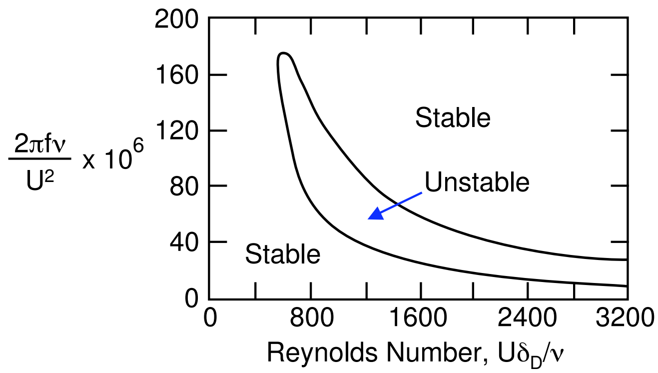
PROBLEM B18

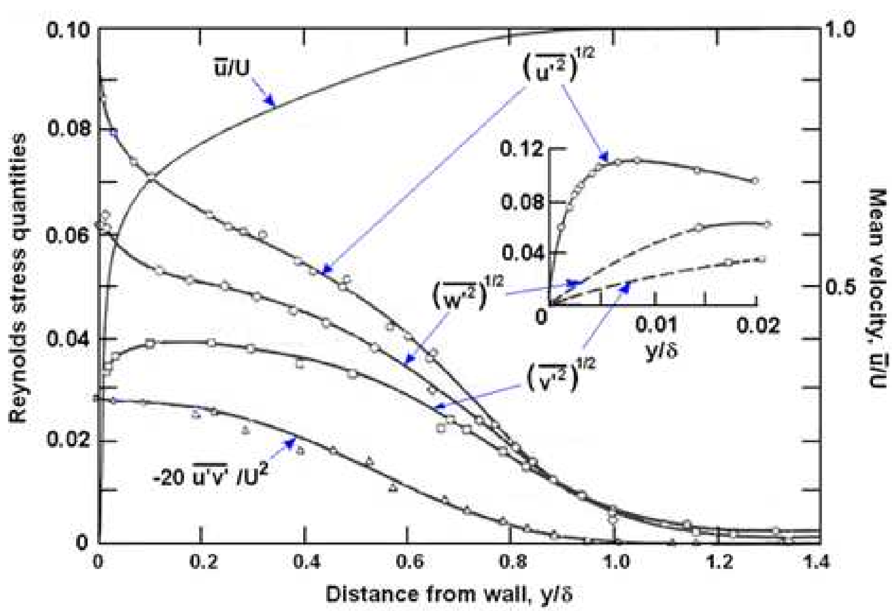
The stability diagram for a laminar boundary layer on a flat plate with zero pressure gradient (Blasius problem) is given below:



Using the solid, theoretical curve find the distance from the leading edge of the plate to the point where transition to turbulence begins for a flow of water ($\nu = 10^{-6}\ m^2/s$) when $U = 2\ m/s$. What is the frequency of the most unstable disturbances (in Hz) under these conditions?

PROBLEM B19

Using the data in the graph below estimate the value of the "universal constant", κ, at positions, $y/\delta = 0.1,\ 0.2,\ 0.4,\ 0.6,\ 0.8$ in the turbulent boundary layer.

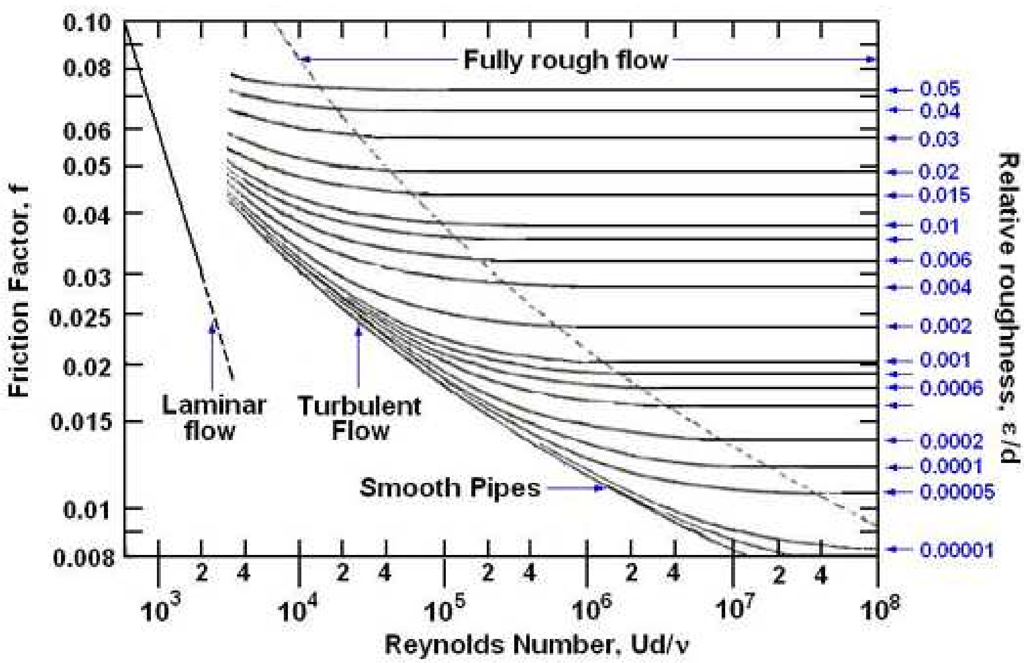


What is the ratio of the mixing length, ℓ, to the boundary layer thickness, δ, at these distances from the wall?

PROBLEM B20

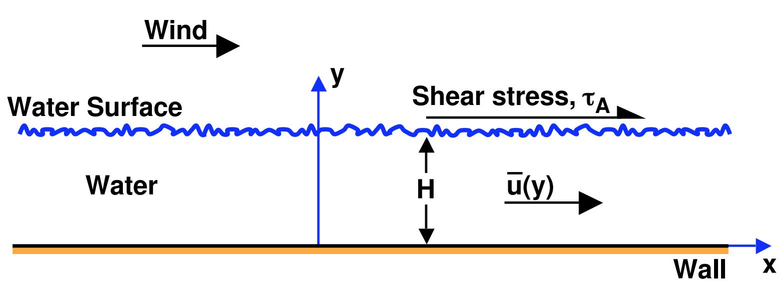
A long ventilation duct is used to transport air at normal temperatures (density, $\rho = 1.2\ kg/m^3$, kinematic viscosity, $\nu = 2.3 \times 10^{-6}\ m^2/s$). The duct has a smooth interior surface, a circular cross-section with a diameter of 0.5 m and is 50 m long. A pressure difference of 1 $kg/m \cdot s^2$ is applied between the two ends of the duct. Using the data in the graph below, find (by trial and error or other means) the average velocity of flow through the duct.

[Note that the friction factor, $f = (-dp/dx)d/\frac{1}{2}\rho U^2$, $Re = Ud/\nu$ where d is the diameter and U is the volumetric average velocity of flow.]



PROBLEM B21

A high wind drives a film of water over a solid surface at such a speed that the flow in the film becomes turbulent. This occurs because the wind applies a shear stress, τ_A, to the surface of the water:



The thickness of the film, H, and the mean water velocity, $\bar{u}(y)$, are constant in time and with position, x. Using the assumptions listed below, find an expression for the mean velocity on the water surface, $\bar{u}(H)$, in terms of τ_A, H, ρ (the water density), ν (the kinematic viscosity of the water), and the Karman universal constant, κ. The assumptions:

- The laminar sublayer next to the solid surface (in which $u^* = y^*$) extends to $y^* = 5$ where the mean velocity is to be matched with that of the turbulent flow in the rest of the water film.
- Outside the laminar sublayer, the Reynolds stresses dominate and the viscous component of the shear stress can be neglected.
- Prandtl's mixing length theory is to be used with a Karman universal constant denoted by κ.

PROBLEM B22

The velocity profile in a turbulent boundary layer of incompressible fluid on a flat plate (U = constant) is to be approximated by the form:

$$u/U = (y/\delta)^{1/7}$$

[Disregard the fact that this does not exactly satisfy one of the constraints usually imposed on laminar boundary layer profiles namely that du/dy should tend to zero as y tends to δ]. Find the profile parameter α for this profile. If the wall shear stress, τ_w, for this turbulent profile is assumed to be given by the empirical formula

$$\tau_w = 0.023\rho U^2(\nu/\delta U)^{1/4}$$

where ρ and ν are the fluid density and kinematic viscosity, then solve the resulting Karman momentum integral equation to obtain an expression for the thickness of the boundary layer, δ, as a function of distance, x, along the plate. Assume that the layer first becomes turbulent at $x = x_0$ where the thickness is δ_0.

[Do not use $\tau_w = \mu(du/dy)_{y=0}$ which is inappropriate in turbulent boundary layer calculations.]