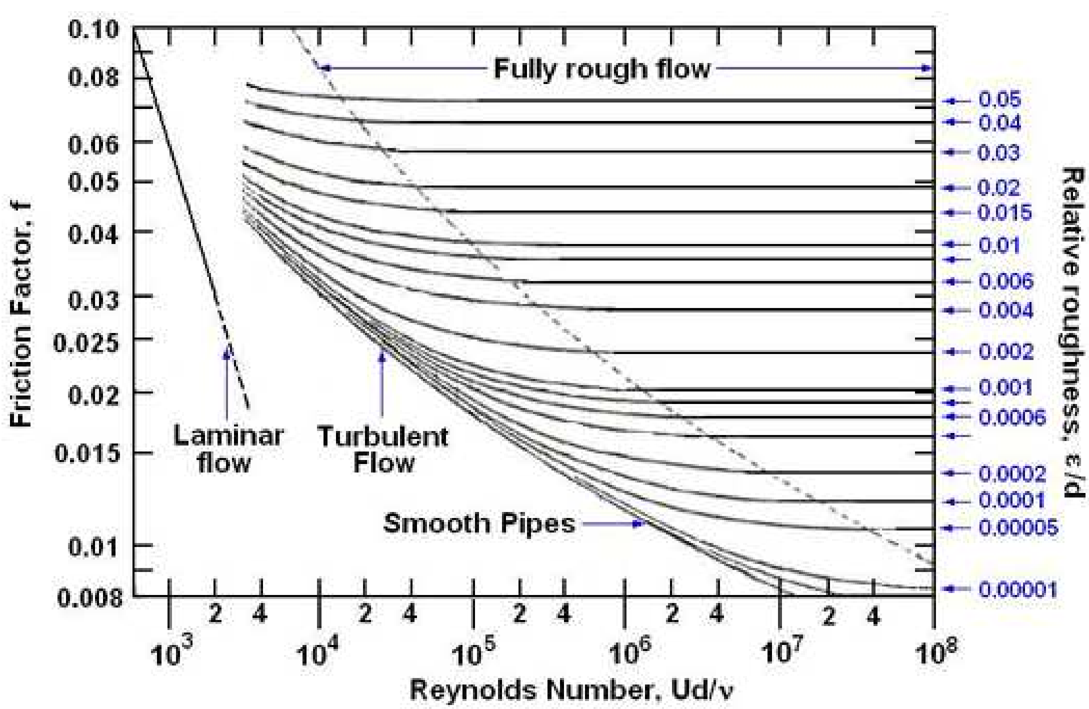
PROBLEM B23

In 1844, to commemorate a visit by the Tsar of Russia, the Duke of Devonshire, the greatest landowner in Britain, wished to construct the tallest fountain ever built in the grounds of his great house at Chatsworth in Derbyshire. He employed the renowned engineer Joseph Paxton to build what was to become known as the Emporer Fountain. That fountain remains the tallest, gravity-fed fountain in the world, with a maximum height of 90.2 m above the pond into which it falls. What Paxton did was to excavate a massive eight acre lake on a nearby hill such that the lake surface was 120 m above the afore-mentioned pond. The pipe to the fountain was 800 m long and had an internal diameter of 0.381 m. (Paxton knew that to maximize the height of the fountain he would have to make the pipe diameter large.) The result was that the maximum flow rate through the pipe (when the control valve was fully opened) was 15000 $liters/min$. Questions:

1. Using the above information find the friction factor for Paxton's pipe.
2. Find the Reynolds number for the flow in Paxton's pipe assuming a water temperature of $15°C$ so that the kinematic viscosity of the water 1.16×10^{-6} m^2/s.
3. What kind of flow is occuring in Paxton's pipe?
4. Using the answers to the first two questions estimate the typical height of the roughnesses in the interior surface of Paxton's pipe.
5. Assuming the same friction factor, what would the maximum height of the fountain have been if Paxton had used a pipe with a half of the above diameter?



PROBLEM B24

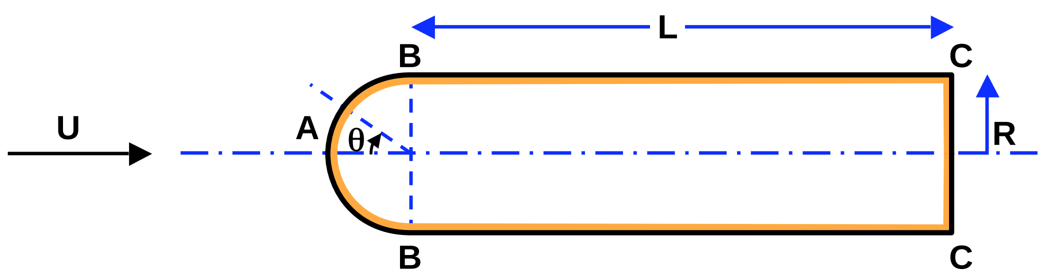
Consider a turbulent boundary layer on a flat plate (constant and uniform velocity and pressure in the flow outside the boundary layer). The plate is very rough, the size of the roughnesses, ϵ, being very much greater than the laminar sub-layer thickness which would occur in the absence of the roughness. It is anticipated that the velocity distribution within the turbulent part of the boundary layer can be approximated by

$$u^* = C(y/\epsilon)^{\frac{1}{7}}$$

where C is some constant, y is the distance from the wall, $u^* = \bar{u}/u_\tau$, where $\bar{u}$ is the mean velocity and the friction velocity, $u_\tau = (\tau_w/\rho)^{\frac{1}{2}}$, τ_w being the wall shear stress and ρ the fluid density. Using approximate boundary layer methods find an expression for the boundary layer thickness, δ, as a function of x, the distance along the plate from the leading edge. Assume initial conditions $\delta = 0$ at $x = 0$; the result includes ϵ, C and the profile parameter $\alpha = 0.0972$.

PROBLEM B25

The sketch below defines the geometry of an axisymmetric underwater body that is quite streamlined in the sense that L/R is large. This body travels through the incompressible water at a velocity, U, parallel to the axis.



It is to be assumed:

- that the velocity distribution over the spherical nose, BAB, is the same as in potential flow, that is to say the velocity outside the boundary layer is $\frac{3}{2}U \sin\theta$.
- that the flow separates at the sharp trailing edge, C, so that the pressure coefficient acting on the circular base, CC, is

  $$C_p = -0.5$$

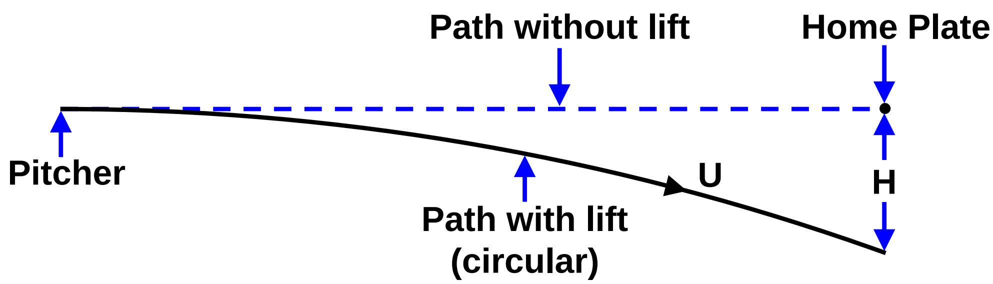
  Remember that the pressure coefficient is defined as, $C_p = (p - p_\infty)/\frac{1}{2}\rho U^2$ where p is the pressure, p_∞ is the pressure far upstream and ρ is the fluid density.
- that the skin friction forces on the spherical nose are negligible.

If the drag coefficient is defined as the drag divided by $\frac{1}{2}\rho U^2$ and the frontal projected area (πR^2) find:

1. The contribution of the form drag to the total drag coefficient (denote this by C_{DF}).
2. An estimate of the contribution of the skin friction on the cylindrical surface of the body (between B and C) to the total drag coefficient, assuming the boundary layer remains laminar. This should be in terms of the Reynolds number, $Re = 2UR/\nu$, where ν is the kinematic viscosity of the fluid.
3. For what aspect ratio, L/R, will the drag be comprised of equal parts of form and skin friction drag if $Re = 10000$?

PROBLEM B26

Suppose that the lift force experienced by a spinning baseball is to be estimated by $\rho U \Gamma a$ where ρ is the air density ($1\ kg/m^3$), U is its forward velocity (say $40\ m/s$), a is its radius ($0.03\ m$) and Γ is a circulation which is estimated as $2\pi a^2 \omega$ where ω is the velocity of spin (take $\omega = 200\ rad/s$). If the path of the baseball between the pitcher's mound and home plate (distance $\approx 20\ m$) is modeled as part of a circle, estimate the distance (in m) between the home plate arrival locations with and without the spin, in other words estimate the distance, H:



Neglect gravity (what!?). The mass of the baseball is $0.145\ kg$.