EXAMPLE PROBLEM 1

Write down the general forms of the conservation equations in integral and differential form, using vector notation. Do not make any assumptions about the fluid or the flow (i.e., do not assume the fluid is Newtonian, or incompressible, inviscid, etc. and do not assume the flow is steady, fully-developed, etc.). If you know the general forms of the equations, you will be on your way to solving most problems you have seen thus far. These equations should be the *first* thing you write down for each problem.

Now, assume only that the fluid is Newtonian so that the shear stress tensor is given by the relation

$$\vec{\vec{\tau}} = \mu \nabla \vec{u}. \tag{1}$$

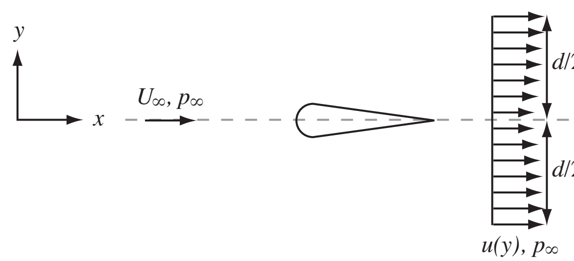
Rewrite the Navier-Stokes equation under this assumption.

EXAMPLE PROBLEM 2

A rocket is moving vertically with speed $u_r(t)$. The mass of the rocket system at time t can be denoted by $M(t)$ and includes the mass of the rocket itself, the unspent fuel, and the exhaust, prior to it exiting. At the nozzle exit, the area is A_e and the pressure and density are p_e and ρ_e, respectively. From stationary tests of the rocket nozzle, the exit velocity of the exhaust is u_e.

Find an expression for the rate of change of mass for the system, and an expression for the acceleration of the rocket. The acceleration can be written in terms of the given parameters, atmospheric pressure p_a, and the drag D.

EXAMPLE PROBLEM 3

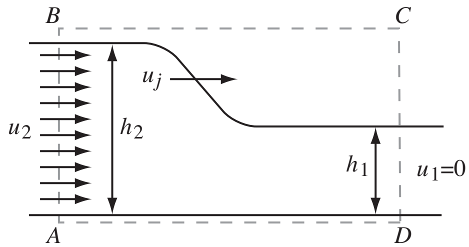


A symmetrical wing is being tested at zero incidence in a wind tunnel. The upstream speed is U_∞ and is uniform across the tunnel, and the pressure is p_∞ (also uniform). At a station far downstream of the wing, the pressure is again uniform at p_∞ and the velocity in the stream-wise direction is

$$u(y) = U_\infty, \quad |y| > d/2; \qquad u(y) = U_\infty - q(1 + \cos(2\pi y/d)), \quad -d/2 \leq y \leq d/2, \tag{2}$$

where y is the distance from the wing centerline (which lies on the x-axis in Cartesian coordinates) and d and q are the wake-defect length and velocity-scale parameters, respectively. Find the drag on the wing per unit meter of span. You can assume that all along streamlines that pass through $y = \pm d/2$ at the downstream station, the pressure is p_∞ and the shear stresses are negligible.

EXAMPLE PROBLEM 4



On a horizontal slope, a large-amplitude surface wave (hydraulic bore or tsunami) is propagating to the right at a constant speed u_j. Far upstream of the bore the (undisturbed) fluid is at rest and is of height h_1. The pressure distribution in the fluid can be considered hydrostatic. Far downstream of the bore (to the left) the fluid velocity distribution can be well approximated by uniform speed u_2 and the pressure under the surface is again hydrostatic. The flow in the region of sudden height change is extremely turbulent. Assume however, that the fluid is incompressible and neglect viscosity.

(a) Using the stationary control volume $ABCD$ and the conservation laws of mass and momentum in integral form for this *unsteady flow*, show that the height ratio h_2/h_1 is given by

$$\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8F_1^2} - 1\right), \tag{3}$$

where $F_1 = u_j/\sqrt{gh_1}$ is the Froude number of the bore and g is the acceleration due to gravity. F_1 is a dimensionless number measuring the ratio of the speed of the bore to the speed of propagation of a long wavelength, small amplitude wave on the surface of the fluid. [Hint: Consider the flow for two different times separated by time Δt.]

(b) Show that the stream-wise pressure-momentum balance for the control voume shown can be written in the form $f(h_1) = f(h_2)$, where

$$f(h) = \frac{gh^2}{2} + \frac{q^2}{h}, \tag{4}$$

and $q = h_1 u_j = h_2(u_j - u_2)$ is the volume flux.

EXAMPLE PROBLEM 5

An incompressible Newtonian fluid is flowing in a long circular pipe of radius R. Write the simplified forms of the continuity and Navier-Stokes equations for this flow assuming steady, fully-developed flow. Derive an expression for the velocity distribution u and evaluate this to determine u_{max} and the average velocity $\overline{u}$.

The flow then encounters a 90° reducing elbow. At the inlet to the elbow, the pressure is p_{in}, the radius is still given by R, and the velocity follows from above. At the outlet, the radius is $R_{out} < R$ and the velocity is assumed to be uniform. The elbow discharges to atmospheric pressure. Determine the force required to hold the elbow in place. You may neglect the weight of the elbow and the fluid.

EXAMPLE PROBLEM 6

A liquid flows down an inclined plane surface (at angle θ) due to gravity in a steady, fully-developed laminar film of thickness h. Simplify the continuity and Navier-Stokes equations to model this flow assuming the fluid is Newtonian. Obtain expressions for the velocity profile in the liquid and the shear stress distribution. Relate the film thickness to the volume flow rate per unit depth of surface normal to the flow.

EXAMPLE PROBLEM 7

Consider a spherical cloud of gas of radius $R(t)$ and total mass M. The cloud is expanding into a vacuum in such a fashion that the mass density ρ remains spatially uniform, i.e., $\rho = \rho(t)$ only. Neglect the influence of gravity.

(a) Compute the divergence $\nabla \cdot \vec{u}$ of the velocity field.

(b) Use this result to show that the fluid velocity within the cloud can be represented by

$$u_r(r,t) = r\frac{\dot{R}}{R} \quad \text{where} \quad \dot{R} = \frac{dR}{dt}. \tag{5}$$

Consider that the cloud remains spherically symmetric and the density is spatially uniform at all times.

(c) Find the location r at time t of a particle that was located at r_0 at time t_0 as a function of $R(t)$ and $R(t_0)$.

(d) The pressure goes to zero at the outer edge of the cloud. Find the pressure distribution within the cloud and the relationship between the pressure at the center of the cloud to the rate of expansion $\dot{R}$.