepted} \]
Protocol

\[ b_1 \rightarrow s_1 \quad b_2 \rightarrow s_2 \quad b_3 \rightarrow s_3 \]

\[ s_1 \rightarrow \frac{1}{4} \quad s_2 \rightarrow \frac{3}{4} \quad s_3 \rightarrow \text{Accepted} \]
Protocol

\[ b_1 \rightarrow s_1 \rightarrow b_3 \]

\[ \frac{1}{2} \]

\[ \frac{3}{4} \]

Rejected

\[ \frac{1}{4} \]

\[ \frac{3}{4} \]

Accepted
Protocol

\[ b_1 \quad b^{R_2} \quad b_3 \]

\[ s_1 \quad S^{R_2} \]
Aside—A different model

One buyer, one seller, size of pie 1.

Buyer proposes with probability $p$.

Seller proposes with probability $q$.

There is a unique perfect equilibrium (Rubinstein, 1982).

Limit payoffs:
Aside—A different model

One buyer, one seller, size of pie 1.

Buyer proposes with probability $p$.

Seller proposes with probability $q$.

There is a unique perfect equilibrium (Rubinstein, 1982).

Limit payoffs:

$$u_i(\delta) \rightarrow \frac{p}{p + q} \quad v_j(\delta) \rightarrow \frac{q}{p + q}$$

Call these payoffs Rubinstein payoffs.
Back to Manea (2011): Some examples

\[ s_1 b_1 \]

\[ u_1 \mapsto \frac{1}{2}, \quad v_1 \mapsto \frac{1}{2} \]

\[ p_{b_1} = \frac{1}{2}, \quad p_{s_1} = \frac{1}{6}, \quad p_{s_2} = \frac{1}{6}, \quad p_{s_3} = \frac{1}{6} \]

\[ s_1 s_2 s_3 \]

\[ b_3 b_2 b_1 \]

\[ p_{s_1} = 0.1, \quad p_{s_2} = 0.1, \quad p_{s_3} = 0.3, \quad p_{b_1} = 0.3, \quad p_{b_2} = 0.1, \quad p_{b_3} = 0.1 \]

\[ v_1 \mapsto \frac{1}{2}, \quad v_2 \mapsto \frac{1}{2}, \quad v_3 \mapsto \frac{2}{3} \]

\[ u_1 \mapsto \frac{2}{3}, \quad u_2 \mapsto \frac{1}{3}, \quad u_3 \mapsto \frac{1}{3} \]
Back to Manea (2011): Some examples

\[ \begin{align*}
    u_1 & \to 1/2 \\
    b_1 & \\
    s_1 & \\
    v_1 & \to 1/2
\end{align*} \]
Back to Manea (2011): Some examples

\[ b_1 \]

\[ s_1 \]
\[ s_2 \]
\[ s_3 \]

\[ u_1 \leftrightarrow \frac{1}{2} \]
\[ v_1 \leftrightarrow \frac{1}{2} \]

\[ p(b_1) = \frac{1}{2} \]
\[ p(s_1) = \frac{1}{6} \]
\[ p(s_2) = \frac{1}{6} \]
\[ p(s_3) = \frac{1}{6} \]

\[ v_1 \leftrightarrow \frac{1}{3} \]
\[ u_2 \leftrightarrow \frac{1}{3} \]
\[ u_3 \leftrightarrow \frac{1}{3} \]
Back to Manea (2011): Some examples

$p_{b_1} = 1/2$

$s_1$
$p_{s_1} = 1/6$

$s_2$
$p_{s_2} = 1/6$

$s_3$
$p_{s_3} = 1/6$
Back to Manea (2011): Some examples

It is as if:

\[ p_{b_1} = \frac{1}{2} \]

\[ u_1 \mapsto \frac{3}{4} \]

\[ v_1 \mapsto \frac{1}{4} \]

\[ v_2 \mapsto \frac{1}{3} \]

\[ u_2 \mapsto \frac{1}{3} \]

\[ u_3 \mapsto \frac{1}{3} \]

\[ p_s = \frac{1}{6} \]

\[ p_{b_1} = \frac{1}{2} \]

\[ p_s = \frac{1}{6} \]

\[ p_{s_3} = \frac{1}{6} \]

\[ p_{b_3} = \frac{1}{8} \]

\[ p_{b_2} = \frac{1}{8} \]

\[ p_{b_1} = \frac{1}{4} \]
Back to Manea (2011): Some examples

\[ u_1 \rightarrow \frac{1/2}{1/6 + 1/2} = \frac{3}{4} \]

\[ v \rightarrow \frac{1/6}{1/6 + 1/2} = \frac{1}{4} \]

\[ p_{b_1} = \frac{1}{2} \]

\[ p_s = \frac{1}{6} \]
Back to Manea (2011): Some examples

\[ p_{b_1} = 0.3 \]
\[ b_1 \]
\[ p_{s_1} = 0.1 \]
\[ s_1 \]

\[ p_{b_2} = 0.1 \]
\[ b_2 \]
\[ p_{s_2} = 0.1 \]
\[ s_2 \]

\[ p_{b_3} = 0.1 \]
\[ b_3 \]
\[ p_{s_3} = 0.3 \]
\[ s_3 \]
Back to Manea (2011): Some examples

\[ p_{b_1} = 0.3 \]

\[ p_{s_1} = 0.1 \]

\[ u_1 \Rightarrow \frac{1}{2} \]
\[ v_1 \Rightarrow \frac{1}{4} \]

\[ u_1 \Rightarrow \frac{3}{4} \]
\[ v_1 \Rightarrow \frac{1}{4} \]

\[ u_1 \Rightarrow \frac{3}{4} \]
\[ v_2 \Rightarrow \frac{1}{3} \]
\[ v_3 \Rightarrow \frac{2}{3} \]

\[ u_3 \Rightarrow \frac{1}{4} \]
\[ v_3 \Rightarrow \frac{2}{3} \]

\[ u_3 \Rightarrow \frac{1}{4} \]
\[ v_3 \Rightarrow \frac{1}{2} \]

\[ u_3 \Rightarrow \frac{1}{2} \]
Back to Manea (2011): Some examples

\[ u_1 \rightarrow \frac{3}{4} \]

\[ v_1 \rightarrow \frac{1}{4} \]
Back to Manea (2011): Some examples

\[ u_1 \xrightarrow{1/2} v_1 \xrightarrow{1/2} u_1 \]

\[ u_1 \xrightarrow{1/6} v_1 \xrightarrow{1/6} u_1 \]

\[ p_{b1} = 1/2 \]

\[ p_{s1} = 1/6 \]

\[ p_{s2} = 1/6 \]

\[ p_{s3} = 1/6 \]

It is as if:

\[ p_{b1} = 1/2 \]

\[ p_{s} = 1/6 \]

\[ u_1 \xrightarrow{1/2} \]

\[ v_1 \xrightarrow{1/4} \]

\[ u_1 \xrightarrow{3/4} \]

\[ v_3 \xrightarrow{1/4} \]

\[ u_3 \xrightarrow{1/4} \]

\[ v_3 \xrightarrow{3/4} \]

\[ u_3 \xrightarrow{1/4} \]

\[ p_{s1} = 0.1 \]

\[ p_{s2} = 0.1 \]

\[ p_{s3} = 0.3 \]

\[ p_{b1} = 0.3 \]

\[ p_{b2} = 0.1 \]

\[ p_{b3} = 0.1 \]

\[ p_{s3} = 0.3 \]

\[ p_{b3} = 0.1 \]

\[ p_{s3} = 0.3 \]
Back to Manea (2011): Some examples

\[ u_3 \rightarrow \frac{1}{4} \]

\[ v_3 \rightarrow \frac{3}{4} \]

\[ u_1 \rightarrow \frac{1}{2}; \quad v_1 \rightarrow \frac{1}{2} \]

\[ u_3 \rightarrow \frac{1}{4} \]

\[ v_3 \rightarrow \frac{3}{4} \]

\[ v_1 \rightarrow \frac{1}{3}; \quad u_2 \rightarrow \frac{1}{3}; \quad u_3 \rightarrow \frac{1}{3} \]
Back to Manea (2011): Some examples

\[ u_1 \rightarrow 1/2 \]

\[ b_1 \]

\[ b_2 \]

\[ b_3 \]

\[ s_1 \]

\[ s_2 \]

\[ s_3 \]

\[ v_3 \rightarrow 1/2 \]
Back to Manea (2011): Some examples

\[
\begin{align*}
p_{b1} &= 1/4 \\
&s_1 \\np_{s1} &= 1/8 \\
b_1 &
\end{align*}
\]

\[
\begin{align*}
p_{b2} &= 1/8 \\
&s_2 \\np_{s2} &= 1/8 \\
b_2 &
\end{align*}
\]

\[
\begin{align*}
p_{b3} &= 1/8 \\
&s_3 \\np_{s3} &= 1/4 \\
b_3 &
\end{align*}
\]


It is as if:

\[
\begin{align*}
p_{b1} &= 1/2 \\
p_{s} &= 1/6 \\
\end{align*}
\]

\[
\begin{align*}
u_1 &\rightarrow 1/2 \\
\end{align*}
\]

\[
\begin{align*}
v_1 &\rightarrow 1/3 \\
u_2 &\rightarrow 1/3 \\
u_3 &\rightarrow 2/3 \\
\end{align*}
\]
Back to Manea (2011): Some examples

\[ u_1 \rightarrow 2/3 \]
\[ b_1 \]
\[ v_1 \rightarrow 1/3 \]
\[ s_1 \]

\[ u_2 \rightarrow 1/3 \]
\[ b_1 \]
\[ v_2 \rightarrow 1/3 \]
\[ s_2 \]

\[ u_3 \rightarrow 1/3 \]
\[ b_3 \]
\[ v_3 \rightarrow 2/3 \]
\[ s_3 \]
Decomposition

Apply Edmonds-Gallai?

Agreements are never reached over these links.
Sometimes enough

\[ \begin{align*}
   v_1 & = \frac{1}{2} v_2 & \frac{2}{3} \\
   u_1 & = \frac{1}{2} & \frac{1}{3} u_2 = \frac{1}{3} & u_3 = \frac{1}{3} \\
   v_3 & = \frac{1}{3} \\
   u_4 & = \frac{2}{3} \end{align*} \]
Sometimes enough

\[ v_1 ' = \frac{1}{2} \quad v_2 ' = \frac{2}{3} \quad u_1 ' = \frac{1}{2} \quad u_2 = \frac{1}{3} \quad u_3 = \frac{1}{3} \quad v_4 ' = \frac{1}{3} \]
Sometimes enough

\[ u_1 \rightarrow \frac{1}{2} \quad b_1 \]

\[ v_1 \rightarrow \frac{1}{2} \quad s_1 \]

\[ u_2 \rightarrow \frac{1}{3} \quad b_2 \]

\[ v_2 \rightarrow \frac{2}{3} \quad s_2 \]

\[ u_3 \rightarrow \frac{1}{3} \quad b_3 \]
Sometimes enough

\[ v_1 = \frac{1}{2}, \quad v_2 = \frac{2}{3}, \quad v_4 = \frac{1}{3}, \quad v_3 = \frac{1}{3} \]

\[ u_1 = \frac{1}{3}, \quad u_2 = \frac{1}{3}, \quad u_4 = \frac{2}{3}, \quad u_3 = \frac{1}{2} \]
Sometimes enough

\[ v_1' = \frac{1}{2} \quad v_2' = \frac{2}{3} \quad v_4' = \frac{1}{3} \]

\[ u_1' = \frac{1}{2} \quad u_2' = \frac{1}{3} \quad u_3' = \frac{1}{3} \]
Sometimes enough

\[ u_1 \rightarrow 1/3 \quad u_2 \rightarrow 1/3 \quad u_3 \rightarrow 1/2 \quad u_4 \rightarrow 2/3 \]

\[ b_1 \quad b_2 \quad b_3 \quad b_4 \]

\[ s_1 \quad s_2 \quad s_3 \quad s_4 \]

\[ v_1 \rightarrow 2/3 \quad v_2 \rightarrow 1/2 \quad v_3 \rightarrow 1/3 \quad v_4 \rightarrow 1/3 \]
But not always

\[ v_2 \rightarrow \frac{3}{4} \; v_3 \rightarrow \frac{2}{3} \]

\[ u_1 = u_2 = u_3 \rightarrow \frac{1}{4} \; u_4 = u_5 \rightarrow \frac{1}{3} \]
But not always

\[ v_2 \Rightarrow \frac{3}{4} v_3 \Rightarrow \frac{2}{3} \]

\[ u_1 = u_2 = u_3 \Rightarrow \frac{1}{4} u_4 = u_5 \Rightarrow \frac{1}{3} \]
But not always

\[ v_2 = \frac{3}{4} v_3 = \frac{2}{3} \]

\[ u_1 = u_2 = u_3 = \frac{1}{4} u_4 = u_5 = \frac{1}{3} \]
But not always

\[
\begin{align*}
1/12 & \quad b_1 \\
1/12 & \quad b_2 \\
1/12 & \quad b_3 \\
1/6 & \quad b_4 \\
1/12 & \quad b_5
\end{align*}
\]

\[
\begin{align*}
& \quad s_1 \\
& \quad 1/3
\end{align*}
\]

\[
\begin{align*}
& \quad s_2 \\
& \quad 1/6
\end{align*}
\]
But not always

\[
\begin{align*}
1/12 & \quad b_1 \\
1/12 & \quad b_2 \\
1/12 & \quad b_3 \\
1/6 & \quad b_4 \\
1/12 & \quad b_5
\end{align*}
\]

\[
\begin{align*}
1/3 & \quad s_1 \quad 1/3 \\
1/6 & \quad s_2 \\
\end{align*}
\]
But not always

\[
\begin{align*}
&\frac{1}{10} \\
&b_1 \\
&\frac{3}{10} \\
&s_1 \\

&\frac{1}{10} \\
&b_2 \\
&\frac{3}{10} \\
&s_1 \\

&\frac{1}{10} \\
&b_3 \\
&\frac{3}{10} \\
&s_1 \\

&\frac{1}{10} \\
&b_4 \\
&\frac{1}{5} \\
&s_2 \\

&\frac{1}{10} \\
&b_5 \\
&\frac{1}{5} \\
&s_2
\end{align*}
\]
But not always

\[ u_1 = u_2 = u_3 \rightarrow 1/4 \]

\[ \nu_2 \rightarrow 3/4 \]

\[ v_3 \rightarrow 2/3 \]

\[ u_4 = u_5 \rightarrow 1/3 \]
An algorithm in the paper shows how to decompose networks.

Deletes a superset of the links deleted by the Edmonds-Gallai decomposition.

After this decomposition, payoffs determined the ratio of buyers to sellers in each component.
Discussion

Alternative options make players more patient yielding bargaining power. But only credible options matter.

Stationarity assumption is strong but delivers a lot of tractability.

Can be motivated as a steady state of a large market (Manea, 2015).

Hard to think about efficiency, or at least mismatch.
Outline

1. Kranton and Minehart, 2001
2. Elliott, 2014
3. Corominas Bosch, 2004
4. Manea, 2011
5. Elliott and Nava, 2016
6. Experiments
Model

Surpluses $s_{ij} \in \mathbb{R}_+$. 

No replication after exit.

Set of active agent (yet to exit) denoted $A$.

Discount factor $\delta < 1$.

Solution concept: Markov perfect equilibria.
Protocol

(i) Player \( i \) selected as proposer with probability \( p_i \).

(ii) Choose who to make an offer to.

(iii) Offer accepted or rejected.

If rejected, move to next round.

If accepted exit, and move to the next round.
Protocol

Diagram of network nodes and connections with labels:

- $b_1$, $b_2$, $b_3$, $s_1$, $s_2$, $s_3$

Connections and weights:

- $b_1$ to $s_1$: 10
- $b_1$ to $s_2$: 17
- $b_1$ to $s_3$: 14
- $b_2$ to $s_2$: 10
- $b_2$ to $s_3$: 15
- $b_3$ to $s_3$: 10
Protocol

\begin{itemize}
\item $b_1$
\item $s_1$
\item $b_2$
\item $s_2$
\item $b_3$
\item $s_3$
\end{itemize}

$10 \rightarrow 15 \rightarrow s_1 \rightarrow 10 \rightarrow 14 \rightarrow s_2 \rightarrow 10 \rightarrow 17 \rightarrow s_3$
Protocol
Protocol
Protocol

$s_1$

$s_2$ $s_3$

$b_1$ $b_2$ $b_3$

10 15

3 3 2

Accepted
Markov Perfect Equilibria (MPE)

**Definition**
The Markov state, in every period, is the set of active players $A$.

Markov Perfection implies that player $i \in A$:
- accepts any offer better than his continuation value.
- if the proposer, choose to make the just acceptable offer that leaves him with the most surplus or else delays.

A MPE exists.
Efficiency

**Definition**

An equilibrium is efficient if it results in the match that maximizes total surplus.

Definition permits delay, but results will be for high $\delta$ and can be shown there is no delay without mismatch.
Example: Is there an efficient MPE?

Figure: All players propose with equal probability.
Example: Is there an efficient MPE?

Figure: Accepted offers in the unique MPE for $y \in (0, 100]$. 
Example: Is there an efficient MPE?

\[ q \in (0, 1) \]

\[ 1 - q \]

\[ 1 \]

\[ a \]

\[ c \]

\[ b \]

\[ d \]

**Figure:** Accepted offers in the unique MPE for \( y \in (100, 143) \).
Example: Is there an efficient MPE?

Figure: Accepted offers in the unique MPE for $y \in [144, 200]$. 
Rubinstein Payoffs

The *Rubinstein Payoff* of player \( i \) is defined as

\[
\sigma_i = \frac{p_i}{p_i + p_{\eta(i)}} s_{i\eta(i)}
\]

where \( \eta(i) \) is \( i \)'s efficient match.
Rubinstein Payoffs

The *Rubinstein Payoff* of player $i$ is defined as

$$\sigma_i = \frac{p_i}{p_i + p_{\eta(i)}} s_{i\eta(i)}$$

where $\eta(i)$ is $i$’s efficient match.

These are the limit payoffs attained by bilateral bargaining with the core matches.
Equilibrium Efficiency (high $\delta$)

**Theorem**

An efficient MPE exists for all $\delta$ close to 1

(i) if Rubinstein payoffs belong to the interior of core:

$$\sigma_i + \sigma_j > s_{ij} \text{ for all } (i, j) \in B \times S \text{ such that } j \neq \eta(i).$$

(ii) only if Rubinstein payoffs belong to the core:

$$\sigma_i + \sigma_j \geq s_{ij} \text{ for all } (i, j) \in B \times S.$$
Outline

1. Kranton and Minehart, 2001
2. Elliott, 2014
3. Corominas Bosch, 2004
4. Manea, 2011
5. Elliott and Nava, 2016
6. Experiments
Laboratory Experiments

Can control the potential gains from trade.

In practice don’t observe counterfactual match productivities.

Charness, Corominas-Bosch and Frechette (2007).

- Find qualitative support for Corominas-Bosch.
- Links far away move payoffs in the direction predicted.
Laboratory Experiments

Agranov and Elliott (2016).

- Bargaining with exit as in Elliott and Nava (2016).
- Find qualitative support for MPE, but inefficiencies are larger.

Sociology (e.g. Cook and Emerson, 1978)

- Unstructured experiments without exit.
- Results roughly in line with the core.
Some References


Some References


Some References


Some References


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